Final Degree Project

Study of advanced materials for thermal insulation in the inner Solar System

Author: Arnau Miró Jané
Supervisor: Dr. Manel Soria Guerrero
Supervisor: Dr. Elena Fantino

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“Imagination will often carry us to worlds that never were. But without it we go nowhere.”

Carl Sagan
Space harsh environmental condition often calls for an adequate thermal regulation and insulation. In this thesis, a thermal model for a greenhouse in Mars is presented. Its objectives are to achieve and maintain suitable temperatures for the growth of the most common vegetable species, only by means of the available solar radiation. The model aims for the assessment of the viability of a bigger structure. The model is thoroughly verified in order to provide reliable data and then it is evaluated a number of times to assess the best parameters that yield the best performance for a set of locations. It is discussed if an active control is needed for sustaining acceptable temperatures inside the greenhouse.
I would like to thank my project adviser Dr. Manel Soria for his enthusiasm and support on this project. His invaluable knowledge and skills have made this work possible. I’m also grateful he provided me with all the resources I needed to achieve the goals set on this project. I would like to thank my co-project adviser Dr. Elena Fantino for her support and her insight on the topic. Her patience with me and her corrections have been most valuable in this work.

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Contents

Abstract ii

Acknowledgements iii

Contents iv

List of Figures viii

List of Tables xi

Abbreviations xii

Physical Constants xiii

Symbols xiv

Introduction xvii

Scope xix

1 State-of-the-art 1

1.1 Thermal regulation on rovers and landers ........................................ 1

1.1.1 Viking landers ............................................................................. 2

1.1.2 Pathfinder lander .......................................................................... 3

1.1.3 Sojourner rover ............................................................................. 4

1.1.4 MER rovers .................................................................................. 5

1.1.5 Phoenix lander ............................................................................... 7

1.1.6 MSL Rover ................................................................................... 7

1.2 Thermal regulation strategies ............................................................ 9

1.2.1 Transparent insulation materials (TIM) ........................................ 9

1.2.2 Aerogel ......................................................................................... 10

1.2.2.1 Heat transfer in aerogels .......................................................... 12

1.2.3 Low-emissivity coatings ............................................................... 14

1.2.4 Multiple layer insulation (MLI) ..................................................... 15

1.2.4.1 Outer cover .............................................................................. 15

1.2.4.2 Reflector layer ......................................................................... 16
1.3 Considerations for a greenhouse in Mars

1.3.1 Threats to viability

1.3.1.1 Micrometeorites

1.3.1.2 Thermal issues

1.3.1.3 Radiation

2 Greenhouse Model

2.1 The model of the Martian environment: MEMM

2.1.1 Convection

2.1.2 Electromagnetic radiation

2.1.2.1 Direct SW radiation

2.1.2.2 Diffuse SW radiation

2.1.2.3 Reflected SW radiation

2.1.2.4 Total SW Radiation

2.1.2.5 Direct LW radiation

2.1.2.6 Diffuse LW radiation

2.1.2.7 Reflected and emitted LW radiation

2.1.2.8 Total LW radiation

2.1.2.9 Interior Radiation

2.2 Insulator cell model

2.2.1 Rayleigh number evaluation

2.2.2 Nusselt number correlation

2.2.3 CO₂ properties correlation

2.2.4 Simple insulator cell

2.2.4.1 Analysis of the shell inertia

2.2.4.2 Energy balance of the insulator cell

2.2.4.3 Cell linearized solution

2.2.4.4 Effects of solar absorption

2.2.5 Multilayer insulator cell

2.2.5.1 Linearized equations

2.2.5.2 Comparison between simple and multiple insulators

2.3 Interior fluid model

2.3.1 Natural convection analysis

2.4 Ground insulation

2.4.1 Liquid thermal inertia accumulator

2.4.1.1 Water thermal properties

2.4.1.2 Water free convection coefficient

2.4.2 Mars ground properties

2.4.3 Soil thermal penetration

2.4.4 Comparison of ground surface temperature with MEMM model

2.5 Global energy balance

2.6 Optimization

2.6.1 fsolve versus linearized algorithms

2.6.2 Matrix solvers
# List of Figures

1.1 Viking lander ........................................... 2
1.2 Pathfinder lander ..................................... 3
1.3 ISA structure cross-section ......................... 3
1.4 ISA structure comparison ............................. 4
1.5 Sojourner rover ........................................ 4
1.6 WEB structure cross-section ......................... 5
1.7 MER Twin Rovers ..................................... 5
1.8 MER insulation simulation ............................ 6
1.9 Phoenix lander artwork ................................ 7
1.10 MSL Rover Artwork ................................... 8
1.11 CO$_2$ conduction-convection ....................... 9
1.12 Block of aerogel ....................................... 10
1.13 Aerogel insulator properties ....................... 12
1.14 Extinction coefficients for fiber and silica aerogel 13
1.15 Characteristics of low-e glass ...................... 14
2.1 Greenhouse geometry wireframe representation .... 20
2.2 Martian environment .................................. 22
2.3 Surface heat transfer coefficient .................... 22
2.4 Electromagnetic radiation overview ................. 24
2.5 Sun zenith angle variation ........................... 25
2.6 Atmospheric gas properties ......................... 29
2.7 $Ra_E$ vs $Ra_M$ ......................................... 31
2.8 Rectangular enclosed cell schematics ............... 31
2.9 Nusselt number correlation .......................... 32
2.10 CO$_2$ thermal conductivity ......................... 33
2.11 CO$_2$ specific heat .................................. 34
2.12 CO$_2$ dynamic viscosity ............................. 34
2.13 Insulator cell scheme ................................ 35
2.14 Insulator cell boundaries ............................. 36
2.15 Insulator cell heat flux comparison ................. 38
2.16 Visible radiation absorption in the insulator cell 41
2.17 Multilayer insulator cell .............................. 41
2.18 Comparison of simple and multilayer insulator cells 44
2.19 Comparison of Nusselt correlations for the enclosed fluid 46
2.20 Natural convection flow patterns .................... 47
2.21 Ground layout scheme ............................... 48
2.22 Thermal wave penetration in Martian soil .......... 52
List of Figures

2.23 Mars ground MEMM comparison for dry soil ........................................... 54
2.24 Mars ground MEMM comparison for wet soil ........................................... 54
2.25 Mars ground MEMM comparison for Leovy’s dry soil .............................. 55
2.26 Mars ground temperature evolution ......................................................... 55

3.1 Crop adaptability ......................................................................................... 61
3.2 Locations of the various locations ............................................................. 62
3.3 Orbit of Mars ............................................................................................... 62
3.4 Ground layout scheme ............................................................................... 64
3.5 Enhanced ground with metallic rods .......................................................... 65
3.6 Time evolution of Valles Marineris for the first set of parameters .......... 70
3.7 Ground emissivity average temperature dependence ............................... 72
3.8 Ground emissivity maximum and minimum temperature dependence ...... 72
3.9 Ground absorptivity average temperature dependence .............................. 73
3.10 Ground absorptivity maximum and minimum temperature dependence ... 73
3.11 Insulator cell emissivity average temperature dependence ....................... 74
3.12 Insulator cell emissivity maximum and minimum temperature dependence 74
3.13 Day-night temperature difference for Valles Marineris ............................ 75
3.14 Seasonal average temperature for Valles Marineris .................................. 75
3.15 Ground layer thickness average temperature dependence ....................... 77
3.16 Ground layer thickness maximum and minimum temperature dependence 77
3.17 Day-night temperature difference for Valles Marineris ............................ 78
3.18 Water layer thickness average temperature dependence .......................... 78
3.19 Water layer thickness maximum and minimum temperature dependence ... 79
3.20 Day-night temperature difference for Valles Marineris ............................ 79
3.21 Active control coating depiction ............................................................... 80
3.22 Daily simulation comparison with and without active control for Valles Marineris .......................................................... 81
3.23 Water layer thickness and insulator emissivity average temperature de- pendency .............................................................................................. 82
3.24 Water layer thickness and insulator emissivity maximum and minimum temperature dependence .............................. 82
3.25 Water layer thickness and insulator emissivity maximum and minimum temperature dependence .............................. 83

B.1 Ground emissivity average temperature dependence .................................. 90
B.2 Ground emissivity maximum and minimum temperature dependence ........ 91
B.3 Ground absorptivity average temperature dependence ................................ 91
B.4 Ground absorptivity maximum and minimum temperature dependence ........ 92
B.5 Insulator cell emissivity average temperature dependence ........................ 92
B.6 Insulator cell emissivity maximum and minimum temperature dependence .... 93
B.7 Day-night temperature difference for Ares Vallis ........................................ 93
B.8 Seasonal average temperature for Ares Vallis ............................................ 94
B.9 Day-night temperature difference for Gale Crater ..................................... 94
B.10 Seasonal average temperature for Gale Crater .......................................... 95
B.11 Day-night temperature difference for Gusev Crater .................................. 95
B.12 Seasonal average temperature for Gusev Crater ..................................... 96
B.13 Day-night temperature difference for Valles Marineris ............................ 96
| B.14 | Seasonal average temperature for Valles Marineris | 97 |
| B.15 | Ground layer thickness average temperature dependence | 97 |
| B.16 | Ground layer thickness maximum and minimum temperature dependence | 98 |
| B.17 | Day-night temperature difference for Ares Vallis | 98 |
| B.18 | Day-night temperature difference for Gale Crater | 99 |
| B.19 | Day-night temperature difference for Gusev Crater | 99 |
| B.20 | Day-night temperature difference for Valles Marineris | 100 |
| B.21 | Water layer thickness average temperature dependence | 100 |
| B.22 | Water layer thickness maximum and minimum temperature dependence | 101 |
| B.23 | Day-night temperature difference for Ares Vallis | 101 |
| B.24 | Day-night temperature difference for Gale Crater | 102 |
| B.25 | Day-night temperature difference for Gusev Crater | 102 |
| B.26 | Day-night temperature difference for Valles Marineris | 103 |
| B.27 | Water layer thickness and insulator emissivity average temperature dependence | 103 |
| B.28 | Water layer thickness and insulator emissivity maximum and minimum temperature dependence | 104 |
| B.29 | Water layer thickness and insulator emissivity maximum and minimum temperature dependence | 104 |
| B.30 | Water layer thickness and insulator emissivity average temperature dependence | 105 |
| B.31 | Water layer thickness and insulator emissivity maximum and minimum temperature dependence | 105 |
| B.32 | Water layer thickness and insulator emissivity maximum and minimum temperature dependence | 106 |
| B.33 | Water layer thickness and insulator emissivity average temperature dependence | 106 |
| B.34 | Water layer thickness and insulator emissivity maximum and minimum temperature dependence | 107 |
| B.35 | Water layer thickness and insulator emissivity maximum and minimum temperature dependence | 107 |
| B.36 | Water layer thickness and insulator emissivity average temperature dependence | 108 |
| B.37 | Water layer thickness and insulator emissivity maximum and minimum temperature dependence | 108 |
| B.38 | Water layer thickness and insulator emissivity maximum and minimum temperature dependence | 109 |
| C.1 | Sample mesh | 113 |
| C.2 | Half control volume | 114 |
| C.3 | Energy balance in the interior of the material | 118 |
| C.4 | Energy balance in the boundary | 119 |
| C.5 | MMS mesh spacing error | 123 |
| C.6 | MMS Mesh Time Error | 124 |
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Electromagnetic radiation overview</td>
<td>23</td>
</tr>
<tr>
<td>2.2</td>
<td>Nusselt correlation</td>
<td>32</td>
</tr>
<tr>
<td>2.3</td>
<td>Nusselt correlation for spherical dome</td>
<td>45</td>
</tr>
<tr>
<td>2.4</td>
<td>Martian soil properties</td>
<td>51</td>
</tr>
<tr>
<td>3.1</td>
<td>Aerogel properties</td>
<td>67</td>
</tr>
<tr>
<td>3.2</td>
<td>Design parameters</td>
<td>68</td>
</tr>
<tr>
<td>3.3</td>
<td>Input parameters for the greenhouse model simulation code.</td>
<td>69</td>
</tr>
<tr>
<td>3.4</td>
<td>Active control</td>
<td>80</td>
</tr>
<tr>
<td>A.1</td>
<td>Budget of the <em>Study of advanced materials for thermal insulator in the inner Solar System</em></td>
<td>89</td>
</tr>
<tr>
<td>D.1</td>
<td>View factor validation</td>
<td>127</td>
</tr>
</tbody>
</table>
## Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Full Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEMM</td>
<td>Mars Environment Multi Model</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
</tr>
<tr>
<td>JPL</td>
<td>Jet Propulsion Laboratory</td>
</tr>
<tr>
<td>MER</td>
<td>Mars Exploration Rovers</td>
</tr>
<tr>
<td>MSL</td>
<td>Mars Science Laboratory</td>
</tr>
<tr>
<td>SCIM</td>
<td>Sample Collection of the Investigation of Mars</td>
</tr>
<tr>
<td>STEP</td>
<td>Satellite Test of the Equivalence Principle</td>
</tr>
<tr>
<td>RTG</td>
<td>Radioisotope Thermal Generator</td>
</tr>
<tr>
<td>RHU</td>
<td>Radioisotope Heater Unit</td>
</tr>
<tr>
<td>ISA</td>
<td>Integrated Structural Assembly</td>
</tr>
<tr>
<td>WEB</td>
<td>Warm Electronic Box</td>
</tr>
<tr>
<td>CCD</td>
<td>Charge Coupled Device</td>
</tr>
<tr>
<td>TIM</td>
<td>Transparent Insulation Materials</td>
</tr>
<tr>
<td>MLI</td>
<td>Multiple Layer Insulation</td>
</tr>
<tr>
<td>LEO</td>
<td>Low Earth Orbit</td>
</tr>
<tr>
<td>ESM</td>
<td>Equivalent System Mass</td>
</tr>
<tr>
<td>PAR</td>
<td>Photosynthetically Active Radiation</td>
</tr>
<tr>
<td>SW</td>
<td>Short Wave (radiation)</td>
</tr>
<tr>
<td>LW</td>
<td>Long Wave (radiation)</td>
</tr>
<tr>
<td>IR</td>
<td>Infrared (radiation)</td>
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<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
</tbody>
</table>
Physical Constants

Stefan-Boltzmann constant

\[ \sigma = 5.670373 \times 10^{-8} \text{ W m}^{-2}\text{K}^{-4} \]

Day/night cycle on Mars

\[ \text{sol} = 24.62 \text{ hours} \]
Symbols

\( T \) temperature \( K \)
\( P \) pressure \( Pa \)
\( q \) heat power \( W \)
\( L \) length \( m \)
\( A \) area \( m^2 \)
\( V \) volume \( m^3 \)
\( m \) mass \( kg \)
\( \dot{q} \) heat flux \( W/m^2 \)
\( g \) gravity acceleration \( m/s^2 \)
\( k \) thermal conductivity \( W/mK \)
\( c_p \) heat capacity \( J/kgK \)
\( h \) superficial heat transfer coefficient \( W/m^2K \)
\( R \) specific gas constant \( J/kgK \)
\( j \) radiosity \( W/m^2 \)
\( \dot{g} \) irradiance \( W/m^2 \)
\( F_{ij} \) view factor dimensionless
\( a \) albedo dimensionless
\( \rho \) density \( kg/m^3 \)
\( \mu \) dynamic viscosity \( Pa s \)
\( \beta \) thermal expansion coefficient at constant pressure \( K^{-1} \)
\( \alpha \) thermal diffusivity \( m^2/s \)
\( \alpha \) thermal absorption dimensionless
\( \varepsilon \) thermal emittance dimensionless
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Nu$</td>
<td>Nusselt number</td>
</tr>
<tr>
<td>$Ra$</td>
<td>Rayleigh number</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandlt number</td>
</tr>
<tr>
<td>$Gr$</td>
<td>Grasshof number</td>
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</tbody>
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To my grandmother, who could not see this work completed. May she rest in peace.
Introduction

This work consists in a numerical model and a preliminary design of a greenhouse for the surface of Mars. The objective of the design is to achieve and maintain temperatures suitable for the growth of the most common vegetable species. Besides, the thermal conditions should be sustained only by means of the available solar radiation, i.e., without any artificial energy supply. However, the aim of this work is not to develop a fully working, large-scale greenhouse but a prototype that can be deployed on the surface of the red planet in the next five or ten years in order to assess the viability of a bigger structure. The methodologies presented are based on current technologies for thermal insulation on the Earth. The materials considered are also currently available.

