APPENDIX A

SHEAR STRENGTH MODELS: OVERVIEW

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A.1. Mohr-Coulomb criterion

Different models describing normal stress vs shear strength relationships exist. The simplest one is the Mohr-Coulomb criterion, which is in fact a failure criterion describing the conditions for which a material will fail. It can be written as:

$$\tau = c + \sigma_n \tan \phi$$  \hspace{1cm} (1.a)

where $c$ is cohesion, $\sigma_n$ the normal stress and $\phi$ the friction angle. Note that this is a linear model, which is not a good enough to describe joint behaviour especially when roughness and imbrication play an important role in it. For plane, smooth surfaces the model may be more accurate if both peak and residual properties are taken into account (figure 1). Then:

peak shear strength: \hspace{1cm} $\tau = c + \sigma_n \tan \phi_p$  \hspace{1cm} (1.b)

residual shear strength: \hspace{1cm} $\tau = \sigma_n \tan \phi_r$ \hspace{1cm} ; \hspace{1cm} $\phi_r < \phi_p$  \hspace{1cm} (1.c)

![Figure 1: Mohr-Coulomb criterion representing the $\sigma - \tau$ relationship for a plane, smooth joint. Note that both peak and residual shear strength have been taken into account.](image)

A.2. Patton's model

Since roughness is of the utmost importance for joint strength, it is essencial for a mathematical model describing joint behaviour to take this aspect into account. In this line, Patton's model (1966) idealizes the plane of discontinuity by assuming a regular shape, such as the one that can be seen in figure 2. Initially, the effect of the inclined planes making up the joint surfaces is added to the original mineral friction angle. Notwithstanding, for higher values of the normal stress, the roughness effect starts to disapear and the residual friction angle should be used to properly describe the behaviour of the altered joint.
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All this is written as:

\[ \tau = \sigma_n \tan (\phi_i + i) \quad \text{if } \sigma < \sigma_T \]  
\[ \tau = c + \sigma_n \tan \phi_r \quad \text{if } \sigma \geq \sigma_T \]

where \( \phi_i \) is the mineral friction angle, \( i \) is the geometrical angle of inclination and \( \sigma_T \) is the normal stress value from which the roughness effect can be neglected.

\[ \tau = C (1 - \exp^{-b\sigma}) + \sigma \tan \phi_r \]

Figure 2: Patton's model representing the \( \sigma - \tau \) relationship for a rough joint.

A.3. Jaeger's model

Non-linear models are able to represent joint behaviour more accurately. One of them is the Jaeger's model (1971), conceptually similar to the aforementioned Patton model, but changing the linear \( \sigma - \tau \) relationship for an exponential one (figure 3):

\[ \tau_p = C_i (1 - \exp^{-b\sigma}) + \sigma \tan \phi_r \]
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A.4. Barton’s model

Another commonly used non-linear $\sigma - \tau$ relationship is Barton’s model (1974), which is in fact a generalisation of the Mohr-Coulomb criterion for rough joints (figure 4):

$$
\tau_p = \sigma_n \tan \left[ R \log_{10} \frac{q_u}{\sigma_n} + \phi \right]
$$

where $q_u$ is the unconfined compressive strength and $R$ is a parameter that accounts for the joint rugosity and that varies from 0° (plane, smooth joint) to 20° (very rough joint). Note that for $R=0°$, the original Mohr-Coulomb criterion is obtained.

Figure 3: Jaeger model representing the non-linear relationship between normal stress and joint shear strength.

Note the asymptote:
$$
\tau_p = C_j + \sigma \tan \phi_r
$$

Figure 4: Barton model representing the $\sigma - \tau$ relationship for a perfectly smooth ($R=0$) and a rough joint.
A.5. Barton & Choubey model

Barton's model can be slightly modified to obtain the **Barton & Choubey model** (1977):

\[
\tau_p = \sigma_n \tan \left( JRC \log_{10} \frac{JCS}{\sigma_n} + \phi \mu \right)
\]

where $JRC$ is the joint roughness coefficient and $JCS$ is the joint wall compression strength. When planes of discontinuity are neither altered nor weathered, $JCS = q_u$.

More complex models, such as the Ladanyi & Archambault model exists, but we are not going to talk about them in the present thesis.