

5. Conclusions

The main purpose of this work has been to extend the formulation of the interior penalty method to the plastic case in order to exploit the beneficial properties of discontinuous Galerkin methods to the more general elastoplastic case.

It has been shown that the interior penalty method results in a consistent and symmetric discontinuous Galerkin method for elasticity. The results obtained for elasticity can be seen in Section 3.7 where it has been shown that the IP method yields an optimal convergence rate, as stated in [8], in the displacement and the stress error for values of the penalty parameter guaranteeing stability. For an unstable method there is still convergence, however the rate is not optimal.

Stability has been studied for the IP method. Only for the simple one dimensional case with elements of identical material and geometrical properties it is possible to determine a minimum value of the penalty parameter that a priori ensures stability. This value has been found equal to one if linear elements are used and equal to three if quadratic elements are used. It would be interesting to see whether this stability condition can be extended to more general cases, even for two-dimensional problems. Further research with the IP method may be done for this purpose.

The IP method method has been compared to the non-symmetric discontinuous Galerkin method. The non-symmetric method is again consistent, however symmetry is lost. Stability is guaranteed for this method for $\eta_K > 0$. The non-symmetric method converges for small values of η_K , however bigger values have to be chosen in order to obtain optimal convergence rate in displacement.

The main objective of Chapter 4 has been to extend the formulation of the interior penalty method to the plasticity case obtaining a consistent, stable and at the same time symmetric discontinuous method. The most significant problem faced in the derivation of the primal form of the IP method has been to maintain symmetry. The consistency and stability terms appearing in the IP method formulation have been extended straightforwardly to the plasticity case, however the extension of the symmetry term is not simple. The problem of obtaining a symmetric method has been solved in two steps. First the consistent but non-symmetric discontinuous Galerkin method has been studied. Then a term has been added resulting in a symmetric and consistent method, which corresponds to the IP method.

There are mainly two problems observed for the resulting interior penalty method if compared to the non-symmetric method. The first one is that instead of using the total strain in the return mapping algorithm only the strain corresponding to the interior of the element, ϵ_{int}^h , is considered. The second strain component corresponding to the boundaries, ϵ_{bound}^h , has been omitted, so the level of yielding does not correspond to the current strain field. However it has been shown that even when low penalty values are used and the contribution of σ_{bound}^h to the total stress is larger, the method yields acceptable results. Bear in mind that the IP method is an approximate method, and will converge to the real

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solution as the number of elements and the penalty values are increased if the method is consistent and stable, which is the case here.

The second observed problem is that the resulting linearization is not exact. This leads to unloading and oscillations of the numerical solution around the final equilibrium state causing for the IP method a greater amount of iterations needed to reach convergence compared to the continuous Galerkin and the non-symmetric method.

No stability condition has been found for the IP method in plasticity. This is caused by the fact that stability is influenced not only by the hardening behavior, but also by the spatial distribution of the yielding level throughout the studied example which at the same time depends on the external load. Since this is not known a priori, even for the simple one-dimensional case, with elements of equal geometrical and materials properties the minimum value of η_K which ensures stability cannot be found. Only after assuming the worse combination of the spatial distribution of yielding level an upper bound can be found, but this has turned to be far from realistic leading to an ill conditioned matrix, very sensitive to round-off errors. The same has been observed for the non-symmetric method where the stability condition, $\eta_K > 0$, valid for elasticity does no longer hold for plasticity. A deeper study of stability conditions in plasticity could be the objective of future research.

For elasticity it is clear that the IP method behaves better than the non-symmetric method for stable values of η_K . The only possible objection arises from the fact that a stability condition for the IP method is not known. However relatively small values of η_K will guarantee stability for general examples.

Although for relatively small increments of η_K , the problems faced by the interior penalty method in plasticity are reduced, the non-symmetric method behaves better. It yields better results and at the same time reaches convergence with fewer iterations. At the same time other discontinuous Galerkin methods, many of them appearing in [8], could be studied.