Space exploration is one of the most ambitious aims of mankind, and to satisfy such ambition several probes and landers have been launched to the planets. Thermal control is needed in every space mission. The often harsh conditions of the space environment represent a challenge to the designer, having to insulate some payload at room temperature from extreme temperatures. This is the case of the present work, in which a greenhouse in Mars is considered. In the inner Solar System it is possible to use the Sun as the main natural heat and energy source. When considering a manned mission to the inner Solar System, Mars is one of the best candidates. The red planet has been explored in the past decade, yielding an increase of knowledge and technology. Moreover, the technologies presented here are also suitable for Earth’s most harsh environments (e.g., Antarctica), space probes or future more efficient space life support systems for manned missions. Such technologies can allow to thermally regulate the temperature in most of Earth’s climates without using external energy (i.e., air conditioning or central heating). It is reasonable to employ such techniques on Earth as the Martian environment presents more severe conditions. Therefore, a better usage of energy resources could be a secondary advantage.
Finally, this work is the culmination of a study aiming at understanding the Martian environment and climate. It uses MEMM - a numerical model of the Martian climate, developed as part of a final year’s project carried out in this same school - to understand and foresee the thermal behavior of a structure located on the Martian surface. Therefore, this work and its antecedent should be considered as two parts of a larger project, albeit they address different topics.

This report is structured as follows:

- Chapter 1 reviews the main rover missions sent to the surface of Mars and analyzes their thermal insulation methodologies. It also analyzes the newest and most efficient methodologies for thermal insulation currently developed on Earth and discusses the latest insulation materials. Finally, it explores other relevant considerations for a greenhouse on Mars.

- Chapter 2 presents the numerical model of the greenhouse in detail. A modular approach has been adopted which consists in dividing the greenhouse into several sections (i.e. shell, enclosed air and ground). The different sections of the greenhouse are presented and then a number of validations are performed to ensure the correct performance of the code. Eventually, some optimization algorithms are presented.

- Chapter 3 explains the design parameters and strategies for a hypothetically viable Martian greenhouse. To this aim, several simulations have been executed involving different scenarios (e.g., different locations, different seasons). The chapter ends by presenting strategies of thermal control by tuning some parameters.

The report ends with a series of conclusions and offers hints for future developments and improvements.
Scope

The scope of this work is:

• To discuss the main strategies and technologies of thermal control used in
  – the main landers that have landed on Mars;
  – the main rovers that are/have been used on Mars;
  – Earth’s most efficient insulation methodologies for outdoor structures.

• To present a consistent thermodynamical model of a greenhouse operating on the
  Martian surface. It shall develop through:
  – The inclusion of MEMM as the model of the Martian environment.
  – The conception of an insulation material for the greenhouse walls.
  – An approach to the air movement inside the greenhouse.
  – The approximation and selection of the most appropriate Martian ground
    properties. In turn, this point is used as a verification of the MEMM model.
  – The formulation of a thermal accumulating ground to counteract the harsh
    difference between day and night.

• To obtain the most suitable greenhouse for a Martian manned base through
  – analyzing various locations in order to find the most appropriate site;
  – assessing the impact of various design parameters on the overall performance
    of the greenhouse;
  – proposing a means of thermal control by tuning some of the design parameters.
The work is focused on the thermal aspects of the greenhouse. However, many more aspects, which are out of the scope of this work, must be considered before reaching a final design. For example:

- The study of the impact of Martian conditions on the crops due to
  - the day and night duration;
  - the length of the Martian seasons;
  - the life cycle of the crops;
  - the influence of reduced solar radiation and temperature conditions on the crops productivity.

- Life support systems and safety considerations such as:
  - the total amount of water to be carried for the crops to grow;
  - the possibility of human life inside the greenhouse;
  - the allowed radiation dose;
  - micrometeorite impacts and dust settlement;
  - the crops CO₂, O₂ and N₂ cycles and their regulation;
  - the need of chemical fertilizers and their availability on Mars.

- Structural considerations such as:
  - material resistance and required stress and strain;
  - resistance to corrosion and near vacuum exposition;
  - holding of the greenhouse on the Martian ground.

- Transportation from the Earth to Mars and in-situ construction of the greenhouse.
Chapter 1

State-of-the-art

This chapter starts with an introduction to the most relevant Martian landers and rovers (Sect. 1.1). For each, some basic data are given and their thermal regulation strategies are discussed. Then, the most advanced insulation materials and techniques used for terrestrial outdoor structures are explained (Sect. 1.2). Finally, some considerations about a Martian greenhouse are made (Sect. 1.3).

1.1 Thermal regulation on rovers and landers

Together with Venus, Mars is the most visited planet in the Solar System. Many probes have been sent to the red planet and some of them carried a lander and/or a rover for ground exploration. The vehicles that successfully landed on the Martian surface are:

- Viking\(^1\) (Lander)
- Pathfinder\(^2\) (Lander)
- Sojourner\(^2\) (Rover)
- MER\(^3\) \textit{Spirit} and \textit{Opportunity} (Rovers)
- Phoenix\(^4\) (Lander)
- MSL\(^5\) \textit{Curiosity} (Rover)

Thermal control is needed in space. In particular, the surface of Mars is characterized by a specific thermal environment which imposes an adequate thermal control. Clearly, the type of thermal control to be designed depends on the nature of the mission itself, the mission duration and the available power. Furthermore, the varying thermal
environment of Mars poses a challenge to the design of the thermal control. The main environmental factors that affect thermal control are temperature variations, atmospheric density and atmospheric opacity. As an advantage, thermal control is not needed in the whole vehicle. The components that need special care are electronics, batteries and the temperature sensitive components of the communications system. These vehicles’ insulation systems are going to be revised, in chronological order, in order to discuss the best insulation strategy for a hypothetical Martian greenhouse.

1.1.1 Viking landers

The Viking landers were part of the Viking mission launched in 1975. Viking 1 landed on the Chryse Planitia at a latitude of 22.3 degrees north and 48.0 degrees longitude. Viking 2 landed at Utopia Planitia (Fig. 1.1) at 47.7 degrees north and 48.0 degrees longitude. Both probes were operated for more than 3.5 years.

The Viking missions used radioisotope thermoelectric generators (RTG) as main power source. They converted heat from decaying plutonium-236 into 70 watts of electrical power. Waste or unconverted heat was conveyed by the thermal switches to the lander’s interior instrument compartment when required. The covers over the RTGs prevented excess heat dissipation into the environment. Moreover, the lander body and exterior assemblies were painted light gray to reflect solar heat and protect the equipment from abrasion. The paint was made of rubber-based silicone. Furthermore, the hexagonal body box was insulated with spun fiberglass and dacron cloth to protect the equipment and reduce the heat loss. The light green coating and the body box can be seen on the left of the image.

![Figure 1.1: Viking 2 picture of the Utopia Planitia, credit by NASA/JPL.](image)
1.1.2 Pathfinder lander

The Pathfinder lander (Fig. 1.2) was sent to Mars as the main part of the Pathfinder mission in 1996. It reached Mars almost one year later. It landed on Ares Vallis, located at 19.33 degrees north and 33.55 degrees west.

![Concept art of the Pathfinder lander at Ares Vallis, credit by NASA/JPL](image)

The primary source of power for the Pathfinder mission was based on solar panels. An insulator structure called Integrated Structural Assembly (ISA) was developed specifically for this mission. The ISA is a static structure that supports the camera and antenna assembly and also acts as a barrier for planetary protection. The ISA is basically a conventional composite honeycomb structure (Fig. 1.3). It consists of graphite-cyanate facesheets with a 2 in nomex core. The core is filled with 2 lb/ft$^3$ of eccofoam pressed into the honeycomb cells. An additional 2-inch thick piece of eccofoam is also bonded in the interior with aluminized kapton as thermal control surface material. Its performance is superior than that of other known insulators (Fig. 1.4).

![Cross-section of the ISA composite material](image)
Figure 1.4: Thermal insulation comparison for the ISA structure [2].

1.1.3 Sojourner rover

The Sojourner rover was launched to Mars along with the Pathfinder mission (Fig. 1.5). Its mission was to assess the feasibility of a Martian rover powered by solar energy.

Sojourner main power system was solar energy. The insulator structure for the Sojourner rover is equivalent on the ISA on Pathfinder lander in terms of thermal regimes. However, different engineering requirements lead to different designs. The Sojourner rover is insulated by means of the Warm Electronic Box (WEB). The WEB is basically designed for mobility. It is basically made of carbon-opacified silica aerogel which
is opaque to both IR and visible light. The WEB structure incorporates glass-epoxy structural spars for structural purposes and a 0.5 \textit{mm} gold coated kapton radiation barrier (Fig. 1.6). Silica aerogel solid block minimizes thermal conduction\cite{2}. Nevertheless, power from batteries at low temperatures still remain a concern. Low temperature primary batteries are becoming available that can operate below -40 degrees centigrade, but currently the lowest temperature secondary rechargeable batteries can operate is -20 degrees centigrade. For this reason, three radioisotope heater units (RHUs) where installed to provide about 1 Watt of thermal power each, thus keeping the electronics inside the WEB warm and ensure the survival of the batteries\cite{7}.

1.1.4 MER rovers

The Mars Exploration Rovers (MER) mission delivered two robots, \textit{Spirit} and \textit{Opportunity}, to the Martian surface (Fig. 1.7). They were launched in 2003 and reached Mars in 2004. Since then they have been exploring the surface. Their landing sites were Gusev Crater and Meridiani Planum, respectively.

![Figure 1.6: Cross-section of the WEB composite material\cite{2}.](image)

![Figure 1.7: Artwork of one of the twin MER rovers, credit by NASA/JPL\cite{6}.](image)
The main energy input of the mission was solar radiation. The insulator structure follows the design of a WEB structure with the objective of maximizing its interior thermal time constant as the items inside the WEB must survive thermal transients. Such transients were driven by the internal power dissipation and the external environment. A considerable amount of effort was put in maximizing the thermal resistance of the WEB structure. The structure exoskeleton design consisted of a stiff box made of aluminum honeycomb and carbon composite face sheets lined on the inside with bricks of carbon-opacified silica aerogel. Such aerogel has extremely low density ($0.02 \text{ g/cm}^3$) and a very low thermal conductivity ($0.012 \text{ W/mK}$). Carbon opacification was added to block the infrared thermal transmission through the material. The aerogel is securely held on the internal walls of the WEB with small Z-spars and protected from abrasion by a thin glass-epoxy composite layer over the sheets. On top of such sheets, a layer of goldized kapton is added with its low emissivity face looking inwards the box to minimize thermal radiation losses. Inn the outside the structure is also finished with a low-emissivity goldized kapton to minimize radiation loss\footnote{3}. Outside the WEB, cameras and actuators needed also thermal control. In general, other rover hardware could survive in the extremes of the Martian night without thermal insulation. Silvered teflon tape was used to protect the CCD cameras from overheating in the sun. For the cameras that were more exposed to the cold sky and actuators and bearings, heaters were used. Insulation simulations were made for both the worst case hot and worst case cold scenarios (Fig. 1.8). The WEB is heated by electric heaters, by the waste heat generated by the electronic components as well as by eight RHUs. Such RHU’s considerably reduce the electric heating load and save battery power particularly in cold winter nights. In addition, the combination of a WEB and RHUs prevents the batteries from being operated at extreme temperatures\footnote{7}.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{mer_web_simulation.png}
\caption{Thermal simulation of the MER WEB structure for the worst cold and worst hot scenarios\footnote{3}.}
\end{figure}
1.1.5 Phoenix lander

The Phoenix mission was launched in 2007 and reached Mars in 2008. It carried a single lander (Fig. 1.9). Its landing site was Vastitas Borealis in the northern pole with the objective of studying the frozen soil of the Martian artic.

![Artwork of the Phoenix lander](image)

Figure 1.9: Artwork of the Phoenix lander, credit by NASA/JPL.

Phoenix’s main energy input was also solar energy. Phoenix had a specially challenging design as it had to withstand the cold temperatures of the Martian artic. Phoenix used heaters to keep its instruments within thermal operation range. It was coated with a low-emissivity coating to prevent loss in the form of thermal radiation. Aluminized tape on internal and external shroud surfaces is devised to reduce radiative exchange to the thermal blanket. Form-fitted fiberglass insulation blankets cover the sensors’ heads. Aluminized kapton is used on the internal surfaces and betacloth is used on the external ones. Such betacloth doesn’t bring any advantages as it increases the radiation exchange with the sky. Its purpose is to prevent solar scattering onto the surface stereo imager. Most of the internal surfaces are gold plated in order to provide a low-emissivity surface finish.

1.1.6 MSL Rover

Mars Science Laboratory (MSL) was launched in 2011 and landed on Mars in 2012. It landed at Gale Crater at 4.5 degrees south and 137.4 degrees east. The MSL mission put the heaviest rover (Fig. 1.10) ever designed for Mars so far on the surface of the red planet. Its name is *Curiosity*: its objective is to assess if the environment was once capable to sustain microbial life.
MSL rover’s main power source is a radioisotope thermoelectrical generator (RTG). *Curiosity* uses the concept of multiple layer insulation (MLI). Conventional vacuum based insulation in MLI is not effective in a gaseous atmosphere, so engineered gaps were employed between the warm rover internal components and the cold rover external structure. Large gaps would lead to more thermal insulation but would also require more volume of the rover. Since the conductivity of low pressure CO$_2$ varies from $0.01 \ W/mK$ to $0.016 \ W/mK$ at room temperature, it serves as a natural insulator. Carefully designed gaps automatically get filled with ambient CO$_2$ upon landing on Mars. The trade-off for the gap design was the minimization of the heat loss from the interior components while minimizing the unused volume of the rover. Such insulator principle is based on the fact that the magnitude of convection is less on Mars than on Earth (Fig. 1.11) and that convection begins in a larger gap thickness on Mars than on Earth\[5\]. MSL represents the latest evolution on Martian insulation technology by being the largest rover ever placed on Mars. For this reason, the insulation structure must be lighter. The thermal insulator design proposed in this work is based on the latest technology and will be further assessed in Chapter 2.
1.2 Thermal regulation strategies

Thermal regulation is needed to achieve a desired temperature inside a body, in particular a greenhouse located in Mars. This section is going to revise some of the state-of-the-art strategies for thermal regulation used both in Earth and in space.

1.2.1 Transparent insulation materials (TIM)

The main aim of thermal insulation on Earth buildings is sustaining a comfortable and hygienic indoor climate at low temperatures. In order to preserve the constructional elements from the damages caused by thermal effect and moisture, thermal insulation is necessary for the exterior walls. The purpose of the thermal insulation in winter is energy conservation which causes a reduction in heating demand. Also, it has a big impact on the protection of the environment. The decrease in the energy consumption for the purposes of heating and air conditioning can be enhanced by transparent insulating materials (TIM) that are utilized for optimization of thermal insulation and transparency. Furthermore, TIMs are used for solar energy on exterior walls. The short wave solar radiation penetrates TIM and heats the wall partition up. The long wave radiation which is emitted from the partition cannot get through the insulation layer which is opaque for infrared radiation. Considering ambient temperatures, heating the exterior surface...
may bring about decreased transmission loss or gain through the wall. In both situations, transparent insulation construction can decrease the demand of heating energy. Among TIMs, aerogel is one of the most efficient material for usage in windows due to its low thermal conductivity. Furthermore, it enables to achieve a high solar energy and daylight transmittance; hence, the annual energy consumption for space-heating in cold climates can be diminished.

1.2.2 Aerogel

Aerogels are rather old materials that were invented back in 1931 by Steven Kistler at the College of the Pacific in Stockton, California. Aerogels are transparent, highly porous, open-cell, low-density foams (Fig. 1.12). Its microstructure, comprised of nanosized pores and linked primary particles, can be tailored by a process known as sol–gel method. As a result of this unique microstructure, these lightweight materials exhibit many interesting and unusual properties. The first aerogels were synthesized from silica gels by replacing the liquid component with a gas. They looked a quite revolutionary type of solid-state materials because of their extremely low density and their outstanding physical properties, especially for thermal and acoustical insulation. As a matter of fact, aerogels have the lowest thermal conductivity, refractive index, sound velocity and dielectric constant of any solid ever tested. Nowadays, a potential exists for aerogels to enter the commercial market for a number of reasons:

- More cost effective production methods are being found;
- Aerogel microstructure can be adapted and optimized for specific applications;
- Aerogels are inert, non-toxic, environmentally friendly insulation materials;
• Aerogel’s performance is superior to that of other foams.

