Abstract

The interest in multimedia wireless networks has increased incredibly due to the need for a better communication system. The purpose of this project is to allocate all users with the optimum configuration; that is, the maximum bit rate possible per user in function of the interference between stations that share the same band and other constraints.

In order to accomplish this objective, an energy function is defined so that the minimum value is the best solution to the problem. Some algorithms are tested and, with a simulated annealing algorithm, some results are evaluated for three cases (single cell, multiple-cell orthogonal access and multiple-cell with interference).

Applying a gradient algorithm with simulated annealing, expected results are obtained for the single-cell and the multiple-cell orthogonal access cases. Otherwise, for the multiple-cell with interference case, the results are not the expected; the algorithm leaves too many users not allocated than the possible ones.
Resum

L'interès en xarxes multimèdia sense fils ha augmentat increïblement degut a la necessitat d'una millora en la comunicació. El propòsit d'aquest projecte és assignar tots els usuaris a la configuració òptima, és a dir, al màxim possible bit ràtio per usuari a la funció d'interferència entre estacions que comparteixen la mateixa banda i altres restriccions.

Per tal de complir aquest objectiu, una funció d'energia es defineix per tal que el valor mínim d'aquesta sigui la millor solució al problema. Es testegen alguns algoritmes i alguns resultats són avaluats per tres casos (una sola cel·la, múltiples cel·les amb accés ortogonal i múltiples cel·les amb interferència).

Aplicant un algorisme de gradient amb 'simulated annealing', s'obtenen resultats esperats pel cas d'una sola cel·la i per múltiples cel·les cas d'accés ortogonal. En canvi, pel cas amb interferència, els resultats no són els esperats; l'algorisme obté un resultat amb masses usuaris sense assignar que els possibles.
Resumen

El interés por las redes multimedia 'wireless' ha aumentado increíblemente debido a la necesidad de una mejora en la comunicación. El propósito de este proyecto es asignar todos los usuarios a la configuración óptima, es decir, a el bit ratio mayor posible por usuario en función de la interferencia entre estaciones que comparten la misma banda y otras restricciones.

Para cumplir con el objetivo, se define una función de energía con valor mínimo en la solución del problema. Se testean algunos algoritmos y se evalúan algunos resultados para tres casos (una sola cela, múltiples celas con acceso ortogonal y múltiples celas con interferencia).

Aplicando un algoritmo de gradiente con 'simulated annealing', se obtienen los resultados esperados para los dos primeros casos. En cambio, para el caso con interferencia, los resultados no son los esperados; el algoritmo obtiene un resultado con más usuarios sin asignar que los posibles.
Acknowledgements

I would like to express the deepest appreciation to my supervisor, Josep Vidal, for his guidance and help during the procedure of this thesis. He assisted me while searching for answers to the project objectives and while adapting the algorithms.

Also, I thank Sandra Lagén Morancho who provided the code for the generation of the wireless deployment and advised me with the adequate scenario.

Finally, I give thanks to my friends and my family for their constant support.
## Revision history and approval record

<table>
<thead>
<tr>
<th>Revision</th>
<th>Date</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>18/12/2014</td>
<td>Document creation</td>
</tr>
<tr>
<td>1</td>
<td>04/02/2015</td>
<td>Document restructure</td>
</tr>
<tr>
<td>3</td>
<td>05/02/2015</td>
<td>Document revision</td>
</tr>
<tr>
<td>4</td>
<td>06/02/2015</td>
<td>Document revision</td>
</tr>
<tr>
<td>5</td>
<td>12/02/2015</td>
<td>Last document version</td>
</tr>
</tbody>
</table>

## DOCUMENT DISTRIBUTION LIST

<table>
<thead>
<tr>
<th>Name</th>
<th>e-mail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cristina Granés</td>
<td><a href="mailto:cristina.granes@estudiant.upc.edu">cristina.granes@estudiant.upc.edu</a></td>
</tr>
<tr>
<td>Josep Vidal</td>
<td><a href="mailto:josep.vidal@upc.edu">josep.vidal@upc.edu</a></td>
</tr>
</tbody>
</table>

## Author and Review

<table>
<thead>
<tr>
<th>Written by:</th>
<th>Reviewed and approved by:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date</td>
<td>Date</td>
</tr>
<tr>
<td>04/02/2015</td>
<td>12/02/2015</td>
</tr>
<tr>
<td>Name</td>
<td>Name</td>
</tr>
<tr>
<td>Cristina Granés</td>
<td>Josep Vidal</td>
</tr>
<tr>
<td>Position</td>
<td>Position</td>
</tr>
<tr>
<td>Project Author</td>
<td>Project Supervisor</td>
</tr>
</tbody>
</table>
Table of contents

Abstract ............................................................................................................................................. 1
Resum .................................................................................................................................................. 2
Resumen .............................................................................................................................................. 3
Acknowledgements ............................................................................................................................. 4
Revision history and approval record ................................................................................................. 5
Table of contents .................................................................................................................................. 6
List of tables ......................................................................................................................................... 7
List of figures ......................................................................................................................................... 7
Glossary .................................................................................................................................................. 8

1. Introduction ..................................................................................................................................... 9
   1.1 Motivation, objectives and deviations ......................................................................................... 9
   1.2 Requirements and specifications ................................................................................................ 10
   1.3 Work plan and Gantt diagram .................................................................................................... 10

2. Radio resources allocation in heterogeneous wireless networks ...................................................... 13
   2.1 Technical scenarios and energy functions .................................................................................... 13
      2.1.1 Multi-RAT single cell case ................................................................................................... 13
      2.1.2 Multiple cells with heterogeneous base stations .................................................................... 15
   2.2 State of the art ............................................................................................................................ 18
      2.2.1 Packet delay-oriented resource allocation ........................................................................... 19
      2.2.2 Resource allocation in a multi-RAT scenario ....................................................................... 19

3. Energy function and solutions .......................................................................................................... 21
   3.1 Hopfield Neural Networks .......................................................................................................... 21
      3.1.1 Energy function and the HNN ............................................................................................. 22
      3.1.2 Discrete-time HNN ............................................................................................................. 25
   3.2 Fast-HNN: projected gradient .................................................................................................... 26
      3.2.1 Introduction .......................................................................................................................... 26
      3.2.2 Updating step ....................................................................................................................... 26
      3.2.3 Updating direction ............................................................................................................... 27
      3.2.4 Summary of the algorithm .................................................................................................. 28
   3.3 Smith’s projected gradient with annealing .................................................................................... 28
      3.3.1 Summary of the algorithm .................................................................................................. 28
      3.3.2 Projection step and annealing .............................................................................................. 29
   3.4 Update of the constraints matrix .................................................................................................. 30

4. Methodology and system evaluation .................................................................................................. 31
   4.1 Technical specifications and scenario .......................................................................................... 31
   4.2 Merit figures .................................................................................................................................. 34
   4.3 Minimization and evaluation of costs .......................................................................................... 35
   4.4 System evaluation ....................................................................................................................... 35
      4.4.1 Single cell .............................................................................................................................. 35
4.4.2. Multi-cell orthogonal access ................................................................. 37
4.4.3. Interference case ....................................................................................... 39
4.5. Conclusions ................................................................................................. 40

5. Budget ............................................................................................................. 41

Bibliography ......................................................................................................... 42

List of tables

- Table 1. Work package 1 .................................................................................. 10
- Table 2. Work package 2 ................................................................................ 11
- Table 3. Work package 3 ................................................................................ 11
- Table 4. Work package 4 ................................................................................ 11
- Table 5. Work package 5 ................................................................................ 12
- Table 6. Work package 6 ................................................................................ 12
- Table 7. Video streaming traffic model parameters [] .................................. 31
- Table 8. Simulation assumptions ...................................................................... 33
- Table 9. Modulation and coding scheme of LTE .............................................. 34

List of figures

- Figure 1. Gantt diagram .................................................................................. 12
- Figure 2. Single cell multi-RAT .................................................................... 13
- Figure 3. Multiple cells without interference .................................................. 15
- Figure 4. Multiple cells with interference (CASE 1) ...................................... 17
- Figure 5. Multiple cells, interference only between small cells (CASE 2) .... 17
- Figure 6. Case 1 on the left (L=4) and Case 2 on the right (L=2) ............... 18
- Figure 7. Example of packets generation ....................................................... 32
- Figure 8. Total lost packets, single cell .......................................................... 36
- Figure 9. Average delay, single cell ................................................................. 36
- Figure 10. Average throughput, single cell .................................................... 37
- Figure 11. Total lost packets, orthogonal multi-cells ...................................... 38
- Figure 12. Average packet delay, orthogonal multi-cells .............................. 38
- Figure 13. Average throughput, orthogonal multi-cells .................................. 39
Glossary

3GPP 3rd Generation Partnership Project
CAC Connection-Admission Control
CDMA Code Division Multiple Access
HNN Hopfield Neuronal Network
JDRA Joint Dynamic Resource Allocation
LTE Long Term Evolution
MBS Macro Base Station
OFDMA Orthogonal Frequency-Division Multiple Access
QoS Quality-of-Service
RAT Radio Access Technology
SCe Small Cell station
SNIR Signal to Noise and Interference Ratio
SNR Signal to Noise Ratio
WP Work Package
1. **Introduction**

1.1. **Motivation, objectives and deviations**

The idea of this project comes from the need of designing distributed radio resource allocation techniques for dense deployments of macro base stations and small cells. This scenario has appeared in the modern mobile and wireless communications systems as a result of the increasing demand of data traffic driven by advanced terminals, like smartphones and tables. The problem can be cast into a quadratic function of binary variables, whose minimization is of NP complexity. In order to propose simpler solutions we plan to apply different optimization techniques including Hopfield Neuronal Networks (HNN) and projected gradients onto a yet not studied scenario.

