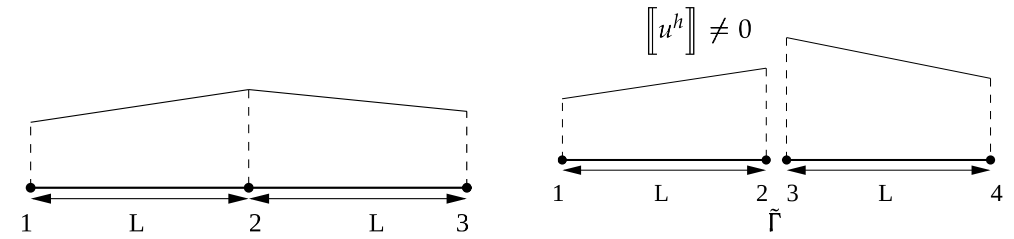
1. Introduction and objectives

The increasing requirements for reliability and durability of structures have stimulated improvement in the description of the elastoplastic behavior of materials. A clear understanding and prediction capacity must be obtained in order to guarantee the established minimum requirements for security. Economic reasons have also influenced this development since exploiting plastic behavior of materials leads to considerable material savings and therefore fewer costs. During the last several decades many works have been devoted to the development of elastoplastic models. However analytical solutions have turned to be extremely difficult if not impossible and that is why the use of the finite element method (FEM) in modeling elastoplastic behavior is essential. Most of the computational methods available combine the FEM with an implicit integration scheme called return mapping algorithm, as described in Simo and Hughes [1]. The FEM ensures that equilibrium at a global level is satisfied, whereas the return mapping algorithm guarantees that the stresses always lie within the established limits of the yield surface.

Commercial finite element packages using traditional continuous Galerkin finite element (cG) method are capable of predicting this elastoplastic behavior. However these face difficulties for certain cases. For consistent implementation of thin Poisson-Kirchhoff plates, shape functions require C^1 continuity. This is however met by few, if any, plate element and the results obtained for these are not completely satisfactory. An alternative is to use continuous C^0 elements and impose the continuity of slope weakly [2]. This procedure is known as discontinuous Galerkin (dG) finite element method. Unlike the traditional cG, dG methods do not require continuity of the approximate functions across the interelement boundary. Instead, in this case, C^1 continuity is enforced weakly by adding a term that penalizes the jump in the normal derivative. This makes it possible to use spaces of discontinuous piecewise polynomials to solve forth order elliptic problems.

Discontinuous Galerkin methods were first introduced for hyperbolic equations in 1973, Reed and Hill [3]. The development of these methods for parabolic and elliptic equations was done independently from the first group in the late sixties when Aubin [4] and Nitsche [5] considered numerical solution of problems with very rough Dirichlet boundary conditions. This could be done by replacing the Dirichlet boundary conditions with approximate boundary condition and penalizing this difference. Through a correct penalty parameter, the obtained solution was unique and the solution converged to the solution of the original problem. From there a number of discontinuous Galerkin methods for elliptic and parabolic equation where proposed, Arnold [6] and Baker [7]. The procedure used for Dirichlet boundary conditions could be used to impose interelement continuity weakly by penalizing the jump in displacement, $\llbracket \boldsymbol{u}^h \rrbracket$, between elements, see figure 1.1. Discontinuous Galerkin methods relax continuity requirements between elements. This makes it possible to use piecewise polynomials which are discontinuous across the inte-



(c) C^0 continuity in continuous Galerkin methods.

(d) Weak enforcement of continuity in discontinuous Galerkin methods.

Figure 1.1.. Continuous and discontinuous Galerkin methods.

rior boundaries. Recently a unified analysis of the different approaches for discontinuous Galerkin methods for elliptic problems has been presented in [8], where most of the references concerning the topic can be found.

At the same time dG methods have proved to have other interesting properties. In a discontinuous mesh, with no shared nodes between elements, every element is totally independent from the rest. This freedom makes dG methods ideally suited for *h*- and *p*-adaptivity [8]. So *h*-refinement can be done in a part of the mesh independently of other unchanged regions as well as *p*-refinement without worrying about the handling of hanging nodes. Another advantage that follows from this freedom is the possibility to combine with a discontinuous interface mesh non-matching grids [8], and therefore analyze them easily as a single model.

However little has been done assuming plastic behavior for the different dG methods, making it difficult to exploit the explained beneficial properties of the more general elastoplastic case. The main purpose of this master thesis is to analyze the Interior Penalty (IP) method and show if the extra flexibility obtained by using discontinuous approximations together with properly chosen stabilization terms result in optimal order convergence for elasticity as well as for plasticity.

In Chapter 2 the general equations for dG method in the elastic case will be derived. From the general equations, the weak form for the IP method and a non-symmetric discontinuous Galerkin method will be obtained in Chapter 3. It will be seen how the IP results in a consistent, symmetric but at the same time conditionally stable method and the non-symmetric method in a consistent but non-symmetric method. These methods will then be implemented using Matlab, and the convergence rate of the different methods examined giving some illustrative numerical examples, first some one dimensional, and then two dimensions numerical examples.

In Chapter 4 the IP and the non-symmetric method for the plasticity case will be derived and implemented. Some examples of the elastoplastic response using these two different methods will be given showing for each the rate of convergence.

Finally in Chapter 5 some concluding remarks and recommendations will be drawn.