2- SWAN

2.1- Introduction

The following chapter provides some explanations on the program used to propagate the wave data from high sea to the nearshore output locations: SWAN, an acronym for Simulating WAves Nearshore. Its purpose is to simulate the evolution of random, short-crested wind-generated waves in estuaries, tidal inlets, lakes and coastal areas in general.

SWAN is based on the discrete spectral action balance and is fully spectral, that is both in directions and frequencies. This enables random short-crested wave fields propagating simultaneously from widely different directions to be accommodated, and calculate their evolution through deep, intermediate and shallow waters, even including currents. The phenomena SWAN takes (can take) into account are wave generation by wind, dissipation due to white-capping, bottom friction and depth-induced wave braking, non-linear wave-wave interactions (quadruplets and triads) and wave blocking by currents.

This is a third-generation program developed at Delft University of Technology (TUDelft), successor of the stationary, second generation model HISWA. Since SWAN has been commonly adopted as a standard for the above mentioned applications, WL/Delft Hydraulics has integrated it into the wider Delft3D model suite, where it can be used (under a different interface) as part of the Delft3D-WAVE. Delft3D is a software package targetting any process involving water in a free surface environment: flow, waves, water quality, ecology, sediment transport and bottom morphology; as well as the interactions among them. The capabilities of SWAN are widely broadened when coupled with the rest of Delft-3D (essentially the flow-module, which enables to study waves in a current). However, it ceases to be on the public domain, which is its original situation at TUDelft (WL / Delft Hydraulics, 2003).

SWAN’s range of applications encompasses areas of up to more that 50 x 50 km and includes estuaries, tidal inlets, lakes, barrier islands with tidal flats, channels and coastal regions, making it suitable both for harbour or offshore design and coastal development and management projects.

The second section takes a look at the general and practical matters that enable the use of the program. The third, briefly provides its physics’ background, whereas its numerical implementation is left for Section 2.2.4.
2.2- **General background**

2.2.1- **Units and coordinate systems**

The international system (S.I.) is the required manner of expressing quantities in Delft3D-Wave: m, kg, s, degrees, etc... The program doesn’t take the curvature of Earth into account and it operates in a flat horizontal plane.

Geographic locations and orientations (e.g. for the different grids) are defined in one common Cartesian coordinate system with a defined absolute origin (0,0). This can be seen in the following figure:

![Figure 2.1: Nautical (left) and Cartesian (right) conventions of orientation.](image)

Directions (of winds and waves) have to be introduced by either the Nautical or the Cartesian convention, always defined relative to the previously outlaid coordinate system, as presented below:

![Figure 2.2: Grid location according to the conventions used by SWAN.](image)
2.2.2- Grids and boundary conditions

Three different sorts of grids may co-exist while using Delft-3D: Input, computational and output grids. These need not coincide, but they must obviously encompass the same area. Therefore, each may have a different origin, orientation and resolution. The transition from one grid to another is done interpolating, which might cause some accuracy loss.

**Input grids**

Input grids provide the information the user has beforehand: bathymetrical -that is, the bottom features of the study area- as well as regarding the friction this bottom might exert, and about current and wind fields (if known). Preferably, they should be bigger than computational grids, so that all possible situations of the later can be covered. The resolution of the input grids (especially of the bottom one) should be fine enough so as to include all the relevant details in the sea bottom, especially sharp ridges. It is important that their minimal depth (shallowest part) is taken into account as points of the grid: otherwise, the calculations will be biased (wrong) because not all pertinent waves will have been clipped by surf breaking when attaining a minimal depth.

**Computational grids**

Computational grids are 4-dimensional: $x$, $y$- and $\theta$, $\sigma$- space. For most of the calculations, where wave conditions are given at high sea (deep water), the grid in $x$, $y$- space ought to be chosen in such a manner that the boundary up-wave sits in deep water or, at least, in a water deep enough so as not to have affected the wave field with refraction. If the boundary conditions, though, are given in such a manner that refraction and other processes have been taken into account (such as when a nested calculation is performed), then this caution needs not be taken.