Structural applications concerning aerogels have some drawbacks. They are very vulnerable to tensile stress, to moisture and to water (i.e., if liquid water comes into contact with the aerogel, it is rapidly deteriorated). As a conclusion, the material has to be effectively shielded from the environment. Luckily, aerogels are very strong in compression, making it possible to use the material in a sandwich construction, e.g., between two sheets of glass.

Interest in space applications for aerogels have been increasing during the last years. A summary of recently used and prospective applications of aerogels in space applications follows:

• Cryopump insulation for space launch systems. Cryopumping is strictly defined as the production of vacuum using low temperatures. In this case, aerogel super insulator properties are being used to insulate the cryopump device using an aerogel-based insulator system.

• Hypervelocity particle capture. Aerogels have been devised as particle collectors in both Stardust and SCIM (Sample Collection of the Investigation of Mars) missions. The Stardust’s mission was to rendezvous with a comet and collect and return particles from a comet trail of one thousand cometary particles, fifteen microns or larger in diameter. Similar to Stardust, SCIM would collect particles from the atmosphere of Mars. The aerogel makes an excellent particle collector as the particles yield to the filaments of aerogel microstructure thus transferring their kinetic energy to the aerogel sheet and slowly braking to a stop.

• Thermal insulator. Aerogel was used in the insulations of the WEB structure for both the Sojourner and MER rovers (Subsect. 1.1). Aerogel was rendered opaque by adding 0.4% by weight graphite. Therefore, a low conductive and convective insulator was achieved which in turn prevented irradiative thermal transport. Aerogel also simplifies the design of RTGs (radioisotope thermal generators) by being formed when the lander reaches its destination (i.e., Mars in that case). As the precursor is a liquid, it can fill all the spaces among the legs and around the module.

• Cryogenic fluid containment. Aerogel super insulator properties can also be used to keep cryogenic instruments within their extreme temperature operation range. Such technology was designed for the STEP (Satellite Test of the Equivalence
Principle) mission. However, it was not selected for flight. Nevertheless, technology remains with the success of the Gravity Probe B which used a similar technology. [13]

1.2.2.1 Heat transfer in aerogels

Aerogels have very low thermal conductivity and are able to withstand steep temperature gradients (Fig. [1.13]). Heat transfer occurs by three mechanisms inside the aerogel:

- Conduction in the solid skeleton;
- Convection in the material gaps;
- Radiation through the material layer.

Vacuum-like properties of aerogel can be achieved at a very moderate vacuum (i.e., $P < 5 \text{kPa}$). Under such a pressure, heat transport through the material occurs mainly by heat conduction through the structure (silica skeleton) and by radiation.

It is not the purpose of this work to deeply treat the heat transfer within aerogels but to get the overall picture of the working process of heat transfer. Therefore, a simple
model is assumed based on simple superposition of solid ($k_s$), gas ($k_g$) and radiation ($k_r$). Then, an effective conductivity is defined as ($k_{eff}$):

$$k_{eff} = k_s + k_g + k_r.$$ (1.1)

Note that in vacuum-like properties $k_g$ is set to zero. There is another factor of major influence regarding radiation: the extinction coefficient. Such coefficient takes into account scattering and absorption of the radiation as it travels through the material. Due to its particular microstructure, some of the incident radiation gets absorbed by the material at a point and the rest of the radiation gets scattered into the material. Moreover, aerogel fibers also emit thermal radiation. Therefore, scattering, absorption and emission inside the material must be taken into account for a correct modeling of the heat transfer mechanism. Nonetheless, such a treatment is not trivial and is out of the scope of this project. A simple way to account for the heat transfer inside the aerogel is presented as follows:

$$\nabla (k_c \nabla T - \dot{q}_r) = 0,$$ (1.2)

where $\dot{q}_r$ is the radiative heat flux and $k_c$ is the solid conduction in the aerogel given by:

$$k_c = k_s + k_g.$$ (1.3)

$k_s$ and $k_g$ depend purely of the materials the aerogel is synthesized and its porosity. $\dot{q}_r$ is computed by writing the radiative heat equation for an optically thick layer:

$$\dot{q}_r = -\frac{1}{3\beta} \frac{\partial G}{\partial x},$$ (1.4)

where $G$ is the incident radiation and $\beta$ is the extinction coefficient. Such coefficient can be retrieved experimentally from graphs as Fig.1.14.

Figure 1.14: Extinction coefficients of the fiber and silica aerogel: (a) spectral extinction coefficients; (b) Rosseland mean extinction coefficients.(From Zhao et al. [14])
1.2.3 Low-emissivity coatings

During the past (i.e., the last century) a great effort to produce coatings with high visible light transmittance and high reflectance for thermal radiation has been done. These films produced on transparent substrates like glass or polyester foil are called low-emissivity (low-e) coatings and are widely used to reduce heating costs especially in cold climates. The visible light transmittance, i.e., in the wavelength range from 400 to 700 nm, should be similar to clear glass but the reflectance in the thermal infrared should exceed 90% to give the low emissivity character (Fig. 1.15). Therefore, low-e coatings can significantly reduce radiative heat losses of glass panes for solar energy use. Literature in the 70s and 80s have discussed low-e coatings for glass covers to improve efficiency of solar collectors. Their effectiveness, however, is strongly dependent on their optical properties, which need to meet the requirements for the specific application. The mission of such coatings is to selectively separate thermal radiation according to its spectrum. Therefore, visible wavelength is permitted through the coating whereas IR wavelength is rejected, thus thermal losses are reduced. Suitable active materials for the coating, due to their physical properties, are either metals (mainly silver, but also copper or gold) or metal oxides (e.g., tin oxide, indium oxide or zinc oxide). These are embedded in appropriate layer systems, which are deposited with different methods.

![Figure 1.15: Solar irradiation spectrum, blackbody radiation spectra and the characteristics for ideal low-e glass. (From Szczyrbowski et. al.]

**Figure 1.15:** Solar irradiation spectrum, blackbody radiation spectra and the characteristics for ideal low-e glass. (From Szczyrbowski et. al.)
on the collector material, which is mainly glass. Commercially available products have been almost exclusively developed for architecture. For various comfort reasons, coatings based on silver are primarily used, which can provide for extremely low emissivity (less than 0.03) and high visible transmittance (up to 0.90). Solar transmittance, however, is rarely higher than 0.60. Such coatings usually consist of thin silver layers sandwiched with dielectric layers. Besides the performance of the coating, its resistance to temperature, humidity, mechanical stress, and their combined effect, play a significant role in the coating performance. Silver coatings for architectural glass, for example, are very sensitive to corrosion and can only be used in a dry and airtight environment such as in the gap of multiple insulating glazing.

1.2.4 Multiple layer insulation (MLI)

Multilayer insulation (MLI) is a type of high-performance insulator which uses multiple radiation-heat transfer barriers to retard the flow of energy. Individual radiation barriers usually are thin polymer films with vapor-deposited metal on one or both sides (Subsect. 1.2.3). NASA provides a series of guidelines in order to select the proper materials for MLI to be effective. Such guidelines state that the blanket must consist of the following layers:

- Outer cover.
- Reflector layer.
- Separator layer.
- Inner cover.

The layers are held together by means of an adhesive layer.

1.2.4.1 Outer cover

The outer cover material is resistant to shedding, flaking and other forms of particulate generation. Materials that are not opaque to UV radiation will have a metallized reflector layer acting as a light block directly under the outer cover with no separator layer. Outer cover materials which are aluminized will have the aluminized side facing the interior of the blanket. Materials for outer cover are betacloth (which can be aluminized), tedlar, kapton and teflon.
1.2.4.2 Reflector layer

Reflector layers need an outer cover for protection from space environment effects, because organic material is heavily attacked by atomic oxygen, thus the effectiveness of the insulation is reduced. Another parameter to be considered in the design is how many reflector layers are needed to achieve the desired thermal effect on the protected surface. A long-term low-Earth orbit (LEO) spacecraft generally uses 15 to 20 reflector layers. Typical materials are aluminized or goldized kapton, aluminized mylar, polyester and teflon.

1.2.4.3 Separator layer

The separator layer is placed between the reflector layer and the inner and outer covers. Dacron and nomex are typical separator layer materials.

1.2.4.4 Inner cover

The inner cover is adjacent to or faces the object to be insulated. Such layer is not always included in MLI designs. The reinforcement in these films and single aluminizations must face the MLI blanket. Often, the inner layer is not metallized in order to reduce the chance of an electrical short. Kapton is a typical material for the inner cover which can be aluminized, double goldized or reinforced with glass fibers.

1.3 Considerations for a greenhouse in Mars

Designs of a life support system to be used on a Mars mission are being made by NASA researchers. A tool to be able to compare the designs and predict the cost of a designed life support system is needed. For these purposes, NASA developed a term called Equivalent System Mass (ESM). The ESM can be defined as the sum of the life support mass and volume, power generation, cooling and crew time for maintaining a crew over the mission duration \([19]\). With the corrections based on available light levels, it can be seen that the ESM of a naturally lit plant growth system significantly differs from an electrically lit system. Artificial lighting makes use of power from generators or solar collectors whereas natural lighting can be used through transparent structures or irradiance collection, transmission and distribution systems. According to Lockheed Martin Space Mission Systems & Services the equation below is used for relating light and edible plant growth:

\[
\text{Edible} = 0.77 \times \text{PAR} - 6.1, \tag{1.5}
\]
where *Edible* is the edible plant mass produced (in g/(m$^2$ day)), *PAR* is the lighting level of photosynthetically active radiation (in mol/(m$^2$ day)). The level of available *PAR* depends on whether natural or artificial lighting is used for edible plant growth. By making assumptions on the deep space radiation and the possibilities as natural light passes through the Martian atmosphere, the amount of natural light can be determined. Furthermore, local dust storms and atmospheric conditions for nominal weather are taken into consideration. Under these assumptions:

- The average daily *PAR* on Mars can be calculated as 20.8 mol/(m$^2$ day);
- When artificial lighting at 400 $\mu$mol/(m$^2$ s) is used, there is 17 mol/(m$^2$ day) of *PAR*.

As Martian seasons change, the amount of *PAR* changes as well. Under the assumption that excess food can be stored during the year, if natural lighting worsens and does not permit the generation of enough food, the stored food can make the difference. Moreover, the lower the transmittance that can actually be achieved, the more comparable the artificial system becomes to the natural greenhouse system. The higher the transmittance, the more favorable the natural greenhouse system is over the artificial system. As a result, an inflatable greenhouse making use of natural lighting is better than an inflatable hybrid system using artificial lighting[20]. Natural solar insulation requires the containment structure which stands between the plants and their light source. This can be achieved either by designing a transparent structure enabling light to pass through the structure efficiently or by collecting and transferring the light through fiber optics to the interior of the structure. A true greenhouse, i.e., a transparent structure permitting natural lighting, needs a minimum of extra equipment with which to capture, concentrate, convert or transmit the light. The general transmittance efficiency of a greenhouse depends on material choice, material thickness, latitude, greenhouse geometry and orientation and the solar zenith angle. By considering the average available insulation for a greenhouse to provide the required light, it needs to be 84% transmittant. It would be hard to obtain with a Mars greenhouse since preserving pressure and surviving in the Mars environment boost the thickness and limit the choice of materials. On Martian surface, plants need an atmosphere of pressure greater than 1.0 kPa. Actually, their survival is possible at pressures as low as 10 kPa but this strongly depends on the partial pressures of oxygen, water, and carbon dioxide. For crop performance, 25 kPa is better. The ideal solution is to employ an inflatable structure due to the difference between the pressure created by the higher internal plant atmosphere and the lower pressure of the Martian surface environment. Inflatable structures have many advantages. Firstly, they can have very high packaging efficiencies. Secondly, they are easy to construct at
remote locations. And thirdly, they weigh less since the pressure difference provides structural stabilization without any need for rigid supports or internal frameworks. Material selection and thickness of the material have a big impact on the transmittance of an inflatable greenhouse. Current terrestrial greenhouse cladding materials lack the strength and resistance to environmental degradation needed to operate as a pressurized structure in the harsh Martian environment. Nonetheless, for greenhouse applications there exists materials used in space applications as thermal blanket materials or actual space inflatables such as polyesters, polyimides, perfluorinated polymers and some emerging materials with new capabilities [21].

1.3.1 Threats to viability

A life supporting structure in the surface of Mars raises questions of reliability and safety. Threat of micrometeorites can yield to moderate to severe damage in the structure or depressurization from a puncture. Diurnal temperature extremes could threaten the plants while both the plants and crew might be exposed to higher radiation. These and many other factors must be evaluated to determine whether or not a transparent inflatable greenhouse is even feasible [21].

1.3.1.1 Micrometeorites

The thin Martian atmosphere provides some protection against micrometeorites, however, they still pose a threat to a structure on the surface of Mars. The flux of meteorites entering Mars’ atmosphere can be estimated as:

\[
\log N = -0.689 \log m + 4.17, \tag{1.6}
\]

where \( N \) is the number of meteorites per year having a mass \( m \) that can impact on an area of \( 10^6 \; km^2 \). Threat to micrometeorites is very dependent on the wall thickness of the greenhouse, the thinner the thickness is the lighter the micrometeorite needs to be to produce a puncture. Obviously, more detailed study needs to be taken into account once designing the final structure to be carried to the red planet.

1.3.1.2 Thermal issues

Mars is a colder place than Earth. It has been determined that a transparent greenhouse maintaining adequate temperature would lose about 900 \( W/m^2 \) in terms of heat dissipation during a mid-summer’s night at the Viking landing site location. Therefore,
nighttime heat losses must be dealt with. Two solutions arise: either generate this energy by means of an active system (i.e., heater) or store the excess heat during the day in an indigenous material (i.e., rocks) or in a phase change material (e.g., water). Moreover, nighttime losses could be reduced with a insulating cover. The eventual thermal control problem could be getting rid of the excess generated heat rather than to produce heat to increase temperature.

1.3.1.3 Radiation

Radiation levels also pose a high threat to a hypothetical Martian greenhouse. On the one hand, seeds need to be transported from Earth, thus they will be exposed to interplanetary radiation. On the other hand, Martian surface radiation levels are higher than on Earth. Generally, seeds are less radiosensitive than vegetative plants, therefore, they are able to withstand interplanetary travel. Vegetative plants' shorter life cycle also prevents radiation to pose a serious threat. Moreover, the greenhouse shell can provide additional protection to radiation, particularly if it is a polymer.
Chapter 2

Greenhouse Model

In this chapter, the model of the thermal insulation system is explained in detail. An analysis of the environment of the Martian surface is performed (Sect. 2.1). The proposed structure consists of a cylindrical shell (Fig. 2.1), composed by a series of insulator cells (Sect. 2.2). The structure is inflated with a gas, assumed to have the composition of the air on Earth (Sect. 2.3). The ground is insulated by using a series of layers (Sect. 2.4). Finally, a global energy balance (Sect. 2.5) is discussed in order to verify the whole model and some optimization criteria (Sect. 2.6) are proposed. The model only considers one dimensional distributions, therefore, the shape of the greenhouse has no effect the results (i.e., a slice is considered). The only exception where the geometry is needed is in the computation of the view factors. In that case, a two dimensional half cylinder has been assumed.

Figure 2.1: Wireframe representation of the proposed greenhouse geometry.
2.1 The model of the Martian environment: MEMM

The environment on Mars is different from that which characterizes our own planet. The atmosphere contains 95.3% of CO$_2$, 2.7% of N$_2$ and 1.6% of Ar. These gases produce a mean pressure at the surface of 6.1 mbar with seasonal fluctuations. Dust particles are also present in the atmosphere. During the Viking mission, optical depths at the lander sites were measured and at no time they fell below 0.18. On a relatively clear day, the optical depth can vary between 0.2 and 0.5. The wind plays an important role in forced convection. In first approximation, a constant stream of 10 m/s can be assumed. Moreover, due to the low atmospheric density (around 0.02 kg/m$^3$) and atmospheric pressure (around 6.9 mbar), the air boundary layer can be considered to be of the order of 1 meter.

