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Title:

Study of optimization for vibration absorbing devices applied on airplane structural elements

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A. Stiffness and mass matrix for a beam element

This annex contains the definitions of the stiffness matrix (section A.1) and the mass matrix (section A.2) as well as the description of their assembly process (section A.3).

The content of this annex belongs to D.Sellés and has been adapted from [1].

A.1 Stiffness Matrix of a beam element

The stiffness matrix of a beam element is formulated by assembling the matrix relationships for axial stiffness (equation A.1), torsional stiffness (equation A.2) and flexural stiffness (equation A.3). The latter is used twice to account for flexure in both radial directions of the local coordinate system (figure A.1):

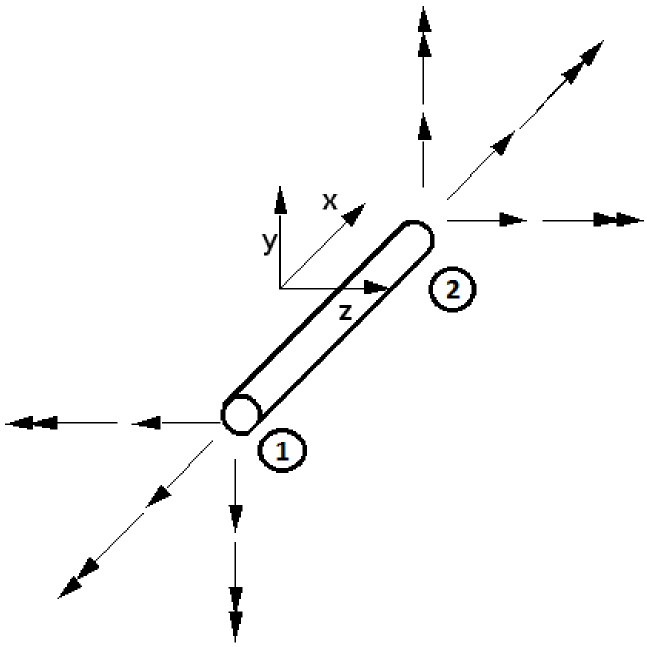

Figure A.1: Local coordinate system

$$\begin{bmatrix} F_x^{(1)} \\ F_x^{(2)} \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x^{(1)} \\ x^{(2)} \end{bmatrix}, \tag{A.1}$$

$$\begin{bmatrix} M_\theta^{(1)} \\ M_\theta^{(2)} \end{bmatrix} = \frac{GJ}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} \theta^{(1)} \\ \theta^{(2)} \end{bmatrix} \tag{A.2}$$

$$\begin{bmatrix} F_y^{(1)} \\ M_\phi^{(1)} \\ F_y^{(2)} \\ M_\phi^{(2)} \end{bmatrix} = \frac{EI_z}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{bmatrix} y^{(1)} \\ \phi^{(1)} \\ y^{(2)} \\ \phi^{(2)} \end{bmatrix}. \tag{A.3}$$

The expression in equation A.3 is rotated 90º to obtain the relationship between *z*,ψ and F_z, M_ψ.

Once those matrices are assembled in the correct order of displacements and twists, the resulting stiffness matrix for the *3D* beam element is matrix $\mathbf{K}$:

$$\mathbf{K^e} = \begin{bmatrix} \mathbf{K_{11}} & \mathbf{K_{12}} \\ \mathbf{K_{21}} & \mathbf{K_{22}} \end{bmatrix}; \tag{A.4}$$

$$\mathbf{K_{ii}} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & \epsilon_i \frac{6EI_z}{L^2} \\ 0 & 0 & \frac{12EI_y}{L^3} & 0 & \epsilon_i \frac{6EI_y}{L^2} & 0 \\ 0 & 0 & 0 & \frac{GJ}{L} & 0 & 0 \\ 0 & 0 & -\epsilon_i \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & 0 \\ 0 & \epsilon_i \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L} \end{bmatrix}$$