In wireless communications, the demands for different types of services (multimedia services) with widely different traffic characteristics have increased. The quality of service expected from these services varies because of the dynamics of the propagation channel, and the non-stationarity of traffic and interference; so wireless networks for multimedia services have to include an efficient connection-admission control (CAC) that operates in real time. Because of these, Ahn gave a solution to provide dynamic connection-admission control for multimedia wireless using the Hopfield neuronal network [1].

The purpose of this project is to go beyond that study and give a solution to the CAC problem with an energy function whose minimum value provides a good Quality-of-Service to users in multimedia heterogeneous LTE wireless networks.

This project starts from the scratch and it is planned to contribute to the project DISNET funded by the *Ministerio de Economía y Competitividad* of the Spanish Government, and European Regional Development Funds (TEC2013-41315-R DISNET). The supervisor, Josep Vidal, provided initial ideas for this project.

The project main goals are:

1. Provide wireless service to users with QoS preserving connection-admission control in multimedia heterogeneous LTE wireless networks, where users can be associated to macro base stations (MBS) or small cells (SCe) depending on the availability and the desired quality per user.
2. Define a distributed and/or centralized solution based, for various MBS and SCe taking into account the interference generated in the downlink.
3. Evaluate and analyse the performance for different traffic conditions, evaluating the average area capacity and user outage rate for different number of stations.

First of all, the idea was to apply HNN, having a set of neurons in each station and all connected by communications between stations. Analysing the activation function of the HNN and its energy function that has to be minimized to find the solution, a dependence on too many parameters was found. Moreover, changing one of the constants would lead to undesired solutions. Consequently, two algorithms to improve the convergence to feasible solutions in HNN were studied and only one had the expected results. However, this algorithm does not apply the idea of a neuronal network, because it just minimizes...
the energy function; so, the calculations need to be centralized and it is not possible to distribute it among the different stations. To accomplish the main objectives of the project, the energy function explained in the article [2] and the algorithm in [3] are used.

1.2. Requirements and specifications

Nowadays, the system in urban areas is more complex and dense than rural areas with less interference, so analysing urban areas is enough. The service should be evaluated to provide the best possible quality to users and give fast solutions, due to the speed needed in the communication systems. As LTE is one of the most used protocols and users do not usually stay still, the following requirements are considered:

Project requirements
- Provide wireless connection for mobile subscribers in dense urban areas with a guaranteed quality-of-service.
- Give fast and distributed solutions to the radio resource management problem, associating the users to the adequate cells and providing them the highest possible quality.
- Take into account moving and still subscribers.
- Adopt solutions for an LTE-like radio access network.
- Evaluate the system performance in a scenario compliant with the evaluation methodology recommendations of 3GPP LTE.

Project specifications
The considered quantitative measures that will determine the system performance are:
- The average area capacity in kbps/km² necessary to give service to the users in the cellblocks will be measured and studied.
- The minimum capacity given to the 5% of the best-effort users (user outage rate).
- The fraction of rejected connections for different traffic densities and SCe densities.

1.3. Work plan and Gantt diagram

Work Packages:

<table>
<thead>
<tr>
<th>Project: Introduction</th>
<th>WP ref.: (WP1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major constituent: Documentation</td>
<td>Sheet 1 of 6</td>
</tr>
<tr>
<td>Short description: Project definition and description of the objectives. Time plan description.</td>
<td>Planned start date: 15/09/2014</td>
</tr>
<tr>
<td></td>
<td>Planned end date: 10/10/2014</td>
</tr>
<tr>
<td></td>
<td>Start event: 15/09/2014</td>
</tr>
<tr>
<td></td>
<td>End event: 10/10/2014</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Internal task T1: Motivation</th>
<th>Internal task T2: Gantt's diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deliverables: Project proposal and work plan</td>
<td>Dates: 10/10/2014</td>
</tr>
</tbody>
</table>

Table 1. Work package 1
### Project: Scenarios and requirements

**Major constituent:** Documentation  
Sheet 2 of 6

**Short description:**  
Scenario description to work with. Definition of the key performance indicators to determine the best solution for our problem.  
Planned start date: 10/10/2014  
Planned end date: 10/11/2014  
Start event: 15/12/2014  
End event: 10/01/2015  

- **Internal task T1:** User scenario: description of scenarios of usage for subscribers and service requirements in which the project is based upon.  
- **Internal task T2:** Technical scenario: propagating conditions, interference, technical system parameters and other characteristics of the final designed system associated to LTE specifications.  
- **Internal task T3:** Figures of merit: definition of the quantitative performance measures to evaluate for each proposed solution.  
- **Internal task T4:** Minimization and evaluation of costs: Criteria for choosing one solution or another, provided that the requirements associated to technical scenario are satisfied.

**Deliverables:**  
- Introduction and scenario description of the final report  
**Dates:**  
5/01/2015

<table>
<thead>
<tr>
<th>Table 2. Work package 2</th>
</tr>
</thead>
</table>

### Project: Theoretical raised solutions

**Major constituent:** Documentation and investigation  
Sheet 3 of 6

**Short description:**  
Detailed description of the theoretical solutions for the problem.  
Planned start date: 22/09/2014  
Planned end date: 10/12/2014  
Start event: 22/09/2014  
End event: 14/12/2015  

- **Internal task T1:** Study of HNN and bibliographic research  
- **Internal task T2:** Application of HNN to the admission control in a single cell multiRAT system.  
- **Internal task T3:** Solution for heterogeneous multiple cells sharing radio resources and hence creating mutual interference.

**Deliverables:**  
- Critical Review  
- Theoretical Solutions in the final report (draft)  
**Dates:**  
21/11/2014  
30/12/2014

<table>
<thead>
<tr>
<th>Table 3. Work package 3</th>
</tr>
</thead>
</table>

### Project: System level evaluation

**Major constituent:** Software development (Matlab code)  
Sheet 4 of 6

**Short description:**  
Implementation and evaluation of the theoretical solutions.  
Planned start date: 20/10/2014  
Planned end date: 15/01/2015  
Start event: 25/10/2014  
End event: 20/01/2015  

- **Internal task T1:** Generation of a Matlab simulation environment with multiple users and multiple base stations compliant with the evaluation methodology of 3GPP.  
- **Internal task T2:** Matlab code for HNN in single cell cases.  
- **Internal task T3:** Matlab code for heterogeneous networks, and various cells with interference.

**Deliverables:**  
- Results document (draft)  
**Dates:**  
02/02/2015

| Table 4. Work package 4 |
Project: Costs evaluation and sustainability study  
**WP ref.:** (WP5)  
**Major constituent:** Documentation  
Sheet 5 of 6  

**Short description:**  
Analyze the cost for every raised solution and check that the solutions are sustainable at environmental, economic and social levels.  
Planned start date: 20/11/2014  
Planned end date: 10/01/2014  
Start event: 04/01/2015  
End event: 30/01/2015  

Internal task T1: Evaluation of costs in terms of the backhaul bandwidth required to support control plane message exchange among base stations.  
Deliverables: Costs document  
Dates: 25/01/2015  

**Table 5. Work package 5**

---

Project: Final writing  
**WP ref.:** (WP6)  
**Major constituent:** Documentation  
Sheet 6 of 6  

**Short description:** Conclusions. Writing of the final report.  
Planned start date: 23/12/2014  
Planned end date: 06/02/2015  
Start event: 28/12/2014  
End event: 20/02/2015  

Internal task T1: Conclusions and recommendations  
Internal task T2: Final report writing and revision  
Internal task T3: Oral presentation preparation  
Deliverables: Final report  
Dates: 06/02/2015  

**Table 6. Work package 6**

---

**Gantt Diagram:**

![Gantt diagram](Figure 1)
2. **Radio resources allocation in heterogeneous wireless networks**

2.1. **Technical scenarios and energy functions**

Nowadays, due to the progress in the communications systems and the need for a more comfortable method to communicate, the interest in multimedia wireless networks has increased incredibly. Also, there has been an increase in the data sent between users in this networks.

In this project, a number of users ($I$) want to transmit video streaming data. Each user will have a number of packets in queue, with different lengths (number of bits), certain arrival time in the queue and a maximum contracted packet delay. Each user $i$ will demand a minimum bit rate $R_{b,\text{Target}_i}$ in order to accomplish the delay constraints.

In video streaming data, it is important to have a minimum quality of the images received and it needs to be in real time. Then, some technique to allocate the best possible bit rate to a given user in the heterogeneous wireless network should be applied.

Due to the importance in the best allocation in order to prevent handovers and losses, a definition of an energy function that represents the different costs to allocate with a given bit rate is a perfect way to find the best solution. The constraints in the energy function will be defined in function of the case: single cell or multiple cells with or without interference.

2.1.1. **Multi-RAT single cell case**

First of all, in order to get used to the energy functions and the algorithm, I applied the algorithm on the admission control in a single cell multi-RAT equation proposed in [2].
In figure 2, an example for single cell multi-RAT with 3 RATs is shown.

In this paper, it is taken into account a system with I users and K possible RATs with J bit rates each. Each user has a queue of packets to send and demands a bit rate $R_{b}$ that depends on the number of bits of the packets in the queue ($l_{i,p}$ for the $p$th packet of the $i$th user), the maximum contracted packed delay $D_{\max}$ and the time in the queue for each packet ($t_{i,p}$).

The output of each neuron $(i,j,k)$ is $V_{ijk}$ and has two possible values: 1 if the neuron is 'ON' and 0 if the neuron is 'OFF'. Also, we consider the possibility of having a bit rate of 0 b/s, which is equivalent to have a user not allocated to any RAT and bit rate, and, therefore, $j$ goes from 1 to $J+1$ ($J$ bit rates + bit rate of 0 b/s).