The most recent and updated atmospheric models and data have been collected and implemented into the Mars Environment Multi Model (MEMM), a macro-model of the Martian environment which provides a description of the thermal properties of the atmosphere on a spatial basis and in time. MEMM is used in this project to model the environment. Since the climate varies with the seasons, a reference epoch - Spring - has been selected to provide the conditions that characterize one typical day.

2.1.1 Convection

MEMM provides ground and atmospheric temperatures as well as the surface convection coefficient $h$. This parameter takes into account natural convection within the atmosphere and forced convection due to the winds. The heat flux $\dot{q}_{\text{conv}}$ that reaches a surface by convection is given by

$$\dot{q}_{\text{conv}} = h \left( T_\infty - T_{\text{surf}} \right),$$

where $T_\infty$ is the fluid temperature that reaches a surface and $T_{\text{surf}}$ is the temperature of the surface. On a typical Martian day, the atmospheric temperature, the ground temperature and the surface heat transfer coefficient $h$ vary as shown in Figs. 2.2 and 2.3. Such values are way lower to that found on Earth, which are around 10 W/m$^2$K.

2.1.2 Electromagnetic radiation

The electromagnetic radiation that reaches the surface can be divided into two spectral bands, i.e., a shortwave (SW) component at visible wavelengths, mainly of solar origin, and a longwave (LW) component in the infrared, mainly of Martian origin (atmosphere
Figure 2.2: Ground, sky and atmospheric temperatures on a typical Martian day.

Figure 2.3: Surface heat transfer coefficient as observed on a typical day.
Each spectral band can be further separated into direct, diffuse, reflected and emitted radiation. Direct radiation comes directly from the Sun. Diffuse radiation is that coming from the Sun but has been scattered somehow through the atmosphere. Reflected radiation comes from the sun but it is reflected back from the ground. Finally, emitted radiation is that the ground emits by the fact that it has a certain temperature.

Table 2.1 summarizes the radiation components and their type as observed at the surface. It also indicates typical day and night values. The abbreviations that appear between parentheses are those adopted in the equations and in the figure legends. Fig. 2.4 shows the evolution of such parameters over a typical day. The analysis of the radiation reaching an arbitrarily tilted surface has been carried out according to the model proposed by Donald Rapp [25].

<table>
<thead>
<tr>
<th>Direct</th>
<th>LW Radiation</th>
<th>SW Radiation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Taken into account inside $LW_d$</td>
<td>Direct radiation of solar origin ($GB$). Typical values: 500 W/m² (day), zero at night.</td>
</tr>
<tr>
<td>Diffuse</td>
<td>Coming from the Sun and scattered in the atmosphere ($LW_d$). Typical values: 50 W/m² (day), 25 W/m² (night)</td>
<td>A direct part coming from around the Sun ($GDB$) and a scattered part coming from the atmosphere ($GDTI$). Typical values: 125 W/m² (day), zero at night.</td>
</tr>
<tr>
<td>Reflected</td>
<td>Diffuse radiation coming from the ground ($LW_u$). It includes the ground-emitted radiation. Typical values: 400 W/m² (day), 80 W/m² (night)</td>
<td>Diffuse radiation coming from the ground due to albedo ($aGH$). Typical values: 100 W/m² (day), zero at night.</td>
</tr>
<tr>
<td>Emitted</td>
<td>Taken into account as $LW_u$</td>
<td>No radiation is emitted as SW by the ground.</td>
</tr>
</tbody>
</table>

2.1.2.1 Direct SW radiation

The direct solar flux is the second most important contribution at the surface, as shown in Fig. 2.4. If we consider an arbitrarily tilted planar surface, the direct solar flux varies according to the solar zenith angle $zt$, i.e., the angle between the normal to the plane and the direction to the Sun. In other words, $zt$ is the solar zenith angle as seen from a
tilted plane and it is defined as follows:

\[
\cos zt = \sin d \sin L - TT \cos d \cos L - TT \cos \left( \frac{2\pi t}{24.6} \right),
\]  

(2.2)

where \( d \) is the Sun’s declination, \( L \) is the latitude of the point of observation, \( t \) is the time in hours since noon and \( TT \) is the tilt angle, i.e., the angle of the given planar surface with respect to the local horizon. In particular, \( TT = 0 \) in Eq. 2.2 yields the conventional solar zenith angle \( z \). Fig. 2.5 shows the evolution of the solar zenith angle for different tilt angles over a typical day. The direct solar flux \( GBT \) at the given tilted plane can be computed as

\[
GBT = GB \cos zt.
\]  

(2.3)

### 2.1.2.2 Diffuse SW radiation

The diffuse solar radiation also plays an important role and it constitutes the third most important contribution at the surface (Fig. 2.4). For high optical depths and large solar zenith angles, the diffuse flux emanates isotropically from the whole sky (\( GTDI \)), whereas, for very low optical depths and small solar zenith angles, it emanates from a zone near the Sun (\( GTDS \)). The approach followed to determine this flux (\( GTD \)) is a
linear combination of these two extreme situations:

\[ GDT = k_1 GDTI + k_2 GDTS. \]  

(2.4)

\( k_1 \) and \( k_2 \) are coefficients to be estimated under the constraint that \( k_1 + k_2 = 1 \). Donald Rapp\cite{25} presents a handy function that seems to make some sense for \( k_2 \):

\[ k_2 = \frac{1}{1 + D / \cos z}, \]  

(2.5)

in which \( D \) is the optical depth. Such a function is the result of the application of various heuristic methods based on the belief that \( D / \cos z \) becomes large. However, there is no absolute way to compute such coefficients. Then \( k_1 \) follows as

\[ k_1 = 1 - k_2. \]  

(2.6)

The flux \( GTDI \) of the diffuse component on a tilted surface with tilt angle \( TT \) is given by

\[ GTDI = GDH \cos^2 \left( \frac{TT}{2} \right), \]  

(2.7)

where \( GDH \) is the diffuse solar intensity on a horizontal surface from sources other than the direct beam. On the same tilted surface, the near-Sun component produces a flux

Figure 2.5: Variation of the solar zenith angle for different tilt angles for a typical day.
\[ GTDS: \quad GDTS = \frac{GDH \cos z t}{\cos z}. \quad (2.8) \]

Summing the two contributions, as specified in Eq. \(2.4\), yields the value of the diffuse flux.

### 2.1.2.3 Reflected SW radiation

In addition to the direct beam and the diffuse component, there is a contribution from the beam that reflects at the ground in the vicinity of the given surface. Such contribution is not as important as the others (Fig. \(2.4\)). Given the albedo \(a\) of the ground, the reflected flux \(GRT\) is obtained as

\[ GRT = aGH \sin^2 \left( \frac{TT}{2} \right). \quad (2.9) \]

### 2.1.2.4 Total SW Radiation

The total SW flux \(q_{SW}\) reaching a titled plane is the sum of the direct, diffuse and reflected terms:

\[ q_{SW} = GBT + GDT + GRT. \quad (2.10) \]

### 2.1.2.5 Direct LW radiation

The direct LW radiation comes from the sun and gets scattered through the atmosphere due to CO\(_2\) and dust clouds. According to the approach of MarsGRAM\[26\] - the Mars Global Reference Atmospheric Model that is built on mathematical models and experimental measurements -, such a component is already taken into account in the LW downwards radiation.

### 2.1.2.6 Diffuse LW radiation

The LW downwards radiation is supposed to emanate isotropically from the atmosphere. As Fig. \(2.4\) shows, it only contributes a very small part to the total radiation flux \(TDR\) received by the tilted plane. This quantity is modelled similarly to the diffuse solar flux:

\[ TDR = LW_d \cos^2 \left( \frac{TT}{2} \right). \quad (2.11) \]
2.1.2.7 Reflected and emitted LW radiation

The MarsRAD [26] - the MarsGRAM module for the computing of electromagnetic radiation - radiation model returns only the LW upwards radiation. This component is the sum of the LW downwards radiation that is reflected at the ground and the thermal radiation that the ground emits. As clearly shown by Fig. 2.4, it constitutes the most important radiation flux. The approach for a tilted surface will be taken similarly to the reflected solar radiation:

\[ TUR = LW_u \sin^2 \left( \frac{T T}{2} \right). \]  

(2.12)

2.1.2.8 Total LW radiation

The treatment for the total LW radiation is equivalent to that followed for the SW radiation. The sum of the different LW contributions yields the total LW flux \( \dot{q}_{LW} \):

\[ \dot{q}_{LW} = TDR + TUR. \]  

(2.13)

2.1.2.9 Interior Radiation

The interior shell of the greenhouse exchanges heat by radiation with itself and with the ground. The following assumptions are made:

- the interior fluid is transparent to radiation;
- the surfaces are isotherm with uniform radiating properties;
- the surfaces are gray.

Then, the radiosity \( j_k \) of the \( k^{th} \) surface in a set of \( N \) is defined as

\[ j_k = \varepsilon_k \sigma T_k^4 + (1 - \varepsilon_k) \hat{g}_k, \]  

(2.14)

where \( \varepsilon_k \) is the thermal emittance of the \( k^{th} \) surface, \( \sigma \) is the Stefan-Boltzmann constant (\( = 5.670373 \times 10^{-8} \)), \( T_k \) is the temperature of the \( k^{th} \) surface and \( \hat{g}_k \) is the irradiance of the \( k^{th} \) surface. The irradiance is computed through means of:

\[ \hat{g}_k = \sum_{i=1}^{N} F_{ki} j_i, \]  

(2.15)
with $F_{ki}$ being the view factor of surface $k$ with respect to surface $i$. The view factor is a geometric property: it takes into account the effects of orientation on radiation heat transfer between two surfaces. It is computed as shown in Appendix D. Combining Eqs. 2.14 and 2.15 for all the surfaces yields a linear system of $N$ equations in $N$ unknowns.

$$
[1 - (1 - \varepsilon_1) F_{1j_1} - (1 - \varepsilon_1) F_{1j_2} - \cdots - (1 - \varepsilon_1) F_{1j_N}] = \varepsilon_1 \sigma T_1^4 \quad (2.16)
$$

$$
-(1 - \varepsilon_2) F_{2j_1} + [1 - (1 - \varepsilon_2) F_{2j_2} - \cdots - (1 - \varepsilon_2) F_{2j_N}] = \varepsilon_2 \sigma T_2^4 \quad (2.17)
$$

$$
\vdots
$$

$$
-(1 - \varepsilon_N) F_{Nj_1} - (1 - \varepsilon_N) F_{Nj_2} - \cdots + [1 - (1 - \varepsilon_N) F_{NN}] j_N = \varepsilon_N \sigma T_N^4. \quad (2.18)
$$

This system can be easily solved by any standard direct method. Once the system is solved, the fluxes are computed using Eq. 2.15. The net radiative heat flux $\dot{q}_{rad,k}$ is computed by

$$
\dot{q}_{rad,k} = j_k - \dot{g}_k = \varepsilon_k (\sigma T_k^4 - \dot{g}_k). \quad (2.19)
$$

### 2.2 Insulator cell model

The shell of the greenhouse was first devised as a layer of aerogel between two glass-like panels. However, for the reasons explained in this chapter and the ones seen in Chapter 1, the insulator system has been designed as a sandwich of materials, resembling a MLI system. Such materials are transparent to the SW radiation but opaque to the LW radiation, in a methodology similar to the one used by Curiosity (Subsect. 1.1.6). The surfaces of the material must be treated to ensure a low IR emissivity, i.e., values between 0.4 to 0.05 (Subsect. 1.2.3). The greenhouse shell is composed of square cells of size $L \times L$. Inside these cells, the atmospheric CO$_2$ is used as insulator (Subsect. 1.1.6). The mean physical properties of the atmosphere at a temperature of -120 degrees centigrade are the following:

$$
R_{CO_2} = 188,9116 \ J/kgK,
$$

$$
c_P = 685.5136 \ J/kgK,
$$

$$
\rho = \frac{P}{R_{CO_2}T} = 0.0173 \ kg/m^3,
$$

$$
\mu = 7.5775 \times 10^{-06} \ Pa \ s,
$$

$$
k = 0.0064 \ W/mK,
$$

where $R_{CO_2}$ stands for the CO$_2$ specific gas constant, $c_P$ stands for the specific heat, $\rho$ stands for the density, $\mu$ stands for the dynamic viscosity and $k$ stands for the thermal conductivity. The variations of these properties are illustrated in Fig. 2.6. The heat flux
\( \dot{q} \) across the gas is

\[
\dot{q} = Nu \frac{k}{L} \Delta T,
\]

(2.20)

where \( Nu \) is the Nusselt number and \( L \) is the characteristic length of the cell, i.e. the cell size. In turn, the Nusselt number is also a function of the Rayleigh and Prandlt numbers, \( Ra \) and \( Pr \), respectively:

\[
Nu = f(Pr, Ra)
\]

(2.21)

The latter is defined by the properties of the gas as

\[
Pr = \frac{c_p \mu}{k},
\]

(2.22)

while the former is defined as

\[
Ra_L = Gr_L Pr.
\]

(2.23)

whereas \( Gr \) is the Grasshof number,

\[
Gr_L = \frac{g \rho^2 \beta \Delta T L^3}{\mu^2}.
\]

(2.24)

\( \beta \) is the thermal expansion coefficient at constant pressure, which for an ideal gas is computed as \( \beta = 1/T \), being \( T \) the gas temperature. \( g \) is the gravity acceleration.
(\(g = 3.71 \text{ m/s}^2\) for Mars) and \(\Delta T\) is the absolute value of the temperature difference between the gas and the surface (\(\Delta T = T_\infty - T_{\text{surf}}\)).

### 2.2.1 Rayleigh number evaluation

The working principle behind this insulation method is that the Rayleigh number for the Earth is much bigger than the Rayleigh number for Mars. If Eqs. 2.22 and 2.24 are combined into Eq. 2.23, the Rayleigh number is obtained as

\[
Ra_L = \frac{g \rho c_P \beta \Delta T L^3}{\mu k}.
\]  

(2.25)

Then, Eq. 2.25 is written separately for Mars and for the Earth and the two expressions are divided. The result is

\[
\frac{Ra_E}{Ra_M} = \left(\frac{g_E}{g_M}\right) \left(\frac{\rho_E}{\rho_M}\right)^2 \left(\frac{c_{PE}}{c_{PM}}\right) \left(\frac{\beta_E}{\beta_M}\right) \left(\frac{\mu_M}{\mu_E}\right) \left(\frac{k_M}{k_E}\right),
\]  

(2.26)

where the subscript \(M\) indicates Martian quantities and the subscript \(E\) indicates terrestrial quantities. The latter parameters have been taken at the standard atmospheric temperature of 20 degrees centigrade. The term \(\Delta T\) has been dropped in Eq. 2.26 as, at both cases, the same difference of temperatures have been taken. Thus, the several ratios appearing in Eq. 2.26 take the following values:

\[
\left(\frac{g_E}{g_M}\right) = 2.64,
\]

\[
\left(\frac{\rho_E}{\rho_M}\right)^2 = 7.65 \times 10^3,
\]

\[
\left(\frac{c_{PE}}{c_{PM}}\right) = 1.37,
\]

\[
\left(\frac{\beta_E}{\beta_M}\right) = 0.665,
\]

\[
\left(\frac{\mu_M}{\mu_E}\right) = 0.545,
\]

\[
\left(\frac{k_M}{k_E}\right) = 0.354.
\]

Eventually, \(Ra_E\) is \(3.56 \times 10^3\) times larger than \(Ra_M\), as illustrated in Fig. 2.7. As a result, thanks to the low atmospheric pressure, the Martian atmosphere itself can be used as a natural insulator.
2.2.2 Nusselt number correlation

Ozoe et al. [27] present a correlation for the Nusselt number for rectangular enclosed cells with tilt angles from 0 to 150 degrees and $2 \cdot 10^3 \leq Ra \leq 1 \cdot 10^5$. Fig. 2.8 shows the scheme of the enclosed cell. Such correlation takes the form

$$Nu = A \, Ra^b,$$  \hspace{1cm} (2.27)

where $A$ and $b$ are values extracted from Table 2 and summarized in Table 2.2. Fig. 2.9 shows the heat flux $\dot{q}$ through the CO\textsubscript{2} and the Nusselt number for a tilt angle of 90 degrees and different values of $L$. The heat flux decreases down to some minimum.
Table 2.2: Correlation of $A$ and $b$ values for the Nusselt number.

