APÉNDICE E

Predicción recursiva del nivel $\hat{y}_i(k+l|k)$ para el controlador de Hayami de 1[er] orden con retardo

La predicción en el intervalo [k,k+λ] puede expresarse del siguiente modo:

$$\hat{y}_i(k+l|k) = \sum_{j=1}^{3} \alpha_{ij}\hat{y}_i(k+l-j|k) + \frac{T}{2A_i}\left[\sum_{j=0}^{2}\beta_{ij}\hat{q}_i(k-1+l-j|k) - \sum_{j=0}^{3}\gamma_{ij}\hat{m}_i(k+l-j|k)\right] \qquad l=1,2,...,\lambda \tag{E.1}$$

Esta predicción puede redefinirse en cada instante de tiempo k con las siguientes condiciones iniciales:

$$\begin{aligned} &\hat{y}_i(k+1-j|k) = y_i(k+1-j), && para\ j=1,...,3 \\ &\hat{q}_i(k-j|k) = q_i(k-j), && para\ j=1,2 \\ &\hat{m}_i(k+1-j|k) = m_i(k+1-j), && para\ j=1,...,3 \end{aligned} \tag{E.2}$$

La predicción en el instante *k+l* depende de los valores en los instantes anteriores *k+l-1*, *k+l-2* y *k+l-3*. Estos valores también deben de predecirse. Para obtener la predicción en el instante *k+l* a partir de los valores en los instantes anteriores a k, se puede utilizar recursivamente la ecuación (E.1) sucesivamente para *l* = 1,2,...,*λ*., teniendo en cuenta las condiciones iniciales (E.2):

- Para *l* = 1

Sustituyendo en (E.1):

$$\hat{y}_i(k+1|k) = \sum_{j=1}^{3} \alpha_{ij}\hat{y}_i(k+1-j|k) + \frac{T}{2A_i}\left[\sum_{j=0}^{2}\beta_{ij}\hat{q}_i(k-j|k) - \sum_{j=0}^{3}\gamma_{ij}\hat{m}_i(k+1-j|k)\right] \tag{E.3}$$

Realizando un cambio de notación (E.5) se obtiene:

$$\hat{y}_i(k+1|k) = \sum_{j=1}^{3} g_{ij}^{(1)}\hat{y}_i(k+1-j|k) + \frac{T}{2A_i}\left[\sum_{j=0}^{2}h_{ij}^{(1)}\hat{q}_i(k-j|k) - \sum_{j=0}^{3}k_{ij}^{(1)}\hat{m}_i(k+1-j|k)\right] \tag{E.4}$$

$$\boxed{\begin{array}{ll} g_{ij}^{(1)} = \alpha_{ij}\,; & j = 1,2,3 \\ h_{ij}^{(1)} = \beta_{ij}\,; & j = 0,1,2 \\ k_{ij}^{(1)} = \gamma_{ij}\,; & j = 0,\ldots,3 \end{array}} \qquad \text{(E.5)}$$

- Para $l = 2$

Sustituyendo en (E.1):

$$\begin{aligned} \hat{y}_i(k+2|k) &= \sum_{j=1}^{3}\alpha_{ij}\hat{y}_i(k+2-j|k) + \frac{T}{2A_i}\left[\sum_{j=0}^{2}\beta_{ij}\hat{q}_i(k+1-j|k)\right. \\ &\left. - \sum_{j=0}^{3}\gamma_{ij}\hat{m}_i(k+2-j|k)\right] = \alpha_{i1}\hat{y}_i(k+1|k) + \sum_{j=2}^{3}\alpha_{ij}\hat{y}_i(k+2-j|k) \\ &+ \frac{T}{2A_i}\left[\sum_{j=0}^{2}\beta_{ij}\hat{q}_i(k+1-j|k) - \sum_{j=0}^{3}\gamma_{ij}\hat{m}_i(k+2-j|k)\right] \end{aligned} \qquad \text{(E.6)}$$

Sustituyendo el valor de $\hat{y}_i(k+1|k)$ en la ecuación (E.6) por el dado por la ecuación (E.4):

$$\begin{aligned} \hat{y}_i(k+2|k) &= \alpha_{i1}\left(\sum_{j=1}^{3}g_{ij}^{(1)}\hat{y}_i(k+1-j) + \frac{T}{2A_i}\left[\sum_{j=0}^{2}h_{ij}^{(1)}\hat{q}_i(k-j)\right.\right. \\ &\left.\left. - \sum_{j=0}^{3}k_{ij}^{(1)}\hat{m}_i(k+1-j)\right]\right) + \sum_{j=2}^{3}\alpha_{ij}\hat{y}_i(k+2-j|k) \\ &+ \frac{T}{2A_i}\left[\sum_{j=0}^{2}\beta_{ij}\hat{q}_i(k+1-j|k) - \sum_{j=0}^{3}\gamma_{ij}\hat{m}_i(k+2-j|k)\right] \end{aligned} \qquad \text{(E.7)}$$

