DECLARATION

Name: Milad Oliaee
Email: milad.oliaee@gmail.com
Title of the Msc Dissertation: Application of a detailed micro modeling technique to the study of the in-plane capacity of masonry walls
Supervisor(s): Pere Roca
Year: 2013

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University: Universitat Politècnica de Catalunya
Date: July 18, 2013
Signature: ___________________________
I dedicate this work to Henry Kagey, Steven Mahen, Nuria Casquero-Modrego, Abolhassan Astaneh and my parents. Without their support and influence, this would not be possible.
Application of a detailed micro modelling technique to the study of the in-plane capacity of masonry walls

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ABSTRACT

The Mediterranean basin is particularly at high public health risk due to the large number of existing unreinforced masonry buildings. Unreinforced masonry structures are particularly vulnerable to dynamic actions, especially seismic action, and should to be reevaluated and hopefully preserved. The material, often viewed as archaic and unpredictable for engineers to apply to modern scientific principles, has shown varying performance in response to past earthquakes. In the 2012 Emilia Romagna earthquake, many soundly built masonry buildings performed well, in stark contrast to others in close proximity that had collapsed. It has become the wider aim of this study to grow comprehension on the ultimate strength of typical masonry walls under bi-axial longitudinal loads whilst sustaining significant bearing load. To this aim, the simulation of in-plane capacity of walls, subject to combined vertical and horizontal loading, will be performed by methods of detailed micro modeling.

Such knowledge could greatly impact the perceived limitations of the structural material. As computation time and cost decrease with the advent of new technology, it is natural that new methods will be explored for modeling the material behavior. In this work, the comparisons to the simplified method among other theories gauge the success of the stated method.

Figure 1: San Paulo Church in Mirobello, Emilia-Romagna after the May 2012 earthquake.
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Resum

La conca mediterrània presenta un important risca causa de la gran quantitat d'edificis de maçoneria no reforçada existents. Les estructures de maçoneria no reforçada són particularment vulnerables a accions dinàmiques, especialment a l'acció sísmica, i han de ser revaluades i esperablement conservades. El material, sovint vist com arcaic i imprevisible per als enginyers en relació a l'aplicació dels principis de la ciència moderna, ha mostrat un rendiment variable en resposta als terratrèmols recentment esdevinguts. En el terratrèmol d'Emilia Romanya de 2012, molts edificis de maçoneria correctament construïts van presentar un bon acompliment, en marcat contrast amb d'altres edificis pròxims que van col·lapsar.

L'objectiu general d'aquest estudi és contribuir a millorar la comprensió de la resistència a la ruptura dels murs de maçoneria comuns sota la combinació de càrregues verticals i horitzontals. Amb aquest objectiu, la simulació de la capacitat de les parets sota l'efecte de càrregues en el seu plànol es durà a terme mitjançant un mètode per a la micromodelització detallada.

Un major coneixement del comportament de les parets de maçoneria podria afectar en gran mesura sobre la percepció de les limitacions d'aquest material estructural. Amb la reducció del temps de càlcul i dels costos que comporta l'adveniment de les noves tecnologies, és esperable que s'intensifiqui l'exploració dels mètodes de càlcul avançats per a la modelització del comportament dels materials. En el present treball, l'anàlisi mitjançant micromodelització detallada s'aplica per a la comprovació de l'acompliment de mètodes de caràcter simplificat.

Figura 1: Església de Sant Pau a Mirobello, Emília-Romanya, després del terratrèmol de maig 2012
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انتزاعی

ساختار های بنایی هستند به خصوص در معرض خطر به اقدامات پیوی به خصوص با توجه به اقدامات لرژه ای. کشته‌های در حوضه مدیرانه به خصوص در معرض خطر با توجه به تعداد زیادی از ساختمان‌های بنایی موجود و ساخت و ساز مداوم با این مواد در منطقه هستند. این مواد، اغلب به عنوان قدمی و ترسناک برای مهندسین استفاده از اصول علمی مدرن، نشان داده است عملکرد های مختلف در پاسخ به زلزله است. در زلزله سال 2012 امیلیا رومانیا به نظر مرسه بسیاری از ساختمان‌های بنایی خوب با نظر دیگران در مرحله نزدیک به تخریب قرار داشتند. این هدف ساخته شدن، در حالی که در مقایسه گسترده‌تر از این مطالعه به رشد درک قدرت نهایی دیواره‌های سنگ تراشی معمولی و پاسخ خود را به بارهای جانبی در توجه به بارهای تحمل شخص تبیین شده است. با این حال، به طور خاص، این پدیده بیش از حد قدرت در فشرده سازی از سنگ تراشی یا ساختارهای بارگذاری خارج از مرکز خواهند شد با استفاده از روش مدل سازی دقیق میکرو تجزیه و تحلیل شده است. چنین داشتن تا حد زیادی می‌تواند به دیدگاه‌های درک فلسفی از مواد ساختاری را تحت تاثیر قرار دهد. همطور که در زمان محاسبات و کاوش هزینه با ظهور فن‌های جدید، طبیعی است که روش‌های جدید برای مدل سازی و فشار مواد بررسی خواهد شد. در این کار، مقایسه روش ساده و نظریه‌های دیگر به موفقیت از روش تجویز سنح.

شکل 1: کلیسای سان پاتولو، امیلیا بعد از زلزله 2012.
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ABSTRACT

I paesi nel bacino del Mediterraneo sono particolarmente ad alto rischio a causa del gran numero di edifici in muratura “non rinforzati” (non adeguati sismicamente) e all’incessante attività di edificazione. Le strutture in muratura non rinforzate sono vulnerabili alle azioni dinamiche, specialmente a quelle sismiche, e necessitano di essere rivalutate e preservate. La muratura, spesso considerato, dagli ingegneri, arcaico ed imprevedibile per l’applicazione di moderni principi scientifici, ha dimostrato prestazioni variabili in risposta ai terremoti del passato. In occasione del terremoto dell’Emilia Romagna del 2012 diverse costruzioni in muratura solidamente edificate hanno risposto bene, in forte contrasto con altre che, invece, sono crollate pur essendo situate nelle immediate vicinanze. L’ampio obiettivo di questo studio è’ diventato quello di approfondire la comprensione della resistenza ultima di tipiche pareti in muratura sotto carichi longitudinali bissiali ed, allo stesso tempo, i carichi portanti. A tale scopo la simulazione della capacità portante nel piano di pareti soggette ad azioni combinate verticali e orizzontali sarà effettuata attraverso metodi di micro modellazione agli elementi finiti.

Tale conoscenza potrebbe notevolmente influenzare i limiti attuali di questo materiale strutturale. Poiché il tempo di calcolo e il costo diminuisce con l’avvento di nuove tecnologie, è naturale che nuovi metodi saranno esplorati per la modellazione del comportamento di questo materiale. Il confronto dei risultati ottenuti in questo lavoro con quelli ottenuti con un metodo semplificato derivato da altre teorie conferma il successo del metodo utilizzato.

Figura 1: Chiesa di San Paolo a Mirabello, Emilia-Romagna dopo il terremoto del Maggio 2012
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1. INTRODUCTION

The behavior and strength of unreinforced masonry structures is challenging for simple engineering tools to predict. The material has a wide array of configurations, joint thicknesses, mortar types and strengths, unit dimensions, and the construction quality is highly variable. Although the wide focus of structural engineers has left this construction material, to this day it's highly prevalent in the environment in which we inhabit. Structural masonry is used in developing countries for small residential projects in rural environments but also serves as partition or infill walls in developed countries. Residential projects in Morocco use masonry as infill for concrete frame construction as seen Figure 2.

Figure 2: Hollow clay tile infill in modern construction, Morocco 2013.

The impact of the material on public safety in construction is largely underestimated. For a six story apartment building with poorly detailed infill frames, a small seismic event may cause serious problems for occupants. From severe damage to global collapse, fatal seismic events are recurring around the globe in China's Sichuan province, Turkey's capital Ankara, rural Iran and L'Aquila’s historical center in southern Italy. Although the issue may not lie solely on poor engineering practice, our limited knowledge of the material behavior in such events might be causing poor decision making for design and intervention. Improving computational modeling of masonry structures is intended.

