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Simultaneous design and planning of supply chains with reverse flows: A generic modelling framework

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ABSTRACT

The increase in societal awareness towards environmental issues has accrued the responsibility of goods producers, which at present came to encompass the entire product life cycle. Recently, the efficient design and operation of supply chains with return flows have, in particular, become a major challenge for many companies, given the high number of factors involved and their intricate interactions.

In this paper, a multi-period and multi-product network model for the simultaneous design and planning of supply chains with reverse flows is proposed. A graph approach based on the conventional concepts of nodes and arcs is employed to model the network, where it is assumed that any network node is a transformation point of inbound into outbound flows, which in the limit may not differ, and that related arcs describe products flows along the chain. In here the formulation of time adopts a management perspective, i.e., the strategic design of the supply chain is dealt simultaneously with the tactical planning of its operation, which covers supply, production, storage and distribution.

An example based on a Portuguese industry case is studied in order to validate both the applicability and adequacy of the model to real world problems.

1. Introduction

The increasing societal concern on issues related with the environment has been the main driver for the growing number of environmental legislation produced essentially by the European Union (EU) and the United States. This legislation sets targets as to the minimization of the environmental impacts that include a mix of waste prevention, material recycling, energy recovery, and disposal options. The recovery of end-of-life products is beginning to be understood as a business opportunity (Guide and Van Wassenhove, 2006). Together with this recent legislation, managers are now getting more and more concerned about their products over the entire life cycle, and as a result, the number of challenges associated with supply chain management has grown considerably. Not only an efficient forward supply chain is required, but also the design and management of a reverse supply chain should be in place. In many cases, the reverse chain depends on the existing forward chain. This is especially true if products are recovered from original equipment manufacturers (Fleischmann et al., 2001). Several questions must be answered in order to deal with this supply chain extended problem such as: Should reverse flows be included in the existing distribution ones? Or should they be separated? If separated, a good coordination between both supply chains must be achieved in order to avoid cost duplication.

Therefore, the design/redesign of the supply chain with return flows has become a challenge for many companies. This is an important area of research as it helps lowering costs, while improving coordination and customer service (Guide et al., 2003). Together with the tightening of the legal framework, the dynamics within the supply chain are increasing and the product life cycle is getting shorter.

When designing a supply chain, numerous decisions have to be taken, which will hold for different time spans. For instance, the opening of a facility is a long-term decision, while the vehicle routing establishment for delivering products to customer is short-term. Therefore, considering the time horizon of planning, decisions can be classified as strategic, tactical and operational. It is often the case that depending on the nature of the business, decisions that hold from more than 3 to 5 years are said to be strategic. On the other hand, if 1 hour, 1 month or one trimester is the time horizon, these decisions are taken as operational. All decisions with an intermediate duration are classified as tactical (Vidal and Goetschalckx, 1997). Note that these time units are merely an indication and may differ from company to company.

To achieve a global and integrated system, strategic decisions should be taken in close relation with the more tactical issues (Goetschalckx et al., 2002b). Integrated decisions with different

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time units may imply significant savings for a company. However, very few models combine, within a single formulation, both strategic supply chain design (SCD) and tactical issues, such as acquisition, production, inventory and distribution planning (Goetschalckx et al., 2002a).

This work contributes to overcome these limitations on existing generic models, since strategic and tactical decisions are now taken together. Extending previous work developed by the authors (Salema et al., 2009), the present work considers a more generic modelling approach where the definition of the network superstructure allows a formulation where entities and constraints are not directly linked with the real entities (such as factories, warehouses, and so on). By means of a global bill-of-materials, where assembly and disassembly activities are defined, all entities are now modelled as capacitated transformation points. Different products flow between each pair of the network entities in capacitated flows. Since product recovery is often imposed by legislation, recovery targets are explicitly modelled. The proposed formulation is very flexible and allows the modelling of a large number of different supply chain structures.

This paper is structured as follows. In Section 2, an overview of the existing literature related with this work is presented. In Section 3, the supply chain representation is described in detail. Then, due to its significance, the time structure developed is discussed (Section 4). A comprehensive description of the problem is given in Section 5. The characterization of the model formulation follows, where variables, parameters and constraints are explained (Section 6). An example based on the Portuguese glass industry is explored in Section 7, which demonstrates the applicability of the model, followed by a sensitivity analysis on some significant parameters in order to assess their impact on the supply chain structure. This section ends with a preliminary study on the model behaviour. Finally, some conclusions and future research directions are drawn.

2. Related literature

Strategic supply chain design models can be divided into three major groups according to the modelled flows: forward flow – the traditional supply chain that ends at final customers; reverse flow – the supply chain that starts at customers and ends in factory/recovery plants; closed-loop supply chains (CLSC) – supply chain that considers simultaneously forward and reverse flows.

Considering only strategic decisions, numerous works have been published for the traditional supply chain. Major reviews can be found in Vidal and Goetschalckx (1997), Beamon (1998), Klose and Drexl (2005) and Shah (2005). Over the last 10–15 years, several models for the design of the reverse supply chains have been proposed. The high number of published case studies shows how this subject is of extreme importance: recycling of construction sand (Barros et al., 1998), carpet recycling (Louwers et al., 1999; Realff et al., 2004), component/module recovery from refrigerators (Krikke et al., 2003) and recycling of LGP-tanks (le Blanc et al., 2004), among others. In de Brito et al. (2004), a comprehensive list of published cases can be found. However, being case dependent, these models may be difficult to adapt in order to fit other problems.

For the case where both forward and reverse structures are accounted for simultaneously, the number of published works is less than for cases where forward or reverse structures are treated independently. Also published works are essentially case dependent (Krikke et al., 1999; Krikke et al., 2003). Nonetheless, some generic models have been proposed. In Fleischmann et al. (2001), a single product two-echelon forward and reverse model is proposed. This model is applied to two cases in order to investigate the advantages and disadvantages of defining the reverse network, having an established forward network against the simultaneous design of both networks. In Beamon and Fernandes (2004), a model for a single product closed-loop SCD problem is used in order to analyse the impact of several parameters on the network structure. Salema et al. (2006) proposed a coordinated warehouse location model with capacity limits for all facilities and multi-product handling. Lu and Bostel (2007) proposed a closed-loop supply chain model for a remanufacturing network. The model considered producers, remanufacturing sites, intermediate centres and customers. The intermediate centres belong exclusively to the reverse network and send their products only to remanufacturing facilities. A Lagrangean heuristic approach was developed and numerical experiments were performed on examples adapted from classical test problems.

The above-mentioned works developed only strategic models. No tactical features were contemplated. On integrating both strategic and tactical characteristics, Fandel and Stammen (2004) proposed a model for the design of forward and reverse networks together with the modelling of some multi-period aspects. Although promising, the model was not tested in any case or example and, therefore, it was not proven solvable. Salema et al. (2009) proposed a multi-product, multi-period model for the design and planning of supply chain with reverse flows. Together with the strategic location of facilities, the planning of production, storage and distribution is performed for a time horizon divided in two-interconnected time scales. However, the proposed formulation depends on the network structure, and although the formulation is general, this dependency creates some complexity on its application to different problems.

The problem addressed in this research follows previous work developed by the authors. More specifically, the primary objective is to propose a generalized model that not only designs the supply chain considering simultaneously the forward and reverse flows, but also provides a tactical plan for acquisition, production, storage and distribution in a predefined time horizon. Using a graph approach, each facility/customer is assumed as a supply chain node with an operational task to perform. The multi-product arcs allow the connection between nodes. Two other new features are considered to this kind of models: travel times are modelled on arcs, and usage/processing times on nodes. Not depending on any case study, the described formulation is generic. Thus, the model can be easily applied to several supply chains structures, ranging from a forward supply chain to a closed-loop one.

3. Closed-loop supply chain representation

A graph representation is used to characterize the CLSC structure that goes from factory to customers and back to factories (the same or a different one). Nodes represent any supply chain entity (such as factories, warehouses, customers, distribution centres and sorting centres, amongst others), while arcs between two nodes define an existing flow. The supply chain is divided into echelons according to the nodes nature. The first echelon is formed by factories and warehouses, the next one by warehouses and customers, and so on. In the assumed representation, a flow can only connect two nodes of the same echelon. For instance, a flow cannot link a factory to a customer. If that is so, then a fictitious node should be created, which in this case would be a warehouse.

Let the supply chain be represented by a graph $G = (V,A)$ with a set of nodes (V) and a set of arcs (A). Consider that the supply chain is composed of n levels, V can then be defined by $V = \bigcup_{i=1}^{n} V_i,\ V_i \cap V_j = \emptyset$ where V_i is a subset of all nodes that belong to level i. Let $A_i = V_i \otimes V_{i+1}$ be the set of arcs that link nodes from V_i to nodes of V_{i+1}. Note that a node from V_i needs not to be connected to all nodes of V_{i+1}, thus operator $\otimes$ represents only the potential connections. Consequently, $A = \bigcup_{i=1}^{n-1} A_i,\ A_i \cap A_j = \emptyset$. As mentioned above, a multi-product supply chain design problem is being studied. Consider M the set of products and let $\overline{A}_i = M_i \otimes A_i$ be an extension of arcs subsets, where the pair product-flow is defined. As for A, M may also be defined as $M = \bigcup_{i=1}^{n} M_i,\ M_i \cap M_j = \emptyset$. In what concerns nodes, two types of products are related to each subset V_i (inbound and outbound products) depending whether the node is on the origin or at the end of an arc. Therefore, two extended subsets of vertices are defined: one for incoming products ($\widehat{V}_i$) and another for outgoing products ($\widetilde{V}_i$) : $\widehat{V}_i = M_i \otimes V_i$ and $\widetilde{V}_i = M_{i+1} \otimes V_i$.