The energy function for this system is:

$$E(x) = -\frac{\mu_1}{2} \sum_{i=1}^{1} \sum_{j=1}^{j+1} \sum_{k=1}^{K} c_{ijk} V_{ijk} + \frac{\mu_2}{2} \sum_{i=1}^{1} \sum_{j=1}^{j+1} \sum_{k=1}^{K} \xi_{ijk} V_{ijk} + \frac{\mu_3}{2} \sum_{i=1}^{1} \sum_{j=1}^{j+1} \sum_{k=1}^{K} \psi_{ijk} V_{ijk} +$$

$$+ \frac{\mu_4}{2} \sum_{i=1}^{1} \sum_{j=1}^{j+1} \sum_{k=1}^{K} V_{ijk}(1-V_{ijk}) + \frac{\mu_5}{2} \sum_{i=1}^{1} \left(1 - \sum_{j=1}^{j+1} \sum_{k=1}^{K} V_{ijk}\right)^2$$

Where:

1. The first term maximizes the cost function (normalized to unity by $C_{\max}$) that is calculated by the expression $C_{ijk} = \frac{R_{b,ijk}}{R_{b,max}} + \alpha_{ijk} + \beta_{ijk}$, with $R_{b} = [R_{b,ijk}]$ the bit rate matrix in b/s, $R_{b,max} = \max_{ijk}(R_{b,ijk})$ in b/s, $\alpha_{ijk} = \{1 \text{ if } R_{b,ijk} \geq R_{b,Target,i} \}, \beta_{ijk}$ the term used to prioritize the different RATs (also used to consider the preferences of each user and operator).

2. The second term, the first constraint, has the capacity constraint matrix $\xi_{ijk}$ defined as $\xi_{ijk} = u\left(\eta_{ijk} - b_{Tik}\right)$, where $u(\cdot)$ is the step function, $\eta_{ijk} = R_{b,ijk} + \sum_{m=1}^{m+1} R_{b,mnk} V_{mnnk}$ is an indicator of the bandwidth utilization and $b_{Tik}$ is the bandwidth available for the RAT $k$. Then, if the bandwidth of a RAT is full, that is $\eta_{ijk} \geq b_{Tik}$, user $i$ will not be able to allocate to RAT $k$.

3. In the third term $\psi_{ijk}$ indicates if the allocation to the bit rate $j$ and the RAT $k$ is feasible ($\psi_{ijk} = 0$) or not ($\psi_{ijk} = 1$) for the $i$th user, depending on its position in the space some RATs or bit rates will not have coverage there.

4. The fourth term is to force the neurons to be either 0 or 1 (a stable state).
5. The fifth term is to allow the activation of only one neuron per user, that is, each user is allowed to allocate to only one RAT and bit rate.

6. And $\mu_i$ are the weighing coefficients of the energy function for each term that shall be weighed correctly as in [1] to allow a good convergence to the desired solution. But, using the algorithm in [3], it is not necessary to define their values ($\mu_i = 1, \forall i$) because in this algorithm there is only one constant to be considered for the restrictions, $\gamma$.

2.1.2. Multiple cells with heterogeneous base stations

Due to the design of new networks in order to accomplish with the expected service for users, there is an increment in the number of base stations and micro-stations and they interfere with each other when they sent data at the same frequency (when they share the band).

![Figure 3. Multiple cells without interference](image)

In figure 3, an example of multiple cells (one MBS and two SCe) without interference is represented. The different curves correspond to three different bit rates (R1, R2 and R3) that each station can provide. Although the curves intersect, there is no interference between the signals (orthogonal access).

The purpose of this project is to take into account the interference between stations in the energy function and select the maximum bit rate possible with this interference.

In order to introduce a new restriction to the energy function, the Shannon-Hartley theorem must be explained. It says that the limit of reliable information rate (bit rate $R$) of a channel depends on bandwidth ($B$) and signal-to-interference-plus-noise ratio (SNIR):

$$R < B \cdot \log_2 (1 + \text{SNIR})$$

(2.3)
As it was explained in the multi-RAT problem, the third term $\psi_{i,jk}$ indicates if the allocation to the bit rate $j$ and the RAT $k$ is feasible ($\psi_{i,jk} = 0$) or not ($\psi_{i,jk} = 1$) for the $i$-th user. So, the restriction can be incorporated into this term. For a user $i$, the allocation to the bit rate $j$, station $k$ and sub-band $l$ is possible if:

$$R_{b,jkl} < B \cdot \log_2 \left( 1 + \frac{B}{I_{lk} + N_0} \right) = B \cdot \log_2 \left( \frac{S_{lk}}{N_0} \right)$$

$$= B \cdot \log_2 \left( \frac{S_{lk}}{N_0} \right) - B \cdot \log_2 \left( 1 + \frac{I_{lk}}{N_0} \right)$$

$$= B \cdot \log_2 \left( \frac{S_{lk}}{N_0} \right) - B \cdot \log_2 \left( 1 + \sum_{v \in \{k \} \setminus \{l \}} I_{lk'} \cdot V_{i,l,k',l} \right)$$

Eliminating the algorithms we obtain:

$$\frac{R_{jkl}}{S_{ik}} < \frac{1}{1 + \frac{1}{N_0} \sum_{v \in \{k \} \setminus \{l \}} I_{lk'} V_{i,l,k',l}}$$

then we can add this in the energy function as $\psi_{ijkl} = u \left( \frac{1}{1 + \frac{1}{N_0} \sum_{v \in \{k \} \setminus \{l \}} I_{lk'} V_{i,l,k',l}} - 1 \right)$ because minimizing $\psi_{ijkl} V_{i,jkl}$ we obtain that for not possible allocations, which do not fulfil the inequality, $\psi_{ijkl} = 1$ and $\psi_{ijkl} = 0$ for possible allocations.

The added term in the energy function will be the following one:

$$\frac{\mu_{\text{term}}}{2} \cdot \sum_{i,j,kl} \psi_{ijkl} V_{ijkl}$$

Another modification in the energy function will be the capacity constraint. In the previous energy function for the single cell case, the capacity constraint was given by the bandwidth utilization in b/s. In this case, we are working in the OFDMA case, so the users will be sending the packets at the same time, at different frequencies. There are two cases to take into account:

- **CASE 1**: The case in which the macro-base station and all the small-cell stations share the band. An example with three possible bit rates per station (R1, R2 and R3) is shown in figure 4. There is interference between all the stations, so, compared to figure 3, all the curves are displaced.

- **CASE 2**: The case in which the macro-base station and the small-cell stations use different bands. An example is represented in figure 5, also with three possible bit rates per station. In this case, only the curves of the SCe are displaced, comparing with figure 3.
Figure 4. Multiple cells with interference (CASE 1)

Figure 5. Multiple cells, interference only between small cells (CASE 2)
In figure 6, an example with 4 sub-bands is shown. For the first case, the whole bandwidth is divided in 4 sub-bands, each one giving service to only one user, and the full bandwidth is used by all the stations (MBS and SCe). In the second case, the total bandwidth is also divided in four sub-bands, but two are used by the MBS and the rest are used by SCe (MBS and SCe do not share the frequency).

![Figure 6. Case 1 on the left (L=4) and Case 2 on the right (L=2)](image)

In both cases, the band will be divided in sub-bands of the same width and each of them will be assigned to one user with the correspondent bit rate (the largest possible).

The capacity constraint would be the sum of users connected in one sub-band of one station that must be 1 or less, which is expressed in the following equation:

\[ \sum_{i,j} V_{ijkl} \leq 1, \text{ for a given station } k \text{ and a given sub-band } l \]  

In the first case the macro-cell station and the small-cells stations will interfere with each other while in the second case there will be interference only between the small-cells stations.

The other constraints (4 and 5) in the energy function in the single cell case would be the same for the heterogeneous multiple cells case.

### 2.2. State of the art

Changing the connection in wireless networks is an important matter because users nowadays move constantly (people use wireless networks while travelling, running...). Also, the size of the cells tends to decrease in order to provide higher capacity for more connections and, as a consequence, there are more frequent handoffs; and this develops into the dynamic connection-admission control (CAC). Dynamic CAC is a validation process with constant change where a check is performed before an establishment of a connection to see if current resources are enough for the proposed connection and it can be used to prevent congestion in connection-oriented protocols.

Hopfield Neuronal Networks are considered very good candidates to design dynamic allocation algorithms, since they can provide feasible solutions to very complex optimization problems within a very short time. Furthermore, the HNN are recurrent
networks that operate in an unsupervised mode and require no training, which is an advantage. In [1] they use the HNN with a dynamic CAC approach that tries to maximize resource utilization while guaranteeing a Quality-of-Service (QoS) in multimedia wireless networks. The multimedia connection consists of three kinds of service (video, voice and data sub-streams) with each of them allowing three possible QoS (high, medium and low) and each user in the network wants one or more of the three services with a determined QoS. The objectives of this article are to maximize the utility factor of scarce wireless resources, to minimize the blocking and dropping probabilities and to provide a fair distribution of resources among the connections with acceptable service grades. To do that, in the energy function of the HNN, they introduce a cost $C_{ij}$ associated to the connection $i$ corresponding to the QoS level $j$, which allows to find the best QoS for each user, and the fairness that is important to have less droppings and connection blockings.

2.2.1. Packet delay-oriented resource allocation

For a data service, the delay of the packets in queue is important in order to have fewer losses, to not accumulate packets in the queue and to obtain a better quality for the user, who usually wants speed and no droppings.

In [4] a delay-centric dynamic resource allocation algorithm maximizes resource utilization of the overall system while minimizing the packet delay. Its objective is to select the optimal amount of radio resources to be allocated for each user. It is the first article to introduce packet delay using the HNN for wireless communication systems, by adding a constraint in the energy function related to the maximum packet delay allowed for each user.

Another, [5], introduces the packet delay constraints to schedule the downlink transmissions in CDMA scenario. The objective is to deliver each packet without exceeding a specific time deadline. In the energy function they use a cost function associated to each bit rate and some downlink restriction, such as fairness and portioning of the total bandwidth. They evaluate the algorithm, compare it to a reference scheme that tries to allocate the optimum bit rate to deliver the packets in the specific delay bound and find that it has a better behaviour in terms of delay and dropping and a higher ability to adapt to the traffic load constraints.

