4. FORMULATION OF AN ANALYTICAL MODEL

The objective of this study is to determine the most effective bus network configuration for the cities of Chicago and Barcelona. In order to do so, an analytic model representing the bus network through its principal parameters is needed. This model needs to include the interests and characteristics of the bus operator as well as the users, so that the evaluation of the social costs associated with the bus network will be possible by means of this model.

The analytical model used in this study will be the one presented by van Nes (1999). It is a model based in the optimization of an objective, which depends on the variables that are the key design parameters in a bus network: stop spacing, line spacing and vehicle frequency. In order to determine the relationships between these parameters of study, this model is developed from the different objectives that the different parties within a bus network have: user, administration, and transportation operator. These objectives are built from blocks that represent the different characteristics of a bus network, such as the operational costs or the travel time (see Table 2).

Table 2. van Nes’ Objectives.

<table>
<thead>
<tr>
<th>Viewpoint</th>
<th>Objective</th>
<th>Access time</th>
<th>In-vehicle time</th>
<th>Waiting time</th>
<th>Total travel time</th>
<th>Operational costs</th>
<th>Revenues</th>
<th>Total travel costs</th>
<th>Total costs</th>
<th>Patronage</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Traveler</strong></td>
<td>O1: Minimizing weighted travel time under fixed frequency</td>
<td>X</td>
<td>X</td>
<td>O</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Traveler</strong></td>
<td>O2: Minimizing weighted travel time under fixed budget</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Operator</strong></td>
<td>O3: Maximizing cost effectiveness</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td><strong>Operator</strong></td>
<td>O4: Maximizing profit</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td><strong>Authority</strong></td>
<td>O5: Minimizing total costs</td>
<td>X</td>
<td>X</td>
<td>O</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>O</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Authority</strong></td>
<td>O6: Maximizing patronage</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

X: Required; O: Optional
A more in-depth description of these blocks and what they represent can be found in section 4.2.

Other analytical models with different approaches to the optimization of a transit network were considered as well. However, they were not the most suitable for the purposes of this study, and therefore were rejected. A review of some of these models along with the considerations taken for their suitability evaluation is following.

4.1 Literature Review

The literature review on transit stop spacing research is very extensive. A few models were considered to be used in this study, which are described in this chapter. However, for different kinds of reasons, they were not selected as the most suitable. A brief description of them and the justification for the model selection is following.

The first study on optimal design of bus routes is the one presented by Newell (1979). The objective of that study was to determine an optimal geometry for bus routes given a particular trip distribution. The mathematical formulation that was used took into consideration factors such as the waiting cost and the access time cost for users, the operational cost for the operator, and transfer costs. The mathematical model was applied to a fictional bus network geometry. The conclusions of the study focused on the variability of the cost depending on the network geometry. However, it also mentioned that the conclusions might not be suited for their application on a real bus network geometry.

Afterward, Ceder et al. (1986) contributed to the work started by Newell by presenting an algorithm that could be used to design new bus routes taking account of both passenger and operator interests. However, the algorithm presented focused only on a single component of the overall bus operations planning process.

Within the current State-of-the-Art for bus network design it is important to mention the contributions made by authors such as Saka (2001), Hasseltröm (1981), and Shi et al. (1994).

4.1.1. Anthony A. Saka’s analytical model (2001)

Saka presents a model that can be used to determine effectively a sub-optimal policy for bus stop spacing in urban areas. His study is divided in two parts, being the first one a sensitivity analyses that demonstrates that proper spacing of stops
can significantly improve the quality of transit service, and decrease travel time, headway, and fleet size. The second part of his study presents the proposed model, which is derived from the fundamental relationships that exist among velocity, uniform acceleration/deceleration, and displacement, and among the average bus operating speed, headway, required fleet size and potential system capacity.

The purpose of this model is to serve as a valuable decision support tool for transit planners in determining suitable spacing of bus stops for prevailing network and traffic conditions.

The model development is based on the assumption that the transit operator is interested in sustaining a policy service-headway with minimal number of transit units or buses.