<table>
<thead>
<tr>
<th>tilt angle $\psi$ (in degrees)</th>
<th>$A$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0981</td>
<td>0.336</td>
</tr>
<tr>
<td>30</td>
<td>0.0735</td>
<td>0.385</td>
</tr>
<tr>
<td>60</td>
<td>0.0832</td>
<td>0.375</td>
</tr>
<tr>
<td>90</td>
<td>0.0919</td>
<td>0.354</td>
</tr>
<tr>
<td>120</td>
<td>0.1754</td>
<td>0.253</td>
</tr>
<tr>
<td>150</td>
<td>0.546</td>
<td>0.0867</td>
</tr>
</tbody>
</table>

Figure 2.9: Heat flux $\dot{q}$ and Nusselt number as functions of the reference distance $L$.

value, then it slowly increases again. The design criterion for the insulator is to let the minimum heat transfer flux across the enclosed Martian atmosphere, therefore, a distance larger than 100 mm is suitable since the increase rate of the heat flux is very low (Fig. 2.9).

2.2.3 CO$_2$ properties correlation

A set of polynomial relations for the thermodynamic properties of CO$_2$ have been extracted from the documentation of MEMM. Such properties are the thermal conductivity $k$, the specific heat $c_P$ and the dynamic viscosity $\mu$. The relations described above are
the following:

\[ k = -9.3828 \times 10^{-11}T^3 + 1.3201 \times 10^{-07}T^2 + 2.5715 \times 10^{-05}T - 2.8962 \times 10^{-4}. \quad (2.28) \]

\[ c_P = -4.0528 \times 10^{-06}T^3 + 2.8289 \times 10^{-03}T^2 + 4.5256 \times 10^{-01}T + 564.4112. \quad (2.29) \]

\[ \mu = 0.01480 \left( \frac{0.555 \times 527.67 + 240}{0.555^2 T} + 240 \right) \frac{2}{527.67^2} \times 10^{-3}. \quad (2.30) \]

Figs. 2.10 to 2.12 show the behavior of Eqs. 2.28 to 2.30. Such adjustments are valid from 175 K to 400 K. However, it is desired to extend their validity range because the temperatures on Mars can easily get lower than 150 K and, while iterating, the solver can find temperatures above 400 K. Figs. 2.10 and 2.12 indicate that the correlation can be extended down to 100 K and beyond 400 K, whereas the specific heat (Fig. 2.11, blue dots) clearly cannot be extended, therefore the value of this parameter has been continued linearly (red dots) to obtain reasonable values. Although such an approach is not entirely correct, it has been implemented due to the lack of information on the properties out of the validity range. In any case, such values are not suitable for operating a greenhouse, and they have been implemented only in order for the solver not to stall.

2.2.4 Simple insulator cell

Fig. 2.13 shows the scheme of an insulator cell. It is composed of two low-density,
Figure 2.11: CO$_2$ specific heat $c_p$ as a function of temperature.

Figure 2.12: CO$_2$ Dynamic viscosity $\mu$ as a function of temperature.
space-proven materials which are transparent to visible radiation and opaque to infrared radiation. On the surface, a treatment of low emissivity has been executed. The ideal material for the insulator cells walls would be glass. Nevertheless, as glass is too heavy, a lightweight alternative is considered by using a material similar to Polymethyl methacrylate (PMMA). Such properties are:

\[ k = 1.93 \text{ W/mK}, \quad \rho = 1.19 \times 10^3 \text{ kg/m}^3, \quad c_P = 1.42 \times 10^3 \text{ J/kgK}, \quad \varepsilon = 0.05. \]

The temperature on the walls of a cell and the heat fluxes involved are described in Fig. 2.13. In steady state, the temperature profile of the walls present a linear behavior. Therefore, the wall temperature map can be written as

\[ T = ax + b. \]  

(2.31)

The problem lies in finding the coefficients \( a \) and \( b \) that solve the boundary conditions problem. For this reason, instead of working with the four coefficients (two per each wall), the wall temperatures will be used. Fig. 2.14 shows the boundary conditions for a simple insulator cell. Heat fluxes are considered positive when they leave the insulator cell. Therefore, for the first boundary

\[ -k \frac{dT}{dx} = \dot{q}_{\text{conv}} + \dot{q}_{\text{rad}}, \]  

(2.32)

where \( \dot{q}_{\text{conv}} \) stands for the convective flux and \( \dot{q}_{\text{rad}} \) stands for the radiative flux. Hereinafter, the subscripts for the temperatures such according to Fig. 2.14. Developing such fluxes yields

\[ -k \frac{T_2 - T_1}{L} = h(T_f - T_1) + \dot{q}_{IR} - \varepsilon \sigma (T_1^4), \]  

(2.33)
or

\[ k \frac{T_2 - T_1}{L} + h(T_f - T_1) + \dot{q}_{IR} - \varepsilon \sigma (T_1^4) = 0, \]  

(2.34)

where \( \dot{q}_{IR} \) denotes the infrared heat flux coming to the wall. For the second boundary condition, it can be written

\[ -k \frac{dT}{dx} = -(\dot{q} + \dot{q}_{rad}), \]  

(2.35)

which, upon developing the expressions, it becomes

\[ -k \frac{T_2 - T_1}{L} = -\dot{q} - \varepsilon (\dot{g}_2 - \sigma T_2^4), \]  

(2.36)

where \( \dot{q} \) denotes the heat flux across the insulator gas as shown in Fig. 2.14. Eventually, equating to zero provides

\[ k \frac{T_2 - T_1}{L} - \dot{q} - \varepsilon (\dot{g}_2 - \sigma T_2^4) = 0. \]  

(2.37)

For the third boundary,

\[ -k \frac{dT}{dx} = -(\dot{q} + \dot{q}_{rad}), \]  

(2.38)

or

\[ -k \frac{T_6 - T_3}{L} = -\dot{q} - \varepsilon (\sigma T_3^4 - \dot{g}_2), \]  

(2.39)

which must be set to zero:

\[ k \frac{T_6 - T_3}{L} - \dot{q} - \varepsilon (\sigma T_3^4 - \dot{g}_2) = 0 \]  

(2.40)

Finally, for the fourth boundary one writes

\[ -k \frac{dT}{dx} = -(\dot{q} + \dot{q}_{rad}), \]  

(2.41)
or
\[-k \frac{T_6 - T_3}{L} = -h(T_f - T_6) - \dot{q}_{IR} + \varepsilon \sigma (T_6^4), \tag{2.42}\]
and
\[k \frac{T_6 - T_3}{L} = -h(T_f - T_6) - \dot{q}_{IR} + \varepsilon \sigma (T_6^4) = 0. \tag{2.43}\]
Two more equations are needed, for the upper and lower wall, respectively. The dominant conduction is expected to occur between the outside wall (denoted with indexes 1 and 2) and the inside wall (denoted with indexes 3 and 6). For this reason, the upper and lower walls are considered adiabatic in the direction set by those two faces (i.e., from 4 to 5). In other words, there is no heat conduction considered from 4 to 2, 3 and 5. The analogous consideration is also considered from 5 to 2, 3 and 4. Therefore,
\[j_i - \dot{g}_i = 0, \tag{2.44}\]
being \(i\) either top or bottom wall. Developing the expression for \(j\) according to Eq. 2.14 gives
\[\varepsilon \sigma T_i^4 - (1 - \varepsilon) \dot{g}_i - \dot{g}_i = 0. \tag{2.45}\]
Therefore, for the upper wall
\[\sigma T_5^4 - \dot{g}_5 = 0, \tag{2.46}\]
and for the lower wall
\[\sigma T_4^4 - \dot{g}_4 = 0. \tag{2.47}\]
The heat flux \(\dot{q}_{IR}\) stands for
\[\dot{q}_{IR} = \varepsilon \sigma T_{sky}^4 + \dot{q}_{lw}, \tag{2.48}\]
where \(T_{sky}\) is the sky temperature, i.e., the temperature the sky must have to thermally radiate the LW downwards radiation, and the \(\dot{q}_{lw}\) denotes the heat flux due to the LW upwards radiation. The radiative heat that comes from the other walls, for both the exterior and in the interior boundary, can be written as
\[\dot{q}_{IR} = \varepsilon \dot{g}_k, \tag{2.49}\]
and the heat flux \(\dot{q}\) that leaves the insulator is
\[\dot{q} = N u \; k_{CO_2} \frac{T_{w_3} - T_{w_2}}{L_c} + \varepsilon (\dot{g}_2 - \sigma T_{w_2}^4). \tag{2.50}\]
The radiosity \(\dot{g}\) present in Eqs. 2.34 to 2.47 is a function of \(T_2, T_3, T_4\) and \(T_5\) and the view factors for a square-shaped enclosure. The view factors have been computed by means of the Hottel’s expression\[ for two-dimensional geometries. Moreover, \(\dot{q}\) is also a function of \(T_2\) and \(T_3\) given by Eq. 2.20. The algorithm for solving such a problem
can be summarized as follows:

1. Computing the view factors;
2. Computing the radiosities using a supposed or previous temperature $T^*$;
3. Solving the nonlinear system composed of Eqs. 2.34 to 2.47 to obtain a new temperature $T$;
4. Iterating points 2 to 3 until $|T - T^*| < \epsilon$ where $\epsilon$ is a sufficient small number, say $10^{-6}$;
5. Computing the flux that leaves the insulator cell by means of Eq. 2.50

2.2.4.1 Analysis of the shell inertia

A transient heat transfer code is run on the shell in Martian-like conditions for a small time step to assess if the thermal inertia is important in the determination of the overall shell temperature. The simulation is carried out over two Martian days and the heat flux across the insulator is plotted in Fig. 2.15. There exists a little offset between

![Figure 2.15: Heat flux across the insulator for the analytic case (blue), the numerical case (green), and the linear case (red).](image)

the transient solution and the permanent solutions. However, it is smaller than the
characteristic response time of the ground. At this point a simplification has been made, i.e., the transitory part has been disregarded. In future iterations of this model, the transitory effect should be taken into account. Moreover, there is no difference between the analytic and the linear solver, therefore, the linear solver can be used without yielding any appreciable error. The hypothesis of the insulator cell being in a steady state will be maintained.

2.2.4.2 Energy balance of the insulator cell

An energy balance in the insulator cell is determined in order to verify the results. As the insulator cell is considered to be in permanent regime, there is no accumulated heat, i.e., all the heat that enters the cell must exit by the other side. In other words, all the fluxes computed for the insulator cell must be equal. There are three interior fluxes as shown in Fig. 2.14. The flux across the gas is given by Eq. 2.50 The conduction fluxes are

\[
\dot{q}_{k1} = k \frac{T_2 - T_1}{L},
\]

\[
\dot{q}_{k2} = k \frac{T_6 - T_3}{L}.
\]

Then, the boundary fluxes are computed by means of

\[
\dot{q}_{\text{bound}1} = h (T_f - T_1) + \varepsilon (\dot{q}_{IR} - \sigma T_1^4),
\]

\[
\dot{q}_{\text{bound}2} = h (T_f - T_{end}) + \varepsilon (\dot{q}_{IR} - \sigma T_{end}^4).
\]

The computations are verified when all the fluxes take the same value.

2.2.4.3 Cell linearized solution

Eqs. 2.34 to 2.47 form a system of nonlinear equations. This fact is due to the terms at the fourth power derived from the radiation treatment. Such a system is long and time-consuming to solve as it cannot be approached using conventional methods, i.e., matrix inversion. For this reason, Eqs. 2.34 to 2.47 are linearly approximated so that the system can be solved in a faster way. An implicit methodology is adopted to ensure convergence. The first consequence of the linearization is that Eqs. 2.46 and 2.47 can be decoupled from the system and solved for directly. Eq. 2.46 takes the following form

\[
T_5 = \left( \frac{\dot{q}_5}{\sigma} \right)^{\frac{1}{3}},
\]

(2.55)
whereas Eq. 2.47 takes the form

$$T_4 = \left( \frac{\dot{q}_4}{\sigma} \right)^{\frac{1}{4}}.$$  

(2.56)

Eqs. 2.34 to 2.43 are linked, therefore they must be solved together. The superscript * indicates that the value is the last known value of the variable. The linear form of Eqs. 2.34 to 2.43 is

$$\left( \frac{k}{L} - h - \varepsilon \sigma T^{*3}_1 \right) T_1 + \frac{k}{L} T_2 = -hT_f - \varepsilon \dot{q}_{IR},$$  

(2.57)

$$-\frac{k}{L} T_1 + \left( \frac{k}{L} + \varepsilon \sigma T^{*3}_2 + Nu \frac{k_{CO_2}}{L_e} \right) T_2 + \left( -Nu \frac{k_{CO_2}}{L_e} \right) T_3 = \varepsilon \dot{q}_2,$$  

(2.58)

$$\left( Nu \frac{k_{CO_2}}{L_e} \right) T_2 + \left( -\frac{k}{L} - \varepsilon \sigma T^{*3}_3 - Nu \frac{k_{CO_2}}{L_e} \right) T_3 + \frac{k}{L} T_4 = -\varepsilon \dot{q}_3,$$  

(2.59)

$$\frac{k}{L} T_3 + \left( \frac{k}{L} + h + \varepsilon \sigma T^{*3}_6 \right) T_6 = hT_f + \varepsilon \dot{q}_{IR}.$$  

(2.60)

The algorithm for solving the linear problem can be summarized as follows:

1. Computing the view factors;
2. Computing $Nu$ and $k_{CO_2}$ using $T^*$;
3. Computing the radiositys using $T^*$;
4. Solving the linear system of Eqs. 2.57 to 2.60;
5. Computing $T_4$ and $T_5$ with Eqs. 2.55 and 2.56;
6. Evaluating if $\Delta T < \epsilon$.

If the answer is yes then stop, otherwise $T^* = T$ and the process restarts from the second step.