2.2.2. Resource allocation in a multi-RAT scenario

As explained above, the advance in technology has ended up to a demand of a more complex communication system. Because people want to communicate in various ways (data, video and voice), various types of access are needed, which are known as Radio Access Technologies (RAT).

In [6] they use HNN to select the optimum RAT for each user and radio resources allocated, subject to certain restrictions in terms of total available resources, QoS requirements (different for each service and user), coverage constraints, ... They propose a generic formulation for packet services with delay constraints to decide the optimal bit rate and RAT allocation. Joint Dynamic Resource Allocation (JDRA) minimizes the number of simultaneous packet-switched connections and, consequently, the overall system capacity. It also tries to guarantee a maximum contracted packet delay and a maximum packet-dropping ratio.
Another article [7] in which they use JDRA based on HNN to decide which RAT serves each user in the next time interval and also the distribution of resources to fulfil the QoS for each user. They use a benefit function that measures the benefit of allocating each bit rate to each user in terms of delay and use as constraints the possible resources per user (depending on their location, for example), the bit rates feasible per user and the delay permitted in the packets they want to send. So, they also decide which RAT is the best for each user in order to provide them the best QoS, taking into account the delay constraints in the heterogeneous wireless network.
3. **Energy function and solutions**

The principal objective of this project is to minimize an energy function, to find the best solution to our problem. The energy function is composed by a cost function and some restrictions. If \( f(x) \) is the cost function we want to minimize and the restrictions can be expressed as the minimum value of a function \( g(x) \), the energy function can be expressed as \( E(x) = f(x) + g(x) \).

3.1. **Hopfield Neural Networks**

The **Hopfield neuronal network (HNN)** is a recurrent and fully connected feedback network. It can be used as an associative memory or to solve optimization problems.

First of all, a study on Hopfield Neuronal Networks was carried out from [8] and the articles in the bibliography. The neuronal networks are mathematic models inspired by the central nervous system of animals that are used to estimate functions (including non-linear functions) from, normally, a large number of inputs. Artificial nodes, which represent the neurons of the system, are connected to form a network and give outputs in function of their inputs and the weights chosen for each connection.

In 1949 Donald Hebb wrote the book The Organization of Behaviour in which he explained a model that captured the idea of the associative memory. His theory is known as the Hebbian theory and the models that use it, such as the HNN, are said to have "Hebbian learning". It says "When an axon of cell A is near enough to excite cell B and repeatedly or persistently takes part in firing it, some growth process or metabolic change takes place in one or both cells such that A's efficiency, as one of the cells firing B, is increased". So, two neurons that are correlated between \( \frac{dw_{ij}}{dt} \) can be approximated with the correlation \( (x_i, x_j) \) them implies that when one has a stimulus and increases its activity, the other neuron correlated to it also has a stimulus and increments (or decrements) its activity.

In the HNN the connections are symmetric and bidirectional, its weights are symmetric \( (w_{ij} = w_{ji} \) for every par of neurons \( i, j \) and there are no self-connections \( (w_{ii} = 0, \) for every neuron \( i \).

In this section the parameters that will be used are:

- \( I \): number of neurons;
- \( w_{ij} \): weight from neuron \( i \) to neuron \( j \);
- \( w_{0i} \): weight from a fictitious neuron \( 0 \) that has a permanent activity \( x_0 = 1 \);
- \( a_i \): activation of neuron \( i \);
- \( a \): vector of neuron activations;
- \( x_i \): activity (output) of neuron \( i \);
- \( x \): vector of neuron outputs;
- \( J (=T) \): weighing matrix of the HNN;
- \( h (=h) \): entropy in the energy function.
The HNN can be binary or continuous:

- **Binary HNN**

  The activity rule of the binary HNN is the updating of every neuron in the network as if there were no other neurons with the threshold activation function:

  \[ x(a) = \theta(a) = \begin{cases} 1 & a \geq 0 \\ -1 & a < 0 \end{cases} \]  

  and the activation computed by one neuron \( i \) is:

  \[ a_i = \sum_j w_{ij} \cdot x_j \]  

  followed by the modification of its state \( x_i = \theta(a_i) \).

  Since the HNN are feedback networks, we must specify if the updating will be synchronous (all the neurons will compute their activations and modify their states simultaneously) or asynchronous (one neuron at a time will compute its activation and update its state). The properties of the HNN may be affected depending on the choice.

  The learning rule specifies the way in which the weights change in time, it is to make a set of desired states \( \{x^{(n)}\} \) be the stable states of activity rule in the HNN (with \( x_i \in \{-1,1\} \)).

  The weights calculation or activation is given by the Hebb rule:

  \[ w_{ij} = k \cdot \sum_n x_i^{(n)} \cdot x_j^{(n)} \]  

  the adequate value for the constant \( k \) may be \( 1/N \) (but is irrelevant in the binary case) to prevent large values.

- **Continuous HNN**

  The states of a continuous Hopfield network are real numbers of \([-1, 1]\). The activation of the neurones is the same as the binary Hopfield network \( a_i = \sum_j w_{ij} \cdot x_j \) and the states updating is given by:

  \[ x_i = \tanh (a_i) \]  

  It can also be synchronous or asynchronous.

  However, the value of \( k \) in the weights activation (equation 3.3) becomes relevant because \( x_i \) takes real values. Usually, \( k \) is fixed and a gain \( \beta \in (0, \infty) \) is used in exchange in the state function: \( x_i = \tanh (\beta \cdot a_i) \).

### 3.1.1. Energy function and the HNN

We want that the HNN converge into a stable and correct state. If we take into account the spin system's energy function:

\[ E(x; J) = -\frac{1}{2} \sum_{m,n} J_{mn} x_m x_n - \sum_n h_n x_n \]  

that has a separable distribution (where \( a = \theta \))
\[ Q(x; a) = \frac{1}{Z_Q} e^{\sum_n a_n x_n} \]  

and we use the variational free energy minimization, we obtain an equivalent function for the HNN that we need to minimize.

The variational free energy is defined as

\[
\beta \cdot F(\theta) = \sum_x Q(x; \theta) \cdot \ln \left( \frac{Q(x; \theta)}{\langle e^{-\beta E(x; J)} \rangle_Q} \right) = 
\beta \cdot \sum_x Q(x; \theta) \cdot E(x; J) - \sum_x Q(x; \theta) \cdot \ln \left( \frac{1}{Q(x; \theta)} \right) = \beta \cdot \langle E(x; J) \rangle_Q - S_Q
\]

where \( \langle E(x; J) \rangle_Q \) is the average energy function under \( Q(x; \theta) \) and \( S_Q \) the entropy of the distribution (with \( k_B = 1 \)).

Using the definition of

\[
P(x|\beta,J) = \frac{1}{Z(\beta,J)} e^{-\beta E(x; J)} \Rightarrow e^{-\beta E(x; J)} = P(x|\beta,J) \cdot Z(\beta,J)
\]

we obtain:

\[
\beta \cdot F(\theta) = \sum_x Q(x; \theta) \cdot \ln \left( \frac{1}{P(x|\beta,J)} \right) - \ln (Z(\beta,J)) = D_{KL}(Q||P) + \beta F
\]

with \( F \) the true energy defined by \( \beta \cdot F = -\ln (Z(\beta,J)) \).

Then, applying the Gibb’s inequality \( D_{KL}(Q||P) \geq 0 \) we have \( F(\theta) \geq F \), equal only when \( Q = P \). So, the objective is to vary \( \theta \) to minimize \( F(\theta) \) and, then, \( F(\theta) \) would be an upper bound for \( F \) (and \( Z = e^{-\beta F(\theta)} \) is a lower bound for \( Z \)).

To evaluate the variational free energy for spin systems we need

\[
S_Q = \sum_n Q(x; a) \cdot \ln \left( \frac{1}{Q(x; a)} \right) \text{ and } \langle E(x; J) \rangle_Q = \sum_n Q(x; a) \cdot E(x; J).
\]

The entropy of the separable distribution is \( S_Q = \sum_n H_2^{(\theta)}(q_n) \) (the sum of the individual spins’ entropies), with \( q_n \) the probability that a spin \( n \) is \( +1 \) \( q_n = \frac{e^{x_n}}{e^{x_n}+e^{-x_n}} = \frac{1}{1+e^{-2x_n}} \) and

\[
H_2^{(\theta)}(q) = q \cdot \ln \left( \frac{1}{q} \right) + (1-q) \cdot \ln \left( \frac{1}{1-q} \right)
\]

If the mean value of \( x_n \) is \( \overline{x_n} = \frac{e^{x_n} - e^{-x_n}}{e^{x_n}+e^{-x_n}} = \tanh(e^{x_n}) = 2 \cdot q_n - 1 \), we have that the mean energy under \( Q \) is given by \( \langle x_n \rangle \) and \( x_n \) independent and \( J_{mn}=0 \) when \( m=n \):

\[
\langle E(x; J) \rangle_Q = \sum_x Q(x; a) \cdot E(x; J) = \langle E(x; J) \rangle_Q
\]

\[
= \sum_x Q(x; a) \cdot \left[ -\frac{1}{2} \cdot \sum_{m,n} J_{mn} x_m x_n - \sum_n h_n x_n \right] = -\frac{1}{2} \sum_{m,n} J_{mn} \overline{x_m} \overline{x_n} - \sum_n h_n \overline{x_n}
\]
Then, the variational free energy is:

\[
\beta \cdot F(a) = \beta \cdot (E(x; J))_Q - S_Q
\]

\[
= \beta \cdot \left( -\frac{1}{2} \sum_{m,n} J_{m,n} \cdot \bar{x}_m \cdot \bar{x}_n - \sum_n h_n \cdot \bar{x}_n \right) - \sum_n \mathcal{H}_2^{(e)}(q_n)
\]

We minimize this function respect to \(a\).

\[
q = \frac{1}{1 + e^{-2a}} \Rightarrow \frac{\partial}{\partial q} H_2^{(e)}(q) = \ln \left( \frac{1 - q}{q} \right) = -2a
\]

\[
\bar{x}_n = 2 \cdot q_n - 1 \Rightarrow \frac{\partial}{\partial a_m} \beta \cdot F(a)
\]

\[
= \beta \cdot \left( -\sum_{m,n} J_{m,n} \cdot \bar{x}_m - h_m \right) \cdot \left( 2 \cdot \frac{\partial q_m}{\partial a_m} \right) - \ln \left( \frac{1 - q_m}{q_m} \right) \cdot \left( \frac{\partial q_m}{\partial a_m} \right)
\]

\[
= 2 \cdot \left( \frac{\partial q_m}{\partial a_m} \right) \cdot \left[ -\beta \cdot \left( \sum_{m,n} J_{m,n} \cdot \bar{x}_m + h_m \right) + a_m \right]
\]

This is equal to zero and, therefore, \(F(a)\) is minimized when \(a_m = \beta \cdot (\sum_{m,n} J_{m,n} \cdot \bar{x}_m + h_m)\)

and \(\bar{x}_m = \tanh(a_n)\) (these are the mean field equations for a spin system). If we update \(\bar{x}_n\) and \(a_n\) with these equations, we guarantee the decrease of \(\beta \cdot F(a)\).