The development process involves two primary steps. First, the bus travel time that determines the fleet size requirement is computed by partitioning the time into four major components: acceleration-deceleration time; dwell time at stops; delay time associated with traffic signals; and unimpeded travel time involving cruise speed (equation (a)). Second, a sensitivity analysis is done by gradually varying the bus-stop spacing until the optimum (lower threshold) bus-stop spacing is determined (equation (b)).

Expected terminal-to-terminal bus travel time is determined as

\[ T_{bus} = T_{a,d} + T_s + T_c + T_o + T_m \]  

(a)

Where

\( T_{a,d} \) = total one-way bus travel time during acceleration and deceleration

\( T_s \) = total one-way delay attributed solely to bus dwell time

\( T_c \) = total one-way delay attributed solely to traffic control devices

\( T_o \) = one-way bus travel time at cruise speed

\( T_m \) = miscellaneous delay

The optimum bus-stop spacing \( (x_o) \), which can be viewed as the minimum spacing to minimize the required fleet size, is determined as
\[
\begin{align*}
\text{minimize } & \quad x_s, \ n \\
\text{subject to } & \quad h = h^*, \ x_s = X
\end{align*}
\]

where

\( x = \text{bus-stop spacing} \)

\( n = \text{required fleet size} \)

\( h^* = \text{policy headway expressed in minutes} \)

\( X = \text{maximum allowable bus stop spacing} \)

The study concludes that this model is expected to serve as a tool in determining the appropriate stop locations on new bus routes or in evaluating the optimality of stop spacing in existing urban bus routes.

### 4.1.2. Hasseltröm’s analytical model (1981)

The model developed by Hasseltröm in 1981 intends to optimize a transit network by maximizing the customer’s benefit. The parameters that the customer’s benefit involves are considered to be, besides the transit fare, the traveling time consumed, the access time towards the transit route, the waiting time, and the inconveniences (transfers, crowded vehicles). According to Hasseltröm, this total cost could be referred to as a “generalized cost” and could be defined as the way to provide a better comfort to the transit user.

The formulation of this model considers a homogeneous demand along the day, so that the frequency of the service remains the same.

Maximizing the customer’s benefit is equivalent to maximizing the number of trips for a given demand function. Consequently, according to Hasseltröm, maximizing the number of trips, considering a constant fare, results in a maximum income for the operator. Therefore, the operator’s benefit will come from providing a transit system that maximizes the operations and the social welfare.

The approach to this model is developed in two optimization levels: in the first level, an initial network design is created, whereas in the second level the network design is detailed.
The first optimization level is divided in three parts: on the first place, the number of passengers that will use the network is estimated; a set of possible routes within the network is created; and an optimal combination of routes and frequencies is selected. This first level provides us with a first approach to the optimal network.

The second level evaluates the routes that have been created previously in the first level. In order to do so, some information is needed:

a) for each route, a list of the bus stops

b) for each bus stop, a list of the bus routes that serve that specific stop

c) for each pair of routes, a list of common bus stops, where the transfers would be located

The model applies linear programming in order to minimize transfers, which are limited to two. A maximum number of routes are selected, and the access time, in-vehicle time, and waiting time are calculated for each one of these routes.

The demand variable is then incorporated to the model, and the bus routes that compose the optimal network are evaluated individually and globally. The routes are supposed to be optimized by breaking them or connecting them, so that the number of transfers and the number of vehicles can be minimized. The frequencies and vehicle types can be optimized as well.

It is interesting to notice that the model presented by Hasseltröm is composed by a set of different objective functions, each one of them containing different algorithms. However, the objectives fail to consider the economical costs that the design network would imply.

4.1.3. Shi’s and Mahmassani’s analytical model (1994)

In this model, Shi introduces the concept of programmed transfers to the network design problem. It is basically an efficient solution to minimize transfer time, optimize the integration between the routes of a transit mode, and optimize the integration between different transit modes.