1.1 General Objective

The objective of the study is to explore new modeling techniques for masonry walls, observe the validity of past theories to predict the in-plane capacity of masonry walls, and weigh the success of more recent, simpler approaches.
1.2 Specific Objectives

- Review of principles for yield surfaces and failure criteria
- Define the numerical modeling approach
- Simulate the response of a masonry wall and observe the effects of boundary conditions
- Compare with classic failure criteria
- Compare with the simple modeling technique
- Obtain conclusions evident from the numerical results

1.3 Methodology

The micro modeling approach is utilized for numerical case studies to consider the behavior of a generic, running bond wall, in respect to a recent study on walletes [1]. The structure with different boundary conditions undergoes in-plane loads, and the influence of eccentricity is closely monitored. The bi-axial stress theories of Mann & Muller [2] and Turnsek & Cacovic [3] are compared to the numerical results, as well as the contemporary simplified mode just introduced by Pere et al [4].

1.4 History

At the 2nd International Brick Masonry Conference in 1970, two researchers from the former Yugoslavia, Turnsek and Cacovic, proposed theoretical considerations on the strength, deformation and failure mechanisms of brickwork walls under axial, eccentric and combined (vertical and horizontal) loading based on experimental results performed in the university lab at Padova, Italy [3].

Mann & Muller republished earlier research in English for the Proceedings of the British Ceramic Society of 1980, introducing three equations to predict the three different failure modes of shear-stressed masonry [2]. The equations contain coefficients that include the influence of individual unit height and width.

In 1993, Riddington and Naom developed a micro finite element program to predict the ultimate compressive strength of masonry [5]. The local failure criteria included in the program are brick tensile strength, mortar tensile strength, brick-mortar interface bond tensile strength and brick mortar interface shear strength. The nonlinear elastic behavior of the mortar is accounted for and the program was used to predict the ultimate compressive strength of panels in good agreement with experimental results. However, the program is limited to two dimensional plane elements.

In June of 2011, Roca et al [4] published in the International Journal of Architectural Heritage to propose a simple equilibrium model to obtain the capacity of unreinforced masonry shear walls for all three regions. The current version assumes a plastic stress distribution at the base equal to the compressive strength of the masonry.
Although there are many other contributions in the field not mentioned, these theories apply explicitly to solid clay masonry and will be of utmost interest in comparison the results of a new numerical model. The model is larger in size and has three dimensions. Dilation, confinement and cracking is considered in the third dimension.
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2. APPLIED MECHANICS

2.1 Bi-Axial Stress Theories

For purposes of recognizing the mathematical formulations implemented in modeling masonry, the following theories are briefly reviewed for both the awareness of the reader and the author as state of the art.

2.1.1 Failure Envelopes

Tresca

The yield condition of Tresca is a maximum shear stress condition expressed in the principal stress space [6]. The equation considers the principle major and minor stresses,

\[ f(\sigma, \kappa) = |\sigma_1 - \sigma_2| - \bar{\sigma}(\kappa) \]  

with \( \bar{\sigma}(\kappa) \) the uniaxial yield strength as a function of the internal state variable \( \kappa \).

![Figure 3: Tresca yield condition (in the π- and rendulic plane) [7].](image)

Mohr-Coulomb

The yield condition of Mohr-Coulomb is an extension of the Tresca yield condition that considers pressure dependent behavior [8]. The conical yield function can be expressed in the principal stress space as

\[ f(\sigma, \kappa) = \frac{1}{2}(\sigma_1 - \sigma_3) + \frac{1}{2}(\sigma_1 + \sigma_3) \sin \varphi(\kappa) - \bar{c}(\kappa) \cos \varphi \]  

with \( \bar{c}(\kappa) \) the cohesion as a function of the internal state variable \( \kappa \) and \( \varphi \) the angle of internal friction, a function of the internal state variable.

Drucker-Prager

The Drucker-Prager yield condition is a smooth approximation of the Mohr-Coulomb yield surface [9], with formulation given by
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![Equation](3)

\[ f(\sigma, \kappa) = \sqrt{\frac{1}{2} \sigma^T \mathcal{P} \sigma + \bar{\alpha} + a_f \pi^T \sigma - \beta \bar{c}(\kappa)} \]

with \(\bar{c}(\kappa)\) the cohesion function of the internal state variable \(\kappa\). The projection matrix \(\mathcal{P}\) is equal to that of the Von Mises yield condition:

\[
\mathcal{P} = \begin{bmatrix}
2 & -1 & -1 \\
-1 & 2 & -1 \\
-1 & -1 & 2 \\
\end{bmatrix}
\]

The projection vector \(\pi\) is given by

\[
\pi = \begin{bmatrix}1 & 1 & 0 & 0 & 0\end{bmatrix}^T
\]

The scalar quantities \(a_f\) and \(\beta\) are given by

\[
a_f = \frac{2 \sin \varphi(\kappa)}{3 - \sin \varphi(\kappa)} \quad \text{and} \quad \beta = \frac{6 \cos \varphi(\kappa)}{3 - \sin \varphi_0}
\]

where \(\varphi\), the angle of internal friction, is a function of the internal state variable, and \(\varphi_0\) is the initial friction angle.

**Rankine Hill**

The Rankine Hill multi-surface failure envelope is proposed by Lourenco and Rots [11], combining the preceding bi-axial stress limit theories of Hill in the bi-axial compression quadrant and Rankine in the remaining quadrants [7]. The Rankine yield criterion reads

\[
f = \sqrt{\frac{1}{2} \xi^T P_t \xi + \frac{1}{2} \pi \xi}
\]

where \(\xi\) is the yield stress with the projection matrix \(P_t\) and vector \(\pi\) given by

\[
\mathcal{P}_t = \frac{1}{2} \begin{bmatrix}1 & -1 & \cdots & -1 \\
-1 & 1 & \cdots & 1 \\
\vdots & \ddots & \ddots & \vdots \\
-1 & \cdots & 1 & 4\alpha_T
\end{bmatrix} \quad \text{and} \quad \pi = \begin{bmatrix}1 \\
1 \\
0
\end{bmatrix}
\]

The parameter \(\alpha_T\) controls the shear stress contribution to failure and can be expressed as

\[
\alpha_T = \frac{f_{t,x} f_{t,y}}{\tau_u^2}
\]

where \(f_{t,x}\) and \(f_{t,y}\) are the tensile strengths in the\( x \) and \( y \) direction respectively, and \(\tau_u\) is the shear strength at zero normal stress. This criterion applies specifically for masonry, having different
strengths parallel and perpendicular to the bed joints. The Hill yield criterion is a rotated centered ellipsoid in stress space [12], expressed as

$$f_c = \frac{1}{\sqrt{2}} \sigma^T p_c \sigma - \bar{\sigma}_c \kappa_c$$

(9)

with the projection matrix

$$p_c = \begin{bmatrix} 2\bar{\sigma}_{c,y} \kappa_c & \beta \\ \bar{\sigma}_{c,x} \kappa_c & \beta \\ \beta & 2\bar{\sigma}_{c,y} \kappa_c \\ \bar{\sigma}_{c,x} \kappa_c & 2y \end{bmatrix}$$

(10)

where $\bar{\sigma}_c$ is the yield value, $\kappa_c$ controls the softening behavior, and $\beta$ rotates the surface around the shear stress axis and may be determined from biaxial compression tests [13].

![Figure 4: Rankine Hill multi-surface yield criterion [7]](image)

Thus the Rankine Hill model is a combined plane stress continuum model which captures different strengths and softening characteristics in orthogonal directions [7] depicted and plotted dependently in Figure 4 above.

### 2.1.2 Failure Criteria Specific for Masonry

**Mann & Muller (1980)**

A shear-failure theory considering brick strength, format, bonding type, friction and cohesion coefficients, Mann & Muller introduces three regions of behavior: friction failure of the bed joint, failure due to cracking of the bricks, and compression failure of the masonry wall. The theory is based on the assumption that no shear stresses can be transferred in the vertical joints between the bricks, not a
completely true assumption, but one that has been found reasonable for the current study and whose qualifications will be assessed later on in this report [2].

Figure 5: Mann and Muller deduction principles [2].