Therefore, if \(J = W\) (weights matrix), \(\bar{x} = x\) and \(h_n = w_{i0}\), the equations of the HNN are equal to a set of mean-field equations that minimize

\[
\beta \cdot F(x) = -\beta \cdot \frac{1}{2} x^T \cdot W \cdot x - \sum_i \mathcal{H}_2^{(e)} \left( \frac{1 + x_i}{2} \right)
\]

\(a_m = \beta (\sum_{i,n} J_{i,m} \bar{x}_n + h_m)\) and \(\bar{x}_n = \tanh(a_n)\) are the iterative equations that minimize the variational free energy and, at the same time, are the equations of activation and state updating in the Hopfield neuronal network.

The function 3.14 is the Lyapunov function; it decreases under dynamical evolution of the system and is bounded below (it has a minimum). If a system has this type of function, it converges to a point (a local minimum of the function or a limit cycle in which the Lyapunov function is a constant) and chaotic behaviour is not possible for this system.

If the HNN activity rules are implemented asynchronously, they have a Lyapunov function. This is a convex function for every activation \(a_i\) and, therefore, the system will always converge to a stable point because of the symmetry in the HNN and only if the activity rules are synchronously implemented.

Therefore, finding the solution of the HNN implemented asynchronously is the same as minimizing the energy function defined by \(J\) and \(h\), where \(J\) is the matrix of the weighing coefficients in the neuronal network.

We use a continuous-time HNN because it prevents the change of the HNN properties from asynchronous to synchronous updates.
If we suppose the activity of a neuron is continuous in time we have:

\[ a_i(t) = \sum_j w_{ij} \cdot x_j(t) \]  \hspace{1cm} (3.15)

Then, the response of the neuron to the activation is supposed to be

\[ \frac{d}{dt} x_i(t) = -\frac{1}{\tau} (x_i(t) - f(a_i)) \]  \hspace{1cm} (3.16)

where \( f(a) \) is the activation function (e.g.: \( f(a) = \tanh(a) \)).

For a stable activation, \( x_i(t) \) tends to \( f(a_i) \) exponentially with the time constant \( \tau \), as it is demonstrated below.

We first express the derivative equation in the Laplace form:

\[ s \cdot X_i(s) = -\frac{1}{\tau} \cdot X_i(s) + \frac{1}{\tau} \cdot f(a_i) \cdot \delta(s) \]  \hspace{1cm} (3.17)

From 2.17 we can isolate \( X(s) \):

\[ X_i(s) = \frac{C \cdot \delta(s)}{s + \frac{1}{\tau}}, \text{ where } C = \frac{1}{\tau} \cdot f(a_i) \]  \hspace{1cm} (3.18)

Then, as \( F(s) \cdot G(s) \leftrightarrow f(t) \ast g(t), \delta(s) \leftrightarrow 1, \frac{1}{s+\tau} \leftrightarrow e^{-t/\tau} \cdot u(t) \) and \( \), we obtain:

\[ x_i(t) = C \ast e^{-t/\tau} \cdot u(t) = C \cdot \int_0^t e^{-\frac{(t-x)}{\tau}} \cdot dx = C \cdot \left[ \frac{e^{-\frac{(t-x)}{\tau}}}{1/\tau} \right]_0^t \]  \hspace{1cm} (3.19)

\[ = C \cdot \left( \tau \cdot e^0 - \tau \cdot e^{-t/\tau} \right) = \frac{1}{\tau} \cdot f(a_i) \cdot \left( \tau - \tau \cdot e^{-t/\tau} \right) = f(a_i) \cdot \left( 1 - e^{-t/\tau} \right) \]

\textbf{Note:} when the weight matrix \( W \) (or \( J \)) is symmetric, the system has the variational free energy as its Lyapunov function.

\subsection*{3.1.2. Discrete-time HNN}

A form to compute the HNN is to update the activations and the outputs asynchronously until the new updated output and the last updated output differ little.

The dynamics of the HNN with \( N \) neurons is represented by:

\[ \frac{dU_i}{dt} = \sum_{k=1}^{N} T_{ik} \cdot V_k - \frac{U_i}{\tau} + I_i \]  \hspace{1cm} (3.20)

Where \( \tau \) is the time constant of the circuit, \( T_{ij} \) represents the weight from the \( i \)-th neuron to the \( j \)-th neuron and \( T \) and \( I \) are the matrix and vector from the energy function \( E(v) = -\frac{1}{2} \sum_i \sum_j T_{ij} v_i v_j - \sum_i I_i v_i \). Then, the equation 3.20 can be written as:

\[ \frac{dU_i}{dt} = -\frac{U_i}{\tau} - \frac{\partial E}{\partial V_i} \]  \hspace{1cm} (3.21)
The activations (or inputs) are updated as:

\[ U_i(t + \Delta t) = U_i(t) + \Delta t \cdot \left\{ -\frac{U_i(t)}{\tau} - \frac{\partial E}{\partial V_i} \right\} \quad (3.22) \]

Where \( \Delta t \) is the updating step.

And the outputs of the neurons are updated from the activations as:

\[ V_i = \frac{1}{1 + e^{-\alpha_i U_i}} \quad (3.23) \]

Where \( \alpha_i \) are the gains in each neuron.

In this section, in order to converge to the expected states, the weighing coefficients of the energy function \( \mu_i \) should be chosen wisely to accomplish this purpose (Annex A).

This case is not consistent because the HNN converges to a completely different solution changing the weighing coefficients while still accomplishing the way to choose them in Annex A.

3.2. Fast-HNN: projected gradient

As explained above, solving the HNN is equivalent to find the minimum in the energy function (equation 3.5). This algorithm (in [9]), knowing this equivalence, is based on the minimization of the energy function with a gradient method and ensuring that all the neurons are confined in the hypercube \([0,1]^N\). As in 3.1.2, the weighing coefficients need to be selected in order to provide feasible solutions.

3.2.1. Introduction

It is supposed that \( T \) and \( i \) define the energy function \( E = -\frac{1}{2} \sum_{m=1}^{N} \sum_{n=1}^{N} T_{mn} \cdot V_m \cdot V_n - \sum_{m=1}^{N} i_m \cdot V_m \), where \( T \) is a symmetric matrix with dimensions \( N \times N \) and \( i \) is a \( N \)-dimensional vector. Also, the network state is defined by the neuron outputs \( V_i(t) \).

The updating of the neuron outputs is given by:

\[ \Delta_i(t) = \beta(t) \cdot d_i(t) \quad (3.24) \]

\[ V_i(t + 1) = V_i(t) + \Delta_i(t) \quad (3.25) \]

where \( d(t) \) is the updating direction at time \( t \) with components \( d_i(t) \) and \( \beta(t) \) is the updating step at time \( t \).

Also, the updated output has to be inside the hypercube, so the updating direction and step must satisfy the following equation:

\[ -V_i(t) \leq \beta(t) \cdot d_i(t) \leq 1 - V_i(t) \quad (3.26) \]

3.2.2. Updating step

First of all, an updating step \( \beta_0(t) \) is chosen to satisfy the equation, given an updating direction, so that the increment is the optimum directional update for \( d(t) \).
A critical point is obtained from
\[
\frac{dE(t + 1)}{d\beta(t)} = 0
\]  
That is:
\[
\beta(t) = \frac{s_1(t)}{s_2(t)}
\]
where:
\[
s_1(t) = -\sum_{i=1}^{N} d_i(t) \frac{\partial E}{\partial V_i(t)}
\]
\[
s_2(t) = -\sum_{i=1}^{N} \sum_{j=1}^{N} T_{ij} \cdot d_i(t) \cdot d_j(t)
\]
Then, there are 4 cases depending on the sign of $s_1$ and $s_2$. From those cases the sign of $\beta$ is selected: $\text{sign}(\beta(t)) = \text{sign}(s_1(t))$ and its value is selected to accomplish equation 3.26.

The result is the following optimum value for the updating step:
\[
\beta_0(t) = \begin{cases} 
-\min\{l_i(t)\} & \text{if } s_1(t) < 0, s_2(t) < 0 \\
\min\left\{\frac{s_1(t)}{s_2(t)}, \min\{l_i(t)\}\right\} & \text{if } s_1(t) > 0, s_2(t) > 0 \\
\min\{l_i(t)\} & \text{if } s_1(t) > 0, s_2(t) < 0 \\
-\min\left\{-\frac{s_1(t)}{s_2(t)}, \min\{l_i(t)\}\right\} & \text{if } s_1(t) < 0, s_2(t) > 0 
\end{cases}
\]
where $I(t)$ is the vector of limits with components $l_i(t) = \begin{cases} 1 - V_i(t) & \text{if } s_1(t) \cdot d_i(t) > 0 \\
-V_i(t) & \text{if } s_1(t) \cdot d_i(t) < 0 
\end{cases}$.