The super-structure of supply chain is then a direct graph $\overline{G} = (V, \overline{A})$ where V is the set of all entities, and $\overline{A} = \bigcup_{i=1}^{n} \overline{A}_i$ is the set of extended vertices, i.e., the set formed by all pairs product-flow.

Nodes represent any kind of entity in a supply chain (a factory, a warehouse, a customer, a distribution centre, a collection centre, a sorting centre, etc.), and are characterized by the following set of properties:

- Input flows: arcs that 'bring' products from the previous network level.
- Output flows: arcs that 'send' products to the next network level.
- Operation: being a multi-product supply chain, any node transforms input into output products. This can be a processing operation, a storage operation or simply a cross-docking operation. The operation varies depending on the function that the entity has on the supply chain (factory, warehouse, customer, etc.). Any operation has also some specific characteristics:
  - Time: any operation has a time of execution, which can take the value of zero if defined as instantaneous.
  - Capacity: the operation may be performed within maximum and minimum pre-established values.

Arcs represent any kind of flows between two entities. They are characterized by:

- Origin: source entity.
- Destination: target entity.
- Material: each flow has one material associated with it. If there are two materials flowing between the same pair of origin and destination, a different flow is defined.
- Time: number of time units needed to go from the origin to the destination.
- Capacity: maximum and minimum limits that can be set for each flow. These are mostly related with transportation modes.

3.1. Example

Consider a supply chain with three levels (Fig. 1). Graph $G = (V,A)$ is defined by $V = \bigcup_{i=1}^{3} V_i$, where $V_1 = \{1,2,3\}$, $V_2 = \{4,5\}$ and $V_3 = \{6,7,8,9\}$, and $A = \bigcup_{i=1}^{2} A_i$, where $A_1 = \{(1,4),(2,4),(2,5),(3,4),(3,5)\}$, $A_2 = \{(4,6),(4,7),(4,8),(4,9),(5,6),(5,7),(5,8),(5,9)\}$.

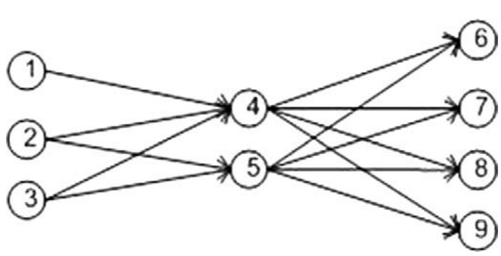

Fig. 1. Graph G.

Now, suppose that entities of V_2 transform material P_1 into materials P_2 and P_3. Then, $M = \bigcup_{i=1}^{2} M_i$, where $M_1 = \{P_1\}$ and $M_2 = \{P_2, P_3\}$. Two different types of extended entities are defined $\widehat{V}_2 = \{(P_1,4),(P_1,5)\}$ and $\widetilde{V}_2 = \{(P_2,4),(P_2,5),(P_3,4),(P_3,5)\}$, which relate the elements of V_2 with inbound and outbound products, respectively.

The extended arcs are defined by the following sets:

$$\overline{A}_1 = \{(P_1,1,4),(P_1,2,4),(P_1,2,5),(P_1,3,4),(P_1,3,5)\},$$

$$\begin{aligned}\overline{A}_2 = \{&(P_2,4,6),(P_2,4,7),(P_2,4,8),(P_2,4,9),(P_2,5,6),\\ &(P_2,5,7),(P_2,5,8),(P_2,5,9),(P_3,4,6),(P_3,4,7),(P_3,4,8),\\ &(P_3,4,9),(P_3,5,6),(P_3,5,7),(P_3,5,8),(P_3,5,9)\}.\end{aligned}$$

The super-structure of the supply chain represented in Fig. 1 is therefore defined by graph $\overline{G} = (V, \overline{A})$, where $\overline{A} = \overline{A}_1 \cup \overline{A}_2$.

4. Time modelling

The model developed considers two main levels of decisions: the design level and the planning level. At the design stage, the closed-loop supply chain structure is defined where the choice of the main entities (e.g. factories) is made. This decision is taken for a given period of time (time horizon), under the assumption that the chain structure remains unchanged. The planning level involves a more detailed time description. Within the time horizon, two-interconnected time scales are considered. A macroscale that is taken as the time horizon discretization, where the demand and return values are to be satisfied, and a microscale that allows for a more detailed planning on how this satisfaction is to be attained. For instance, over a time horizon of 5 years, a macroscale of 1 year is defined and each year is then discretized into months (microscale). The time scales can be year/month, year/trimester or month/day or any other combination that suits the problem in study.

The interconnection between these two time scales is depicted in Fig. 2. Consider H as the time horizon, $t \in T$ an element of the macrotime set and $t' \in T'$ an element of the microtime set. For each $t \in T$, n elements in T' exist. Both sets have to be ordered.

In order to establish the relation between the two time scales, an operator is defined where a current time unit is related to one that has already taken place (backward operator). It allows a connection between two different time units, within the same constraint. This is required when there is an event on the chain that starts in a certain macroperiod and continues over the next macroperiod.

Consider $t \in T$ and $t' \in T'$, let (t,t') be the current time instance. Suppose that one wants to relate the current time unit with one that has occurred τ microtime units before. The backward time operator Υ is defined as:

$$\Upsilon(t, t'-\tau) = \begin{cases} (t, t'-\tau), & \text{if } t'-\tau \geqslant 0,\\ (t-\omega, \omega n + t' - \tau), & \text{if } t'-\tau < 0 \wedge t \geqslant \omega, \end{cases}$$

where $\omega \in \mathbb{Z}$ is the smallest integer greater or equal than $\frac{\tau - t'}{n}$, $\omega = \lceil \frac{\tau - t'}{n} \rceil$ and $n = |T'|$.

For instance, let $T = \{1,2,\ldots,12\}$ and $T' = \{0,1,\ldots,21\}$, and suppose that the current time unit is $(t,t') = (6,10)$. Consider $\tau = 5$ then $\Upsilon(6, 10-5) = (6,5)$ as $t' - \tau \geqslant 0$. Suppose now that $\tau = 30$, then

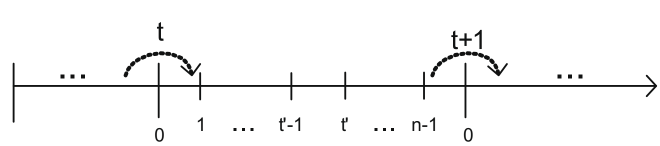

Fig. 2. Interconnection between macro and microtime scales.

$\omega = \lceil \frac{30-10}{22} \rceil = 1$. Thus, $\Upsilon(6, 10-30) = \Upsilon(6-1, 1 \times 22 + 10 - 30) = (5, 2)$.

5. Problem description

As mentioned above, the extended supply chain should not only comprehend a manufacturing system where raw materials are transformed into final products and delivered to end customers, but it should also integrate the recovery system where end-of-life products are returned to recovery facilities (Fig. 3). The supply chain consists of three integrated processes: Production and Inventory, Logistics and Distribution, and Reprocessing.

In each of these processes, there can be one or more entities involved: manufacturers, warehouses, distribution centres, customers, collection centres and recovery facilities, amongst others. In each facility, there may be several materials flowing that may undergo a transformation when passing it through.

The network in study is formed by four echelons: plants, warehouses, end customers and disassembly centres. Between each echelon, one or more products are transported. The problem can be generally described as follows:

Given

- a possible superstructure for the location of the supply chain entities,
- the investment costs,
- products' bills of materials,
- the relation between forward and reverse products,
- travel time between each pair of interacting network agents,
- the minimum disposal fraction,
- the minimum usage time for each return product,
- forward product return fractions,
- the maximum and minimum flow capacities,
- the maximum and minimum acquisition and production capacities,
- the maximum storage capacities,
- the initial stock levels,

and for each macroperiod and product

- customer's demand volume,
- the unit penalty costs for non-satisfied demand and return,

and for each microperiod and product

- the unit transportation cost between each pair of interacting network agents,
- the factory acquisition and production unit costs,
- each facility unit storage cost,
- the unit disposal cost.

Determine

- the network structure,
- the production and storage levels,
- the flow amounts, and
- the non-satisfied demand and return volumes.

So as to minimize the global supply chain cost.