Therefore, a torque on each unit originates from an imbalance of vertical compressive forces on each half of the unit. This torque must be, in turn, balanced by traction forces on the top and bottom faces of the unit to achieve equilibrium. This equation of equilibrium gives way to three separate expressions for which each different failure mode governs a particular region.

\[ \sigma_{1,2} = \sigma_x \pm \tau \cdot \frac{2\Delta x}{\Delta y} \] (11)

The failure mode at low compressive stresses is governed by friction failure of the bed joint. However, Mann & Muller introduce modified constants in the Coulomb friction law. These constants are improved by unit geometry characteristics as seen below in equations (13) & (14).

\[ \tau = \bar{k} + \bar{\mu} \cdot \sigma_x \] (12)

with the 'reduced' cohesion

\[ \bar{k} = \frac{1}{1 + \mu \frac{2\Delta x}{\Delta y}} \] (13)

and the 'reduced' friction coefficient

\[ \bar{\mu} = \mu \frac{1}{1 + \mu \frac{2\Delta x}{\Delta y}} \] (14)

A stepped failure region occurs at moderate compressive stresses. Because it is assumed no shear stresses can be transferred in the vertical joints, the bricks must transfer double the shear force, and
the units crack with the onset of principal tensile stresses in the brick exceeding the tensile strength $\beta_{zst}$ [2].

$$\sigma_1 = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (2.3r)^2} = -\beta_{zst}$$  \hspace{1cm} (15)

Turnsek's failure criterion, based on the tensile stress of the masonry, is given by

$$\tau = \frac{\beta_{zst}}{2.3} \cdot \sqrt{1 + \frac{\sigma_x}{\beta_{zst}}}$$  \hspace{1cm} (16)

At very high compressive stresses, compression failure occurs, namely when a half unit is subject to a principle stress approaching the compressive strength of the masonry $\beta_M$. Rearranging equation (11) gives

$$\tau = (\beta_M - \sigma_x) \cdot \frac{2\Delta x}{\Delta y}$$  \hspace{1cm} (17)

Thus the enveloping curve is outlined by the above equations. Any overlapping functions shall take the minimum value for two intersecting regions. In extreme cases, the parabola of equation (15) can fall completely under the curve of equation (12) for very low tensile strength units with high cohesion in the mortar layers.

**Turnsek & Cacovic (1970)**

Turnsek's theory diffidently begins with the constitutive relations of plane stress.

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\tau_{xy}^2 + \left(\frac{\sigma_x - \sigma_y}{2}\right)^2}$$  \hspace{1cm} (18)

Applying Airy's [14] equations concerning plane distribution of stress within the wall, the following equation results:

$$\sigma_n = \sigma_0 \left[ -0.50 + \sqrt{\left(\frac{\tau_0}{\sigma_0}\right)^2 + 0.25} \right]$$  \hspace{1cm} (19)

For wider walls with greater eccentric loads, St. Venant's principle no longer holds true [15]. However, for both cases, the maximum tensile stress at the center of the wall panel may be expressed as
Although the theory prescribes the quantity $b$ taken constant for panel height-to-width ratios greater than 1.5, for panel aspect ratios equaling unity, the following equation ensues:

$$b = 1.543 - 0.478\left(\frac{\tau_0}{\sigma_0}\right)$$

The coefficient falls in size with increasing eccentricity of loading ratios. Plotting the parameter across a range of compressive stresses with shear stress held constant demonstrates the deviation for the original constant value of 1.5 is negligible.

For purposes of simplifying the comparison of Turnsek’s envelope to numerical results, the coefficient $b$ is kept constant at the original value of 1.5. As seen, for increased compressive stress holding shear constant, the coefficient has small influence on the coefficient beyond 10 MPa compressive stress. It is noted that the value of Turnsek’s $b$ coefficient has also been reviewed by Tomazevic.

**Simplified Method (Roca et al., 2011)**

The simplified method considers the global equilibrium of resultant forces acting on the wall panel. From the base reaction to the top resultant point load, a compression strut is formed with an associative angle. Depending on the aspect ratio of the wall, the slope of any compression line will be limited by the friction angle of the unit-mortar interface. Struts can be either discrete or smeared depending on the type of loads and reactions (i.e., concentrated or distributed). Fans represent the converging compression field stresses with rays that project from the concentrated base reaction to the larger area of the applied load [4]. The resultant horizontal forces may be taken as follows:

$$H = V \frac{b - m}{2h} = V \frac{b}{2h} (1 - \nu)$$
where \[ v = \frac{m}{b} = \frac{V}{b f_c} \] (23)

For compression struts that exceed the maximum allowed slope according to the Mohr-Coulomb criterion, the wall is prescribed by a model of fan and parallel struts. Therefore, the resisting forces are divided into two actions, as indicated by \( V_1 \) and \( V_2 \)

\[
H = V_1 \frac{\tan \alpha}{2} + V_2 \tan \alpha = V \tan \alpha \left[ 1 - \frac{h}{2b(1 - v)\tan \alpha} \right]
\] (24)

For concentrated loading, the slope of the strut will gradually increase until the strut expands to fully cover the wall length, in which case the failure is due to the compression at the toe [4].

\[
\tan \xi = \frac{b - m + 2e}{2h}
\] (25)

If the maximum slope at the boundaries of the strut approach tangent \( \alpha \), the resulting failure is due to frictional sliding, and the maximum horizontal force becomes

\[
H = V \tan \xi
\] (26)

The latter version of the simplified model is more appropriate for fixed rotation boundary conditions as presented in Figure 7(c). Depending on the friction modulus and wall geometry, typical distributed eccentric loads may fall into two categories shown in Figure 7 (a) & (b).

![Figure 7: Free body diagrams for the Simplified Method [4].](image)

**2.2 Compression Failure Theories**

For walls with dimensions within the buckling slenderness ratio, the failure of masonry is dependent upon the interaction of stresses and strains between the units and the mortar. Depending on the
elasticity of the composite masonry, the unrestrained height, and effective wall thickness, buckling may be an issue, especially if the wall leaves separate. However, for a wall six courses high of clay brick masonry, by observation one can infer buckling failure is not expected.

Considering a brick-mortar prism subjected to an axial compressive load, equilibrium and compatibility relations prescribe the extensional strains in the brick and adjacent mortar beds [16]. Assuming elastic behavior within the brick units,

\[ e_{xb} = \frac{1}{E_b} \left[ \sigma_{xb} + v_b (\sigma_y - \sigma_{zb}) \right] \]  \hspace{1cm} (27)

\[ e_{zb} = \frac{1}{E_b} \left[ \sigma_{zb} + v_b (\sigma_y - \sigma_{xb}) \right] \]  \hspace{1cm} (28)

and similarly in the mortar joints

\[ e_{xm} = \frac{1}{E_m} \left[ -\sigma_{xm} + v_m (\sigma_y - \sigma_{xm}) \right] \]  \hspace{1cm} (29)

\[ e_{zb} = \frac{1}{E_b} \left[ -\sigma_{zb} + v_m (\sigma_y - \sigma_{zm}) \right] \]  \hspace{1cm} (30)

![Figure 8: Stresses in brick-mortar composite [16].](image)

The total lateral tensile force in the brick is equal the total lateral compressive force in the mortar.

\[ \sigma_{xm} = \alpha \sigma_{xb} \quad \text{and} \quad \sigma_{zm} = \alpha \sigma_{zb} \quad (31-32) \]

where \( \alpha \) is the ratio of the height of the brick to the thickness of the mortar bed. As the lateral strains in the bricks and mortar are the same, substituting the above equations gives
Application of a detailed micro modelling technique to the study of the in-plane capacity of masonry walls

$$\sigma_{xb} = \sigma_{zb} = \frac{\sigma_s(\beta y_m - v_b)}{1 + \alpha \beta - v_b - \alpha \beta y_m}$$

(33)

where $\beta = E_b/E_m$. Assuming a linear relationship between ultimate longitudinal compressive stress and lateral tensile stress as well as neglecting $(1 - \nu)$ gives the relationship

$$\sigma_{xb} = \sigma_{zb} = \frac{\sigma_s(\beta y_m - v_b)}{1 + \alpha \beta - v_b - \alpha \beta y_m}$$

(34)

Applying Hilsdorf’s tri-axial strength equation for concrete to mortar as

$$f'_1 = f'_2 + 4.1\sigma_2$$

(35)

The magnitude of local stress at failure is therefore given by

$$\sigma_y = f'_b \left( f'_u + \alpha f'_j \right)$$

(36)

where $j$ is the mortar bed thickness, $b$ is the height of the brick, $f'_j$ is the uniaxial compressive strength of mortar and $\alpha = j/4.1b$. The average masonry stress at failure is then taken as the local stress at failure divided by a coefficient of non-uniformity, $U_u$, which Hilsdorf has established experimentally for various brick-mortar combinations [17]. The coefficient varies with brick-work strength, but for cement mortar it has been shown to have a value of around 1.3 in the medium strength range [16]. This approach was developed by Khoo and Hendry [18], who investigated the behavior of brick material under a state of biaxial compression-tension and of mortar under a state of tri-axial compression. Although the limit state of masonry behaves nonlinearly at failure, the elastic principals behind orthogonal stress interaction reasonably approach the correct solution.
Application of a detailed micro-modelling technique to the study of the in-plane capacity of masonry walls

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NUMERICAL METHODS

2.3 Finite Element Method

For the finite element model, three dimensions are chosen to account for the out-of-plane stresses in the mortar which influence compressive strength of the joint. The confinement effects will influence the ultimate local compressive strength within the mortar joints loaded in biaxial tension and vertical compression.