### 3.2.3. Updating direction

The HNN of N neurons has M linear constraints defined by $A \cdot v = b$ ($A$ is a MxN matrix and $b$ is a M-dimensional vector). Then, the ideal updating direction should be in the nucleus of the space, that is $d(t)$ should accomplish $A \cdot d(t) = 0$. Then, the updated output would comply the following equation:
\[
A \cdot v(t + 1) = A \cdot (v(t) + \beta(t) \cdot d(t)) = A \cdot v(t) + \beta(t) \cdot A \cdot d(t) = b
\]  

Consequently, a projection matrix $P$ is used to project the direction into the nucleus mentioned ($A \cdot P \cdot v = 0, \forall v \in \mathbb{R}^N$). Since the computation of this matrix is not efficient, a better approach can be derived from the analysis of the null space of $A$, $P$ is the orthogonal projector onto the null space of $A$. Then, if $Q$ is a NxM matrix whose columns constitute an orthonormal basis of the row space of $A$ ($QQ' = I$ and $\text{span}(Q) = \text{span}(A')$), the projector can be written as $P = I - Q \cdot Q'$. Before obtaining the optimal updating step value, the updating direction vector needs to be found. To find its value, the energy gradient ($\nabla E = -T \cdot v - I$) is used and the updating direction is evaluated as the following equation:
\[
d(t) = -\nabla E + Q \cdot Q' \cdot \nabla E
\]
But this is not enough, since high or low values may obtain results that are not confined in the hypercube. If there is a neuron that would go out of the hypercube; that is, if \( V_i(t) = 0 \) and \( d_i(t) < 0 \) or \( V_i(t) = 1 \) and \( d_i(t) > 0 \), new constraints are added so that there is no update in those directions and the projection matrix \( P = I - \mathbf{Q} \cdot \mathbf{Q}' \) is changed accordingly.

### 3.2.4. Summary of the algorithm

- **STEP 1**: Initialize \( \mathbf{A} \) and derive \( \mathbf{Q} \). Define a random vector \( \mathbf{v}(t) \) as the initial outputs (t=0) of the neurons that is confined in the hypercube and accomplishes the linear constraints.
- **STEP 2**: Calculate the energy gradient as \( \nabla E = -\mathbf{T} \cdot \mathbf{v} - \mathbf{i} \).
- **STEP 3**: Update the direction as in equation 3.33 and check if all the neurons will be confined in the hypercube. If all the neurons are confined, continue with step 5.
- **STEP 4**: While there is one neuron that will not be confined in the hypercube, add new constraints to \( \mathbf{A} \) and derive the new columns of \( \mathbf{Q} \). Actualize \( \mathbf{d}(t) \).
- **STEP 5**: Calculate \( s_1(t) \), \( s_2(t) \), \( l(t) \) and \( \beta(t) \) (equations 3.28 - 3.31).
- **STEP 6**: Update neuron states: \( \mathbf{v}(t+1) = \mathbf{v}(t) + \beta(t) \cdot \mathbf{d}(t) \).
- **STEP 7**: Update time and repeat from step 2 until a termination criterion is met.

**Note**: the termination criteria would be, for example, until \( \max\{\text{abs}(\mathbf{v}(t) - \mathbf{v}(t+1))\} < \delta \).

This algorithm has problems in some cases: when \( s_2 \) is 0 and where the values are too small for the sensibility of zero in Matlab.

### 3.3. Smith’s projected gradient with annealing

Smith, Palaniswami and Krishnamoorthy in [3] proposed an algorithm to compute the discrete HNN. It also uses the same equivalence (solving the HNN is the same as minimizing the energy function), but uses the simulated annealing that is a generic probabilistic meta-heuristic for the global optimization problem of locating a good approximation to the global optimum of a given function in a large search space.

#### 3.3.1. Summary of the algorithm

The functioning of the algorithm can be resumed with the following steps:

- **STEP 1**: Initialize the parameters of the network (\( \mathbf{T} \) and \( i \)) as
  \[
  \mathbf{T} = -2 \cdot \mathbf{Q} + \gamma \cdot (\mathbf{P} - \mathbf{I}) \]
  \[
  i = \gamma \cdot s - c \]

  \( x_i \approx 0.5 \), initialize the time step \( \Delta t \) and an epsilon \( \varepsilon \), \( L = 0 \), \( U = 1 \), and \( \tau \).

- **STEP 2**: Update \( k(t) = 1 - 2 \cdot e^{-t/\tau} \) and generate \( \alpha(t) \) randomly from \([k(t),1]\).
- **STEP 3**: Update the neurons according to

  \[
  x_i = x_i - \Delta t \left( \alpha(t) \frac{\partial f}{\partial x_i} \right) \]
• **STEP 4**: Project $\mathbf{x}$ back onto the constraint plane and within the unit hypercube.

**STEP 5**: Update $L = L + \varepsilon$ and $U = U - \varepsilon$. Repeat from step 3 for one Markov chain length (or the number of random walks permitted in the multidimensional space).

• **STEP 6**: Increase $t$ and repeat from step 2 until $k(t) = 1$ and $\frac{dx_i}{dt} = 0, \forall i$.

The parameters of the network, $\mathbf{T}$ and $\mathbf{t}$, are defined as in step 1 because the energy function $E(\mathbf{x}) = -\frac{1}{2} \sum_i \sum_j T_{ij} x_i x_j - \sum_i t_i x_i$ is expressed as the following equation:

$$E(\mathbf{x}) = f(\mathbf{x}) + \frac{\gamma}{2} \cdot ||\mathbf{x} - (\mathbf{P} \cdot \mathbf{x} - \mathbf{s})||^2 \tag{3.37}$$

Where

$$\mathbf{P} = \mathbf{I} - \mathbf{A}^T (\mathbf{A} \mathbf{A}^T)^{-1} \mathbf{A} \tag{3.38}$$

$$\mathbf{s} = \mathbf{A}^T (\mathbf{A} \mathbf{A}^T)^{-1} \mathbf{b} \tag{3.39}$$

With $f(\mathbf{x}) = c^T \mathbf{x} + \mathbf{x}^T \mathbf{Q} \mathbf{x}$ the function to minimize subject to the constraints $\mathbf{A} \mathbf{x} = \mathbf{b}$ and $x_i \in \{0, 1\} \ \forall i = 1, \ldots, n$, and the second term a measure of the deviation of the vector $\mathbf{x}$ from the constraint plane given by $\mathbf{A} \mathbf{x} = \mathbf{b}$.

### 3.3.2. Projection step and annealing

Once marked the direction with the cost function and $\alpha(t)$, the solution, $\mathbf{x}$, should be projected into the constraint plane and the unit hypercube.

Given the constraints $\mathbf{A} \mathbf{x} = \mathbf{b}$, this defines an hyper-plane in the space. The projected solution, $\mathbf{x}_p$, is given by:

$$\mathbf{x}_p = \mathbf{P}^T \cdot \mathbf{x} + \mathbf{s} \tag{3.40}$$

Where $\mathbf{P}$ is the projection matrix and $\mathbf{s}$ is the solution of minimum norm in the constraint plane, defined as in equations 3.38 and 3.39.

In the multiple cell case, there are constraints with inequalities and the activated neurons. So, in each iteration, the constraints $\mathbf{A} \mathbf{x} \leq \mathbf{b}$ that are not accomplished are added as $\mathbf{A} \mathbf{x} = \mathbf{b}$.

Then, project the solution into the constraints' space and into the unit hypercube depending on the values of $L$ and $U$ as:

$$x_i = \begin{cases} 
0 & \text{if } x_i \leq L \\
\frac{x_i - L}{U - L} & \text{if } L < x_i < U \\
1 & \text{if } x_i \geq U 
\end{cases} \tag{3.41}$$

As the values $x_i$ are constrained in $[0,1]$, the constraints might not be accomplished; so the projection is made periodically until the error is less than a limit or until a maximum number of iterations is carried out.
3.4. **Update of the constraints matrix**

In both algorithms (3.2 and 3.3), the constraint matrix is modified in each iteration.

In the first algorithm, the linear constraints are the rows in the matrix $\mathbf{A}$ of the first step (initial matrix of constraints). In an iteration, before actualizing the direction in step 4, the rows corresponding to the inequality constraints that are not accomplished must be added in the matrix $\mathbf{A}$; and afterwards derive the new $\mathbf{Q}$ to update the direction.

In the second algorithm, the matrix $\mathbf{A}$ will be modified in each projection iteration. That is, if a lineal constraint or an inequality constraint is not fulfilled, the corresponding rows are added to the matrix. The number of iterations projecting $\mathbf{x}$ will depend on the lineal constraints error, the inequality constraints error (stop the iterations when the errors are minimized) and a threshold value for the maximum number of iterations permitted. The errors are defined as:

$$ ||\mathbf{A}_{lc}\mathbf{x} - \mathbf{b}_{lc}||_2 $$ (3.42)

$$ \mathbf{A}_{ineq}\mathbf{x} - \mathbf{b}_{ineq} $$ (3.43)

where $\mathbf{A}_{lc}$ is the matrix corresponding to the rows of lineal constraints not accomplished, $\mathbf{b}_{lc}$ is the value of $\mathbf{A}_{lc}\mathbf{x}$ should take, $\mathbf{A}_{ineq}$ is the matrix corresponding to the rows in the inequality constraints not accomplished, $\mathbf{b}_{ineq}$ are the maximum values of $\mathbf{A}_{lc}\mathbf{x}$. 

4. **Methodology and system evaluation**

4.1. **Technical specifications and scenario**

The proposed algorithm will be tested in simulated environment with habitual conditions by means of computer simulations. There will be a number of users (I) that want to transmit video streaming data. Each user will have a number of packets in queue, with different lengths (number of bits), certain arrival time in the queue and a maximum contracted packet delay.