When designing a supply chain, the company may want to evaluate if the set of customers is economical viable or not. Traditionally, this option is not available in supply chain design models since they consider end customers as existing sites whose demands have to be satisfied. In the present model, it is assumed that there is a set of end customers with demands and returns to be satisfied. However, they may or not be all chosen to integrate the supply chain and even if they are demand and return may not be entirely satisfied. Penalty costs are incurred either if a customer is left out of the network or if there is some demand or return that is not satisfied. This latter cost can be viewed as a measurement of the "quality of service" that the company wants to guarantee to its customers, since a higher cost will force a larger number of customers to be fully served.

In terms of distribution, customers' demands are satisfied by plants through warehouses, which may perform a postponement operation. A transportation cost is also incurred. In addition, when products are stocked in warehouses, a holding cost is considered.

Returns are collected from customers by disassembly centres. After disassembly and inspection operations, some components/materials appropriate to be reused are sent to plants where they are reprocessed and integrated in the chain. Others that are not in good conditions are sent to other entities in order to be properly processed or just sent to disposal (outside the boundaries of the supply chain considered in the model). Again, holding costs are incurred when components are stored in disassembly centres.

Some assumptions are taken in the model:

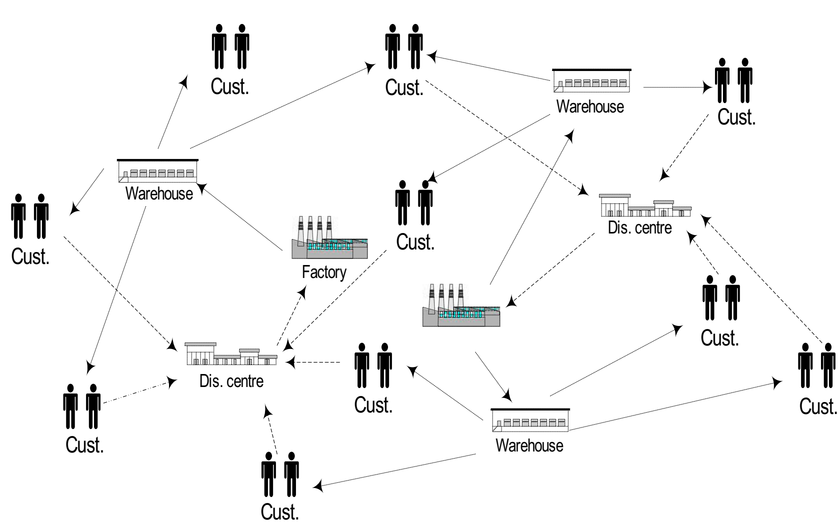

Fig. 3. Generic network representation.

- *New and remanufactured products are not differentiated.* Due to the strategic nature of the model, it is assumed that used products sent by disassembly centres are integrated as products along with new ones.
- *Supplier contract limits.* The supply of new materials/products may lie between a maximum and a minimum level imposed by contracts.
- *Storage capacities have a maximum limit.* All facilities where storage is allowed (plants, warehouses, and take-back centres) have a maximum limit for storage.
- *Returns volumes are a fraction of the forward products supplied.* Returns are demand dependent since it can only collected what have been supplied. The model defines the amount of products that are collected by take-back centres. The total collection of returns at the end customers is not mandatory. However, if any such return is not collected, a penalization cost is incurred in the objective function.
- *Limits are imposed on flows.* Since transportation modes have limited capacities, a maximum limit is imposed on all network flows. This limit may differ depending on the kind of transportation used. Minimum flow limits are also imposed. However, they are modelled differently depending on the sites involved. For example, an inbound or outbound of a plant may not occur in all time units, but if it occurs, a minimum limit must be satisfied. If it is an inbound or outbound flow at a customer, it is assumed that a minimum value has to occur in every time unit.
- *Disposal costs are considered for the case of non-recovered products.*

Other features are also assumed:

- *Travel time* is defined as the number of microtime units needed for a product to flow from its origin to its destination, and it is modelled between network levels.
- *Product transformation time* and *product time usage at a costumer* are defined as the minimum number of microtime units that a product is being processed/used in any facility/costumer.

6. Model formulation

The proposed formulation models the problem described above. The supply chain involves a four-echelon structure, which means that, for instance, customers cannot be directly supplied by factories, nor can they send returns directly to be reprocessed without going through disassembly centres.

6.1. Sets

Sets are one of the major features of this modelling approach. They allow the definition of the network superstructure. Under this structure, a set of straightforward equations defines all constraints that must be satisfied. Consider the following indices: i, j as entity and $m, \bar{m}$ as product.

6.1.1. Entities

Each level of the supply chain is defined by just one kind of entity (e.g., factory, warehouse, customer, etc.), thus node set V is divided into subsets, each referring to a different level. Moreover, no entity can belong to two different subsets:

I_f possible locations for factories, $i \in I_f \subseteq V$
I_a possible locations for warehouses, $i \in I_a \subseteq V$
I_c locations of customers, $i \in I_c \subseteq V$
I_r possible locations for disassembly centres, $i \in I_r \subseteq V$
I_0 disposal option, $i \in I_0 \subseteq V$

Set $I = I_f \cup I_a \cup I_r$ contains all entities for which a fixed cost is incurred if the choice is to open/use a facility in a particular location. Although customers may or not be selected as part of the network, there is no fixed cost associated with such a choice.

6.1.1.1. Extended entities. Entities and products are related. Extended entities are defined by the pair product–entity. Since two entities in consecutive levels may relate to the same product, two different sets must be defined. Thus, consider the following subsets of M, each one referring to a different product:

M_f factories outbound products, $m \in M_f \subseteq M$
M_a warehouses outbound products, $m \in M_a \subseteq M$
M_c customers outbound products, $m \in M_c \subseteq M$
M_r disassembly centres outbound products, $m \in M_r \subseteq M$

The extended entities are then defined by the following sets:

$$\breve{V}_f = \{(m,i) : m \in M_f \wedge i \in I_f\}, \quad \breve{V}_a = \{(m,i) : m \in M_a \wedge i \in I_a\},$$
$$\breve{V}_c = \{(m,i) : m \in M_c \wedge i \in I_c\} \quad \text{and} \quad \breve{V}_r = \{(m,i) : m \in M_r \wedge i \in I_r\}.$$

Due to the transformation activity, there are two products associated with each node. There will be constraints where some entities need a second product–entity set, in order to establish the relation between these entities and their inbound products. So, consider $\widehat{V}_f = \{(m,i) : m \in M_r \wedge i \in I_f\}$ and $\widehat{V}_c = \{(m,i) : m \in M_a \wedge i \in I_c\}$ as the inbound product sets for factories and customers, respectively.

Consider also set $\overline{V} = \widehat{V}_f \cup \breve{V}_a \cup \breve{V}_r \cup \breve{V}_c$, which will be needed when defining the objective function.

6.1.2. Flows

Flows are the edges in the supply chain graph. They establish the connection between the supply chain levels. They are defined by a pair entity–entity. The modelled network has four echelons that are defined as follows:

$$A_{f_1} = \{(i,j) : i \in I_f \wedge j \in I_a\}, \quad A_{f_2} = \{(i,j) : i \in I_a \wedge j \in I_c\},$$
$$A_{r_1} = \{(i,j) : i \in I_c \wedge j \in I_r\} \quad \text{and} \quad A_{r_2} = \{(i,j) : i \in I_r \wedge j \in I_f\}.$$

Let A be the set of all network flows, $A = \bigcup_{k \in K} A_k$, $K = \{f_1, f_2, r_1, r_2\}$.

Suppliers and a disposal option are two other features integrated in this model. For each one of them, a fictitious entity is created allowing its formulation as an additional flow. Let $A_s = \{(i,i) : i \in I_f\}$ and $A_d = \{(i,j) : i \in I_r \wedge j \in I_0\}$ be the sets for suppliers and disposal, respectively. The first is modelled as a factory internal flow and the latter is defined by a flow from any disassembly centre to a fictitious factory.

6.1.2.1. Extended flows. As for entities, flows are also extended when products are considered. As before, each flow relates to a different product. The following sets define each network echelon:

$$F_{f_1} = \{(m,i,j) : m \in M_f \wedge (i,j) \in A_{f_1}\},$$
$$F_{f_2} = \{(m,i,j) : m \in M_a \wedge (i,j) \in A_{f_2}\},$$
$$F_{r_1} = \{(m,i,j) : m \in M_r \wedge (i,j) \in A_{r_1}\} \quad \text{and}$$
$$F_{r_2} = \{(m,i,j) : m \in M_c \wedge (i,j) \in A_{r_2}\}.$$

Supplying and disposal lead to the definition of two other sets:

$$F_s = \{(m,i,i) : m \in M_c \wedge (i,i) \in A_s\} \quad \text{and} \quad F_d = \{(m,i,j) : m \in M_c \wedge (i,j) \in A_d\}.$$

6.1.2.2. Network super-structure. Finally, the super-structure of the network is defined by set F: $F = \bigcup_{k \in K} F_k$, $K = \{f_1, f_2, r_1, r_2, s, d\}$

6.1.3. Time

Consider t, t' as indices for macro and microtime, respectively. Let sets T and T' be defined as $T = \{t_1, t_2, \ldots, t_h\}$ and $T' = \{t'_1, t'_2, \ldots, t'_n\}$, respectively. Let $\overline{T} = \{(t, t') : t \in T \wedge t' \in T'\}$ be the set of all time units.