Solid Three Dimensional Elements

Solid elements are general purpose elements that have the tendency to produce large systems of equations. Some elements may have as many as 20 nodes, such as the CHX60 element in the finite element program Diana [7]. Typical applications of solid elements are the analysis of voluminous structures, thick walls and floors, and soil masses.

Figure 9: Generic solid element in Diana [7].

The displacements in the nodes yield deformations of an infinitesimal part $\delta_x$, $\delta_y$, $\delta_z$ of the element. From these deformations, Diana derives the Green-Lagrange strains as

$$\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{yy} & \varepsilon_{zz} & \gamma_{xy} & \gamma_{yz} & \gamma_{zx} \end{bmatrix}^T$$

(39)

$$\varepsilon_{xx} = \frac{\delta u_x}{\delta x}, \quad \varepsilon_{yy} = \frac{\delta u_y}{\delta y}, \quad \varepsilon_{zz} = \frac{\delta u_z}{\delta z}$$

(40)

$$\gamma_{xy} = \frac{\delta u_x}{\delta y} + \frac{\delta u_y}{\delta x}, \quad \gamma_{yz} = \frac{\delta u_y}{\delta z} + \frac{\delta u_z}{\delta y}, \quad \gamma_{zx} = \frac{\delta u_z}{\delta x} + \frac{\delta u_x}{\delta z}$$

(41)

The Green-Lagrange strains are derived for all integration points and may be extrapolated to the nodes. The sign convention holds that elongation yields positive strain at every node at the brick corners and midpoints [7]. The Gaussian integration points do not coincide with nodes, and the interpolation functions may often result in wide variation of nodal forces for dense meshes. Smoothing
functions automatically generate color mapped contours within the medium to contrast stress states, strain states, and more. These visual tools aid in the assessment of the model's validity, as well as the verification of resulting phenomena the user would expect from the fundamental principles describing nature.

![Figure 10: Solid Three Dimensional CHX60 element in Diana [7].](image)

The CHX60 element is a twenty-node isoparametric solid brick element based on quadratic interpolation and Gauss integration. The polynomials for translations can be expressed as

\[
u(\xi, \eta, \zeta) = a_0 + a_1 \xi + a_2 \eta + a_3 \zeta + a_4 \xi \eta + a_5 \xi \zeta + a_6 \xi^2 + a_7 \eta^2 + a_8 \zeta^2 + a_9 \xi \eta^2 + a_{10} \xi \zeta^2 + a_{11} \xi^2 \eta + a_{12} \xi^2 \zeta + a_{13} \xi^2 \eta + a_{14} \xi \eta^2 \zeta + a_{15} \eta^2 \zeta^2 + a_{16} \eta^2 \xi^2 + a_{17} \eta^2 \xi \zeta + a_{18} \eta^2 \xi^2 \eta + a_{19} \eta^2 \xi^2 \zeta + a_{20} \eta^2 \xi^2 \zeta^2\]

(42)

Solid elements are general purpose elements that have the tendency to produce large systems of equations. However, they prudently serve to capture three dimensional local phenomena otherwise overlooking by simpler finite elements.

**Interface Elements**

The structural interface elements describe the interface behavior in terms of a relation between the normal and shear tractions, and the normal and shear relative displacements across the interface [7].

Typical applications for structural interface elements are elastic bedding, nonlinear-elastic bedding, discrete cracking, friction between surfaces, masonry, etc. The chosen modeling technique employs plane interface elements between face of three-dimensional elements.

![Figure 11: Variables of the line-solid interface elements in Diana [7].](image)
Point-solid structural interfaces are oriented in the local xyz axes. The normal traction \( t_x \) is perpendicular to the interface; the shear tractions \( t_y \) and \( t_z \) are tangential to the interface as shown in Figure 11. Although it is a feasible option, the compression failure is not modeled to insure the crushing and cracking will occur within the individual mortar and unit elements.

**Total Strain Crack Model**

The total strain crack model includes constitutive properties for tensile behavior, shear behavior, compressive behavior, and lateral influence [7]. The lateral influences may be applied within a concept based on total strain to describe the effect of lateral cracking or lateral confinement, including Vecchio’s modified compression field theory [19].

The total strain based crack models follow a smeared approach for the fracture energy. Four softening functions based on fracture energy are implemented, a linear softening curve, an exponential softening curve, and two non-linear softening curves, all related to a crack bandwidth as is usual in smeared crack models. Tensile behavior which is not directly related to the fracture energy can also be modeled. The three-dimensional extension to this theory is proposed by Selby & Vecchio (1993).

Vecchio presented an analytical model capable of predicting the load deformation response of reinforced concrete elements subjected to in-plane shear and normal stresses [19]. The model treats cracked concrete as a new material with its separate stress-strain characteristics and has been implemented in Diana for masonry. The modified compression field theory is capable of prediction the response of cementitious material by allowing tensile stresses to develop normal to cracking planes.

The model includes lateral expansion effects due to poisson’s ratio. The Poisson effect is taken into account via the equivalent strain concept. In case of linear-elastic behavior the constitutive relationship in three-dimensional stress-strain state is given by

\[
\begin{pmatrix}
\bar{\varepsilon}_1 \\
\bar{\varepsilon}_2 \\
\bar{\varepsilon}_3
\end{pmatrix} = \begin{pmatrix}
\frac{1 - \nu}{(1 + \nu)(1 - 2\nu)} & \frac{\nu}{(1 + \nu)(1 - 2\nu)} & \frac{\nu}{(1 + \nu)(1 - 2\nu)} \\
\frac{\nu}{(1 + \nu)(1 - 2\nu)} & \frac{1 - \nu}{(1 + \nu)(1 - 2\nu)} & \frac{\nu}{(1 + \nu)(1 - 2\nu)} \\
\frac{\nu}{(1 + \nu)(1 - 2\nu)} & \frac{\nu}{(1 + \nu)(1 - 2\nu)} & \frac{1 - \nu}{(1 + \nu)(1 - 2\nu)}
\end{pmatrix} \begin{pmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3
\end{pmatrix}
\]

with the equivalent uniaxial strain vector defined by

\[
P = \begin{pmatrix}
\frac{1 - v^2}{\Delta'} & \frac{v_{yx} + v_{zx}v_{yz}}{\Delta'} & \frac{v_{yx} + v_{zx}v_{yz}}{\Delta'} \\
\frac{v_{xy} + v_{zx}v_{yz}}{\Delta'} & \frac{1 - v^2}{\Delta'} & \frac{v_{xy} + v_{zx}v_{yz}}{\Delta'} \\
\frac{v_{zx} + v_{xy}v_{yz}}{\Delta'} & \frac{v_{yz} + v_{yx}v_{xx}}{\Delta'} & \frac{1 - v^2}{\Delta'}
\end{pmatrix}
\]
The concept is applied to the nonlinear material model implement in Diana via a tangent stiffness sub-matrix and an orthotropic formulation adapted for Poisson’s ratios [7].

2.4 Implementation

Detailed Micro Modeling

A method once thought too computationally intensive has regained attention in the scientific foreground for analyzing unreinforced masonry structures. Disassembling the complexity of the heterogeneous material and allowing simple elements to capture local behavior numerically allows the user to observe many phenomena otherwise invisible to the user. Each bed joint, head joint, and masonry unit has its own entity are modeled by solid elements (in three dimensional analyses) and interfaces by the fundamental laws of elasticity, plasticity and friction.