### 2.2.4.4 Effects of solar absorption

The insulator cell is not entirely transparent to the visible radiation. A part of the visible radiation will be absorbed in each wall and then re-irradiated inside and outside the insulator cell. The higher the number of insulator cells that comprise the shell, the larger the amount of visible radiation is absorbed. This factor is estimated to be small because the thickness of the material is also small. Fig. 2.16 shows this effect. Such effect decreases the efficiency of the insulator cell. For this model, it will be considered that all the absorbed visible radiation is re-irradiated towards the outside. This assumption is conservative because a worst-case scenario is being assumed.
2.2.5 Multilayer insulator cell

The multilayer insulator cell consists in concatenating two simple insulator cells in order to reduce the heat loss through the insulator. The multilayer insulator is composed of two cavities of Martian atmosphere and three walls of material. The scheme of such insulator is shown in Fig. 2.17. Similarly to the simple insulator cell, the multilayer configuration works in permanent state, therefore, the temperature on the walls is expected linear. Moreover, all the fluxes (those in the cavity and those in the material) must have the same value. The unknowns are the temperatures of the surface of the material. Using the same balances for the simple insulator cell, the equations for the multilayer layout
are:

\[ k \frac{T_2 - T_1}{L} + h(T_f - T_1) + \varepsilon \dot{q}_{IR} - \varepsilon \sigma T_1^4 = 0, \]  
(2.61)

\[ k \frac{T_2 - T_1}{L} - \dot{q}_1 - \varepsilon (\dot{g}_{1,2} - \sigma T_2^4) = 0, \]  
(2.62)

\[ k \frac{T_6 - T_3}{L} - \dot{q}_1 - \varepsilon (\sigma T_3^4 - \dot{g}_{1,3}) = 0, \]  
(2.63)

\[ k \frac{T_6 - T_3}{L} - \dot{q}_2 - \varepsilon (\dot{g}_{2,2} - \sigma T_6^4) = 0, \]  
(2.64)

\[ k \frac{T_{10} - T_7}{L} - \dot{q}_2 - \varepsilon (\sigma T_7^4 - \dot{g}_{2,3}) = 0, \]  
(2.65)

\[ k \frac{T_{10} - T_7}{L} - h(T_f - T_{10}) - \varepsilon \dot{q}_{IR} + \varepsilon \sigma T_{10}^4 = 0, \]  
(2.66)

\[ \sigma T_5^4 - \dot{g}_{1,5} = 0, \]  
(2.67)

\[ \sigma T_4^4 - \dot{g}_{1,4} = 0, \]  
(2.68)

\[ \sigma T_8^4 - \dot{g}_{2,5} = 0, \]  
(2.69)

\[ \sigma T_9^4 - \dot{g}_{2,4} = 0, \]  
(2.70)

where the heat fluxes \( \dot{q}_1 \) and \( \dot{q}_2 \) in the cavity also depend on the wall temperatures in the form of

\[ \dot{q}_1 = Nu_1 k_{CO_2,1} \left( \frac{T_3 - T_2}{L} \right), \]  
(2.71)

\[ \dot{q}_2 = Nu_2 k_{CO_2,2} \left( \frac{T_7 - T_6}{L} \right). \]  
(2.72)

The total heat flux across the insulator cavities is then

\[ \dot{q}_{t,1} = Nu_1 k_{CO_2,1} \left( \frac{T_3 - T_2}{L_c} \right) + \varepsilon (\dot{g}_{1,2} - \sigma T_2^4), \]  
(2.73)

\[ \dot{q}_{t,2} = Nu_2 k_{CO_2,2} \left( \frac{T_7 - T_6}{L_c} \right) + \varepsilon (\dot{g}_{2,2} - \sigma T_6^4). \]  
(2.74)

The gas properties \( Nu \) and \( k \) are evaluated for the mean temperature between the walls. The algorithm of solution is analogous to the case of the simple cell:

1. Computing the view factors for the two cavities;
2. Computing the heat fluxes \( \dot{q}_1 \) and \( \dot{q}_2 \) using a supposed or previous temperature \( T^* \);
3. Computing the radiosities \( \dot{g}_1 \) and \( \dot{g}_2 \) for the two cavities using a supposed or previous temperature \( T^* \);
4. Solving the system of nonlinear Eqs. 2.61 to 2.70;
5. Iterating points 2 to 3 until \( |T - T^*| < \epsilon \);
6. Computing the flux that leaves the insulator cell;

### 2.2.5.1 Linearized equations

Eqs. 2.61 to 2.70 are linearly approximated in the same fashion as those of the simple cell. Eqs. 2.61 to 2.66 are linked through the heat fluxes $\dot{q}_1$ and $\dot{q}_2$ and therefore must be solved together:

\[
\left( \frac{k}{L} - h - \varepsilon \sigma T_1^* \right) T_1 + \frac{k}{L} T_2 = -hT_f - \varepsilon \dot{q}_{IR} \tag{2.75}
\]

\[
-\frac{k}{L} T_1 + \left( \frac{k}{L} + \varepsilon \sigma T_1^* - Nu_1 \frac{k_{CO_2}}{L_c} \right) T_2 = -\varepsilon \dot{g}_{1.2} \tag{2.76}
\]

\[
Nu_1 \frac{k_{CO_2}}{L_c} T_2 + \left( \frac{k}{L} - \varepsilon \sigma T_2^* - Nu_1 \frac{k_{CO_2}}{L_c} \right) T_3 + \frac{k}{L} T_4 = -\varepsilon \dot{g}_{1.3} \tag{2.77}
\]

\[
-\frac{k}{L} T_3 + \left( \frac{k}{L} + \varepsilon \sigma T_3^* + Nu_2 \frac{k_{CO_2}}{L_c} \right) T_6 = -\varepsilon \dot{g}_{2.2} \tag{2.78}
\]

\[
Nu_2 \frac{k_{CO_2}}{L_c} T_6 + \left( \frac{k}{L} - \varepsilon \sigma T_6^* - Nu_2 \frac{k_{CO_2}}{L_c} \right) T_7 + \frac{k}{L} T_10 = -\varepsilon \dot{g}_{2.3} \tag{2.79}
\]

\[
-\frac{k}{L} T_7 + \left( \frac{k}{L} + h + \varepsilon \sigma T_7^* - Nu_2 \frac{k_{CO_2}}{L_c} \right) T_10 = hT_f + \varepsilon \dot{q}_{IR} \tag{2.80}
\]

The superscript $\ast$ indicates that the variable takes the last known value. Eqs. 2.67 to 2.70 can be decoupled from the system and solved directly:

\[
T_5 = \left( \frac{\dot{g}_{1.5}}{\sigma} \right)^{\frac{1}{3}} \tag{2.81}
\]

\[
T_4 = \left( \frac{\dot{g}_{1.4}}{\sigma} \right)^{\frac{1}{3}} \tag{2.82}
\]

\[
T_8 = \left( \frac{\dot{g}_{2.5}}{\sigma} \right)^{\frac{1}{3}} \tag{2.83}
\]

\[
T_9 = \left( \frac{\dot{g}_{2.4}}{\sigma} \right)^{\frac{1}{3}} \tag{2.84}
\]

The algorithm of solution is analogous to the nonlinear system. Eqs. 2.75 to 2.84 are solved using an iterative implicit method to ensure convergence. The process consists in

1. Computing the view factors for the two cavities;
2. Computing the gas properties $Nu$ and $k_{CO_2}$ using $T^*$;
3. Computing the radiosities for the two cavities using $T^*$;
4. Solving the linear system of Eqs. 2.75 to 2.80;
5. Computing the upper and lower wall temperatures using Eqs. 2.81 to 2.84;
6. Checking if $\Delta T < \epsilon$. 
If yes the solution is found, otherwise set $T^* = T$ and the process is restarted from the second step.

### 2.2.5.2 Comparison between simple and multiple insulators

The simple insulator cell and the multilayer are compared by running the cell in a Martian-like environment. The multilayer insulator cell offers a better performance than the simple type, as illustrated in Fig. 2.18. However, the simple insulator cell is easier to deploy and dissipates less visible radiation. The model will further analyze if a simple insulator cell is sufficient for maintaining a suitable temperature, otherwise, the multilayer layout will be used.

### 2.3 Interior fluid model

The greenhouse will enclose a volume $V$. Inside such volume, there will be terrestrial air at a constant pressure $P$. Natural convection occurs between the enclosed air, the ground and the walls. Such phenomenon results in a very complex movement of the fluid which makes impossible to define a meaningful averaged fluid temperature to be
used with a suitable heat transfer coefficient to evaluate a heat flow. For this reason, a Computational Fluid Dynamics (CFD) simulation would be needed to correctly model the buoyancy convection of the confined air. However, the addition of a CFD model inside the greenhouse and, in particular, the associated complex boundary conditions are out of the scope of this project. This is the reason why a different approach has been adopted.

2.3.1 Natural convection analysis

Let us see how the natural convection in the greenhouse occurs. The natural convection constitutes the link between the ground and the wall. A two-dimensional approach, as proposed by Fujii et al.\textsuperscript{[29]} where an experimental study is described concerning natural-convection heat transfer from a plate with arbitrary inclination, is not applicable to this case because by reducing the characteristic length of the wall the convection coefficient would tend to zero. Consequently, a correlation for the convection coefficient for hemispherical cavities is needed. The present deduction is based on the correlations provided by Shiina et al.\textsuperscript{[30]} and Baïri\textsuperscript{[31]}:

\[ \overline{Nu} = k \, Ra^n, \]  

(2.85)

where \( \overline{Nu} \) is the averaged Nusselt number and \( Ra \) is the averaged Rayleigh number. The definition by Shiina et al. of the Rayleigh number is similar to Eqs. 2.22, 2.24 and 2.23. However, Baïri\textsuperscript{[31]} adopts a different definition for the Rayleigh number:

\[ Ra = \frac{g \beta R^4 \rho}{\mu k \alpha} (T_g - T_w), \]  

(2.86)

where \( R \) is the radius of the hemispherical dome. Both authors use the dome radius as the reference length and the difference between the dome and the ground as the temperature difference. Table 2.3 shows the values of \( k \) and \( n \) and their range of validity, while Fig. 2.19 compares the range of validity of the correlations. The convection coefficient \( h \) between the ground and the walls is computed with the definition of the Nusselt number:

\[ h = \frac{\overline{Nu} \, k}{L}. \]  

(2.87)
In some cases, the Rayleigh number computed for the interior fluid might be out of the range of the correlation. The reason is that on Mars the conditions are different from those found on the Earth (i.e., the gravity is approximately one third that measured at the surface of the Earth). In such cases, the Rayleigh number will be set at the minimum specified by the correlation, and the convection coefficient will be computed using the minimum value of the Rayleigh number. With such a convection coefficient, the temperature of the fluid is not needed for coupling the dome and the ground. Such coefficient changes the boundary conditions of both the shell and the ground. Note that in Subsects. 2.2.4 and 2.2.5, convection has been considered for both the shell and the ground with the air. Such discussion is kept for general purposes, i.e., coupling with a CFD code. In order to use the present definition, the convection coefficient of the air has to be changed with that deduced in this part and the air temperature set as the shell temperature or the ground temperature, respectively. Shiina et al. also provide images of how the fluid moves inside the spherical dome. Fig. 2.20 clearly illustrates why a meaningful average fluid temperature cannot be computed. From Figs. 2.20a and 2.20b it is evident that the fluid near the ground will have a temperature similar to the ground and the stream will come upwards from the center of the dome, leaving the shell colder. Nevertheless, the fluid in the greenhouse will present a behavior more similar to Fig. 2.20c which is in turbulent regime. For this reason, the simplification made will

\[
\begin{align*}
0.382Ra^{0.253} & \\
0.370Ra_t^{0.25} & \\
0.0658Ra_t^{0.33} &
\end{align*}
\]
yield a better approximation.

Note that this treatment (computing a heat transfer coefficient between the dome and the ground) does not allow to compute the air temperature. Such temperature is a key parameter of relevance for the greenhouse since it indicates its performance. By taking into account that the fluid will never be hotter than the ground nor colder than the shell, the fluid temperature has been arbitrarily evaluated as the arithmetic average of the two:

\[ T_f = \frac{T_w + T_g}{2}, \]

being \( T_f \) the fluid temperature, \( T_w \) the shell mean interior surface temperature and \( T_g \) the ground surface temperature.

### 2.4 Ground insulation

The ground is considered as the part in which the greenhouse stands. The most important losses occur through the ground, therefore, a thorough modeling is needed. This is carried out with a one-dimensional transient heat transfer algorithm. Such algorithm is fully detailed in Appendix C. The main role of the ground is to accumulate heat during the day and release it during the night thus making the night and day temperature difference smoother. The layout for the greenhouse ground is as described in Fig. 2.21.
Chapter 2. The greenhouse model

Such layout includes a first thin layer with the role of absorbing solar energy. This layer controls the optical surface properties of the ground, i.e., the thermal emissivity \( \varepsilon \) and the solar absorptivity \( \alpha \). Then a layer of a certain material, which can be solid or liquid, with moderate thermal conductivity will act as accumulator. Its role is to have a high thermal inertia to provide heat during the night and accumulate it during the day. In order to prevent this material from losing heat through the ground, a layer of low density space-proven insulator is added. Finally, a thick enough Martian ground layer is added so that the top heat transfer is independent of the bottom boundary conditions.

2.4.1 Liquid thermal inertia accumulator

Water in liquid state is needed for the plants to survive, however, it can also be employed as a thermal inertia accumulator, as water is an excellent and sensitive heat storage material. The temperature of the water is computed by performing a simple energy balance on itself. The heat \( q_u \) that comes from the upper layer is

$$ q_u = h (T_u - T_f), $$

(2.89)

the heat \( q_b \) that comes from the bottom layer is

$$ q_b = h (T_b - T_f). $$

(2.90)

Here \( T_u \) is the temperature of the upper layer, \( T_b \) is the temperature from the lower layer, \( T_f \) is the water temperature and \( h \) is the convection coefficient between the layers and the water. Then, the energy balance reads

$$ \rho c p V \frac{dT}{dt} = \dot{q}_b + \dot{q}_u, $$

(2.91)
in which \( \rho \) and \( c_P \) refer to the water. By numerically integrating Eq. 2.91 one obtains
\[
\rho c_P V \frac{T_f - T_f^{i-1}}{\Delta T} = h (T_b - T_f) A + h (T_u - T_f) A.
\] (2.92)

By calling \( C \) the following expression
\[
C = \frac{\rho c_P V}{hA\Delta T},
\]
Eq. 2.92 can be written as
\[
C \left( T_f - T_f^{i-1} \right) = T_b + T_u - 2T_f.
\] (2.93)

Therefore,
\[
T_f = \frac{T_b + T_u + CT_f^{i-1}}{C + 2},
\] (2.94)
directly obtaining the temperature of the fluid.

### 2.4.1.1 Water thermal properties

Water inside the cavity of the greenhouse is considered to be in liquid state. Therefore, correlations for water properties in liquid state are needed. Nevertheless, liquid water is not possible due to Mars low temperatures which are usually under 0 degrees centigrade. Then water starts as a solid and melts to liquid state as the greenhouse begins heating itself. Therefore, correlations for ice are needed as well. Such a treatment would be too complicated for a first analysis. For this reason, water properties are considered fixed at the ones found at 15 degrees centigrade:
\[
\begin{align*}
    k &= 0.563 \text{ W/mK}, \quad (2.95) \\
    \rho &= 1000 \text{ kg/m}^3, \quad (2.96) \\
    c_P &= 4210 \text{ J/kgK}, \quad (2.97) \\
    \beta &= 0.000214 \text{ 1/K}, \quad (2.98) \\
    \mu &= 1.307 \text{ Pa s}. \quad (2.99)
\end{align*}
\]

Note that such an approach should be refined in further iterations of this model.

### 2.4.1.2 Water free convection coefficient

An empirical correlation for the water free convection coefficient is obtained from Incropera [32]. The convection case is modeled as two separate horizontal plates. Two cases
arise:

- Hot surface up or cold surface down;
- Lower surface of hot plate or upper surface of cold plate.

For the first case, two correlations are considered. The first,

$$\bar{Nu}_L = 0.54Ra_L^{\frac{1}{4}},$$

is valid for $10^4 \leq Ra_L \leq 10^7$ and $Pr \geq 0.7$. The second,

$$\bar{Nu}_L = 0.15Ra_L^{\frac{1}{3}},$$

is valid for $10^7 \leq Ra_L \leq 10^{11}$ and any value for $Pr$. The following correlation is considered for the second case with the validity limits of $10^4 \leq Ra_L \leq 10^9$ and $Pr \geq 0.7$:

$$\bar{Nu}_L = 0.52Ra_L^{\frac{1}{2}}.$$  \hfill (2.102)

The length $L$ is defined as follows:

$$L = \frac{As}{P},$$

where $As$ is the area of the flat plate and $P$ is its perimeter. Finally, the convection coefficient $h$ is retrieved from the Nusselt number through its definition:

$$Nu = \frac{hL}{k}.$$  \hfill (2.104)

### 2.4.2 Mars ground properties

Martian ground properties are a big uncertainty for the model. Thermal properties are needed in order to run the heat transfer code. The literature suggests that the thermal properties of regolith and megaregolith should closely approximate those of terrestrial frozen soil and basalt.\(^{[33]}\) Thermal diffusivity is also uncertain, the values of density and specific heat being largely unknown. It has been assumed an average value of $10^{-6}$ m\(^2\)/s.\(^{[33]}\) Literature on this topic is conflicting and there is not a uniform criteria on which to base the properties. Table\(^{[2.4]}\) summarizes the different properties obtained from the literature.
### Table 2.4: Thermal properties of the Martian ground.