6.1.4. Parameters

τ_{ij} travel time between entities i and j,
ϕ_m processing/usage time of product m,
$\xi_{mi} = f(\tau_{ij}, \phi_m)$ function of both travel and processing times, giving the earliest microtime unit a flow of product $m \in M$, with origin in entity $i \in I$, may occur.

Time independent parameters

α_m recovery target for product m set by legislation, $\alpha_m \in [0,1]$,
$\beta_{m\bar{m}}$ relation between product m and $\bar{m}$,
ρ_m return fraction of product $m \in M_a$,
s_{mi0} initial stock of product m in entity $i \in I$,
f_i investment cost of entity $i \in I$,
c_i cost of leaving customer i out of the supply chain,
g_i^s maximum storage capacity of entity $i \in I$,
g_i^p and h_i^p maximum and minimum supplying limit of entity $i \in I_f$,
g_i upper bound value for flows leaving entity i,
h_i^f lower bound value for flows leaving entity $i \in I_f$,
h_{mi}^c lower bound value of product m flow leaving entity $i \in I_c$.

Macrotime parameters

d_{mit} product m demand for entity i for macroperiod t, $i \in I_c$,
c_{mit}^u unit variable cost of non-satisfied demand/return of product m to entity $i \in I_c$, for macroperiod t.

Microtime parameters

$c_{mijt'}$ unit transportation cost of product m from entity i to entity j, at time t',
$c_{mit'}^s$ unit storage cost at entity i, at time t'.

6.1.5. Variables

Continuous variables

$X_{mijt'}$ amount of product m served by entity i to entity j, at time t',
$S_{mit'}$ amount of product m stocked in entity i, over period t',
U_{mit} non-satisfied amount of product m of entity $i \in I_c$, over macroperiod t.

Binary variables

$Y_i = 1$ if entity i is opened/served; 0 otherwise
$E_{ijt'}$ auxiliary variable that allows the modelling of minimum limits imposed on model flows; $E_{ijt'} = 1$ if the flows between entity i and entity j occurs at time t'.

6.1.6. Model formulation

The model formulation is derived taking into account the problems characteristics and using the above sets, parameters and variables:

$$\text{Min} \quad F = \sum_{i \in I} f_i Y_i + \sum_{i \in I_c} c_i (1 - Y_i) + \sum_{mij:(m,i,j) \in F} \sum_{t' \in T_m} c_{mijt'} X_{mijt'} + \sum_{mi:(m,i) \in N_c} \sum_{t \in T} c_{mit}^u U_{mit} + \sum_{mi:(m,i) \in N \setminus N_c} \sum_{t' \in T'} c_{mit'}^s S_{mit'} \tag{1}$$

$$\text{s.t.} \quad S_{mi\Upsilon(t,t'-1)} + \sum_{\bar{m}j:(\bar{m},j,i) \in F} \beta_{m\bar{m}} X_{\bar{m}ji\Upsilon(t,t'-\tau_{ji}-\phi_m)} = \sum_{\bar{m}j:(\bar{m},i,j) \in F \setminus F_s} \beta_{m\bar{m}} X_{\bar{m}ijt'} + S_{mit'}, \quad (m,i) \in \overline{V} \wedge (t,t') \in \overline{T}, \tag{2}$$

$$\sum_{j:(m,j,i) \in F} \sum_{t' \in T'} X_{mji\Upsilon(t,t'-\tau_{ji})} + U_{mit} = d_{mit} Y_i, \quad (m,i) \in \breve{V}_c \wedge t \in T, \tag{3}$$

$$\sum_{j:(m,i,j) \in F} \sum_{t' \in T'} X_{mijt'} + U_{mit} = \sum_{\bar{m}j:(\bar{m},j,i) \in F} \sum_{t' \in T'} \beta_{m\bar{m}} X_{\bar{m}ji\Upsilon(t,t'-\tau_{ji}-\phi_m)}, \quad (m,i) \in \widehat{V}_c \wedge t \in T, \tag{4}$$

$$\sum_{j \in I_0} \sum_{t' \in T'} X_{mijt'} \leqslant (1 - \alpha_{\bar{m}}) \sum_{\bar{m}j:(\bar{m},j,i) \in F} \sum_{t' \in T'} \beta_{m\bar{m}} X_{\bar{m}ji\Upsilon(t,t'-\tau_{ji})}, \quad (m,i) \in \widehat{V}_r \wedge t \in T, \tag{5}$$

$$\sum_{m:(m,i,j) \in F_s} X_{mijt'} \leqslant g_j^p Y_j, \quad (i,j) \in A_s \wedge (t,t') \in \overline{T}, \tag{6}$$

$$\sum_{m:(m,i,j) \in F_s} X_{mijt'} \geqslant h_j^p Y_j, \quad (i,j) \in A_s \wedge (t,t') \in \overline{T}, \tag{7}$$

$$\sum_{m:(m,i) \in N \setminus N_c} S_{mit'} \leqslant g_i^s Y_i, \quad i \in I \wedge (t,t') \in \overline{T}, \tag{8}$$

$$\sum_{m:(m,i,j) \in F} X_{mijt'} \leqslant g_{ij} E_{ijt'}, \quad (i,j) \in A \wedge (t,t') \in \overline{T}, \tag{9}$$

$$\sum_{m:(m,i,j) \in F} X_{mijt'} \geqslant h_{ij} E_{ijt'}, \quad (i,j) \in A_{f_1} \cup A_{r_2} \wedge (t,t') \in \overline{T}, \tag{10}$$

$$2E_{ijt'} \leqslant Y_i + Y_j, \quad (i,j) \in A \wedge (t,t') \in \overline{T}, \tag{11}$$

$$\sum_{j:(m,i,j) \in F} X_{mijt'} \geqslant h_{mi}^c Y_i, \quad (m,i) \in \widehat{V}_c \cup \breve{V}_c \wedge ((t,t') \in \overline{T} : t' \geqslant \xi_{mi} \vee t \geqslant t_2), \tag{12}$$

$$X_{mijt'},\ S_{mit'},\ U_{mit} \in \Re_0^+ \quad Y_i,\ E_{ijt'} \in \{0,1\}. \tag{13}$$

The model objective is to minimize the total supply chain cost, (1). This involves a fixed cost for opening/use of facilities (first term); a penalization cost for leaving a customer out of the supply chain (second term); a flow variable cost proportional to the amount transported (third term), which also includes a supplying cost when $(m,i,j) \in F_s$ and a disposal cost for $(m,i,j) \in F_d$; a penalization to any non-satisfied demand or return (fourth term), and finally, a penalization for any stock left in any entity except at customers (fifth term). Other objective functions can be considered such as profit maximization, sales maximization, and return on investment maximization, among others.

Eq. (2) is the material balance constraint where it is assured that in any microperiod, in any entity and for each product, the inbound flow must equal the outbound flow plus the difference between the existing and the new retained stocks. The quantity $S_{mi\Upsilon(1,0)}$ corresponds to the initial stock level of product m in entity i (s_{mi0}).

Eq. (3) is the equation for the demand. A customer selected to be part of the network has a demand value set for each macrotime unit. This demand can be entirely or partially satisfied in one or more micro time units. If it is only partially satisfied then the missing part is retained by variable U that is penalized in the objective function.

Eq. (4) assures customer returns. The total volume of returns available at each customer depends on the supplied amount. Again, as in (3), the return can be entirely or partially collected during the macrotime unit.

European legislation sets targets for the recovering of materials. Therefore and having this in mind, constraint (5) assures that disassembly centres can only send to disposal less than a pre-assumed fraction of their collected products. This fraction can vary with the products.

Constraints (6) and (7) model contractual clauses since most supplying contracts have defined maximum and minimum limits for supplied volumes. These limits are modelled differently for each opened/used factory allowing for different management policies.

Constraint (8) sets limits on total storage capacities that are defined to all entities except for customers, where the concept of stock is meaningless.

Constraints (9)–(12) are the flow constraints. Constraint (9) establishes a maximum limit on each flow that may occur between entities *i* and *j*. It is assumed that the limit is set for total amount of products transported between these entities. In terms of minimum flows, two different approaches are implemented: the first one assumes that a flow when occurring must have at least some pre-established value (constraint (10)); the second is set only for customers and assures that a minimum flow must reach and leave ((12)) each customer in every microperiod. The demand and return satisfactions are performed during the macroperiod, in several microtime units. In this way, one gives to the company a more detailed supply plan (than the one given by strategic supply models) that will help management to take an adequate decision.

7. Case study

A Portuguese glass company wants to design and plan its forward and reverse networks simultaneously. The supply chain comprises four entities: factories, warehouses, customers, and sorting centres. Supply and disposal are not viewed as entities once they have no geographical location. After use, the glass returns to the factories.

In terms of time, a 5-year horizon is set as the strategic time. The macrotime unit is considered as 1 year and the microtime as 2 months.