$$i \in \{1, 2\},$$

$$\mathbf{K_{21}} = \begin{bmatrix} -\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{12EI_z}{L^3} & 0 & 0 & 0 & -\frac{6EI_z}{L^2} \\ 0 & 0 & -\frac{12EI_y}{L^3} & 0 & +\frac{6EI_y}{L^2} & 0 \\ 0 & 0 & 0 & -\frac{GJ}{L} & 0 & 0 \\ 0 & 0 & -\frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & 0 \\ 0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L} \end{bmatrix},$$

$$\mathbf{K_{12}} = \mathbf{K^t_{21}},$$

where

E = Longitudinal elasticity modulus,

G = Transversal elasticity modulus,

I_i = Moment of inertia on the i axis,

A = Cross section area,

L = Beam length,

J = Torsion constant,

$\epsilon_1 = +1 \;\; , \;\; \epsilon_2 = -1.$

A.2 Mass matrix of a beam element

The mass matrix of a *3D* beam element in local coordinates (see figure A.1) is formed by combining the matrix relationships of the beam element for the the axial (equation A.5), torsional (equation A.6) and flexural (equation A.7) effects:

$$\begin{bmatrix} F_x^{(1)} \\ F_x^{(2)} \end{bmatrix} = \frac{\bar{m}L}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \ddot{x}^{(1)} \\ \ddot{x}^{(2)} \end{bmatrix}, \tag{A.5}$$

$$\begin{bmatrix} M_\theta^{(1)} \\ M_\theta^{(2)} \end{bmatrix} = \frac{\bar{m}I_0L}{6A} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}^{(1)} \\ \ddot{\theta}^{(2)} \end{bmatrix}, \tag{A.6}$$

$$\begin{bmatrix} F_y^{(1)} \\ M_\phi^{(1)} \\ F_y^{(2)} \\ M_\phi^{(2)} \end{bmatrix} = \frac{\bar{m}L}{420} \begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \begin{bmatrix} \ddot{y}^{(1)} \\ \ddot{\phi}^{(1)} \\ \ddot{y}^{(2)} \\ \ddot{\phi}^{(2)} \end{bmatrix}. \tag{A.7}$$

Following the same methodology and order used in the stiffness matrix, the coupled mass matrix for a beam element is:

$$\mathbf{M^e} = \begin{bmatrix} \mathbf{M_{11}} & \mathbf{M_{12}} \\ \mathbf{M_{21}} & \mathbf{M_{22}} \end{bmatrix}; \tag{A.8}$$

$$\mathbf{M_{11}} = \rho AL \begin{bmatrix} \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{13}{35} & 0 & 0 & 0 & \frac{11L}{210} \\ 0 & 0 & \frac{13}{35} & 0 & \frac{-11L}{210} & 0 \\ 0 & 0 & 0 & \frac{I_y+I_z}{3A} & 0 & 0 \\ 0 & 0 & \frac{-11L}{210} & 0 & \frac{L^2}{105} & 0 \\ 0 & \frac{11L}{210} & 0 & 0 & 0 & \frac{L^2}{105} \end{bmatrix},$$

$$\mathbf{M_{12}} = \rho AL \begin{bmatrix} \frac{1}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{9}{70} & 0 & 0 & 0 & \frac{-13L}{420} \\ 0 & 0 & \frac{9}{70} & 0 & \frac{-13L}{420} & 0 \\ 0 & 0 & 0 & \frac{I_y+I_z}{6A} & 0 & 0 \\ 0 & 0 & \frac{-13L}{420} & 0 & \frac{-L^2}{140} & 0 \\ 0 & \frac{13L}{420} & 0 & 0 & 0 & \frac{-L^2}{140} \end{bmatrix},$$

$$\mathbf{M_{21}} = \rho AL \begin{bmatrix} \frac{1}{6} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{9}{70} & 0 & 0 & 0 & \frac{13L}{420} \\ 0 & 0 & \frac{9}{70} & 0 & \frac{-13L}{420} & 0 \\ 0 & 0 & 0 & \frac{I_y+I_z}{6A} & 0 & 0 \\ 0 & 0 & \frac{13L}{420} & 0 & \frac{-L^2}{140} & 0 \\ 0 & \frac{-13L}{420} & 0 & 0 & 0 & \frac{-L^2}{140} \end{bmatrix},$$