Every packet call (100ms), 8 packets will be generated with lengths and inter-arrival times following the distributions mentioned in table 1. The algorithm proceeds to assign a bit rate every 20ms to each user that contains packets in its queue and is chosen so that it minimizes the energy function. Users that do not contain packets in the queue are assigned the bit rate 0, the cost function for these users changes to: $C_{ijkl} |_{j=1} = 2$ and $C_{ijkl} |_{j=2:j+1} = 0$.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Statistical characterization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inter-arrival time between the beginning of each frame</td>
<td>Deterministic at 100 ms (10 frames per second)</td>
</tr>
<tr>
<td>Number of packets (slices) in a frame</td>
<td>Deterministic</td>
</tr>
<tr>
<td>Packet (slice) size</td>
<td>Truncated Pareto distribution, Mean = 10 Bytes, Maximum = 250 bytes (before truncation)</td>
</tr>
<tr>
<td></td>
<td>$f_x = \frac{a^\alpha}{x^{\alpha+1}}$, $k \leq x &lt; m$, $f_x = \left(\frac{k}{m}\right)^\alpha$, $x = m$</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 1.2$, $k = 20$ bytes, $m = 250$ bytes</td>
</tr>
<tr>
<td>Inter-arrival time between packets (slices) in a frame</td>
<td>Truncated Pareto distribution, Mean = $m = 6$ ms, Maximum =12.5 ms (before truncation)</td>
</tr>
<tr>
<td></td>
<td>$f_x = \frac{a^\alpha}{x^{\alpha+1}}$, $k \leq x &lt; m$, $f_x = \left(\frac{k}{m}\right)^\alpha$, $x = m$</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 1.2$, $k = 2.5$ ms, $m = 12.5$ ms</td>
</tr>
<tr>
<td>Maximum contracted packet delay</td>
<td>100 ms</td>
</tr>
</tbody>
</table>

**Table 7. Video streaming traffic model parameters [10]**

In figure 7, an example for the generation of packets is pictured.
Therefore, each user $i$ will demand a minimum bit rate $R_{b, \text{Target}, i}$ in order to accomplish the delay constraints, which is evaluated as:

$$R_{b, \text{Target}, i} = \max_j \left( R_{b, j}^i \right) = \max_j \left( \frac{\sum_{p=1}^{j} l_{i,p}}{D_{\text{max}} - t_{ij}} \right)$$ \hspace{1cm} (4.1)$$

Where $R_{b, j}^i$ is the bit rate associated to packet $j$, $l_{i,p}$ is the length of the $p$-th packet of user $i$, $D_{\text{max}}$ is the maximum contracted packet delay and $t_{ij}$ is the time that the $j$-th packet will stay in the queue of user $i$.

Also, each user will be situated in a different position in space and that will lead to constrictions of possible bit rates, stations and/or bands in terms of SNIR.

The simulated scenarios will follow the settings in 3GPP [11], a quasi-static system level simulator is used according to the parameters for Small Cell Enhancement Scenario #2a. Some relevant parameters are in the following table:
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Setting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deployment scenario</td>
<td>SCE Scenario #1</td>
</tr>
<tr>
<td>Network Layout</td>
<td>500m macro-layer inter-site distance</td>
</tr>
<tr>
<td>Cell layout</td>
<td>1 macro-sites with 3 sectors per site (3 macro-cells)</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>10MHz</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>Macro-eNB: 2GHz; Small-eNB: 2GHz</td>
</tr>
<tr>
<td>UE deployment</td>
<td>from 10 to 60 users per macrocell geographical area</td>
</tr>
<tr>
<td>UE placement</td>
<td>2/3 UEs inside the cluster; the remaining UEs are uniformly distributed within the macro-cell area; 80% users indoor; 20% users outdoor</td>
</tr>
<tr>
<td>Transmit power</td>
<td>Macro-eNB: 46dBm; Small-eNB: 30dBm</td>
</tr>
<tr>
<td>Antenna system</td>
<td>DL 1x1; UL 1x1</td>
</tr>
<tr>
<td>Antenna gain</td>
<td>Macro-eNB: 17 dBi; Small-eNB: 5 dBi; UE: 0 dBi</td>
</tr>
<tr>
<td>Antenna pattern</td>
<td>Macro-eNB: 3D; Small-eNB and UE: Omni</td>
</tr>
<tr>
<td>Antenna Height</td>
<td>Macro-eNB: 25m; Small-eNB: 10m; UE: 1.5m</td>
</tr>
<tr>
<td>Path loss</td>
<td>Macro-eNB to UE: ITU UMa; Small-eNB to UE: ITU UMi</td>
</tr>
</tbody>
</table>
| Penetration losses               | For outdoor UEs: 0dB  
For indoor UEs: 20dB+0.5d_{in} (d_{in} : independent uniform random value between [ 0, min(25,d) ] for each link, being d the distance between eNB and UE) |
| Shadow fading                    | Macro-eNB to UE: ITU UMa; Small-eNB to UE: ITU UMi                                                                                                                                                   |
| Fast fading channel              | Rayleigh                                                                                                                                                                                             |
| Number of clusters per macro     | 1                                                                                                                                                                                                     |
| Number of small cells per cluster| 4                                                                                                                                                                                                     |
| Radius for small cell dropping in a cluster | 50m                                                                                                                                                                                                 |
| Radius for UE dropping in a cluster | 70m                                                                                                                                                                                                  |
| Minimum distance (2D distance)   | Small-eNB to Small-eNB: 20m  
Small-eNB to UE: 5m  
Macro-eNB to small cell cluster center: 105m  
Macro-eNB to UE : 35m  
Cluster center to cluster center: 2 x Radius for small cell dropping in a cluster = 100m                                                                 |
| UE noise figure                  | 9dB                                                                                                                                                                                                   |

Table 8. Simulation assumptions
Using this assumptions, the Matlab function in the annexes is used to simulate the channel, calculate the path losses, SNRs for each user, the position of each user, among others.

The possible assigned bit rates are the ones that follow the following modulation and coding scheme:

<table>
<thead>
<tr>
<th>CQI index</th>
<th>modulation</th>
<th>code rate x 1024</th>
<th>efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>out of range</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>QPSK</td>
<td>78</td>
<td>0.1523</td>
</tr>
<tr>
<td>2</td>
<td>QPSK</td>
<td>120</td>
<td>0.2344</td>
</tr>
<tr>
<td>3</td>
<td>QPSK</td>
<td>193</td>
<td>0.3770</td>
</tr>
<tr>
<td>4</td>
<td>QPSK</td>
<td>308</td>
<td>0.6016</td>
</tr>
<tr>
<td>5</td>
<td>QPSK</td>
<td>449</td>
<td>0.8770</td>
</tr>
<tr>
<td>6</td>
<td>QPSK</td>
<td>602</td>
<td>1.1758</td>
</tr>
<tr>
<td>7</td>
<td>16QAM</td>
<td>378</td>
<td>1.4765</td>
</tr>
<tr>
<td>8</td>
<td>16QAM</td>
<td>490</td>
<td>1.9141</td>
</tr>
<tr>
<td>9</td>
<td>16QAM</td>
<td>616</td>
<td>2.4063</td>
</tr>
<tr>
<td>10</td>
<td>64QAM</td>
<td>466</td>
<td>2.7305</td>
</tr>
<tr>
<td>11</td>
<td>64QAM</td>
<td>567</td>
<td>3.3223</td>
</tr>
<tr>
<td>12</td>
<td>64QAM</td>
<td>666</td>
<td>3.9023</td>
</tr>
<tr>
<td>13</td>
<td>64QAM</td>
<td>772</td>
<td>4.5234</td>
</tr>
<tr>
<td>14</td>
<td>64QAM</td>
<td>873</td>
<td>5.1152</td>
</tr>
<tr>
<td>15</td>
<td>64QAM</td>
<td>948</td>
<td>5.5547</td>
</tr>
</tbody>
</table>

Table 9. Modulation and coding scheme of LTE

Where the bit rates can be calculated as bandwidth (Hz) \cdot efficiency (b/s/Hz).

4.2. Merit figures

The measures to evaluate each solution in order to follow with the mentioned specifications in the introduction are:

- **Packet delay** $D_{j,i,u}$ (expressed in seconds) is defined as the temporal difference between the arrival time $T_{j,i,u}^{arr}$ of the $j$-th packet of the $i$-th packet call destined for user $u$ arrives at the user queue and the delivery time $T_{j,i,u}^{del}$ to the station as

  \[
  D_{j,i,u} = T_{j,i,u}^{del} - T_{j,i,u}^{arr}
  \]  

- **The average packet delay** $D_{avg,u}$ (in seconds) for user $u$ is

  \[
  D_{avg,u} = \frac{\sum_j D_{j,i,u}}{N \cdot T_{call}}
  \]  

  i.e., the average interval between packets originated at the source station (user queue) and received at the destination station in a system for a given packet call duration.

- **The 5%-tile packet delay** is the maximum packet delay for the 5% of packets with lowest delay.
• The **packet-dropping ratio** or **packet loss ratio** is the ratio between the number of lost packets (not delivered in time) and the total transmitted packets.

\[
\text{PLR} = \frac{\text{Number of Loss Packets}}{\text{Total Number of Transmitted Packets}}
\]  

(4.4)

• The **average throughput**, which is the rate of successful message delivery in the communication channel. Measured in bits/second. It is defined as:

\[
R_u = \frac{1}{T_{\text{simulation}}} \sum_{ij} b_{ij,u}
\]  

which represents the ratio of the number of information bits successfully received divided the total simulation time, \(i\) is the number of packet calls, with \(j\) packets for the \(i\)-th down-link packet call and \(b_{ij,u}\) bits for the \(j\)-th packet.

• The **5%-tile user throughput** that is defined as the maximum throughput for the worst 5% of the users.