Due to the strategic nature of the study, customers are grouped into clusters and assigned to the district capital. Eighteen customers are considered: Aveiro, Beja, Braga, Bragança, Castelo Branco, Coimbra, Évora, Faro, Guarda, Leiria, Lisboa, Portalegre, Porto, Santarém, Setúbal, Viana do Castelo, Vila Real, and Viseu.

This company has already a factory operating in Leiria where supplying contracts of raw materials vary between 1000 and 2000 thousand units.

The management board has decided to open new facilities. Four cities were chosen as candidates to locate new factories: Évora, Lisboa, Porto, and Setubal. If opened, these should produce at least 250 thousand units every 2 months. In terms of warehouses and sorting centres, the board has pointed out eight locations: Braga, Coimbra, Leiria, Lisboa, Porto, Santarém, Setubal, and Viseu.

Customers have different product needs and, thus, some products are selectively supplied. Demand volumes, for the six products leaving the warehouses, differ among customers and are assumed proportional to the number of inhabitants of each location. Among all customers, the first year demand of each product varies according to the values given in Table 1. For the remaining 4 years, an estimated factor, ranging from 0.98 to 1.05, is applied in order to calculate the demand volumes.

Table 1
Maximum and minimum values for the first year demand (in thousand units).

	A_1	A_2	A_3	A_4	A_5	A_6
Maximum	802	314	748	508	264	207
Minimum	25	18	11	12	49	20

Table 2
Products return fraction.

	A_1	A_2	A_3	A_4	A_5	A_6
Return fraction	0.45	0.7	0.5	0.8	0.4	0.9

In terms of return, each product has a different return fraction that is given in Table 2. These values remain constant over the time horizon.

Concerning products, factories produce three different types of glassware, which are differentiated mainly by their colour: green (F_1), white (F_2) and brown (F_3), see Fig. 4. These products are sent to associated warehouses where they are subjected to a postponement operation (i.e., packaging) resulting in six different products (A_1–A_6).

The final products are supplied to customers according to their demand. After use, all products are considered simply as glass (R_1) that is collected by sorting centres. A sorting activity is performed in the sorting centres, separating glass into white and non-white glass (C_2 and C_1, respectively). Factories or proper disposal are the final destination of these two types of glass. M_1 and M_2 are raw materials used to product brand-new glass.

7.1. Results

The optimal supply chain is shown in Fig. 5. Together with the factory in Leiria, two new factories (Lisboa and Porto) will supply three warehouses located in the same cities. For the reverse network, the same three locations are chosen to open sorting centres. All 18 customers integrate the optimal supply chain. This figure shows the major structure of the supply chain. However, flows do not have to occur in all microperiods. Both warehouses and sorting centres are connected with the factory located in the same city.

In order to show some of the planning information given by the model, Santarém customer will be closely observed. This customer is supplied by two warehouses (Lisboa and Leiria), and sends its returns to Lisboa sorting centre. Firstly, the flow between Leiria factory and Leiria warehouse will be shown, and then the supplying plan of Santarém customer will be presented, followed by its return plan. Lastly, the connection between Leiria centre and the Leiria factory will be illustrated. In terms of inventory, a zero stock policy is proposed.

With an assumed maximum limit of 2000 thousand units, the flow between Leiria factory and warehouse is shown in Fig. 6. Although no minimum limit was imposed, this flow occurs in every microtime unit.

Santarém customer has its demand fulfil by the two warehouses, Leiria and Lisboa (Fig. 7). It was assumed that at least 1% of customers' total demand should be supplied in each microperiod. Therefore, even if it is only a small amount, in each microperiod

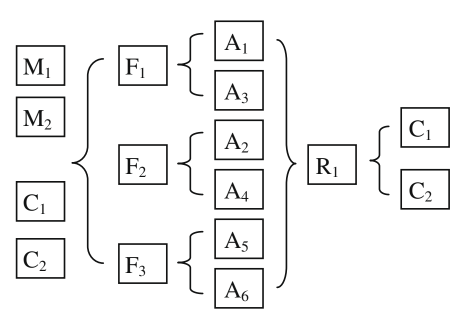

Fig. 4. Product relations.

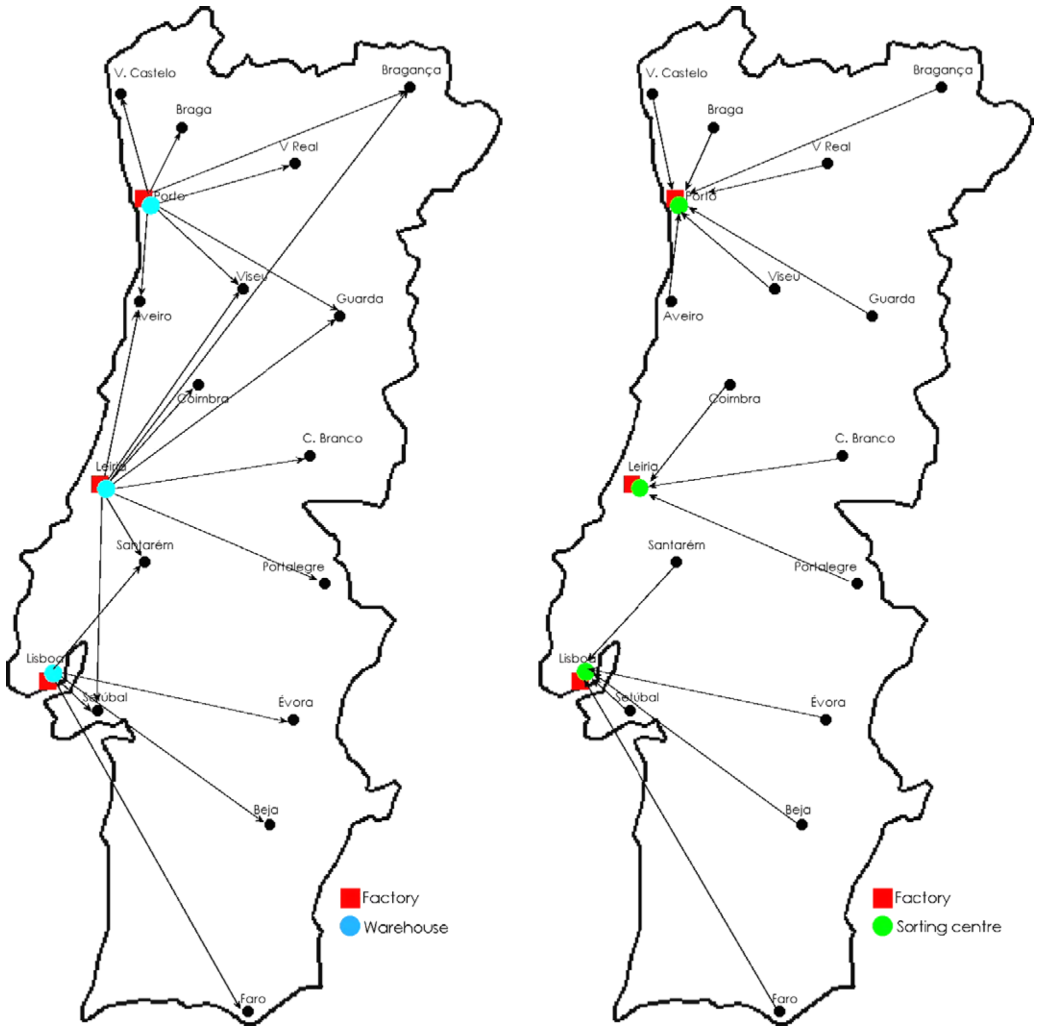

Fig. 5. Forward and reverse networks.

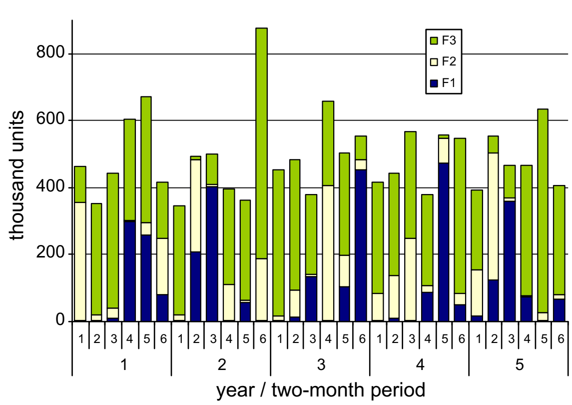

Fig. 6. Flow between Leiria factory and Leiria warehouse.

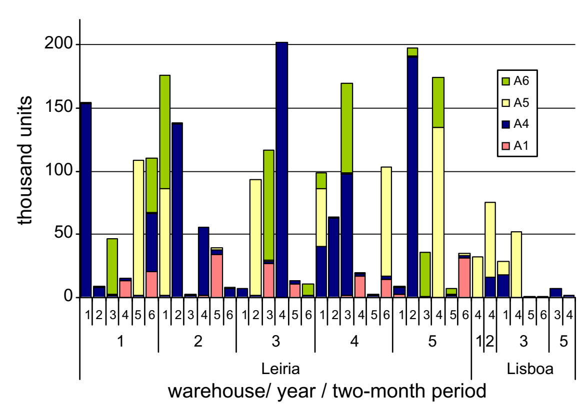

Fig. 7. Santarém customer supplying plan.

there is at least one of the four products supplied to the customer. Note that Lisboa warehouse fulfils only part of the demand for two (A_4 and A_5) out of four products.