![Figure 12: 3D finite element mesh for a detailed micro model of a masonry prism [20].](image)

It is recommended that each mortar layer be modeled by at least two solid elements with intermediate nodes via Gaussian integration points. The mesh shall be uniform in thickness in the units as well. Each interface element needs correct access orientation to perform as intended; therefore, model creation is relatively cumbersome and should be meticulously monitored.

The constitutive relation for the solid elements in compression is modeled by a parabolic curve in Diana as a formulation based on the fracture energy, according to Feenstra [21]. The parabolic curve is described by three characteristic values. The strain $\alpha_{c/3}$, at which one third of the maximum compressive strength $f_c$ is reached, is

$$\alpha_{c/3} = -\frac{1}{3} \frac{f_c}{E}$$  \hspace{1cm} (43)

The strain $\alpha_c$ at which maximum compressive strength is reached, is

$$\alpha_c = -\frac{5}{3} \frac{f_c}{E} = 5\alpha_{c/3}$$  \hspace{1cm} (44)
Note that $\alpha_{cr/3}$ and $\alpha_e$ are determined irrespective of the element size or compressive fracture energy.

Lastly, the ultimate strain $\alpha_u$, at which the material is completely softened in compression, is

$$\alpha_u = \alpha_e - \frac{3}{2} \frac{G_c}{h f_c}$$  \hspace{1cm} (45)

The parabolic compression curve is plotted in figure

![Figure 13: Parabolic compression curve in Diana.](image)

The constitutive relation for the solid elements in tension is conditional and linear. The relation of the crack stress is given by

$$\sigma_{nn}^{cr}(\varepsilon_{nn}^{cr}) = \begin{cases} 
1 - \frac{\varepsilon_{nn}^{cr}}{\varepsilon_{nn, ult}^{cr}} & \text{if } 0 < \varepsilon_{nn}^{cr} < \varepsilon_{nn, ult}^{cr} \\
0 & \text{if } \varepsilon_{nn, ult}^{cr} < \varepsilon_{nn}^{cr} < \infty
\end{cases}$$  \hspace{1cm} (46)

![Figure 14: Linear tension softening in Diana [7].](image)

The shear behavior is not explicitly modeled, but rather indirectly addressed by the interaction of the three orthogonal components of the strain tensor defined by (37). However, the final effect is in itself similar and deemed sufficient.
Simplified Micro Modeling

To increase the efficiency of the modeling technique, researchers have introduced a simplified modeling technique. The method omits thickness or material properties for the mortar joints and contains its effects within a more complex interface element. Interfaces may also be interleaved diagonally with each unit to capture specific failure mechanisms.

![Diagram](image)

Figure 15: Simplified micro-modeling configuration for masonry units [22]

Although the simplified micro-modeling has been successful to simulate the behavior of masonry wall with combined in-plane loads, the compressive strength must be modeled within the interface element because the mortar joint is omitted. Instead of specifying the compressive strength a priori, micro modeling contains the compressive strength indirectly within the analysis.
3. MODEL CREATION

One quarter of a single unit with surrounding mortar is connected via plane interfaces. The quarter unit is then mirrored twice and the remaining half is omitted by symmetry. Each half unit is arrayed the length of a single course, offset half a unit length and stacked above. Each double layer shall be arrayed upward the height of the wall panel. Finally, a rigid beam is placed above the running bond wall to insure distributed loading. All forces and displacements will be imparted on the beam and transferred to the wall below. Before execution, a compression test serves as one of many model checks and determines an elastic modulus of the masonry of 19,922 MPa and a compressive strength, 40.3 MPa, relatively close to the experimental values obtained by the Riddington study [5].

3.1 Riddington Specimen

Riddington, Naom (1994)

The Riddington study developed a finite element program to model the compressive strength of masonry wallets. In the model, constitutive models for brick and mortar are proposed considering compression, shear and tension independently. Iterative methods for solving non-linearity due to progressive failure and the material non-linearity are dealt with simultaneously in order to reduce computer time requirements. For purposes of validating the program, experiments on wallets built with half scale bricks were conducted and found to come in good agreement with the model’s predictions. Each wallette was analyzed using two versions of the finite element program: with and without mortar material non-linearity. Tabulated results of the experiments and numerical models are shown below.

Table 1: Riddington predicted failure load compared with experimental results

<table>
<thead>
<tr>
<th>Batch</th>
<th>Predicted ultimate load (KN)</th>
<th>Ultimate test load (KN)</th>
<th>Loading arrangement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Without mortar non-linearity</td>
<td>With mortar non-linearity</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>606.7</td>
<td>477.8</td>
<td>480.0</td>
</tr>
<tr>
<td>2</td>
<td>117.5</td>
<td>93.0</td>
<td>98.4</td>
</tr>
<tr>
<td>3</td>
<td>156.1</td>
<td>128.3</td>
<td>149.0</td>
</tr>
</tbody>
</table>
Geometry

In total, the wall dimensions are 325 mm long, 270 mm high and 49 mm thick. Each unit has dimensions 30 mm x 49 mm x 105 mm, and each joint has a 10 mm thickness. There are six courses, each three units long in running bond. The length-to-height ratio of each unit approaches three.

![Figure 16: Wall panel dimensions.](image)

Material Properties

The material properties of the individual mortar and unit samples are obtained from the Riddingon study. The input parameters are the elastic modulus, shear modulus, Poisson’s ratio, tensile strength, tensile fracture energy, interface in-plane shear strength, in-plane shear fracture energy, compressive strength, and compressive fracture energy. The material properties of the masonry constituents are listed in Table 2 below.

<table>
<thead>
<tr>
<th>Table 2: Riddington Material Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>E (MPa)</strong></td>
</tr>
<tr>
<td>------------</td>
</tr>
<tr>
<td>22,000</td>
</tr>
<tr>
<td>0.15</td>
</tr>
<tr>
<td>61</td>
</tr>
<tr>
<td>61</td>
</tr>
<tr>
<td>10.7</td>
</tr>
<tr>
<td>0.2</td>
</tr>
<tr>
<td>-</td>
</tr>
</tbody>
</table>
The brick mortar interface properties, brick and mortar strength properties, elastic moduli, and Poisson’s ratios are taken from lab tests within the Riddington study. The masonry composite properties vary depending upon mortar joint thickness and observing the effect of measured brick tensile strength. For the three different specimens tested, the ultimate compressive strength of the masonry for the geometry is found to range between 30-40 MPa. Larger mortar joint thicknesses result in lower compressive strengths whereas larger brick tensile strengths result in higher compressive strengths.

The tangent modulus of friction for the masonry interface elements is listed as 0.6, corresponding to a friction angle of $34^\circ$. The tensile bond strength and shear bond strength are listed as 0.523 MPa and 1.02 MPa, respectively. These values are hardwired into the Diana model with an assumed 1 mm crack width to calculate fracture energy values. The rigid beam elastic properties are set at an arbitrarily high value to insure proper application of loading.

The nonlinear properties input into solid elements are discrete cracking parameters. That is, any instance of the tension, shear, or compressive stress exceeded will trigger linear softening behavior prescribed by the user. For the nonlinear interface elements, the algorithm preforms at each node and does not explicitly consider the friction tangent modulus or the dilatancy angle.

**Boundary Conditions**

As noted previously, one plane of symmetry is utilized down the middle of the masonry wall to conserve computational resources wisely. Along the mirror plane, rollers restricting the translational displacement of the nodes ensure appropriate boundary conditions. Halving the number of elements and degrees of freedom, decrease the stiffness matrix size four-fold.

![Supports](image)

**Figure 17:** Boundary conditions of the model in Diana for free rotation case.

All nodes at the base are pinned in the three translation direction. The nodes at the top of the rigid beam have boundary conditions depending on the model. The model with free rotation at the top has
no translational restraints whereas the model with restrained rotation at the top has vertical translational restraint.