$$\mathbf{M_{22}} = \rho AL \begin{bmatrix} \frac{1}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{13}{35} & 0 & 0 & 0 & \frac{-11L}{210} \\ 0 & 0 & \frac{13}{35} & 0 & \frac{11L}{210} & 0 \\ 0 & 0 & 0 & \frac{I_y+I_z}{3A} & 0 & 0 \\ 0 & 0 & \frac{11L}{210} & 0 & \frac{L^2}{105} & 0 \\ 0 & \frac{-11L}{210} & 0 & 0 & 0 & \frac{L^2}{105} \end{bmatrix},$$

where

ρ = material density,

A = Cross section area,

L = Beam length,

I_i = Moment of inertia on the i axis,

$\bar{m}$ = distributed mass.

A.3 Super-element assembly

In order to obtain a valid expression in the form of:

$$\underline{\mathbf{F}}_b = \left(\mathbf{K} - \omega^2 \mathbf{M}\right) \underline{\mathbf{X}}', \tag{A.9}$$

both $\mathbf{M}$ and $\mathbf{K}$ elemental matrices have to be expressed in global coordinates, and assembled so that the expression is true for the vectors $\underline{\mathbf{F}}$ and $\underline{\mathbf{X}}$ in the following form:

$$\underline{\mathbf{F}} = \begin{bmatrix} \underline{\mathbf{F}}_e^{(1)} \\ \underline{\mathbf{F}}_e^{(2)} \\ \vdots \\ \vdots \\ \underline{\mathbf{F}}_e^{(n)} \end{bmatrix} ; \qquad \underline{\mathbf{X}} = \begin{bmatrix} \underline{\mathbf{X}}_e^{(1)} \\ \underline{\mathbf{X}}_e^{(2)} \\ \vdots \\ \vdots \\ \underline{\mathbf{X}}_e^{(n)} \end{bmatrix} . \tag{A.10}$$

Recalling equation A.4 and equation A.8 and rewriting them in global notation for a beam of nodes *i j* :

$$\mathbf{B}^{e'} = \begin{bmatrix} \mathbf{B}_{ii}^{'} & \mathbf{B}_{ij}^{'} \\ \mathbf{B}_{ji}^{'} & \mathbf{B}_{jj}^{'} \end{bmatrix} , \tag{A.11}$$

where the prime notation indicates that the matrices are written in local coordinates, and B is either matrix $\mathbf{K}$ or matrix $\underline{\mathbf{M}}$. In order to express the matrices in global coordinates, each sub-matrix has to be rotated separately. In block matrix notation, this can be written as:

$$\mathbf{B}^e = \begin{bmatrix} \mathbf{R}^t & 0 & 0 & 0 \\ 0 & \mathbf{R}^t & 0 & 0 \\ 0 & 0 & \mathbf{R}^t & 0 \\ 0 & 0 & 0 & \mathbf{R}^t \end{bmatrix} \mathbf{B}^{e'} \begin{bmatrix} \mathbf{R} & 0 & 0 & 0 \\ 0 & \mathbf{R} & 0 & 0 \\ 0 & 0 & \mathbf{R} & 0 \\ 0 & 0 & 0 & \mathbf{R} \end{bmatrix} \tag{A.12}$$

Matrix $\mathbf{R}$, in the particular case of beams connecting only nodes of the same plane perpendicular to the *z* axis, is simplified to

$$\mathbf{R} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} . \tag{A.13}$$