Desarrollando los términos de los sumatorios y utilizando las condiciones de contorno (E.2) se llega a la siguiente expresión:

$$\begin{aligned} \hat{y}_i(k+2|k) &= \alpha_{i1}g_{i1}^{(1)}y_i(k) + \alpha_{i2}(y_i(k) + g_{i2}^{(1)}y_i(k-1)) + \alpha_{i3}(y_i(k-1) + g_{i3}^{(1)}y_i(k-2)) \\ &+ \frac{T}{2A_i}\left[\beta_{i0}\hat{q}_i(k+1|k) + \alpha_{i1}h_{i0}^{(1)}\cdot\hat{q}_i(k|k) + \beta_{i1}\hat{q}_i(k|k) + \alpha_{i1}h_{i1}^{(1)}\cdot q_i(k-1) + \beta_{i2}q_i(k-1)\right. \\ &+ \alpha_{i1}h_{i2}^{(1)}q_i(k-2) - \gamma_{i0}\hat{m}_i(k+2|k) - \alpha_{i1}k_{i0}^{(1)}\hat{m}_i(k+1|k) - \gamma_{i1}\hat{m}_i(k+1|k) \\ &\left. - \alpha_{i1}k_{i1}^{(1)}m_i(k) - \gamma_{i2}m_i(k) - \alpha_{i1}k_{i2}^{(1)}m_i(k-1) - \gamma_{i3}m_i(k-1) - \alpha_{i1}k_{i3}^{(1)}m_i(k-2)\right] = \\ &= (\alpha_{i1}g_{i1}^{(1)} + \alpha_{i2})y_i(k) + (\alpha_{i1}g_{i2}^{(1)} + \alpha_{i3})y_i(k-1) + \alpha_{i1}g_{i3}^{(1)}y_i(k-2) + \left[\beta_{i0}\hat{q}_i(k+1|k)\right. \\ &+ (\alpha_{i1}h_{i0}^{(1)} + \beta_{i1})\hat{q}_i(k|k) + (\alpha_{i1}h_{i1}^{(1)} + \hat{\beta}_{i2})q_i(k-1) + \alpha_{i1}h_{i2}^{(1)}q_i(k-2) - \gamma_{i0}\hat{m}_i(k+2|k) \\ &- (\alpha_{i1}k_{i0}^{(1)} + \gamma_{i1})\hat{m}_i(k+1|k) - (\alpha_{i1}k_{i1}^{(1)} + \gamma_{i2})m_i(k) - (\alpha_{i1}k_{i2}^{(1)} + \gamma_{i3})m_i(k-1) \\ &\left. - (\alpha_{i1}k_{i3}^{(1)} + \gamma_{i4})m_i(k-2)\right]\frac{T}{2A_i} \end{aligned}$$

(E.8)

Aplicando un cambio de notación:

$$\begin{aligned}
&\hat{y}_i(k+2|k) = g_{i1}^{(2)} y_i(k) + g_{i2}^{(2)} y_i(k-1) + g_{i3}^{(2)} y_i(k-2) + \frac{T}{2A_i}\left[h_{i0}^{(1)}\hat{q}_i(k+1|k)\right.\\
&+ h_{i0}^{(2)}\hat{q}_i(k|k) + h_{i1}^{(2)} q_i(k-1) + h_{i2}^{(2)} q_i(k-2) - k_{i0}^{(1)}\hat{m}_i(k+2|k)\\
&\left.- k_{i0}^{(2)}\hat{m}_i(k+1|k) - k_{i1}^{(2)} m_i(k) - k_{i2}^{(2)} m_i(k-1) - k_{i3}^{(2)} m_i(k-2)\right] =\\
&= \sum_{j=1}^{3} g_{ij}^{(2)} y_i(k+1-j) + \frac{T}{2A_i}\left[\sum_{j=1}^{2} h_{ij}^{(2)} q_i(k-j) - \sum_{j=1}^{3} k_{ij}^{(2)} m_i(k+1-j)\right.\\
&\left.+ \sum_{j=1}^{2} h_{i0}^{(2+1-j)}\hat{q}_i(k-1+j|k) - \sum_{j=1}^{2} k_{i0}^{(2+1-j)}\hat{m}_i(k+j|k)\right]
\end{aligned} \tag{E.9}$$

Las ecuaciones (E.10) y (E.11) muestran el cambio de notación realizado.