The collecting plan for Santarém customer is depicted in Fig. 8. In every microperiod, this customer has at least six thousand units collected, which is the minimum assumed limit imposed on outbound flows for the customer.

With the maximum limit set on 2000 thousand units, the outbound flows from Leiria sorting centre is shown in Fig. 9. This flow occurs in all microperiods either to Leiria factory for recycling, or to disposal.

To finish this planning example, two more cases are shown: raw material supplying plan for Leiria factory and customers that are not fully supplied.

For the entire time horizon, the factory located in Leiria is supplied with the minimum contractual volume, which could vary from 1000 to 2000 thousand units (Fig. 10). All remaining needs are satisfied by the disassembly centre.

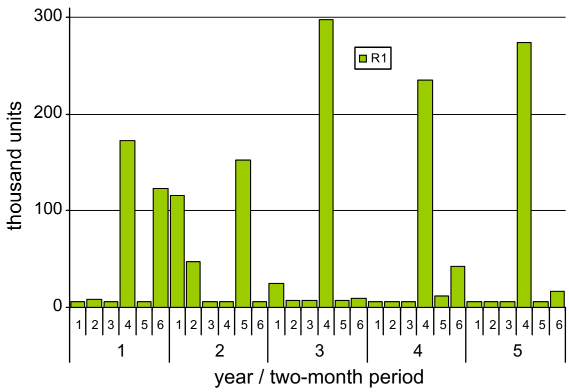

Fig. 8. Santarém collection plan.

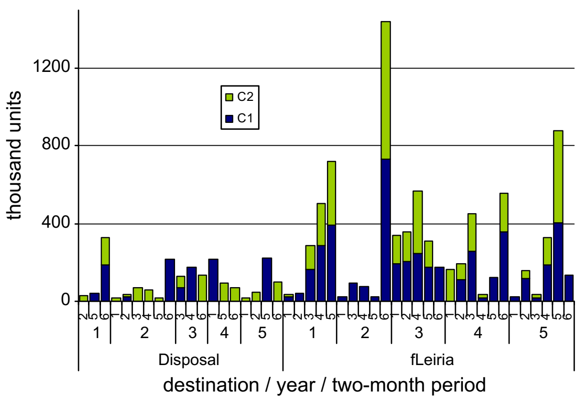

Fig. 9. Outbound flows from Leiria centre.

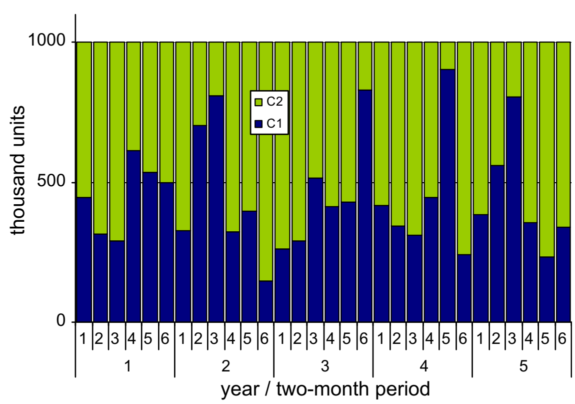

Fig. 10. Raw material supplying plan for Leiria factory.

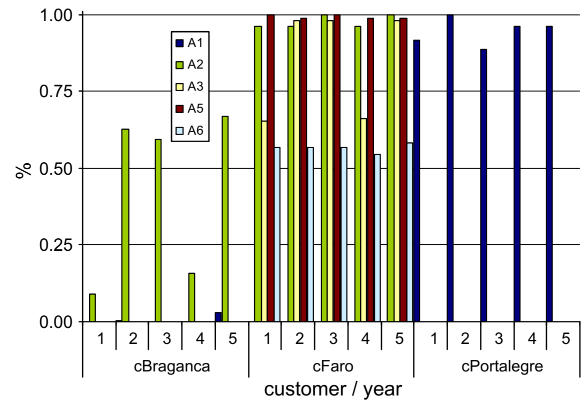

Fig. 11. Fraction of customer non-satisfied demand.

Finally, the customers that are not fully supplied are shown in Fig. 11. Three customers have some demand not satisfied. On average, Bragança customer has 2% and 43% of products A_1 and A_2, respectively, not supplied. Portalegre has just one product that is not fully supplied and on average only 5% of the demand is satisfied. The worst case is found for Faro customer. This customer is located in southern Portugal, which places it far apart from all warehouses. The demand for all four products is not fully supplied.

Although it may be questionable that a company would find reasonable to have three of its customers with non-satisfied demand, these results may lead management to raise several questions: are penalty costs correctly estimated? If affirmative, might these customers be strategic to the supply chain? What is the cost of fully satisfying them? Should a contract option be considered to the storage and supplying these three customers? The proposed model can provide some answers to all these questions.

The resulting MILP model was solved by GAMS/CPLEX (built 22.2), in a Pentium 4, 3.4 GHz. The model is characterized by 51,495 variables (11,078 binary) and 32,666 constraints, and took about 2540 CPU seconds to solve to optimality. The optimal value found is for 4216×10^3 m.u.

7.2. Supply chain robustness

In order to analyse the impact of changes in some parameters on the network structure, a sensitivity analysis was performed. The primary objective was to assess the effect that some changes on parameter values have on facilities number and location, and on major planning decisions. Six different analyses were performed, which are characterized in Table 3. The test ranges are related to the initial values used in the previous section.

7.2.1. Return fraction

This analysis is related with parameter ρ_m. The model is tested for this parameter over the range 10–100% of its initial value, by increments of 10% (Fig. 12).

In run 1 where the return fraction is 10% of its initial value, no customer integrates the network (Fig. 12), therefore the SC is not designed. However, there are eight warehouses opened. Why open warehouses if there is no customer to be supplied? The reason is that Leiria factory is operating already. This facility has a minimum production level to be met and a maximum storage capacity. So, as there is no customer to be supplied and in order to meet the minimum level of production, storage as to be shared with warehouses. The main conclusion is that if the return fraction is only 10% of the expected values, then there is no business opportunity.

In run 2 (parameter is 20% of its initial value), the network is composed of the 18 customers, served by three factories, three warehouses, and two sorting centres. The number of sorting centres rises to three when the parameter is 50% of the initial value but the remaining facilities remain within the same number of locations. From this point onwards, the network structure does not suffer any modification.

One may conclude that, if returns are not as high as expected, the network is over capacitated only if these returns are less than 50% of the total estimated volumes.

7.2.2. Disposal cost

This analysis is related with parameter $c_{mijt'}$. The model is tested for this parameter from 0% to 300% of its initial value by increments of 25%.

Table 3
Tests performed for the sensitivity analyses.

Parameter		Base value	Range of test	Increments
Return fraction	ρ_m	In Table 2	10–100%	10%
Disposal cost	$c_{mijt'}$ $(i,j) \in A_d$	0.0005	0–300%	25%
Non-satisfied demand/return cost	c^u_{mit}, $i \in I_c$	0.2/0.8	0–100%	10%
Cost of leaving a customer out of the supply chain	c_i	800	0–100%	10%
Recovery target	α_m	0.8	0–100%	10%
Investment cost	f_i	Between 50 and 200	0–200%	25%
Demand	d_k	In Table 1	−10% to 10%	5%

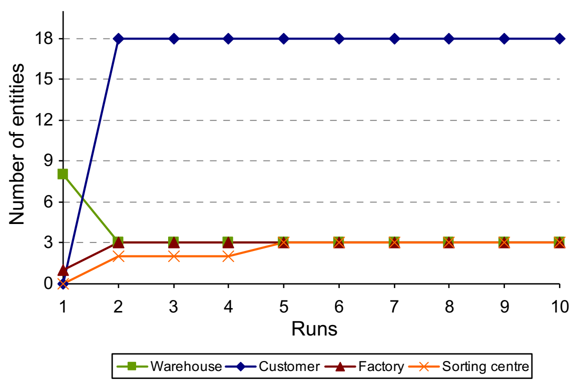

Fig. 12. Network entities.

For all runs, the network structure is the same as in Fig. 5: three factories, three warehouses, and three sorting centres satisfying 18 customers.

The chart in Fig. 13 shows the relationship between total transportation cost from sorting centres to factories (CstXr2), total cost of disposal (CstD) and the fraction of collected products that is sent to disposal (Disposal volume (%)).

From run 1 to run 4, the disposal volume is at its maximum. It is assumed a legal target of 80% of recovery, which means that a maximum of 20% of collected products can go to disposal. When the parameter is at its initial value (run 5), the disposal volume starts to decrease (from 20% to 17%). After run 9, when the parameter is twice the initial value, the disposal volume reaches the level of 3%. It remains unchanged until the end of the test. In terms of total disposal cost (CstD), it increases until run 8. After a decrease in run 9, it starts to rise again. With respect to disposal volume and total disposal cost, it is possible that further increase in the latter will cause a disposal fraction of 0%.