**Loading Sequence**

A vertical load is applied to the top nodes of the wall in increments until the desired compressive stress is obtained. Nodal forces at every node atop the rigid beam achieve this end. Immediately following, the top of the wall is displaced horizontally in increments until divergence is reached or the wall has reached 1 mm displacement. The horizontal load is applied by deformation of the top most nodes in unison via displacement controlled nonlinear analysis. Divergence may occur when the discrete cracking mechanisms within the elements have become so severe that the stiffness matrix is rendered inutile for the nonlinear solver to continue.

![Conceptual loading scheme diagram.](image)

**Figure 18: Conceptual loading scheme diagram.**

**Verification**

When introducing non-linear interface elements into the model, it is important to orient the local axes correctly to insure proper behavior. Because the element properties may be reversed, improper axis rotation will induce tensile rather than compressive stresses, immediately causing crack openings in every node tied to the interface. If not executed appropriately, the model would become a completely disconnected arrangement of solid plates at low stress states.

To insure the proper orientation of the non-linear interface elements, a pure tensile test in the vertical direction is performed also. Because the interface tensile fracture energy is the smallest of the other solid elements, it is expected the masonry tensile strength is equal to or slightly larger than the tensile strength of the interface elements. The interface elements are stronger in shear than tension, so each head joint could theoretically resist more force if the failure plane crosses more than one course. However, that is not the case, and the wall yields in tension at 0.521 MPa. The value falls just below the 0.523 MPa interface tensile strength. The post yielding is output by the solver quite easily because linear softening has been elected. The full stress-strain curve is shown in Figure 19.
A horizontal tension test is also done to capture the behavior of the interface elements. However, the result is the strength approaching the tensile strength of mortar. The interface elements in the head joints open in tension, but the elements in shear along the bed joints transfer force to the solid elements and yield the mortar.

In addition to tension tests, pure compression tests are conducted to verify the model, in the vertical and horizontal direction. For the vertical compression test, the full scale tests in the Riddington study show ultimate compressive strengths ranging between 30-35 MPa. The numerical model finds compression strength of 40.3 MPa at 0.0028 shortening strain. The results are satisfactory, and the stress-strain curve is shown on the following page in Figure 20.
Figure 20: Vertical compressive test of masonry wall.
4. RESULTS

Two models with 13 loading schemes provide adequate results to map the failure surface of the Riddington specimen. 12 vertical load cases with displacement control are accompanied by an displacement controlled compression analysis. Every ultimate load reach in the numerical results signifies the maximum resisted shear force in the displacement based analysis. Post-peak behavior is successfully captured and shown on stress-lateral drift ratio curves. Some cases having relatively high and low compressive loads (0 MPa and 36 MPa) have no post-peak behavior in the stress-strain curves, but rather serve as extremities to obtain a complete failure surface. In reality, these cases are impractical because there exists no massless wall nor any lateral force resisting system with such high static bearing loads.

4.1 Interaction Diagrams

The vertical-horizontal force interaction diagrams follow a parabolic shape rotated in the first quadrant, lining the ordinate axis. Due to some inherent tensile strength in the masonry, the toe offsets at the abscissa, however, far less than observed for reinforced concrete walls. Each ultimate state of the load cases per model are plotted in comparison with the initial compression force on the wall along with the moment induced by the horizontal force atop the wall. The moment, therefore, is simply the maximum horizontal force with moment arm about the base of the wall, equaling the wall height.

Notice the drastic effect of the boundary conditions in Figure 21 above. The two boundary conditions contrast drastically with the influence of restraining the top easily recognizable. The shear strength at low compressive stresses increases greatly with the increased restraints, probably due to restrained uplift from the unloaded toe. At the opposing end of the spectrum, the shear strength decreases
rather sharply. Perhaps allowing the mortar to crush at the bed joint nearest the loaded toe adds ductility.

The stress field failure surface follows a rounded parabolic curve, certainly for that of the rotation constrained model as seen above. The rising branch closely matches the theories of Turnsek & Cacovic (1970) and Mann & Muller (1980); however, issues occur on the descending branch where over-strength properties have been known to have occurred in other studies.

Comparing the free rotation numerical model to the uniform distributed simplified model has great success. Except for two points at high compressive stresses, the simplified model matches the numerical results very closely.

![Figure 22: Results comparison, free rotation, uniform base distribution.](image)

Because the simplified model neglects of diagonal tension failure, the ascending branch of the curves will have some disagreement approaching the peak shear capacity on the envelope. Within this region, Mann & Muller as well as Turnsek & Cacovic have better predictions with adjusted values of the input parameters. However, using experimental values for these parameters would cause poorer correlation because they have been adjusted within reasonable limits for the best curve fit.

It is clear that all models perform better in particular regions and with different parametric assumptions. There is not one model that relentlessly performs better than the others. Boundary conditions, assumed tensile strength properties, and base reaction distribution all effect the outcome when comparing results. For the free rotation numerical model to the triangular distributed simplified model, the latter has less success on the descending branch as evident in Figure 23 on the following page.
Figure 23: Ultimate capacity, free rotation, triangular distribution.

Rationally the results should have improvement on the descending branch where high compressive stresses will likely cause trapezoidal base reaction stresses. Unfortunately, the results go against this logical premise, as there is an offset gap between the simplified model and numerical results. However, the simplified model is still the preferred model in respect to the two others. The underestimation of shear strength at high compressive stresses might in fact be due to the over-strength phenomenon for eccentric loading [22]. A parametric study is needed for more evidence.

The fixed rotation numerical model and the regular simplified model are compared in Figure 24 below.

Figure 24: Ultimate capacity, fixed rotation, uniform distribution.
The strong disagreement in the frictional sliding range may be explained by boundary conditions effects on the tensile cracking perpendicular to the bed joints in the unloaded bottom corner. Sliding interfaces consider cracking criteria at every load step; however, high tensile stresses in the bed joints that might trigger cracks are inhibited by the restraint at the top. Because the true failure mode may not occur, the simulation continues to load the structure to higher horizontal loads until another failure mode occurs. These results are therefore invalidated.

The ascending branch at lower compressive stresses changes quite dramatically. As briefly mentioned before, the restrained rotation at the top must inhibit tensile cracking of the bed joints, attributing the wall to have an exaggerated and unlikely cohesion resistance. Nevertheless, the peak of the envelope and the descending branch match rather well. The triangular distributed simplified model largely underestimates the numerical results and is not recommended with constrained rotation.

Assuming plastic distribution, the zenith of the envelope, at mid-range compressive stresses rise near to the simplified method for all other cases; although equivalence depends on the assumption of distribution of stresses at the base. At high compressive stress, the plastic based approach is best, whereas the triangular distribution may be more appropriate at low compressive stresses.

### 4.2 Observations

**Linear vs. Non-Linear Interfaces**

A comparison between the free rotation model for nonlinear and linear interface elements exposes the participation of the interface elements in various failure mechanisms across the three regions. For tensile cracking, the interfaces do not participate in the shearing of units. Also for compressive crushing failure, the data points are in agreement. However, for low compressive stresses are
sporadic, the interface elements participate in frictional sliding within the wall. At the zenith of interaction curves, the ultimate shear stress increases by introducing nonlinearity in the interface elements. This could be to relaxation of stresses allowed by frictional sliding of the elements.

**Figure 26: Linear and nonlinear interface elements influence.**

The interaction diagram of for global vertical force and overturning moment is analogous to that of the stress yield surface. Shown below in fi, the two highest resisting moments are realize even greater shear strength with combined loading. However, shear strength decreases at low compressive stresses for nonlinear interface elements because the element has a lower cohesion than the unit or mortar solid elements.

**Figure 27: Influence of nonlinear interface elements on N-M interaction diagram.**
Again, it is noted the moment is the product of the summation of the global horizontal force applied and the height of the masonry wall. The height of the masonry wall is always taken as the original height, 270 mm, and vertical deformations are not accounted for. However, the maximum vertical deflection is observed as 0.3 mm downward, just a tenth of a percent difference.

**Failure Mechanisms**

The failure mechanisms observed are frictional sliding, tensile cracking (including diagonal indirect tension) and compressive crushing failure. Cracking sequences at low compressive stresses are sporadic within the wall and concentrated also at boundary edges. Mid-range compressive stresses cascade down the wall diagonally, outlining the compressive strut forming within the heterogeneous material at a maximum friction angle. At high initial compressive stresses, toe crushing is prevalent and plasticity is concentrated at the base near the edge of the wall, indicated by wracking of the deformed shape as shown in Figure 28.