Once the $\mathbf{K}^e$ and $\mathbf{M}^e$ matrices are rotated from local beam coordinates to global coordinates, they can be assembled to a general stiffness and mass matrix following the scheme in equation A.14:

$$\mathbf{B} = \begin{bmatrix} \sum_{e=1}^{n_b} \mathbf{B}_{11}^{(e)} & \sum_{e=1}^{n_b} \mathbf{B}_{12}^{(e)} & \sum_{e=1}^{n_b} \mathbf{B}_{13}^{(e)} & \cdots & \sum_{e=1}^{n_b} \mathbf{B}_{1n}^{(e)} \\ \sum_{e=1}^{n_b} \mathbf{B}_{21}^{(e)} & \sum_{1}^{n_b} \mathbf{B}_{22}^{(e)} & \cdots & \cdots & \\ \sum_{e=1}^{n_b} \mathbf{B}_{31}^{(e)} & \vdots & \ddots & & \\ \vdots & \vdots & & & \\ \sum_{e=1}^{n_b} \mathbf{B}_{n1}^{(e)} & & & & \sum_{e=1}^{n_b} \mathbf{B}_{nn}^{(e)} \end{bmatrix} \tag{A.14}$$

B. Data of the simple optimization of a plane plate

In this section the data of the simplest test performed in this study which results are discussed in section 6.4, section 9.2.6 and section 9.3.4 of the report is presented.

The problem presented is the optimization of a plane plate where it is only possible to place one type of DVA and the optimization is at a frequency of 302Hz. The plate material properties and dimensions are summarized in table table B.1.

Lx	Ly	Thickness	Material	Damp. coef.	ρ	ν	E
0.3m	0.38m	0.00192m	Steel	0.01	7850 $\frac{\text{kg}}{\text{m}^3}$	0.27	$2.0\cdot 10^{11}$ Pa

Table B.1: Physical properties of the plane plate studied.

In table B.2 there are the physical properties defining the available DVAs for this optimization. The data is given in International System of Units.

Type	m [kg]	k [$\frac{\text{N}}{\text{m}}$]	c [$\frac{\text{N s}}{\text{m}}$]	Tuning ω [Hz]
1	0.150	540090	9.4876	302

Table B.2: Physical properties of the available DVAs.

The boundary conditions applied to the model are restrictions in all three degrees of freedom of displacement in the perimeter nodes of the plate.

The locations where a device can be placed are presented in table B.3 and the external force is applied in the node number 9 with a modulus of 250N.

Code of the node	x coordinate [m]	y coordinate [m]
1	0.045	0.235
2	0.100	0.195
3	0.205	0.105
4	0.280	0.075
5	0.200	0.170
6	0.135	0.235
7	0.275	0.160
8	0.235	0.235
9	0.305	0.230

Table B.3: Coordinates of the possible locations.

C. Input files format

In this annex the format of the input files required by the optimization tools developed in this study are described and illustrated.

C.1 FRF files

The FRF matrix of the structure is presented in two different text files.

- *FRFMatrixR.txt* contains the real part of all the elements in the matrix.
- *FRFMatrixI.txt* contains the imaginary part of all the elements in the matrix.

The matrix are presented in a text file with their columns separated by a blank space and their rows by an end line ($"\backslash n"$).

Both rows and columns must be sort following the next sequence:

1. Objective nodes (nodes in $\mathcal{O}$).
2. Possible nodes where a device can be placed (nodes in $\mathcal{D}$) sorted for the intrinsic codification of the algorithm.
3. Points that don't belong to $\mathcal{O}$ or $\mathcal{D}$ but where an external force is applied.

In figure C.1 is presented a piece of a FRF input file as example.

```
1.5557e-008 -3.9747e-009 -1.0046e-008 3.2643e-012 3.2969e-011 2.435e-012
-3.9747e-009 4.2696e-008 -7.6602e-008 3.8231e-011 1.9932e-010 4.4435e-012
-1.0046e-008 -7.6602e-008 2.3137e-007 1.4544e-010 1.0499e-009 5.7393e-012
3.2643e-012 3.8231e-011 1.4544e-010 2.5241e-012 -1.0862e-012 -1.7026e-015
3.2969e-011 1.9932e-010 1.0499e-009 -1.0862e-012 9.1863e-011 -2.1771e-014
2.435e-012 4.4435e-012 5.7393e-012 -1.7026e-015 -2.1771e-014 5.3195e-013
3.84e-009 -8.0291e-010 -5.67e-009 2.0229e-012 2.0069e-011 5.6325e-012
```

Figure C.1: Sample of a *FRFMatrixR.txt* or *FRFMatrixI.txt* file.

