

# Chapter 4

## Concluding remarks

The application of the common finite element method to solve the stationary Stokes problem in an incompressible fluid using linear elements derives in instabilities in the solution. Therefore, the use of a stabilization method is required in order to use linear elements instead of the quadratic ones. Linear elements are more appropriate because the computational cost is lower than quadratic elements.

One of the most important aspects in the implementation of the finite element method is the type of mesh selected. The results of this work show that the use of both quadrilateral or triangular elements in two dimensions or hexahedral or tetrahedral elements in three dimensions do not change substantially the obtained solution. Though, there is another basic aspect related to the type of mesh: the distribution of the elements. The results of the Cavity flow problem prove that the elements should be concentrated where the gradient of the variables is higher to achieve the highest accuracy in the solution.

The analysis of the results of the stationary Stokes problem with analytical solution in two dimensions leads to the conclusion that the most decisive factor in the stabilization parameter is the measure of the element size. For the analyzed problems, the most convenient measure is the minimum side of the element. In addition, the comparison of the analytical solution with the numerical solution confirms the predicted error behavior: a quadratic approximation for the velocity error and a linear one for the pressure field.

The stationary Stokes problem in three dimensions has a different behavior than the two dimensional case. The stabilization parameter depends strongly on the fluid viscosity. The stabilization parameter has to be multi-

plied by a constant value depending on the fluid viscosity in order to obtain an acceptable solution to the problem.

## 4.1 Future lines of research

The analysis performed in this work can be extended in several ways. Firstly, an analysis of the stabilization method implemented in the general Navier-Stokes equations. The addition of the viscous terms and the time dependency may alter the behavior and the accuracy of this stabilization method.

In addition, the translation of the code generated in Matlab to a language with a better memory management could allow the solution of the problem using finer meshes and therefore, more accurate results, mainly in the three dimensional case.

Besides, this paper only includes one stabilization method. There are several types of stabilization methods that could be analyzed to compare their accuracy and computational cost with the Galerkin Least-Squares method.