7.2.3. Non-satisfied demand/return cost

This analysis is performed on the parameter c^u_{mit}. The model is tested varying this parameter from 0% to 100% of the initial value, by increments of 10%. Irrespective of the value, all possible customers integrate the SC. In terms of facilities, when this parameter takes the value of zero, only one warehouse and sorting centre are opened (Fig. 14). These facilities allow the connection between the 18 customers and the Leiria factory, which already exists. An increase of 10% of the initial value causes the opening of two factories, two warehouses, and one sorting centre. The third sorting centre opens when the cost of non-satisfied demand/return reaches 30% of its initial value.

In terms of costs, Fig. 15 shows that total penalization cost (line with triangles) grows until the parameter reaches 50% of the initial value. After this point, the cost decreases. The volume of non-satisfied demand and return (line with cross) has its maximum value when the parameter has the value of zero. Then, it decreases until almost zero in the last test.

Some final remarks: in terms of facilities, the network structure shown in Fig. 5 is attained when the parameter reaches 30% of its initial value; in terms of costs, all runs reported different solutions. However, costs seem to converge to a constant value when the parameter reaches 80% of the initial value. In the objective function, the trade-off between the supply of this unsatisfied demand and the incurred costs generates a marginal gain.

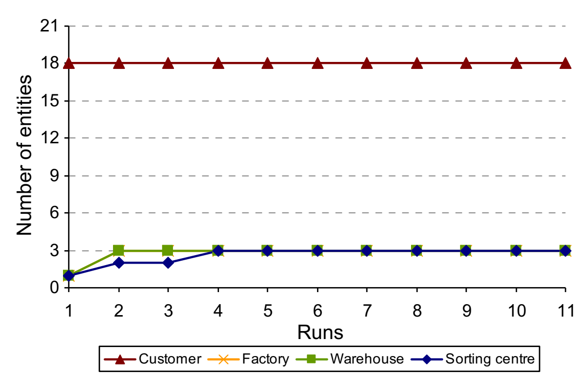

Fig. 14. Network entities.

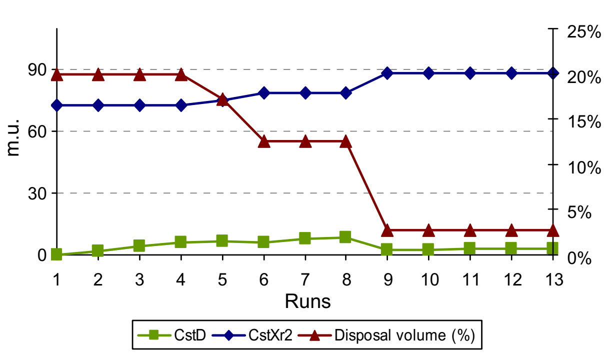

Fig. 13. Relation between costs and disposal volume.

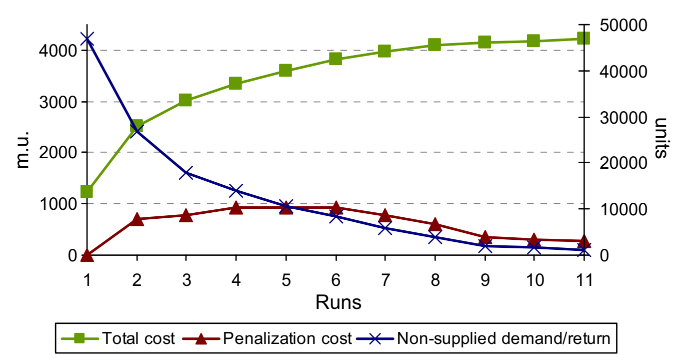

Fig. 15. Relation between costs and demand/return non-satisfied volumes.

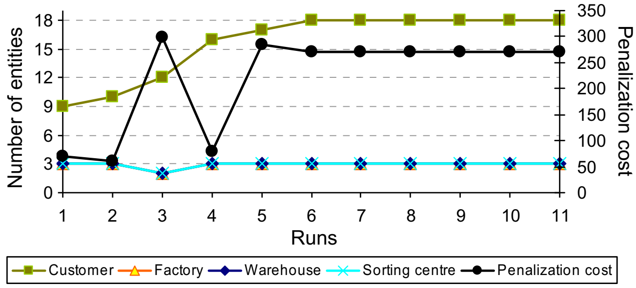

Fig. 16. Relation between network entities and non-satisfied demand penalization cost.

7.2.4. Cost of leaving a customer out of the supply chain

The primary objective of this analysis is to assess the impact that the cost of leaving one customer out of the SC has on the number of customers that will integrate it. This parameter varies between 0% and 100% of its assumed value, by increments of 10%.

In Fig. 16, as expected, it is found that the number of customers in the SC grows together with this cost. However, the maximum number of customers is reached when the cost is 50% of the initial value (400 m.u.).

It is interesting to observe that when the cost is of 160 m.u. (run 3), the number of customers grows from 10 to 12 but the number of the other entities in the SC reduces from 3 to 2. This corresponds to the highest penalization cost, meaning that more customers have seen their demand not fully satisfied. In run 2, the non-satisfied demand volume is about 250 thousand units. This volume rises to 1360 thousand units in run 3 and drops again to 360 thousand units in run 4.

In terms of optimal value, the chart below (Fig. 17) shows that it increases until run 6. In all the remaining runs, the optimal solution is the same.

Some final remarks:

1. If there is no penalization, the SC is composed of 9 customers and 3 factories, warehouses and sorting centres. The location of these facilities is the same of Fig. 5.
2. The increase of customer penalization cost increases the number of customers in the SC. The number and location of facilities remains unchanged, except for one run (run 3).
3. In run 3, the opening of only 2 facilities of each kind causes an increase in the non-satisfied demand.
4. The structure of the supply chain changes until the customer penalization cost reaches 50% of its initial value. When it grows above 400 m.u., the optimal solution remains unchanged.

7.2.5. Recovery target

The primary objective is to see the impact parameter α_m has on the network structure. The model is tested varying this parameter from 0% to 100% of the initial value, by increments of 10%. This means that when the parameter assumes the value of 100%, the total amount of product collected by the sorting centre has to be sent for remanufacturing.

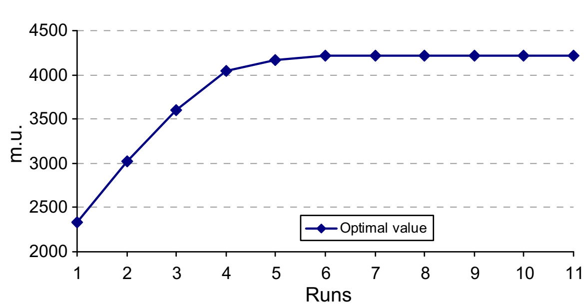

Fig. 17. Objective function values.

Fig. 18 shows that the recovery target only affects the number of opened sorting centres. In runs 3 and 8 there is a decrease of the satisfied demand. This leads to a smaller return volume, causing the need of less sorting centres.

In terms of costs, Fig. 19 shows that total cost increases as the disposal cost decreases. The reason is that recovery of products is

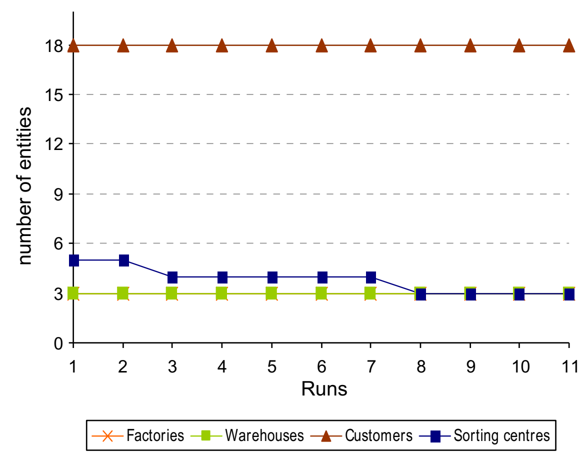

Fig. 18. Network entities.

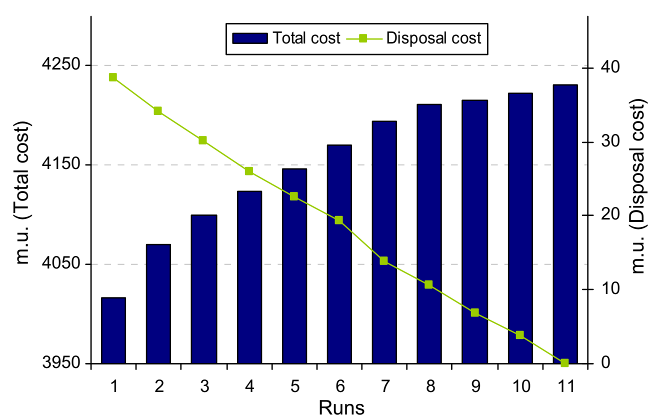

Fig. 19. Total cost versus disposal cost.