![Figure 28: Normal cracks, E.knn1, 20 MPa compressive stress, free rotation.](image)

**Simulation**

The micro modeling technique with the total strain crack model visually displays the crushing of mortar bed joints, lateral expansion of the head joints, shear crushing of individual units and the tensile cracking at opposite corners. The deformed shape is a magnified simulation of the real behavior of the wall, a unique attribute to the micro modeling approach. The results of the analyses are quite educational for the engineer not accustomed to the interworking of the brick-mortar lattice. In Figure 29, plasticity of the unit at the toe three courses from the bottom is clearly visible. Also the crushing of the mortar bed joints at the base, as well as the separation of the head joints above and along the diagonal is noticeable. Nonlinearity at the unloaded bottom corner show extreme nonlinearity, with some nodes separating from the wall after tensile cracking has occurred. They have become detached from the rest of the structure and cause numerical difficulties in the iteration procedure.
gentle reminder that each degree of freedom for all twenty nodes of all 22,620 elements must be accounted for and solved every iteration, every load step.

![Figure 29](image)

**Figure 29:** Ultimate deformed shape, magnified 50 times, 20 MPa, free rotation.

### 4.3 Base Reaction Distribution

Observing the base vertical reaction distribution may give light to the varying error within the different bi-axial stress regions. The distribution widely varies and depends on the eccentricity of the load. At 28 MPa, low eccentricity causes the vertical base reaction distribution to be triangular. Due to the inter-element nodes and interpolation functions mentioned earlier. The base reaction forces vary widely from node to node; however, the shape formed by the vectors acting on base reveal the distribution for the three different loading conditions displayed on the following page.

![Figure 30](image)

**Figure 30:** Base reaction vectors, 28 MPa, free rotation.

At higher compressive stress and less eccentric load, the distribution becomes trapezoidal, for the unloaded corner is required to support additional vertical load.
In contrast, the vertical reaction distribution at low compressive loads and high eccentricity show agreement with the original simplified model assumption. A majority of the vertical load is taken by the corner edge, close to a uniform magnitude (Figure 32).

The simplified method normally assumes a plastic distribution and does so effectively. However, the trapezoidal assumption for specific load eccentricities improves the prediction of the ultimate shear strength of the wall. Modifying the stress distribution based on eccentricity is open for further study.

### 4.4 Crack Patterns

The cracking pattern varies for each load case depending on the compressive stress applied. At high compressive stresses, the cracking smears near the bottom but shows signs of a diagonal strut causing indirect tension. Figure 33 shows distributed cracking spreading from the diagonal compression strut. Only the normal cracking of the units is shown in the figure for clarity. The crushing of the mortar at the corners and edges occurs before the tension in the bricks follow suit.
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Figure 33: Ultimate cracked state, 32 MPa, free rotation.
The ensuing bed at mid-range compressive stresses, the diagonal cracking is clearly visible.

Figure 34: Ultimate cracked state, 16 MPa, free rotation.
At low compressive stresses, the cracking is more sporadic and occurs near boundary edges.

Figure 35: Ultimate cracked state, 4 MPa, free rotation.
At low compressive stresses, cracking in the mortar is severe at ultimate loads. In Figure 36 below, there is severe cracking in both the bed and joints.

![Figure 36: Cracking in the mortar, fixed rotation, 8 MPa.](image)

**4.5 Cracking Sequence**

Snap shots of normal cracking states in the units only are displayed in Figure 36.

![Figure 36: Cracking sequence, free rotation, 20 MPa.](image)

Cracks start at the base where tensile forces are expected as well as corners of units along the diagonal strut. These areas spread and grow in severity until failure.

**4.6 Strength Degradation**

With the discrete cracking model, solid and interface elements may close, open, or remain plastic continuously throughout the analysis. The nonlinear global behavior becomes extremely difficult for
the solver to meet a proper convergence criteria set by the user. Therefore, larger initial load steps converge the solution without undermining path dependent deformation and convergence criteria.

![Stress vs. Drift, Free Rotation](image)

**Figure 37: Stress vs. drift, free rotation, runs 0-16 MPa.**

At lower compressive stress levels, the wall provides less ductility and the analysis prescribes smaller load steps. For instance, the load-deformation curve of the 2 MPa load case highly contrasts that of 16 MPa. In fact, the displacement ductility of the latter approaches three times the magnitude.

The maximum strength and ductility of the wall occurs with a 20 MPa initial compressive stress before displacements are incurred (see Figure 38). Thus, the maximum shear strength occurs when the masonry wall is bearing half the masonry compressive strength.

There is no snap back behavior observed. The arc length method is abstained from in the non-linear analysis procedure. Instead an automatic displacement based analysis with load step cut back is elected. It reduces the load step if convergence criteria are not met within the maximum allowed iterations.

![Stress-Strain Diagram, Free Rotation](image)

**Figure 38: Stress vs. drift, free rotation, runs 20-36 MPa**
Although the final data set is sound, preliminary runs created difficulties in achieving post-peak behavior with small load steps. Applying energy based increments and other alternative load increment procedures following the last converged step were unsuccessful for solving the next step.

4.7 Plasticity

Plasticity may occur in the bed joints, head joints, and units depending on the load combination and the state of the wall. Masonry is known to exhibit most of its plastic behavior in compression within the mortar bed joints. The micro model agrees with this notion as seen in Figure 39 on the following page. The largest principal plastic strains are mostly contained within the mortar layers whereas the units undergo far less strain due to higher compressive strength, tensile strength and modulus of elasticity.

![Figure 39: Principle plastic strain, free rotation, 36 Mpa.](image)

4.8 Assessment of Assumptions

Mann & Muller's assumption that no shear stresses are transferred in the vertical joints agree with Figure 40. At a mid-range compressive stress (24 MPa), the shear forces in the head joints are low and double the shear force is taken by each unit half along the diagonal. Three bed joints just below the diagonal have shear stresses reaching 10 MPa, whereas adjacent head joints fall below 1 MPa.

![Figure 40: Shear stresses in the mortar for 24 MPa pre-compression load case.](image)
However, the shear stresses in the head joints at the top and bottom wythes do show stresses reaching 6 MPa. Whether this influences the ultimate capacity of the wall is up to debate and further investigation. Judging by the deformed shape for this load case, the failure is not influenced by the shear in the head joints, but rather crushing of the toe and tensile along the diagonal.

![Deformed shape at failure for 24 MPa pre-compression.](image)

**Figure 41: Deformed shape at failure for 24 MPa pre-compression.**

The advantage of micro-modeling is clearly visible, in that, all phenomena occurring within the heterogeneous structure may be captured. These assumptions may be accurately assessed for all bi-axial stress conditions. With the failure mechanism and deformation behavior varying widely across different bear loads, the sophisticated model generates respectable results to base sound research off of. The detailed micro modeling method enables engineers discover knowledge regarding behavior of unreinforced masonry walls though still may be too computationally demanding for application to entire buildings.
Application of a detailed micro-modelling technique to the study of the in-plane capacity of masonry walls

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5. CONCLUSIONS

The micro modeling technique considers the stress state of every element within every bed joint, head joint, and masonry unit. Unlike other modeling techniques, micro modeling exposes all relevant behavioral phenomena for unreinforced masonry walls. In comparison, limit analysis may be an effective, fast approach to attain the global state of a wall structure but may overlook overstressed local regions risking progressive collapse. Micro modeling is by no means a replacement for sound engineering judgment chiseled by years of practical experience and intuitive knowledge. Rather, it’s seen as a concrete basis for numerically validating strength and safety of complex problems not easily detected by simple observation or graphical approaches.

The three separate regions of behavior each have unique characteristics and respond to unit/mortar properties. Discovering a single model to accurately assess all three regions has still not been achieved to the full extent possible. The performance of the numerical model in these three regions does quite well, however, in triggering the appropriate failure mechanisms. The third region, especially, has shown numerical difficulties to simulate accurately the crushing of the material in compression while satisfactorily estimating the wall shear capacity.

The influence of the boundary conditions on the response of the walls is very significant. In particular, the influence of a restrained rotation at the top of the wall is very noticeable. The first region and third region results are undermined by the tensile cracking being inhibited by the restraint top edges. The vertical-horizontal force interaction diagram is smoother for the rotation restrained model; and the same peak horizontal forces are not as defined as the free rotation. The real boundary conditions of masonry walls fall somewhere between the two and relate to floor bearing loads, load transfer and connection detailing.