<table>
<thead>
<tr>
<th>Soil</th>
<th>$k$</th>
<th>$\rho$</th>
<th>$cp$</th>
<th>$\alpha$</th>
<th>Author</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry</td>
<td>0.144 W/m$^2$K</td>
<td>-</td>
<td>-</td>
<td>$10^{-6}$ m$^2$/s</td>
<td>Gori, Corasaniti[33]</td>
</tr>
<tr>
<td>Frozen</td>
<td>2.70 W/m$^2$K</td>
<td>-</td>
<td>-</td>
<td>$10^{-5}$ m$^2$/s</td>
<td>Gori, Corasaniti[33]</td>
</tr>
<tr>
<td>Dry</td>
<td>4.2 $10^{-4}$ cal/cm$s$K</td>
<td>2 g/cm$^3$</td>
<td>0.15 cal/gK</td>
<td>-</td>
<td>Leovy[34]</td>
</tr>
<tr>
<td>Mean</td>
<td>0.37 W/m$^2$K</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Urquhart[35]</td>
</tr>
</tbody>
</table>

#### 2.4.3 Soil thermal penetration

An important parameter for the soils is the thermal wave penetration, i.e., how the heat propagates from the surface to the interior. Such a problem has an analytic solution by several methods[36]. One of such methods is the amplitude equation presented here.

$$T(z, t) = \overline{T} + T_0 \exp \left( -z \sqrt{\frac{\omega}{2\alpha}} \right) \sin \left( \omega t - z \sqrt{\frac{\omega}{2\alpha}} \right),$$  \hspace{1cm} (2.105)

subject to the following boundary conditions:

$$T(0, t) = \overline{T} + T_0 \sin \omega t$$  \hspace{1cm} (2.106)

$$T(\infty, t) = \overline{T}.$$  \hspace{1cm} (2.107)

$\overline{T}$ is the average soil temperature, $T_0$ is the amplitude of the surface temperature wave and $\omega$ the radial frequency equal to $2\pi/P$ with $P$ being the period of the fundamental cycle. $\alpha$ is the thermal diffusivity of the soil defined as

$$\alpha = \frac{k}{\rho c_p}.$$  \hspace{1cm} (2.108)

The analytic solution is then compared to the solution given by the one-dimensional conduction algorithm for the same parameters of $\overline{T}$, $T_0$ and $\omega$ in order to check its validity. Fig. 2.22 shows the results of such test for various depths and compares analytic results with numerical results. These results validate some previous assumptions:

- Firstly, the conduction algorithm is working properly as the results obtained for both numerical and analytic methods are practically the same, therefore, the proposed heat transfer code has been verified to provide correct and accurate answers.

- Secondly, an insulator layer is needed. Fig. 2.22 shows that the ground temperature at 10 cm depth is still changing, thus heat is being dissipated to the ground. To avoid such a fact, an insulator layer at about this depth is needed to retain heat to be released at night.
Figure 2.22: Thermal wave penetration in the Martian soil for $T$ of 25 deg. centigrade, $T_0$ of 25 deg. centigrade and $\omega$ that of $P = 24$ hours and various depth $z$. Numerical results in top graph and analytic results in bottom graph.

2.4.4 Comparison of ground surface temperature with MEMM model

To assess the thermal wave penetration of the Martian soil and to verify the computation from the data obtained by MEMM, the algorithm presented in Appendix C is used to obtain the temperature at a certain depth. The factors of the Martian ground that produce more uncertainty are the convection coefficient $h_c$ and the surface emissivity $\varepsilon$, apart from the ground properties themselves.

Such verification is made by recomputing two outputs from MEMM using a heat conduction code. Such outputs are the surface temperature and the emitted infrared radiation $LW_u$. The boundary conditions for the heat transfer code are the radiation on the surface with

$$\dot{q}_{vis} = GB + GDH,$$  \hspace{1cm} (2.109)

$$\dot{q}_{IR} = (1 - \varepsilon)Lw_d - \varepsilon \sigma T^4.$$  \hspace{1cm} (2.110)
The value of the emissivity is computed by using data outputs from MEMM. The infrared radiation balance yields

$$LW_u = \varepsilon \left( \sigma T_{\text{surface}}^4 + LW_d \right). \quad (2.111)$$

Therefore,

$$\varepsilon = \frac{LW_u}{\sigma T_{\text{surface}}^4 + LW_d}, \quad (2.112)$$

where $T_{\text{surface}}$ is the surface temperature obtained by MEMM. The other boundary condition, at a certain depth $d$ is considered adiabatic.

Simulations are run through a Martian year and repeated many times to ensure having reached a steady state. The tests to be made are as follows:

- Mars ground properties and $h_c$ by checking that the resulting surface temperature equals the output from MEMM;
- Mars ground properties and $\varepsilon$ by checking that the computed $LW_u$ according to Eq. 2.111 equals the output $LW_u$ from MEMM;

Figs. 2.23 to 2.25 show the comparison between the output from MEMM and the results of running the heat transfer code for various materials of Table 2.4. Therefore, the material used for modeling the Martian soil will be the dry one proposed by Gori as it is that which best approximates the output results from MEMM (see Fig. 2.23). An interesting result is to compute the temperature evolution of the Martian ground along one Martian year. The mean Martian ground temperature will be interesting to be used as a starting temperature for further computations. Fig. 2.26 shows the ground temperature evolution for various depths. At a depth of 10.50 cm (cyan), the temperature varies from -20 to -80 degrees centigrades. For this reason, a mean temperature of -50 degrees centigrades will be taken as input for further greenhouse simulations.

## 2.5 Global energy balance

Up to this point the model has been thoroughly explained. Validations in the form of energy balances have been executed in all the modules. Nevertheless, a means to assess the validity of the whole greenhouse model is needed, therefore, a global energy balance is devised. Such balance is easy to be explained: all the energy that comes into the greenhouse must be absorbed by the greenhouse ground, i.e., the ground accumulator, the water layer and the Martian ground. Simplifications made so far and ground deep
Chapter 2. The greenhouse model

Figure 2.23: Mars ground temperature comparison with MEMM (green) and according to Table 2.4 (blue) for dry soil.

Figure 2.24: Mars ground temperature comparison with MEMM (green) and according to Table 2.4 (blue) for wet soil.
Figure 2.25: Mars ground temperature comparison with MEMM (green) and according to Table 2.4 (blue) for dry soil proposed by Leovy.

Figure 2.26: Evolution of the Martian ground temperature over one year at various depths.
adiabatic boundary condition have yielded into such result. The energy $Q_{in}$ that enters the greenhouse is the direct visible solar radiation, convection with the Martian atmosphere and infrared radiation to the Martian ground and atmosphere:

$$
\dot{Q}_{in} = \alpha \dot{q}_{GBT} A_{ground} - h_{Mars} (T_w - T_{atm}) A_{walls} - \varepsilon_w (\sigma T_w^4 - \dot{g}_w),
$$

(2.113)

where $\alpha$ and $\varepsilon$ refer to the ground. $h_{Mars}$ refers to the Martian atmosphere. $\dot{q}_{GBT}$ is the direct SW heat flux explained in Sect. 2.1 $A_{ground}$ is the area of the ground and $A_{walls}$ is the area of the walls. $T_w$ refers to the greenhouse walls and $T_{atm}$ to the atmosphere. The energy $Q_{abs}$ that is absorbed by the ground is

$$
\dot{Q}_{abs} = \rho_g c_{P_g} k_g \frac{T_g - T_{ant}^g}{dt} A_{ground},
$$

(2.114)

where $\rho_g$ $c_{P_g}$ and $k_g$ refer to the ground as well as $T_g$. $T_{ant}^g$ is the ground temperature at the previous time instant and $dt$ is the time step. Eq. 2.114 is only valid for a solid ground. If there is a water layer on the ground, the absorbed part for the water must be also taken into account, thus, a more general expression would be

$$
\dot{Q}_{abs} = \sum \dot{Q}_{abs-g_i} + \dot{Q}_{abs-f},
$$

(2.115)

where $\dot{Q}_{abs-g_i}$ is computed using Eq. 2.114 for a certain ground layer and

$$
\dot{Q}_{abs-f} = \rho_f c_{P_f} V_f \frac{T_f - T_{ant}^f}{dt} A_{ground},
$$

(2.116)

where the subscript $f$ denotes the properties refer to the water. Finally, the balance closes when $\dot{Q}_{abs}$ is equal to $\dot{Q}_{in}$.

### 2.6 Optimization

Once the code is written, there are some tweaks that can be done in order to optimize it. The code has been implemented in Matlab. This platform provides robust tools for code debugging. These are:

- **Code analyzer:** it analyzes the written code and returns in either orange or red the sections of the code which can or must be improved. A highly-optimized code would show green in the code analyzer.

- **Tic-Toc:** `tic` and `toc` compute the elapsed execution time of a chunk of code starting with `tic` and ending with `toc`. 
• **Profiler**: the Profiler debugs the code and computes the amount of time dedicated to run each line and section of the code. It also returns how many times a chunk of code is called.

When assessing the performance of a code, it is important to take into account the following issues:

• how much time a chunk of code is executed;
• how many times a chunk of code is called.

So, a function that takes 0.001 seconds but is called 10000 times is more critical than one that takes 1 second but is called only once, therefore, a slight improvement of a few milliseconds in a function can yield a great overall improvement. In general, the following actions can be used to improve the overall performance of a code:

1. **Pre-allocating**: Matlab has no problem in working with dynamically-allocated matrices. However, such an approach supposes that Matlab is constantly deallocating and re-allocating the variables, which is an expensive operation. Besides, it is executed many times. Working with pre-allocated vectors yields a great boost of performance.

2. **Vectorization**: Matlab is designed to work using matrices. Matlab’s matrix libraries and algorithms are highly developed to be fast and reliable. On the other hand, *for* loops are expensive to compute. They should only be used when no other option is possible. Code vectorization consists in rewriting such loops using Matlab’s native matrix and array operations.

3. **Avoid code and operation redundancy**: avoiding multiple calls of highly-used functions, i.e., `length` or `optimset`, when they can be called once also improves code. Also, making the correct use of Matlab intrinsic functions helps.

4. **MEX functions**: when a function needs to be really fast but it has reached the maximum execution speed with Matlab, a possibility is to use a MEX function. MEX functions are code compiled in FORTRAN or C into Matlab. When compiled code is being executed, it is generally faster. Furthermore, Matlab *Coder* can also be used to generate MEX functions from Matlab functions and scripts.

5. **Parallel computing**: parallel computing is not always the best solution and it should only be used when the serial code is fully optimized. Parallel computing becomes extremely useful in batch simulations such as genetic algorithms.
Chapter 2 The greenhouse model

For the model presented, generally options 1 to 3 have been used extensively. However, the use of Profiler still showed some bottlenecks which have been assessed independently.

2.6.1 fsolve versus linearized algorithms

Matlab’s fsolve routine is really easy to use, but it yields long execution times. To avoid such a drawback, the previously presented linear algorithms are used. Such algorithms enable to solve a nonlinear system of equations by simple matrix solving but have to be iterated to find the correct solution and avoid unwanted oscillations. A sample case is run using the simple insulator cell and the multilayer insulator to assess their execution times. For the simple insulator layer using the fsolve routine the execution time is of 2.64 seconds whereas for the linear algorithm on the same problem the execution time is of 1.30 seconds, thus, linear algorithms generally offer better performance.

2.6.2 Matrix solvers

A really good approach for a matrix solver is the use of Matlab’s intrinsic operation ”\”. Such operation stands for

\[ x = A^{-1}b \]

and reads

\[ x = A \backslash b. \]

Such operation uses Matlab’s hyper threading and highly optimized matrix computations to yield really low execution times (of orders of 0.0001 sec). However, in the case of a sparse matrix a workaround must be sought. Appendix C shows Thomas’ algorithm for fast resolution of sparse matrices. Such algorithm has been programmed both in Matlab and in FORTRAN and its execution times have been compared with a Gauss-Seidel solver. The computation of the following simple system of three equations

\[
\begin{align*}
100 \; x_1 &= 52 \; x_2 + 11250 = 0 \quad (2.117) \\
110 \; x_2 &= 45 \; x_1 + 60 \; x_3 + 10000 \quad (2.118) \\
150 \; x_3 &= 35 \; x_2 + 30000 \quad (2.119)
\end{align*}
\]

took 0.000136 seconds with Gauss-Seidel, 0.000080 seconds with the TDMA solver and 0.000056 seconds with the MEX TDMA solver. Therefore, the MEX solver is faster and helps reducing the bottleneck of a function that is called many times.
Chapter 3

The greenhouse: design and simulations

This chapter is going to illustrate the thermal design parameters of a hypothetical greenhouse in Mars. The analysis methodology implemented is such that the parameters will be computed to assess a feasible greenhouse. Then materials with similar properties are going to be analyzed. Firstly, the design variables are going to be described in Sect. 3.1. Secondly, an optimization methodology is going to be proposed by successively trying different parameters (Sect. 3.2). A means of thermal regulation for the greenhouse is going to be suggested in Sect. 3.2.2. In the whole chapter, the results obtained from running the code are going to be thoroughly discussed.

3.1 Design parameters of a Martian greenhouse

The model presented in Chapter 2 is used to evaluate the thermal feasibility of a hypothetical greenhouse in Mars, therefore, a number of parameters appear that have an impact in the behavior of the greenhouse. These parameters are presented in this section. It must also be noted that the design has been refined while the results were being obtained, i.e., water was added when the lack of thermal inertia was noticed.

3.1.1 Crop selection

The design of the greenhouse consists in finding the right set of parameters that yield an adequate operative temperature on the interior of the greenhouse for the crops to develop, if such a fact is indeed possible. Fig. 3.1 shows that crops of group I require an
optimum temperature of 15 to 20 degrees centigrade, however, they are able to withstand temperatures of 5 to 30 degrees centigrade. As crops of group I are the most basic ones (i.e. potato, tomato, cabbage, etc.), the operate temperature of such group is going to be considered as design criterion. Nevertheless, the design can be adjusted for other crop groups.

3.1.2 Location

Location of the greenhouse has a major influence on the received solar flux as well as a minor effect in convection and ground composition. The martian climate (Fig. 3.2) indicates that the equatorial latitudes are the most reasonable place to set up the greenhouse, therefore, these latitudes have been selected for subsequent analyses. Among the many options, the following four sites have been considered:

- **Ares Vallis**, which is the landing site of the Pathfinder mission;
- **Gale Crater**, which is the landing site of the Curiosity rover;
- **Gusev Crater**, which is the landing site of the Spirit and Opportunity rovers;
- **Valles Marineris**, which is a rather interesting equatorial location because it is a huge valley with many possible suitable places where to hide from solar radiation.

3.1.2.1 Martian seasons

Seasons strongly depend on the orbit of Mars. The planet has a rather eccentric orbit and an obliquity of 25.19 degrees. Such characteristics yield seasons similar to that on Earth. MEMM[24] defines the seasons according to the northern hemisphere, therefore, summer is defined when the planet is in the aphelion while winter is defined when the planet is in the perihelion (Fig. 3.3). Likewise, for the southern hemisphere winter occurs when the planet is on the perihelion and summer occurs when the planet is on the aphelion. For this reason, temperatures during summer in the southern hemisphere are low whereas temperatures during winter are high. Moreover, temperature variations on the northern hemisphere are milder than they are in the southern hemisphere, albeit average temperatures in the northern hemisphere are generally colder than in the southern. All these aspects must be considered in order to select a suitable location.
### Crop Adaptability Inventory

<table>
<thead>
<tr>
<th>Photosynthesis characteristics</th>
<th>Crop group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>Photosynthesis pathway</td>
<td>C_3</td>
</tr>
<tr>
<td>Radiation intensity at max. photosyn. [J / (cm²-min)]</td>
<td>0.8 - 2.5</td>
</tr>
<tr>
<td>Max. net rate of CO₂ exchange at light saturat. [mg / (dm²-h)]</td>
<td>20 - 30</td>
</tr>
<tr>
<td>Temperature response of photosynthesis</td>
<td></td>
</tr>
<tr>
<td>optimum temp. operative temp.</td>
<td>15 - 20 °C</td>
</tr>
<tr>
<td></td>
<td>5 - 30 °C</td>
</tr>
<tr>
<td>Max. crop growth rate [g / (m²-day)]</td>
<td>20 - 30</td>
</tr>
<tr>
<td>Water use efficiency g / g</td>
<td>400 - 800</td>
</tr>
<tr>
<td>Crop species</td>
<td>Field mustard, potato, oat, tomato, rye, grape, potherum, sugarbeet, bread wheat, chickpea, french bean, arabic coffee, sunflower, olive, barley, cabbage, lentil, linseed</td>
</tr>
</tbody>
</table>

**Figure 3.1:** Crop adaptability parameters for various crop groups (from Eckart).
Figure 3.2: Map depicting average temperature (in Kelvin) of Martian atmosphere with topographic markings. Locations are marked on the map. (From MEMM [24]).