Similarly, if boundary conditions are only given in one of the ridges of the grid (the up-wave one), a lack of energy at the other ‘lateral’ ridges will cripple the results at points near them, since there, energy comes not only from up-wave but also laterally, the more so the bigger the width of the directional energy distribution is. Therefore, older, swell, waves will have the least area contaminated in the grid.
In order to resolve relevant details of the wave field the a certain spatial resolution of the computational grid is needed. Choosing it to be the same as those of the input grids usually suffices. There is a maximum number of nodes SWAN can operate with (under a standard configuration)

The computational spectral grid has to be provided by the user too. A minimum and a maximum frequency coupled with the frequency resolution which is proportional to the frequency itself \((\Delta f = 0.1f)\) define the frequency space. The user determines all these by choosing the lowest and highest frequencies as well as their total number. Advisable values for the extremes are a lowest frequency smaller than 0.6 times the value of the lowest peak frequency expected and a highest one of about 3 times the highest peak frequency expected (usually smaller than or equal to 1 Hz).

The directional range is the full 360º unless specified otherwise. Doing so might save computer time and space, but this should only be performed when there’s the certainty that waves approach the coast only from within a limited sector smaller than 180º. The directional resolution is determined by the number of discrete directions choosen. This should be bigger for swell seas, since the directional spreading around the mean wave direction is smaller. Possible values might fit with a 2º resolution for sea and 10º for swell.

**Output grids**

The results of the calculations are presented via output grids. These results can be given at the points where they were performed (thus coinciding with those of the computational grids) or elsewhere, in which case they are obtained via spatial interpolation. However, no output grid has to be defined used per se. In a stationary mode, SWAN calculates the effects of any given wave situation (input/boundary condition) all over the study area (that is, the computational grid). It is up to the user to ask for “results” (a.k.a. “output”) at
particular points. He might even ask for none, for example in case the computation at hand is being used barely to furnish with data a subsequent, nested computation. If this happens to be the situation, SWAN inputs the result of the general computation into the more particular one automatically. If output is asked for a point that belongs to the area covered by a nested grid, the results provided by Delft 3-D will be those resulting from the more detailed, nested computations.

2.3- Physical background

The waves are described in SWAN with two-dimensional wave action density spectrums. This is done even in highly non-linear situations such as when dealing with the surf zone, because nevertheless enough accuracy is believed to be attained at calculating this spectral distribution of the second order moment of the waves (as opposed to sufficiently to a full statistical description of the waves). SWAN deals with the action density spectrum $N(\sigma, \theta)$ rather than the energy density one $E(\sigma, \theta)$ because in the presence of currents action density is conserved whereas energy density not (Whitman, 1974). Its independent variables are the relative frequency $\sigma$ (as observed in a frame of reference moving with the action propagation velocity) and the wave direction $\theta$, which is the direction normal to the wave crest of each spectral component). The action density is equal to the energy density divided by the relative frequency: $N(\sigma, \theta) = E(\sigma, \theta) / \sigma$.

The spectral action balance equation in Cartesian coordinates that SWAN uses to describe the evolution of the wave spectrum is (Hasselmann et al., 1973):

$$\frac{\partial}{\partial t} N + \frac{\partial}{\partial x} c_x N + \frac{\partial}{\partial y} c_y N + \frac{\partial}{\partial \sigma} c_\sigma N + \frac{\partial}{\partial \theta} c_\theta N = \frac{S}{\sigma} \quad \text{(eq. 2.1)}$$

Where the first term at the left-hand side accounts for the local rate of change of action density in time. The second and third terms represent the propagation of action in geographical space (with $c_x$ and $c_y$ being propagation velocities in $x$- and $y$- space respectively). The fourth represents shifting of the relative frequency due to variations in depths and currents (with propagation velocity $c_\sigma$ in $\sigma$- space). The fifth one represents refraction, both depth and current induced (with propagation velocity $c_\theta$ in $\theta$- space). Linear wave theory provides for the expressions for these propagation speeds (e.g., Whitman, 1974; Mei, 1983; Dingemans, 1997). At the right-hand side $S$ is the source term in terms of energy density representing the effects of generation (by wind $S_{gw}$), dissipation (by white-capping $S_{ds,w}$; bottom friction $S_{ds,b}$ and depth-induced breaking $S_{ds,br}$) and non-linear wave-wave interactions (quadruplets $S_{nl4}$ and triads $S_{nl3}$), each of which will be shortly explained below. A complete formulation for these source terms can be found on appendix A at the end of this work.
2.3.1- Wind generation \((S_{in})\)

The transfer of energy from wind to waves is modelled with a resonance \((Phillips, 1957)\) and feedback mechanisms \((Miles, 1957)\), so that the source term can be described as the sum of linear and exponential growth:

\[
S_{in}(\sigma, \theta) = A + BE(\sigma, \theta) \tag{eq. 2.2}
\]

where both \(A\) (linear growth) and \(BE\) (exponential growth) depend on wave frequency and direction and wind speed and direction. Current effects are taken into account thanks to the use of apparent local wind speed and direction.