$$\begin{aligned}
&g_{i1}^{(2)} = \hat{\alpha}_{i1} g_{i1}^{(1)} + \hat{\alpha}_{i2} = \hat{\alpha}_{i1} g_{i1}^{(1)} + g_{i2}^{(1)} \qquad g_{i2}^{(2)} = \hat{\alpha}_{i1} g_{i2}^{(1)} + \hat{\alpha}_{i2} = \hat{\alpha}_{i2} g_{i1}^{(1)} + g_{i2}^{(1)}\\
&g_{i3}^{(2)} = \hat{\alpha}_{i1} g_{i3}^{(1)} + \hat{\alpha}_{i4} = \hat{\alpha}_{i3} g_{i1}^{(1)} + g_{i2}^{(1)}
\end{aligned} \tag{E.10}$$

$$\begin{aligned}
&h_{i0}^{(2)} = \hat{\alpha}_{i1} h_{i0}^{(1)} + \hat{\beta}_{i1} = g_{i1}^{(1)}\hat{\beta}_{i0} + h_{i1}^{(1)} \qquad k_{i0}^{(2)} = \hat{\alpha}_{i1} k_{i0}^{(1)} + \hat{\gamma}_{i1} = g_{i1}^{(1)}\hat{\gamma}_{i0} + k_{i1}^{(1)}\\
&h_{i1}^{(2)} = \hat{\alpha}_{i1} h_{i1}^{(1)} + \hat{\beta}_{i2} = g_{i1}^{(1)}\hat{\beta}_{i1} + h_{i2}^{(1)} \qquad k_{i1}^{(2)} = \hat{\alpha}_{i1} k_{i1}^{(1)} + \hat{\gamma}_{i2} = g_{i1}^{(1)}\hat{\gamma}_{i1} + k_{i2}^{(1)}\\
&h_{i2}^{(2)} = \hat{\alpha}_{i1} h_{i2}^{(1)} + \hat{\beta}_{i3} = g_{i1}^{(1)}\hat{\beta}_{i2} + h_{i3}^{(1)} \qquad k_{i2}^{(2)} = \hat{\alpha}_{i1} k_{i2}^{(1)} + \hat{\gamma}_{i3} = g_{i1}^{(1)}\hat{\gamma}_{i2} + k_{i3}^{(1)}\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad k_{i3}^{(2)} = \hat{\alpha}_{i1} k_{i3}^{(1)} + \hat{\gamma}_{i4} = g_{i1}^{(1)}\hat{\gamma}_{i3} + k_{i4}^{(1)}
\end{aligned} \tag{E.11}$$

- Para $l = 1,2,...,\lambda$

Una vez se han obtenido las predicciones para $l = 1$ y $l = 2$, se puede generalizar para $l = \lambda$.

$$\begin{aligned}
&\hat{y}_i(k+l|k) = \sum_{j=1}^{3} g_{ij}^{(l)} y_i(k+1-j) + \left[\sum_{j=1}^{2} h_{ij}^{(l)} q_i(k-j)\right.\\
&- \sum_{j=1}^{3} k_{ij}^{(l)} m_i(k+1-j) + \sum_{j=1}^{l} h_{i0}^{(l+1-j)}\hat{q}_i(k-1+j|k)\\
&\left.- \sum_{j=1}^{l} k_{i0}^{(l+1-j)}\hat{m}_i(k+j|k)\right]\frac{T}{2A_i}\\
&para \quad l = 1,2,...,\lambda
\end{aligned} \tag{E.12}$$

Con los siguientes coeficientes:

$$
\begin{array}{|l|}
\hline
g_{ij}^{(l)} = g_{i1}^{(l-1)}\hat{\alpha}_{ij} + g_{i,j+1}^{(l-1)}\,; \quad j = 1,...,3; \quad l = 2,...,\lambda \\
h_{ij}^{(l)} = g_{i1}^{(l-1)}\hat{\beta}_{ij} + h_{i,j+1}^{(l-1)}\,; \quad j = 0,...,2; \quad l = 2,...,\lambda \\
k_{ij}^{(l)} = g_{i1}^{(l-1)}\hat{\gamma}_{ij} + k_{i,j+1}^{(l-1)}\,; \quad j = 0,...,3; \quad l = 2,...,\lambda \\
\\
g_{ij}^{(1)} = \hat{\alpha}_{ij}\,; \quad j = 1,...,3 \\
h_{ij}^{(1)} = \hat{\beta}_{ij}\,; \quad j = 0,...,2 \\
k_{ij}^{(1)} = \hat{\gamma}_{ij}\,; \quad j = 0,...,3 \\
\\
g_{i,4}^{(l-1)} = h_{i,3}^{(l-1)} = k_{i,4}^{(l-1)} = 0\,; \quad l = 2,...,\lambda \\
\hline
\end{array}
\qquad \text{(E.13)}
$$