### 4.3. Minimization and evaluation of costs

The minimization of the cost function with the proposed algorithm will minimize the costs. The cost function is given by the first term; that is:

\[
- \sum_{ijkl} \frac{C_{ijkl}}{C_{\text{max}}} V_{ijkl}
\]  

(4.6)

Therefore, minimizing this function will lead to the optimal bit rate. This optimal bit rate will be the higher possible that accomplishes the restrictions (if possible, equal or higher that the targeted bit rate).

### 4.4. System evaluation

#### 4.4.1. Single cell

For the single cell case (\(K=1\)), the energy function will consist on the cost function, the capacity constraint and the last two constraints that limit the number of activated neurons per user and limit the solutions inside the hypercube.

As there is no restriction among the bit rate because there is no interference and it is considered that the capacity of the system to sent data is high, the maximum bit rate will be given to all users that have the possibility to allocate (depending on the capacity of the cell). The function to minimize is the cost function 4.6 and the constraints are

\[
\sum_{ijkl} V_{ijkl} = 1, \forall i
\]  

(4.7)

\[
\sum_{ijkl, j \neq 2} V_{ijkl} \leq 1, \forall k, l
\]  

(4.8)

\[
V_{ijkl} \in \{0, 1\}, \forall i, j, k, l
\]  

(4.9)
For example, with L=4 (maximum 4 users per call, every 20 ms) and possible bit rates 
[0, 380750, 942500, 2192500] bits/second (same case as in figure 2, with one RAT and 
three possible bit rates for this RAT), the average packet delay, dropping and threshold in 
function of the number of users are shown in the following graphics:

Figure 8. Total lost packets, single cell

Figure 9. Average delay, single cell
Figure 10. Average throughput, single cell

As expected, the number of lost packets and the throughput increases when the number of users increases. And the packet delay increases with users and, from a threshold value, it starts to decrease because the number of lost packets increases.

Moreover, as the maximum number of users that can be allocated to the station per call is 4, when the number of users increases, the number of lost packets increases. The number of lost packets increases from 10 users onwards because for 10 or less users, the system can provide enough service to these users by allocating different 4 users every 20ms.

4.4.2. Multi-cell orthogonal access

In the multi-cell orthogonal access, there is no interference between users because the access is orthogonal. Then, the cost function will contain the same term as in the single cell case plus the constraint of feasible bit rates (equation 3.4 without interference due to orthogonal access) because the bit rate constraint is an inequality without equality and cannot be expressed in the projection matrix, that is:

\[
E = - \sum_{i,j,k} \frac{C_{ijkl}}{C_{\text{max}}} \cdot V_{ijkl} + 10 \cdot \sum_{i,j,k} u \left( \frac{1}{S_k \cdot N_k} \frac{1}{R_{ijkl}} - 1 \right) \cdot V_{ijkl}
\] (4.10)

We multiply by 10 the second term because we want any unfeasible bit rate allocated, so we maximize the cost of choosing them. The constraints are the same as in the single cell case (equations 4.7 - 4.9).
The example shows the results for a multi-cell orthogonal access with three possible bit rates per station, all the stations sharing the band (as in figure 3).

The results for the same bit rates as in 4.4.1 for $L = 2$, $K = 5$ and different values of $I$ (number of users), are:

![Figure 11. Total lost packets, orthogonal multi-cells](image)

![Figure 12. Average packet delay, orthogonal multi-cells](image)
The results obtained are as expected. The curves have the same behaviour as in 4.4.1. The total of lost packets starts to increase when the number of users is higher than 10 (equal to the number of possible bands a user can allocate; that is the multiplication between the number of stations and the number of sub-bands a station can have).

4.4.3. Interference case

In this case, the cost function will be the same as in 4.10 taking into account the interference level when evaluating the possible transmission rates:

$$E = - \sum_{ijkl} \frac{C_{ijkl}}{C_{\text{max}}} \cdot V_{ijkl} + 10 \cdot \sum_{ijkl} u \left( 1 + \frac{1}{N_o} \sum_{v \neq i} \sum_{k} \sum_{l} 1_{lk} \cdot V_{vlj,k} \cdot \frac{S_{lk} \cdot R_{ijkl}}{N_o^2 - B} - 1 \right) \cdot V_{ijkl} \quad (4.11)$$

And the constraints will also be the same (equations 4.7, 4.8 and 4.9).

For the case $R=0$ (bit rate 0), the value of the second term (interference term) is 0 because a user that allocates to bit rate 0 has no effect on which station or sub-band and the interference does not interfere neither.

The minimization of the cost function with Smith's simulated annealing algorithm was not good enough and, at the end of the iterations, the solution did not accomplish the Shannon-Hartley Theorem (equation 2.3).
Then, an alternative solution was proven with the same algorithm. The alternative solution is to maximize the transmission rate (equation 4.12):

\[
F = \sum_{ijkl} \log(1 + \text{SNIR}) \cdot V_{ijkl} = \sum_{ijkl} \log \left( 1 + \frac{S_{ijk} \cdot 2^{R_{ijkl}}}{N_0 + \sum_{V \setminus i \setminus k} V_{i,k,l}} \right) \cdot V_{ijkl}
\]

with the restrictions (4.7 - 4.9) and, afterwards, allocate to the maximum bit rate possible with the given interference. If there is no possible bit rate for one user, it allocates to bit rate 0 and the algorithm is redone to optimize the rest of allocations.

In fact, the function to minimize is \( \sum_{ijkl} \log(w \cdot (1 + \text{SNIR})) \cdot V_{ijkl} \), respect to \( V \), where \( w \) is an \( N \)-dimensional vector (the same dimension as \( V \)) with components \( w_i \) that depend on the targeted bit rate per user \( i \) (when higher the targeted bit rate, higher the value of \( w_i \)). For example, using \( w_i = \frac{R_{b,\text{Target},i}}{\max_i(R_{b,\text{Target},i})} \) so that \( w_i \in [0, 1] \).

With this solution, the result is that too many users are allocated to the bit rate 0, because the algorithm does not find the exact minimum of the function but a local minimum. Then, there are, for example, with \( I = 30 \) users, \( L = 2 \) sub-bands, \( J = 3 \) possible bit rates per station, 1 MBS and 4 SCe, all of them sharing the same band; the solution is that only 3 users are allocated to a bit rate different to 0 while more users can be allocated to the different stations.

Therefore, the proposed solution is not correct or not enough to find the best allocations.

### 4.5. Conclusions

The minimization of the energy function is a sensible way to minimize resource allocation for wireless users, but the cost to find a solution is computationally high (it is a NP problem) as a result of the binary values of these functions.

We have evaluated several solutions to optimise the energy function. The Smith’s projected gradient with simulated annealing is a good algorithm for a small dimension \( N \) of the vector (or a small number of neurons), but when increasing the number of restrictions and the dimension, the algorithm is slow to minimize well the energy function.

The solutions for the single cell and the orthonormal multiple cells cases are the expected ones, as mentioned in 4.4.1 and 4.4.2, but for the case with interference (4.4.3) the algorithm does not converge into the expected solutions. That may be because the cost function is too complex (scalar function or logarithm function) and there are not enough iterations to find the minimum optimum value or because the method to apply the equation 4.12 is not chosen correctly.

Future studies should centralize on how to find a quickest solution to minimize the energy function and analyse the solution for case with interference. Also, it is important to choose the correct constraints and the simplest functions possible.
5. **Budget**

The number of hours approximated dedicated to the thesis is 440 hours and if the cost of a junior engineer is of 8€/hour, the total cost is of 3520€.

The number of approximated hours dedicated to the thesis by the supervisor is of 25 hours (average 1h meeting per week) and supposing a cost of 22€/hour, the total cost is of 550€.

The Matlab license for academic use individually is 500€ [12]. Therefore, the total cost of this project is of 4020€.
Bibliography

[1] Chang Wook Ahn and R.S. Ramakrishna “QoS Provisioning Dynamic Connection-
Admission Control for Multimedia Wireless Networks Using a Hopfield Neural

[2] David Gómez-Barquero, Daniel Calabuig, José Monserrat, Nuria García and Jordi
Pérez-Romero "Hopfield Neural Network-based Approach for Joint Dynamic Resource
Allocation in Heterogeneous Wireless Networks", Mobile Communications Group –
Polytechnic University of Valencia, Spain; University Pompeu Fabra, Spain; Radio
Communication Research Group – Polytechnic University of Catalonia, Spain, IEEE,
2006.

[3] Kate Smith, Marimuthu Palaniswami and Mohan Krishnamoorthy, "Neural Techniques
for Combinatorial Optimization with Applications”, IEEE transactions on neural
networks, vol. 9, no. 6, November 1998.

Delay-Centric Dynamic Resource Allocation Algorithm for Wireless Communication
Systems Based on HNN", IEEE Transactions on Vehicular Technology, vol. 57, no. 6,
November 2008.

Oriented Services based on Hopfield Neural Networks Methodology " Universitat
Pompeu Fabra (UPF), Barcelona, Spain, Universitat Politècnica de Catalunya (UPC),
Barcelona, Spain, IEEE Communications Society, publication in the WCNC 2006.

[6] Daniel Calabuig, Jose Monserrat, David Gomez-Barquero and Narcis Cardona,
"Hopfield Neuronal Network Algorithm for Dynamic Resource Allocation in WCDMA
Systems", Polytechnic University of Valencia (UPV)-Mobile Communications Group,
Spain, 2006.

Joint Dynamic Resource Allocation for Coupled Heterogeneous Wireless Networks. A
New Hopfield Neural Network-based Approach", Institute of Telecommunications and
Multimedia Applications (iTEAM), Polytechnic University of Valencia (UPV), Spain.
, September 2007.


of Service Support in Heterogeneous Wireless Networks, Ph. D. thesis, Universidad
Politècnica de Valencia, February 2010, p. 28-40

[10] ICT-318784 STP TROPIC. Distributed computing, storage and radio resource


4 February 2015]