2.3.2- Dissipation \((S_{ds})\)

The dissipation term is the summation of three different contributions: white-capping \(S_{dr,w}(\sigma, \theta)\), bottom friction \(S_{dr,b}(\sigma, \theta)\) and depth-induced breaking \(S_{dr,br}(\sigma, \theta)\).

**Whitecapping**

The waves’ steepness controls whitecapping. In third-generation wave models such as SWAN, white-capping formulations rely on a pulse-bade model \((Hasselmann, 1974)\), adapted by the WAMDI group \((1988)\) as:

\[
S_{dr,w}(\sigma, \theta) = -\Gamma \frac{k}{\bar{\sigma}} E(\sigma, \theta) \tag{eq. 2.3}
\]

Where \(\Gamma\) is a coefficient dependent on steepness, \(k\) is the wave number and \(\bar{\sigma}\) and \(\bar{k}\) denote a mean frequency and a mean wave number (cf. the WAMDI group, 1988).

**Bottom friction**

Depth-induced dissipation may be caused by bottom friction, bottom motion, percolation or back-scattering on bottom irregularities \((Shemdin et al., 1978)\). In continental shelf seas with sandy bottoms, however, bottom friction turns out to be the principal mechanism \((Bertotti and Cavalieri, 1994)\). It can be formulated as:

\[
S_{dr,b}(\sigma, \theta) = -C_{bottom} \frac{\sigma^2}{g^2 \sinh^2(kd)} E(\sigma, \theta) \tag{eq. 2.4}
\]

Where \(C_{bottom}\) is a friction coefficient dependent on the bottom orbital motion \((U_{rms})\).
Depth induced breaking

The formulation of a spectral version of the bore model by Eldeberky and Battjes to account for the total dissipation (1995) is used in SWAN as a substitute for the process of depth-induced wave-breaking, which is still poorly understood (and little is known about its spectral modelling):

\[ S_{ds,br}(\sigma, \theta) = \frac{D_{tot}}{E_{tot}} E(\sigma, \theta) \]  

(eq. 2.5)

Where \( E_{tot} \) is the total energy and \( D_{tot} \) its rate of dissipation due to wave breaking (Battjes and Janssen, 1978).

2.3.3- Non-linear wave-wave interactions (\( S_{nl} \))

Quadruplet wave-wave interactions dominate the evolution of the spectrum in deep water, transferring energy from the spectral peak to lower frequencies (thus lowering the peak frequency) and to higher frequencies (where whitecapping dissipates the energy).

Triad wave-wave interactions in very shallow water transfer energy from lower to higher frequencies resulting often in higher harmonics (Beji and Battjes, 1993). Low frequency energy generation by triad wave-wave interactions is not considered here. The Lumped Triad Approximation (LTA) derived by Eldeberky (1996) is the model used in SWAN amongst many attempting to describe the triad wave-wave interaction, since it pictures fairly well the energy transfer from the primary peak of the spectrum to the harmonics.

2.4- Implementation

The action balance equation has been implemented in SWAN with finite difference schemes in all five dimensions (time, geographical and spectral spaces). Time is however omitted from the equations in Delft-3D because SWAN is applied in a stationary mode. The geographical space is discretised with a rectangular grid with constant resolutions \( \Delta x \) and \( \Delta y \) in \( x \)- and \( y \)-directions respectively. As for the spectrum, it is discretised with a constant directional resolution \( \Delta \theta \) and a constant relative frequency resolution \( \Delta \sigma/\sigma \) (logarithmic frequency distribution). The discrete frequencies are defined between fixed low and high cut-off values (the prognostic part of the spectrum), where spectral density is unconstrained. Below the low-frequency cut-off (typically \( f_{min} = 0.04 \) Hz for field conditions) the spectral densities are assumed to be zero. Above the high-frequency cut-off (usually 1 Hz) a diagnostic \( f^{-m} \) tail is added (used to compute non-linear wave-wave interactions at the high frequencies and compute integral wave parameters). The reason for using this and not a dynamic cut-off frequency is that, in mixed sea states, the latter might fail to account for the characteristics of one of the seas composing it. For example, it might be too low to properly account for a local wind-generated sea state in a coastal region which is superimposed on a simultaneously occurring swell, albeit unrelated. The value of
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$m$ should be between 4 and 5 (e.g., Phillips, 1985). If the Komen et al. (1984) formulation for wind input is chosen, SWAN uses $m = 4$.