C.2 DVA properties

The properties of the different devices required for the algorithms are presented in the text file *TMA.txt*.

This file must contain a line for each different type of device that can be placed in the structure. In each of this rows there must be three values separated by a blank space. This values must be (from left to right) the mass of the device (m_i), the spring stiffness (k_i) and the damping coefficient (c_i). These values must be coherent with the units used in the problem. In this study it is worked with IS units.

In order to discern point masses from DVAs the optimization tools developed in this study recognise that code 999999999 entered as a value of spring stiffness for a device means that it is a point mass and treat it accordingly.

In figure C.2 an example of the correct format is presented.

```
1   0.150 63165.47 9.4876
2   0.250 63165.47 9.4876
3   0.500 999999999 0
```

Figure C.2: Sample of a *TMA.txt* file.

C.3 Beam properties

The beam properties are stored in a text file named *beam_properties.txt* with a specific order. Each row of the text file represents the set of properties of a kind of beam. The end of the row is marked by an end line and each value is separated by a blank space.

The rows need to be in the order considered for the K M generation function. That is, the first row correlates with beam type 1 from the algorithm, the second with beam type 2 and so on.

The order in which the properties need to be stored in the rows of the text file is the following:

E	G	I_x	I_y	I_z	A	L	J	ρ	$\bar{m}$

The system of units used must be consistent between properties and with the whole problem data. Figure C.3 presents a sample of this file format.

```
211e9 82e9 0 3.413e-6 3.413e-6 0.0064 1 5.76e-6 7874 1
70e9 26e9 0 3.413e-6 3.413e-6 0.0064 1 5.76e-6 2700 1
```

Figure C.3: Sample of a *coordinates.txt* file.

C.4 Input force

The external force array applied to the structure has to be presented in a text file called *F.txt*.

This input file contains a line with the value of the input force in the corresponding degree of freedom of the FRF matrix. The number of lines of this file must agree with the size of the FRF matrix. The units must be consequent with the system of units chosen for the problem. In this study it is SI.
Figure C.4 presents a sample of this file format.

```
0
0
10000
0
0
0
0
0
-10000
0
0
0
0
```

Figure C.4: Sample of a *F.txt* file.

C.5 Coordinates file

The coordinates file has to include one row for each node of interest in the problem, respecting the order in which the algorithm works. The first rows have to contain the information about the coordinates of the objective points (points in $\mathcal{O}$), followed by the coordinates of the points of possible allocation of DVAs or beams (points in $\mathcal{D}$), with the last rows for points of force application which do not belong to $\mathcal{O} \cup \mathcal{D}$ (if any). Each value has to be separated by a blank space and each row has to be separated by an end line.

In order to define the order of the rows, the user needs to keep in mind which is the longitudinal axis of the geometry of study. The software calculates the angles between the second and third rows of the text file.

- x as main axis: The order is **x coord. y coord. z coord.**
- Y as main axis: The order is **y coord. z coord. x coord.**
- z as main axis: The order is **z coord. x coord. y coord.**

Figure C.5 presents a sample of this file format.

```
1 1.7923 -0.74241
1 1.9027 -0.37848
1 1.94 0
1 1.9027 0.37848
1 1.7923 0.74241
1 1.6131 1.0778
1 1.3718 1.3718
1 1.0778 1.6131
1 0.74241 1.7923
1 0.37848 1.9027
1 0 1.94
1 -0.37848 1.9027
1 -0.74241 1.7923
1 -1.0778 1.6131
1 -1.3718 1.3718
1 -1.613 1.0778
1 -1.7923 0.74241
1 -1.9027 0.37848
2.65 -1.9135 0.58034
2.65 1.9132 0.58039
```

Figure C.5: Sample of a *coordinates.txt* file.

D. Bibliography

[1] D. Sellés Alseldà. Study of coupling of vibration countermeasures applied on airplane structure elements. *Etseiat UPC*, TFG, 2014.