

# Chapter 1

## Introduction

### 1.1 Goals of the thesis

This thesis is concerned with the numerical approximation to the solution of the Navier-Stokes equations in viscous incompressible fluids when the time dependency and the convective terms can be neglected. That is, the stationary Stokes problem. A finite element method is implemented in order to solve the differential equations in a discrete domain.

First, a brief description of the problem involved in the analysis of incompressible flows is presented in chapter 2. Some basic concepts needed to understand the Navier-Stokes equations are introduced. Moreover, a brief discussion about these equations and the variables that are used to solve them is presented. Then, the stationary Stokes problem is developed in more detail. In particular, the strong form and the weak form associated to the Galerkin formulation are analyzed. The discretized equations associated to the weak form can be expressed as a matrix problem. Therefore, they can be solved as a system of linear equations. Once the stationary Stokes problem is studied, the need of a stabilization appears. In this case the stabilization method selected is the GLS (Galerkin Least Squares).

Chapter 3 presents a serie of numerical applications and examples of the stationary Stokes problem. These examples are divided into two main cases. The first one, is related to the stationary Stokes problem in two dimensions. Two different options are studied: one with an analytical solution and the cavity flow problem, a well-known problem analyzed by a large amount of researchers (see Huerta(2003)). Then, the problem is extended to three dimensions where the case with analytical solution is used to calibrate the

method accuracy.

Finally, an analysis of the numerical results is made in chapter 4, as well as some important remarks regarding the stabilization parameter. In addition, some future lines of investigation about this issue are suggested.

## 1.2 Main difficulties in numerical simulation of incompressible flow problems

The numerical methods used in this thesis face two main difficulties. The first one is related to the incompressibility condition of the fluid. It is important to point out that it appears when certain type of elements are used in the interpolation of the velocity and pressure fields. These elements produce instabilities that are independent of the fluid viscosity. The LBB condition governs this behavior. That is, the elements that pass the LBB condition do not produce these instabilities.

The second major source of problems occurs when studying fluids with high viscosity. These type of fluids have very important instabilities although an appropriate element interpolation is used. Several stabilization methods have been developed to solve these instabilities. One of them is the GLS method. It is important to point out that the stabilization parameter related to this method depends on the fluid viscosity.

## 1.3 Drawbacks of some finite elements

There are some issues to be commented concerning the several types of finite elements that can be used to solve this problem. On one hand, the finite elements passing the LBB condition, that is, the elements that are related to a stable solution of the stationary Stokes problems produce large systems of equations. The solution of these systems implies a high computational cost. Hence, the size of the elements cannot be as little as desired. On the other hand, the use of linear elements is inadequate due to the instability of the obtained solution. These elements have a lower computational cost than the quadratic ones but this advantage cannot be used.

Therefore, a stabilization procedure is needed in order to use low computational cost elements (i.e. linear elements) and do not generate unstable

solutions. The application of a stabilization method to the linear elements implies a computational cost lower than the one obtained with quadratic elements and higher than the one using linear elements.