The usage of micro modeling techniques for generating an accurate response of masonry walls subject to in-plane shear is recommended. Although simplified micro modeling and macro modeling techniques have been extensively studied to reduce computational resources required for analyses, it is believed the advent of new technology will eventually diminish the aforementioned concerns. If the most accurate simulation of behavior is desired, not only in regards to ultimate force and displacement, but also the path leading to failure, then micro modeling would seem the most prospective method. For three dimensions, model construction and run time will take substantially more time. Optimistically, one day this method will be implemented for nonlinear time histories of entire buildings without so much effort.

Assuming a plastic distribution at the base, the simplified model matches the numerical results successfully. The Mann & Muller model [2] also predicts the first two regions rather well; however, the third (compressive cracking) linear function has a slope far beyond anything observed for the case.
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The influence of the unit geometry for this example may over-influence the Mohr-Coulomb coefficients.

Turnsek and Cacovic [3] do not consider the third region, but prudent modification of the masonry tensile strength aids the curve to fit the numerical results quite nicely. It is rather difficult to obtain the masonry tensile strength in the lab and is not listed in the Riddington study [5]. Assumed values around 1/20 compressive strength are deemed acceptable. However, for the best results, the final assumption for tensile strength in Turnsek’s model is 1/40.

In answer to concerns about the simple model negligence of diagonal cracking, the simple model may be combined with the Turnsek and Cacovic one [3]. A failure envelope consisting of the two theories minimum value is rather straightforward. The flat, parabolic square root function of Turnsek and Cacovic would cut the round ascending curve of the simplified model for associated range of compressive stress. A form of Turnsek’s equation exists in the Eurocode and Italian code already, so the change would not be exceptionally drastic.

With computing power constantly increasing, it is believed the impediments of the micro modeling method will disappear soon. The possibility of accurately modeling full structures with the method in response to earthquakes is drawing ever nearer. At the moment, other modeling methods may highly over simplify the response of these types of buildings. Close numerical matching to experimental hysteretic behavior is yet to be discovered by the author. However, the model herein served as the most accurate results available without performing experiments. The agreement with the simplified models and other theories support the idea of the micro model coming close to reality. Hopefully, the technique will have a positive impact on the modern construction practices involving masonry as well as preservation and rehabilitation efforts for heritage and historic structures.
6. RECOMMENDATIONS & PROPOSALS

6.1 Continuing Study

A parametric study varying the mortar and brick properties, unit dimensions, and wall aspect ratios should be carried out to validate the simplified model. Furthermore, comparison to experimental results with individual tests for each material property is advised before trying to improve failure criteria and the simplified model. The micro modeling poses difficulties in wide matrix data acquisition. Gaining adequate results for a parametric study could take weeks or even months for computers to run. Once they are obtained, the equations provided herein for the simplified model may be modified to improve prediction. The introduction of a new parameter or modification of assumptions could realize this end as seen fit. The effect of varying failure criteria input for the discrete cracking model should be monitored as well. More sensitive input values may be revisited to optimize the numerical model predictions.

6.2 Unreinforced Masonry Wall Performance Based Design

If performance based assessments of masonry buildings becomes popular concept among clientele and engineers, it will become absolutely necessary to accurately model the strength degrading behavior of the material. The benefits would be realized on all sides. Engineers could regain a decision making role in the rehabilitation of the buildings with their clients, and clients could assess risk in terms they easily understand.

Seismic engineering research within the past decade has exhausted probabilistic and stochastic based seismic hazards of tall buildings. If the same procedures can be applied for masonry buildings of any type, the public safety gains would be wide spread. The impact would be immense. Engineers would regain the novel role of protecting the public. The goal would be life safety instead of limiting damage loss.
7. REFERENCES


ANNEX

Foreword

The following select figures come from a large database created by Diana's post-processing program. The analyses considered vary at intervals of 8 MPa for compressive stress, starting at 4 MPa. The first and last models, 4 MPa and 36 MPa, have from both the free and fixed rotation boundary results presented, where differences between the two analyses contrast the most. For each model, the deformed shape showing the outline of each masonry unit is magnified 39 times for clarity. An accompanying image of deformation at the mesh level is also included on the same page. This allows the user to observe local behavior within each masonry unit for crushing or other non-linear behavior.

Next, the principle major and minor stresses within the masonry units are shown with equivalent color spectra for all analyses. Therefore, direct comparisons may be made between various models with different compressive stresses and boundary conditions. The last two figures for each model are the shear stresses in the X-Y plane for the mortar and the vertical base reaction distribution conveyed by vectors acting at the bottom nodes. These diagrams also have equivalent color spectra for consistency.

The results in the annex accompany the claims made in the results section of the dissertation. They go further to reinforce the ideas and concepts presented herein. The images are very descriptive even without words and may be found useful without further written prose.
ANNEX

Figure A1: Deformed shape, 4 Mpa, free rotation.

Figure A2: Deformed shape w/mesh, 4 Mpa, free rotation.
ANNEX

Figure A3: Major principle stress, 4 Mpa, free rotation.
ANNEX

Figure A4: Minor principle stress, 4 Mpa, free rotation.

Figure A5: Shear Stress in the x-y plane, 4 MPa, free rotation.

Figure A6: Base reaction vectors, 4 MPa, free rotation.
ANNEX

Figure A7: Deformed shape, 4 Mpa, fixed rotation.

Figure A8: Deformed shape w/mesh, 4 Mpa, fixed rotation.
ANNEX

Figure A9: Major principle stress, 4 MPa, fixed rotation.

Figure A10: Minor principle stress, 4 MPa, fixed rotation.
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ANNEX

Figure A11: Shear Stress in the x-y plane, 4 MPa, fixed rotation.

Figure A12: Base reaction vectors, 4 MPa, fixed rotation.
ANNEX

Figure A13: Deformed shape, 12 MPa, free rotation.

Figure A14: Deformed shape w/mesh, 12 Mpa, free rotation.
ANNEX

Figure A15: Major principle stress, 12 Mpa, free rotation.

Figure A16: Minor principle stress, 12 Mpa, free rotation.
ANNEX

Figure A17: Shear Stress in the x-y plane, 12 MPa, free rotation.

Figure A18: Base reaction vectors, 12 MPa, free rotation.
ANNEX

Figure A19: Deformed shape, 20 Mpa, free rotation.

Figure A20: Deformed shape w/mesh, 20 Mpa, free rotation.
ANNEX

Figure A21: Major principle stress, 20 Mpa, free rotation.

Figure A22: Minor principle stress, 20 Mpa, free rotation.
ANNEX

Figure A23: Shear Stress in the x-y plane, 20 MPa, free rotation.

Figure A24: Base reaction vectors, 20 MPa, free rotation.
ANNEX

Figure A25: Deformed shape, 28 Mpa, free rotation.

Figure A25: Deformed shape w/mesh, 28 Mpa, free rotation.
ANNEX

Figure A27: Major principle stress, 28 Mpa, free rotation.

Figure A28: Minor principle stress, 28 Mpa, free rotation.
ANNEX

Figure A29: Shear Stress in the x-y plane, 28 MPa, free rotation.

Figure A30: Base reaction vectors, 28 MPa, free rotation.
ANNEX

Figure A31: Deformed shape, 36 Mpa, free rotation.

Figure A32: Deformed shape w/mesh, 36 Mpa, free rotation.
ANNEX

Figure A33: Major principle stress, 36 Mpa, free rotation.

Figure A34: Minor principle stress, 36 Mpa, free rotation.
ANNEX

Figure A35: Shear Stress in the x-y plane, 36 MPa, free rotation.

Figure A36: Base reaction vectors, 36 MPa, free rotation.
Figure A37: Deformed shape, 36 Mpa, fixed rotation.

Figure A38: Deformed shape w/mesh, 36 Mpa, fixed rotation.
ANNEX

Figure A39: Major principle stress, 36 Mpa, fixed rotation.

Figure A40: Minor principle stress, 36 Mpa, fixed rotation.
ANNEX

Figure A41: Shear Stress in the x-y plane, 36 MPa, fixed rotation.

Figure A42: Base reaction vectors, 36 MPa, fixed rotation.