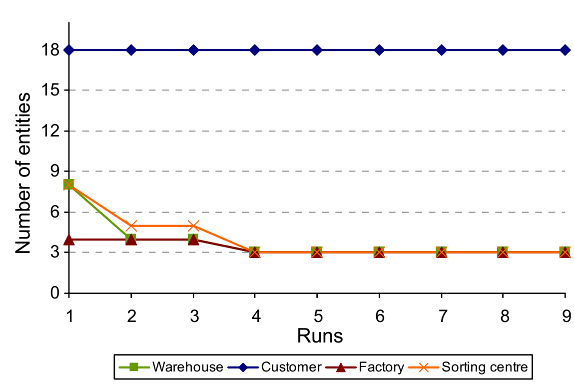

Fig. 20. Network entities.

more expensive than disposal. Thus, the model sends to disposal the largest volume allowed.

7.2.6. Investment cost

In this section, the focus is on parameter f_i. The model was tested from 0% to 200% of its initial value, by increments of 25%.

When there is no investment cost (run 1), all locations for warehouses and sorting centres are opened to serve the 18 customers (Fig. 20). In terms of factories, four out of five locations are opened. The increment of 25% caused the reduction of warehouses from 8 to 5 and sorting centres from 8 to 4. The network structure in terms of entities remains unchanged when the cost reaches 75% of the initial value (run 4).

In terms of cost, Fig. 21 shows the total SC cost (line with squares), the total investment cost (line with diamonds) and the difference between these two costs (line with triangles). After the fourth run, the difference between costs remains constant. This means that the SC design and plan are unchanged from run 4 onwards. The increase in the cost difference from run 3 to run 4 is caused by the increase of the total cost of non-satisfied demand (not shown), due to the decrease of one factory.

Some final remarks:

1. Even with no investment cost, only four out of five factory locations are chosen, which means that this fifth location is, for certain, not a good choice.
2. After run 4, the optimal network is the same, in terms of locations and flow amounts per microtime unit.

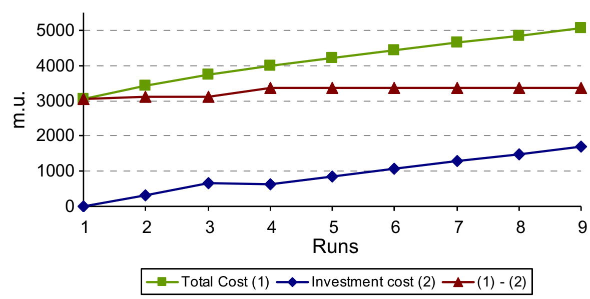

Fig. 21. Relation between total and investment costs.

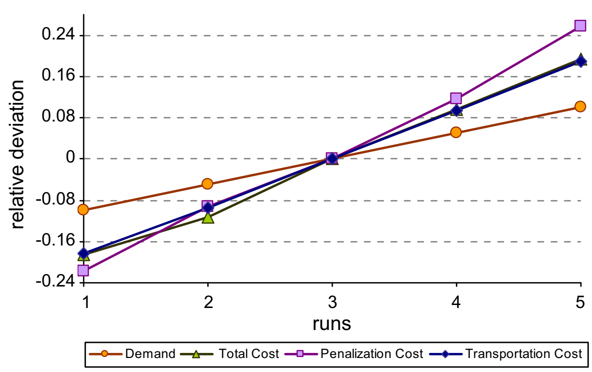

Fig. 22. Relation between demand and costs variation.

7.2.7. Demand uncertainty

Demand uncertainty is one of the major problems in the supply chain design. In this section we aimed at analyzing the impact of demand volumes on the network structure. The model is tested by varying parameter d_k from -10% to 10% by a step of 5%.

The results showed that the network structure remained unchanged in terms of facility location. The same three factories, three warehouses, and three sorting centres are opened. The difference in the five supply chains are on the flow and supplying volumes. Cost values are a good indicator of these differences. Fig. 22 shows the relative deviation of demand and costs relatively to the case shown in Section 7.1 (run 3). A change of about 10% in the demand volume leads to almost 20% change in total cost. This is justified by the multi-echelon structure of this network. The penalization cost is the one that reacts more to demand fluctuations. In all five runs only 2% of total demand is not supplied which represents about 6% of total cost. Therefore, small variations in the absolute value of this variable have a large impact on its relative deviation.

7.2.8. Global analysis

After this analysis, we can conclude that the design network is fairly robust. The number and location of facilities is stable near the assumed values. However, some parameters are more critical than others. There is a business opportunity if the return fractions are at least 20% of the estimated values. The cost of leaving one customer out of the network has a great impact on the network structure if the penalty cost is half of the estimated value. Another parameter that causes changes in the network structure is the investment cost. If it falls below 75% of its initial value, the network exhibits a different design.

7.3. Performance test

In order to evaluate model performance, several randomly generated examples were run. Their main difference is on the number of entities as shown in Table 4. The number of products remained unchanged since the definition of a coherent bill-of materials is a demanding operation, which would only be reflected on the

Table 4
Number of entities and time periods in the superstructure.

	Number of facilities				Time periods	
	Factories	Warehouses	Sorting centres	Customers	Macro	Micro
Ex. 1	5	10	10	50	5 periods	2 periods
Ex. 2	5	10	10	50	5 periods	3 periods
Ex. 3	5	10	10	150	5 periods	2 periods
Ex. 4	5	10	10	150	5 periods	3 periods
Ex. 5	5	10	10	150	5 periods	4 periods
Ex. 6	10	25	25	150	5 periods	2 periods
Ex. 7	10	25	25	150	5 periods	3 periods
Ex. 8	10	25	25	200	5 periods	2 periods
Ex. 9	10	25	25	250	5 periods	2 periods
Ex. 10	10	25	25	250	5 periods	3 periods

Table 5
Number of entities and time periods.

	Total variables	Binary variables	Total constraints	Iterations	CPU (seconds)	Opt. gap (%)	Relaxation ($\times 10^3$)	Obj. function value ($\times 10^3$)
Ex. 1	52,031	11,075	30,106	52,383	366	0.70	58,029	58,779
Ex. 2	77,131	16,575	44,231	324,955	1555	0.90	58,023	58,303
Ex. 3	146,631	31,175	81,606	580,401	4490	1.20	175,295	179,507
Ex. 4	217,231	46,675	119,731	1,210,475	10,076	1.70	175,240	180,269
Ex. 5	Out of memory							
Ex. 6	364,866	80,210	185,506	275,545	7124	4.20	145,504	153,019
Ex. 7	544,566	120,210	275,506	1,588,811	35,042	3.07	145,470	152,388
Ex. 8	479,666	105,260	241,256	321,910	8360	4.10	189,091	198,352
Ex. 9	594,466	130,310	297,006	1,278,475	26,706	2.00	233,440	241,199
Ex. 10	887,166	195,310	441,006	Out of memory			233,369	–

number of continuous variables. Therefore, the focus of this study is on the number of entities and time slots considered.

The computational results are given in Table 5. The results show that the model performs well under large instances. With the exception of examples 5 and 10, all instances took between 6 minutes and 9 hours to be solved. Given the strategic nature of this model and the instances size, these computational times can be considered as very good.

Examples 6–9 define very large supply chains with more than 150 customers to supply and collect. The number of binary variables varies between 80 and 130 thousand. Note that the increase of 50 customers in the example 8, with regard to example 7, leads to a problem that is faster to solve. This is explained by the decrease of one microtime period in example 8. This shows that the increase in the number of time periods may lead to extremely large problems in terms of computer memory usage. Note that a 5-period macrounit and 4-period microunit lead to 20-time periods, if a single unit scale was considered.

In order to show how large problems solved by the model can be, the number of time periods and entities was increased. GAMS/CPLEX was not able to generate the problem defined in example 5. In example 10, the relaxed model was solved but not the branch and bound. For cases like these, an algorithm or heuristic procedure needs to be developed.

8. Conclusions

In this work, a strategic and tactical supply chain design model is proposed. The primary objective is to present a general model that, within a single formulation, designs an extended supply chain, while simultaneously optimising the associated aggregate planning of purchases, production, storage, and distribution for a given time horizon. Time is modelled along a management perspective: for strategic decisions, a given time horizon is divided into strategic time units, which are then further divided into smaller units, where tactical decisions (planning) are to be taken.

Several features of real supply chains are modelled: flows travel times, facilities processing times, products' bill-of-materials, and products' disassembly structures. In addition, environmental targets imposed by legislation are explicitly considered in the model.

An example based on the Portuguese glass industry is solved in order to prove models' applicability and adequacy. The results obtained show that the proposed model deals satisfactorily with problems presenting a considerable degree of detail and complexity.

While dealing with MILP formulations, it is expected that problems with large dimensions will result in a significant increase in problem complexity and therefore in accrued solving difficulties. As future work, improvement of model performance by means of alternative solution techniques, such as a Benders decomposition based approach (Uster et al., 2007) and/or the development of valid inequalities (Cordeau et al., 2006; Thanh et al., 2008) will be explored.

The authors are also studying the application of the proposed modelling framework to different supply chains in order to evaluate the impact of the simultaneous design of forward and reverse chains versus the design of independent networks.

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