Being the main purpose of the model the one described above, there is also a procedure included that is used in the evaluation of the results. This procedure is called Network Analysis Procedure (NETAP), and is used to: (a) allocate the demand and calculate network and route parameters; (b) determine capacities and service frequency for each route; (c) calculate a set of performance
variables. This procedure determines 8 different information types from the system:

1. Descriptive information about the system, subdivided in three categories: node, arch and route;
2. Demand: total trips in the system;
3. User’s cost: total traveling time, considering access time, waiting time, transfer time, and in-vehicle time;
4. Quality of service: frequency, vehicle capacity, and crowding factor for each route;
5. Operator’s cost: operational cost and required fleet size;
6. Carburant cost: total carburant consumption;
7. System utilization: ratio between the kilometers used by the kilometers of routes covered by the system;
8. Network shape: classification of the network according to its shape (radial, longitudinal, grid) and according to the number of centers.

The NETAP procedure was tested on the transit networks designed previously and the results verified the consistency of these networks, in terms of frequency and vehicle capacity.

This model is characterized by a detailed formulation and analysis. However, this fact does not imply an increase of the complexity of the model. Therefore, it is considered to be understandable and accessible for its study and application.

4.2. van Nes’ analytical model

The model presented by van Nes analyzes the problem of transit network design as an opposition between the different parties’ interests. Basically, the model assumes that the traveler wants to travel at any time and as fast as possible to his destination, whereas the operator would like to have a network of profitable lines only. This means that according to the traveler, the main characteristic of a transit network is the total travel time, which would be minimal in a high service density network with high frequencies as well. Obviously, this network is not the optimal for the operator, who would choose the revenues and operational costs as main characteristics of the transit network.

The third party considered in the model is the local authority. It might favor the traveler, for instance by subsidizing transit, or it might try to find a balance between the traveler’s and operator’s interest using the concept of social welfare.

All of these network characteristics are described using a set of basic building blocks. The assumptions made in order to describe a general bus network design start with an area of service of one square kilometer (see Figure 1). It is also
assumed that one or more linear transit lines serve this unit area, each having a constant stop spacing $D_s$. They have parallel routes and offer transit to and from the city center over a distance $D_c$. The line spacing would be $D_l$, providing a homogeneous coverage of the area.

![Image of study area and layout of the transit lines.](image)

**Figure 1. Study area and layout of the transit lines.**

Using this study area, the mathematical formulation of the building blocks for the traveler, the operator and the local authorities can be defined.

### 4.2.1 Building Blocks

A. Blocks affecting travelers.

The building blocks for the traveler are divided into access time, in-vehicle time (which depends on the maximum speed and the time lost at stops), waiting time (which depends on the frequency of the services, which on its turn is determined by the vehicle density), and total time.

**Acces time $T_a$:**

$$T_a = \frac{f_a \cdot (D_s + D_l)}{v_a}$$

(1)

where:

$D_s$ = stop distance

$D_l$ = line distance
$f_a =$ routing factor for the actual access distance, function of $D_s$ and $D_l$

$v_a =$ access speed

**Waiting time $T_w$:**

$$ T_w = \frac{f_w}{F} \quad (2) $$

where:

$f_w =$ factor for the waiting time

$F =$ frequency of the public transport system

**In-vehicle time $T_i$:**

$$ T_i = \frac{D_c}{D_s} \left( \frac{D_s}{v} + T_s \right) \quad (3) $$

where:

$D_c =$ travel distance

$v =$ maximum speed

$T_s =$ time lost at stops

Therefore, the total traveling time, $T_c$, is assumed to be the following:

$$ T_c = w_a \cdot T_a + w_w \cdot T_w + T_i + w_e \cdot T_e \quad (4) $$

where:

$T_c =$ total weighted travel time

$w_x =$ weight for time element x

$T_e =$ egress time, assumed to be fixed
B. Blocks affecting the operator.

The blocks that affect the operator are the operational costs of the network (in terms of cost per unit area served) and the revenues that the service provides.

