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The behavior of a fluid can be modelled using the Navier-Stokes equations. This set of differential equations govern the velocity and pressure fields in a fluid under some external forces. In order to be solved, a set of boundary conditions is required. In general, these equations do not have analytical solution. Therefore, a numerical approximation to the solution is needed. The finite element method is one of the more used techniques to solve this problem using a numerical procedure.

This work is related to the solution of the stationary Stokes problem, that is, the Navier-Stokes problem in viscous incompressible fluids when the time dependency and the convective terms can be neglected. The application of a finite element method to solve the stationary Stokes problem is not trivial. There are some critical aspects that have to be carefully addressed. For instance, several types of elements guide to an unstable solution. This fact implies the necessity to implement a stabilization method in order to avoid these drawbacks. One of the most well-known stabilization methods is the Galerkin Least-Squares or GLS. This thesis studies the application of the GLS to the solution of the Stokes problem.

There are two big sources of difficulties to solve numerically this problem. The first one is related to the incompressibility condition. It appears when certain type of elements is used in the interpolation for the velocity and pressure fields. This behavior is governed by the LBB condition that filters which elements can be used and which not. Basically, linear elements lead to unstable solutions. The second source of problems relies in the analysis of the behavior of a fluid with high viscosity.

This thesis includes an analysis of all these issues and the behavior of the stabilization method in both two and three dimensions. For that purpose, a code using MATLAB software has been developed. In conclusion, the use of a stabilization method is very useful in elements not passing the LBB condition. It allows to obtain a stable solution with a computational cost lower than the cost of quadratic elements.