2.4.1- Propagation

Robustness, accuracy and economy are the basis for the numerical schemes in SWAN. Thus, for a basic equation like where the state in a grid point is determined by the state in the up-wave grid points, an implicit upwind scheme (both in geographical and spectral space) would be the ideal choice, since it’s the most robust scheme. With “implicit” it is meant that all derivatives of action density ($x$ or $y$) are formulated at one computational level, $i_x$, or $i_y$, except the derivative in the integration dimension for which the previous or up-wave level is used too ($x$ or $y$ in stationary mode). For such a scheme the value of space steps, $\Delta x$ and $\Delta y$ would be mutually independent. An extra advantage of such a scheme in economical terms is that it is unconditionally stable, therefore allowing larger time steps in computations than with explicit schemes in shallow water. Thanks to the experience acquired with years of using the second-generation HISWA shallow water wave model (Holthuijsen et al., 1989) it is now known that a first-order upwind difference scheme I accurate enough for the geographical space, whereas not for the spectral. Thus, SWAN has implicit upwind schemes in both geographical space and spectral spaces, this last one supplemented with a second-order central approximation.

In the geographical space the state in a point of the grid is determined by that of the up-wave points (as defined by the propagation direction): Thus, decomposing the spectral space in four quadrants is possible. In each one of them the computations can be carried out independently from the rest except for the interactions between them due to refraction and wave-wave interactions (correspondingly formulated as boundary conditions between quadrants). The wave components in SWAN are propagated in geographical space with the first-order upwind scheme in a four forward-marching sweeps sequence (one per quadrant). The computations are carried out iteratively at each time step so as to properly account for the boundary conditions between the four quadrants. The discretisation of the action balance equation is (for positive propagation speeds; including the computation of the source terms but ignoring their discretisation):
$$\begin{align*}
\left[\frac{c_jN}{\Delta x}\right]_{x-1}^n + \left[\frac{c_jN}{\Delta y}\right]_{y-1}^n + \\
\left[\frac{(1-\nu)c_jN}{2\Delta \sigma}\right]_{x+1}^n + 2\nu\left[c_jN\right]_x^n - (1+\nu)\left[c_jN\right]_{x-1}^n + \\
\left[\frac{(1-\eta)c_jN}{2\Delta \theta}\right]_{y+1}^n + 2\eta\left[c_jN\right]_y^n - (1+\eta)\left[c_jN\right]_{y-1}^n = \left[\frac{S}{\sigma}\right]_{\xi,\eta,\Delta \sigma, \Delta \theta}^n
\end{align*}$$

(eq. 2.6)

With:

\(i_x, i_y, i_\sigma, i_\theta\) : grid counters

\(\Delta x, \Delta y\) : increments in geographic space

\(\Delta \sigma, \Delta \theta\) : increments in spectral space

\(n\) : iteration index

\(n^*\) : iteration index for source terms (equal to \(n\) or \(n-1\))

The degree to which the scheme in spectral space is central or upwind determined by the coefficients \(\nu\) and \(\eta\):

Values of \(\nu = 0\) or \(\eta = 0\) correspond to central schemes (which have the largest accuracy: numerical diffusion \(\gg 0\)), whereas either \(\nu\) or \(\eta\) equalling 1 correspond to upwind schemes. These are somewhat more diffusive (and therefore, less accurate) but more robust.

The propagation scheme is implicit as the derivatives of action density (in \(x\) or \(y\)) at the computation level (\(i_x\) or \(i_y\), respectively) are formulated at that level except in the integration dimension (\(x\) or \(y\), depending on the direction of propagation) where the up-wave level is used too. The values of \(\Delta x\) and \(\Delta y\) are therefore still mutually independent.

Boundary conditions for wave energy that is leaving the computational domain or crossing a coast line are fully absorbing in SWAN, both in the geographical and the spectral space. The user needs to prescribe the incoming wave energy along open geographical borders, although for coastal regions doing so only along the deep-water boundary may suffice. This implies that erroneous lateral boundary conditions are propagated into the computational area (see figure 2.3) from the apexes of the deep-water boundary conditions, and spreading towards the shore with the one sided width of the directional distribution of the incoming wave spectrum (that is, the spreading is smaller for swell conditions, and can reach up to 45° for wind sea). In order to avoid the propagation of such an error into the interest area, lateral boundaries should be sufficiently far away.