Operational costs $C_o$, in terms of cost per unit area served.

$$C_o = c_o \cdot v_d$$  \hspace{1cm} (5)

Where:

$c_o$ = operating costs per vehicle per hour

$v_d$ = vehicle density per square kilometer

The vehicle density can be found depending on the frequency as follows (the factor 1000 appears to account for the unit area of a square kilometer):

$$v_d = F \cdot \frac{1000}{D_s} \cdot \frac{1000}{D_l} \left( \frac{D_s}{v} + T_s \right)$$  \hspace{1cm} (6)

Revenues $R_o$:

$$R_o = (r_s + r_t) \cdot P$$  \hspace{1cm} (7)

Where:

$r_t$ = fare paid by the traveler

$r_s$ = subsidy paid by the authorities per traveler

$P$ = number of passengers

C. Blocks affecting the Authority.

As has been stated earlier, local authorities might try to find a balance between the traveler’s and the operator’s interests. One way to do so is to determine the net benefit of the transit system. The following approach is chosen to represent this net benefit, where the cost associated with traveling $C_t$ is defined as:

$$C_t = c_t \cdot T_c \cdot P + r_t \cdot P$$  \hspace{1cm} (8)
Where:

\( c_t = \text{value of time for travelers} \)

Since the fares paid by the traveler reduce the costs for the operator, they can be excluded from the total costs \( C \) for traveling, which then becomes:

\[
C = C_0 + c_t \cdot T_c \cdot P
\]

(9)

**D. Transport Demand.**

Transport demand \( P \) is a parameter in equations (7) and (8) and may be assumed to be fixed or to depend on the quality of the services offered. A common way to describe the relationship between the service level and the level of demand is the logit-mode-choice function:

\[
P = P_0 \cdot \frac{\exp(-\alpha \cdot T_c)}{\exp\left(-\alpha \cdot T_c + \sum_{m=1}^{n} \exp(-\alpha \cdot T_m) \right)}
\]

Where:

\( P_0 = \text{total demand for transport per square kilometer} \)

\( \alpha = \text{coefficient for public transport} \)

\( n = \text{number of modes excluding public transport} \)

\( a_m = \text{coefficient for mode } m \)

\( T_m = \text{weighted travel time for mode } m \)

**4.2.2 Objectives**

As mentioned before, the model presented by van Nes uses the different parties’ interests to determine different objectives for a network design. These objectives will be composed of the building blocks presented above. A total of six objectives, two for each party, are presented and analyzed in this section.

Regarding the traveler, his main objective is to minimize his perceived total traveling time. From this point of view, two alternatives can be distinguished:
• O1: minimizing weighted travel time in the case that the frequency is fixed. In this scenario, the design problem is to find a balance between access time and in-vehicle time. The key variable for the network design will therefore be stop spacing, since line spacing only influences access time.
• O2: Minimizing weighted travel time given a fixed operational budget.

From the operator’s point of view, a feasible objective is maximizing the cost effectiveness (O3), defined as the ratio of the total revenues and the operational costs. Another objective is profit maximization (O4), that is, total revenues reduced by operational costs.

Finally, the authority’s objectives can be minimizing total costs (O5) or maximizing patronage (O6). Since travel demand depends on the travel time, maximizing patronage is similar to minimizing travel time. Another role of the authorities is defining constraints from a social point of view, such as maximum access distance or minimum frequency.

These six objectives can be defined mathematically using the building blocks, as shown in Table 3.

Table 3. Mathematical formulation of the objectives.

<table>
<thead>
<tr>
<th>Objective</th>
<th>Mathematical Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>O1: MIN</td>
<td>[ w_a \cdot \frac{f_a \cdot (D_s + D_f)}{v_a} + w_w \cdot \frac{f_w}{F} + \frac{D_c}{D_s} \cdot \left( \frac{D_s}{v} + T_s \right) + w_c \cdot T_c ]</td>
</tr>
<tr>
<td>O2: MIN</td>
<td>[ T_c = w_a \cdot \frac{f_a \cdot (D_s + D_f)}{v_a} + w_w \cdot \frac{f_w}{F} + \frac{D_c}{D_s} \cdot \left( \frac{D_s}{v} + T_s \right) + w_c \cdot T_c ]</td>
</tr>
<tr>
<td>O3: MAX</td>
<td>[ \frac{(r_i + r_f) \cdot P}{c_o \cdot v_d} ]</td>
</tr>
<tr>
<td>O4: MAX</td>
<td>[ (r_i + r_f) \cdot P - c_o \cdot v_d ]</td>
</tr>
<tr>
<td>O5: MIN</td>
<td>[ c_i \cdot T_c \cdot P + c_o \cdot v_d ]</td>
</tr>
<tr>
<td>O6: MAX</td>
<td>[ P ]</td>
</tr>
</tbody>
</table>
4.2.3. Discussion and selection

Objectives O1 and O6, i.e. minimizing travel time and maximizing patronage, are concerned with the same dilemma: access time versus in-vehicle time. Given the building blocks described in section 5.1.1, the results will be the same. Besides, O1 can be qualified as a rather naïve objective, since the interests of the operator have no influence at all in the network design. Therefore, we can apply the same qualification to objective O6.

Objective O2, minimizing travel time under a fixed budget, is the only one that uses the waiting time as a block in the formulation of the objective. Therefore, it is the only one that considers the frequency as a variable. Thus, O2 might be used to improve the performance of a network once the operational costs have been determined using one of the other objectives.

Objectives O3 and O4, maximizing cost effectiveness and maximizing profit, also deal with a similar dilemma. In O3, though, the ratio that is used has the disadvantage that in the case that the travel demand is less sensitive for changes in the network that the operational costs, no optimum can be found. O4 might therefore be a more robust objective.

O5 has the interesting feature that a fixed demand level may be assumed.

Given these remarks, van Nes concludes that objective O4, maximizing operator’s profit, and O5, minimizing total costs, are the most realistic objectives for the network design problem. O2 might be used to improve the quality of the network given the constraints derived using one of the other two objectives.

In this study, objective O5 will be used to determine the optimal bus stop and line spacing, since the demand level will be fixed and equal to the current demand level in Barcelona and Chicago. Objective O2 will be used to improve these results, considering as a design parameter the vehicle frequency as well.

4.3. Selection of the analytical model

After a review on the literature related to urban transportation network design, focused on the bus network system, there are some factors that can be mentioned.

An interesting reference is Alexandre Barra Vieira’s research study Roteirização de Ônibus Urbano, where the analysis of the procedures for urban bus network design adopted in different cities of Brazil, USA, and Canada, leads to the following conclusions:
a) Demand based models: this criterion is proved to be indispensable, since the methods that don’t include the demand as a variable in the model tend not to be adequate for their application in complex transportation systems.

b) Multi-modal transportation system models: it is essential to consider a systematic vision of the transportation networks in order to provide an adequate efficiency level for the associated demand at an adequate cost level.

c) Need for a computerized system: a computerized system involves several advantages such as the possibility of a better visualization and graphic comparison, or a facilitation of the calculations and analyses.

According to these conclusions, the model presented by Saka lacks the demand as a design variable. It doesn’t take under consideration the costs associated with the transit user, such as the access time cost, or the waiting time cost. It is therefore not the most realistic model to be used in the evaluation and optimization of a bus network design and the quality of the services that the system provides.

On the other hand, the model presented by Hasseltröm is completely based on the user’s point of view and interest. It is an interesting solution to the network design problem, although it can be considered rather naïve and unrealistic since the economical costs for the operator are completely disregarded.

The third model reviewed in this chapter offers a new point of view to the problem, taking into consideration the whole lot of urban transportation systems that can be found in a network, not only the bus service. In this case, the user’s interests as well as the operator’s interests are taken into consideration in the evaluation of the final network design. It is a well defined and systematic model that has been proved valid for its results. However, this study focuses on the bus network design only. Therefore, the model presented by Shi and Mahmassani can’t be applied in this case, although it remains as a valuable reference.

In conclusion, the model needed for this study, which has the purpose of analyzing the suitability of the current bus network design in the cities of Chicago and Barcelona and optimizing it if that is the case, is a model that needs to take under consideration the costs related to the bus service user as well as the bus service operator.

Being those two conditions indispensable, the model presented by van Nes was found to be the most adequate, since it is based in the formulation and optimization of the interests of the parties mentioned, and it provides an analytic formulation to measure the operational and service parameters. Besides, it is a simple model that can be easily computerized and it provides clear and understandable solutions for the design variables.