Figure 3.3: Orbit of Mars. Summer season is located on the aphelion and winter season is located on the perihelion.
3.1.3 Greenhouse shell

The main mechanism of heat transfer is via thermal radiation to the environment. For this reason, it is reasonable to believe that the thermal emissivity of the surfaces of the insulator cell presented in Chapter 2 must be as low as possible. In Subsect. 1.2.3 such materials were described, and it was pointed out that thermal emissivity can be as low as 0.05, therefore, such value is assumed in a first attempt at setting the thermal emissivity of the insulator cell. In addition, a fraction of solar radiation will be bounced back outside the greenhouse or absorbed by the insulation materials (Subsect. 2.2.4). Such an effect is modeled by the parameter $f$. This parameter is indicative of the efficiency of the insulator cell performance as it reduces the direct SW radiation that gets to the greenhouse ground. The insulator cell must be made of a low density and transparent material in order to let solar radiation pass through (Subsect. 2.2.4). The requirement of low density is essential as the materials will have to be transported to Mars. The solution proposed in Subsect. 2.2.4 is the usage of a lightweight material with glass-like optical properties. Such material is considered to have properties similar to Polymethyl Methacrylate (PMMA). Thermal inertia on the insulator cell has been neglected to avoid further complications on the model (Subsect. 2.2.4.1). A longitude of 1 mm is considered for the insulator cell walls in order to reduce structural weight but it should be a compromise with the structural needs due to pressure (which have been also neglected). Eventually, the choice between the simple or the multiple insulator cell is proposed. The decision between one type or the other is also a design consideration and it is related to the average interior greenhouse temperature.

3.1.4 Interior

The interior fluid is decoupled from the wall and ground temperatures (Sect. 2.3). The only design parameter left is the interior fluid pressure. It is desired for the plants to have similar conditions as in Earth, therefore, it is supposed that the interior pressure is equal of that on Earth’s surface, which is of 1 bar. Interior fluid pressure has a major impact on the ground-shell heat transfer coefficient though as the air properties are needed in order to compute the Prandtl and Rayleigh numbers.

3.1.5 Greenhouse ground

The ground insulator layout (Fig. 3.4) has been fully discussed in Sect. 2.4. The superficial layer of the ground consists in a very thin coating which sets the optical properties
of the ground, i.e., thermal emissivity and solar absorptivity. These parameters are key in regulating the greenhouse interior temperature.

![Diagram of insulator layout for the ground](Image)

**Figure 3.4:** Scheme of the insulator layout for the ground.

### 3.1.5.1 Absorption layer

The absorption layer consists of a very thin coating that helps absorbing as much Solar radiation as possible. It is devised as a black paint cover or volcanic ashes. This layer is very thin, therefore, it is treated as a coating for the further layer (i.e., it only affects the optical properties).

### 3.1.5.2 Accumulator layer

The accumulator layer’s mission is to soften the day-night temperature difference by absorbing heat during the day and releasing it during the night. It has been thought that this layer will have a thickness $L_g$ of 2 cm as a starting value. Two possible solutions are formulated:

- The first solution consists in a section of Martian soil which has its conductivity enhanced by the addition of some metallic rods on it (Fig. 3.5). As a result, its properties (i.e., density and specific heat) are also increased. Such an addition is needed in order to use the native soil as accumulator, otherwise the heat conductivity is not high enough to transfer the heat at an adequate rate. Other materials could have been added, but weight constitutes a serious drawback, therefore, the thin metal rods seem to be a suitable option. Thermal conductivity $k_{mat}$ is computed assuming a one dimensional temperature distribution and that the heat across the material $q_{mat}$ is equal to the heat across the ground $q_g$ plus the heat across the metal rods $q_{rod}$:

$$q_{mat} A_{mat} = q_{rod} A_{rod} + q_g A_g,$$  \hspace{1cm} (3.1)
where $\dot{q}$ is the heat flux and $A$ is the area. Subscripts refer to the composite material, metal rods and ground respectively. By developing Eq. (3.1)

$$
k_{mat} \Delta T_{mat} = k_{rod} \Delta T_{rod} + k_g \Delta T_g; \tag{3.2}
$$

$$
k_{mat} = \frac{k_{rod} A_{rod} + k_g A_g}{A_{mat}}. \tag{3.3}
$$

Eq. (3.3) is used to compute the thermal conductivity of the enhanced soil. The density of the material is computed through the definition of density:

$$
\rho_{mat} = \frac{m_{mat}}{V_{mat}}, \tag{3.4}
$$

where $\rho$ stands for the density, $m$ for the mass and $V$ for the volume. The mass of the composite material can be written as:

$$
m_{mat} = \rho_{rod} A_{rod} L_g + \rho_g A_g L_g, \tag{3.5}
$$

then,

$$
\rho_{mat} = \frac{\rho_{rod} A_{rod} L_g + \rho_g A_g L_g}{A_{mat} L_g}, \tag{3.6}
$$

$$
\rho_{mat} = \frac{\rho_{rod} A_{rod}}{A_{mat}} + \frac{\rho_g A_g}{A_{mat}}. \tag{3.7}
$$

Eq. (3.7) is used to compute the density of the enhanced soil. Eventually, the specific heat $c_P$ by using:

$$
c_{Pmat} = \frac{dH}{m_{mat}dT}, \tag{3.8}
$$
where \( dH \) is the enthalpy jump produced by the increase of temperature \( dT \). On the other hand,

\[
\frac{dH}{m_{mat}} = \frac{\rho_{rod} A_{rod} L_g c_{P_{rod}} dT + \rho_{g} A_{g} L_{g} c_{P_{g}} dT}{\rho_{mat} A_{mat} L_{g}}. \tag{3.9}
\]

By putting both expressions together, the following relation is obtained:

\[
c_{P_{mat}} = \frac{\rho_{rod} A_{rod} c_{P_{rod}} + \rho_{g} A_{g} c_{P_{g}}}{\rho_{mat} A_{mat}}. \tag{3.10}
\]

Eq. 3.10 is used to compute the specific heat of the enhanced soil. Eqs. 3.3, 3.7 and 3.10 depend on the area relations \( A_{rod}/A_{mat} \) and \( A_{g}/A_{mat} \). \( A_{mat} \) is the total area of the enhanced ground whereas \( A_{rod} \) and \( A_{g} \) stand for the areas of the components. \( A_{rod}/A_{mat} \) is related to the weight that will be carried to Mars. For design considerations, it has been decided not to surpass 40 kg, which is the weight of a 2 cm column of water. The weight of the rods is computed by means of the density of the metal. Such metal must be an excellent heat conductor. For this reason, copper is used as the material to manufacture the metallic rods. The weight of one rod is computed by using the density of copper which is 8960 kg/m\(^3\), therefore, the area relation \( A_{rod}/A_{mat} \) is 0.1. The area relation \( A_{g}/A_{mat} \) is computed as

\[
A_{g}/A_{mat} = 1 - A_{rod}/A_{mat}, \tag{3.11}
\]

and takes a value of 0.9. By using Eq. 3.3, the thermal conductivity of the enhanced soil is of 40.23 W/mK. For the density and specific heat, only the product is known, as it is computed through the thermal diffusivity:

\[
\rho c_{P} = \frac{k}{\alpha}. \tag{3.12}
\]

For the Martian soil \( k \) is 0.144 W/mK and \( \alpha \) is \( 10^{-6} \) m\(^2\)/s, therefore, the \( \rho c_{P} \) product is \( 1.44 \times 10^5 \). It has been estimated that the density of the Martian soil is around 1500 kg/m\(^3\), thus, the specific heat takes the value of 96 J/kgK. Then Eq. 3.7 is used to estimate the density of the enhanced ground, which yields 2246 kg/m\(^3\). Finally, Eq. 3.10 is used to compute the specific heat which yields 213.3 kg/m\(^3\).

- If the results prove that the enhanced ground accumulator properties are not enough to face the day-night temperature difference in Mars, the enhanced ground will be replaced by water. Liquid water is needed for watering the plants and its part of their life cycle, therefore, liquid water must be carried to Mars anyway. In this analysis, it has not been taken into account that water is, in fact, a phase change material (i.e., at 0 degrees centigrade it changes between solid and liquid).
Such a change absorbs or releases energy, therefore, temperature does not change when water is in phase change status. The energy that water needs to turn from liquid to solid is 334000 J/kg. When divided by water’s specific heat, 4210 J/kgK, the temperature that water absorbs or releases when changing state is obtained. This value is nearly 80 K, thus, changing state is a huge barrier when increasing or decreasing temperature and it increases dramatically the performance of water as a thermal inertia accumulator. For this reason, the enhancement of the model to incorporate this effect is in the future actions list.

### 3.1.5.3 Insulator layer

An insulator material layer follows with a thickness of 1 cm. The material envisaged for such insulator is aerogel for its low density and high insulator properties. A summary of the thermal properties of an aerogel is given in Table 3.1. The specific heat $c_p$ has been taken as the mean between the average maximum temperature (300 K) and the average minimum temperature (150 K). For the thermal conductivity $k$, values for maximum and minimum temperatures are close to $10^{-3}$ W/mK, therefore this value has been selected as the thermal conductivity of aerogel. Finally, the density $\rho$ of the material has been set to the value of silica aerogel.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>0.001</td>
<td>W/mK</td>
</tr>
<tr>
<td>$\rho$</td>
<td>124</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$c_p$</td>
<td>670</td>
<td>J/kgK</td>
</tr>
</tbody>
</table>

### 3.1.5.4 Martian ground layer

Finally, a layer of ground of 1 m is considered. This layer, which has to be simulated in order to evaluate the heat lost at the bottom, has the properties of the Martian soil that are described in Sect. 2.4.4.

### 3.1.6 Summary of design parameters

All the discussed design parameters have been collected and summarized in Table 3.2. Moreover, Table 3.3 shows the simulation parameters for the first run of the model.
Table 3.2: Summary of the greenhouse design parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Part in the Greenhouse</th>
<th>Influence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insulator conductivity ( k )</td>
<td>Insulator cell</td>
<td>Fixed by the material in use. Little effect on greenhouse.</td>
</tr>
<tr>
<td>Insulator emissivity ( \varepsilon )</td>
<td>Insulator cell</td>
<td>Fixed at 0.05 to avoid heat loss.</td>
</tr>
<tr>
<td>Insulator thickness</td>
<td>Insulator cell</td>
<td>Fixed at 1 mm to avoid transitory.</td>
</tr>
<tr>
<td>Factor of solar radiation across the insulator ( f )</td>
<td>Insulator cell</td>
<td>Maximum and minimum temperatures.</td>
</tr>
<tr>
<td>Fluid pressure</td>
<td>Interior</td>
<td>Heat transfer between walls and ground.</td>
</tr>
<tr>
<td>Ground emissivity ( \varepsilon_g )</td>
<td>Ground</td>
<td>Little effect on mean temperature.</td>
</tr>
<tr>
<td>Ground absorbivity ( \alpha )</td>
<td>Ground</td>
<td>Maximum and minimum temperatures.</td>
</tr>
<tr>
<td>Ground first layer ( L, k, \rho ) and ( c_P )</td>
<td>Ground</td>
<td>Fast thermal response</td>
</tr>
<tr>
<td>Ground insulator layer ( L, k, \rho ) and ( c_P )</td>
<td>Ground</td>
<td>Aerogel. Major impact in overall temperature.</td>
</tr>
<tr>
<td>Ground water layer ( L )</td>
<td>Ground</td>
<td>Heat accumulator to reduce distance between maximum and minimum temperature.</td>
</tr>
<tr>
<td>Parameter</td>
<td>Value</td>
<td></td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>----------------</td>
<td></td>
</tr>
<tr>
<td>Greenhouse dome diameter [m]</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>No. of insulator cells</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>No. of day repetitions</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

**Insulator cell parameters**

- Shell thickness [m] 1.00E-03
- Material conductivity [W/mK] 1.93
- Thermal emissivity 0.05
- Optical transparency \( f \) 0.95
- Multilayer insulator? [Activate/Deactivate] 0

**Air parameters**

- Pressure [Pa] 1.00E+05

**Ground parameters**

- Water layer? [Activate/Deactivate] 0
- Thermal emissivity 0.3
- Solar absorptivity 0.95

<table>
<thead>
<tr>
<th>Layer setup</th>
<th>thick [m]</th>
<th>mesh</th>
<th>( k )</th>
<th>( \rho )</th>
<th>( c_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accumulator</td>
<td>0.02</td>
<td>100</td>
<td>40.23</td>
<td>2246</td>
<td>213.3</td>
</tr>
<tr>
<td>Insulator</td>
<td>0.01</td>
<td>100</td>
<td>0.001</td>
<td>100</td>
<td>670</td>
</tr>
<tr>
<td>Martian ground</td>
<td>1</td>
<td>50</td>
<td>0.144</td>
<td>0.144e6</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table 3.3:** Input parameters for the greenhouse model simulation code.

### 3.2 Parametric design of the greenhouse

In this section, the results of simulating the model proposed in Chapter 2 are presented and thoroughly discussed. The four locations presented in Sect. 3.1.2 are considered. Simulations are run during typical 30 sols, i.e., 30 Martian days, by concatenating the typical day to allow the Martian ground to reach its final equilibrium temperature; otherwise the simulation results are not representative of a typical daily cycle. This issue should be taken into account because the plants should not be deployed until this phase has been completed. The first step of the design (Subsect. 3.2.1.1) is to obtain a suitable average interior temperature (i.e., within the crop’s operating range), despite seasonal maximum and minimum temperatures being too high. To do so, the shell’s thermal emissivity is changed. The second step (Subsect. 3.2.1.2) is to minimize the difference
between seasonal maximum and minimum temperatures, while maintaining the average temperature, so as to match the operating temperature range of the crops. For this reason, the water layer thickness is changed or an active control is used (Subsect. 3.2.2).

### 3.2.1 Analysis of the design parameters

The program is run the first time with the set of parameters proposed in Sect. 3.1 Table 3.3 A stable solution is obtained for a simulation of 30 sols (Fig. 3.6). Such figure, however, gives little information on the dependence of the design parameters with the resulting interior temperature. For this reason, various simulations are executed by tuning only one design parameter at a time in order to assess the influence of such parameters on the design of the greenhouse. The relevant parameters are the average internal temperature, the seasonal maximum temperature and the seasonal minimum temperature. Such temperatures are plotted with the values of the parameters to assess the effect of the parameter on the design.

![Figure 3.6: Time evolution of the greenhouse temperature for various seasons in Valles Marineris for the first set of parameters.](image)
3.2.1.1 Parameter determination for reaching acceptable $T_{avg}$

The ground emissivity has minimum impact on the average temperatures (Fig. 3.7) yielding temperatures that range from 460 $K$ to 430 $K$, which is clearly out of the operational temperatures for the crops of group $I$. Moreover, such parameter has no effect on the maximum seasonal temperature, however, it affects the seasonal minimum temperature (Fig. 3.8). Such change occurs when the emissivities are between 0.2 and 0.3. Values of 0.3 and higher yield higher seasonal temperatures - thus helping the greenhouse to heat up - while values of 0.2 and lower yield lower minimum temperatures, thus helping the greenhouse to cool.

On the other hand, the ground absorptivity has a major impact on the greenhouse design. Average temperatures are shown to decrease as the ground absorptivity is reduced (Fig. 3.10). Maximum and minimum temperatures exhibit the same behavior (Fig. 3.8). The reason for this performance lies in the design itself: the Sun heats the ground which in turn heats the air and the walls. The plots show that, in order to obtain the operational temperatures, the ground absorptivity must be set at 0.2 or less. However, seasonal maxima are still too high. Moreover, such a low ground absorptivity is not considered as it decreases the capability of self heating of the greenhouse.

The shell emissivity has also a major impact on the greenhouse design because the most important means of heat exchange is by radiating to the atmosphere. Average temperatures are shown to decrease while the emissivity increases (Figs. 3.11 and 3.12). The choice of the shell emissivity must be made with great care: a lower emissivity will yield higher temperatures but in turn will help the greenhouse heat faster, whereas a higher emissivity will have exactly the opposite effect.
Figure 3.7: Dependence of average temperature with ground emissivity for various locations.

Figure 3.8: Dependence of maximum temperatures (solid lines) and minimum temperatures (dashed lines) with ground emissivity for various locations.
Figure 3.9: Dependence of average temperature with ground absorptivity for various locations.

Figure 3.10: Dependence of maximum temperatures (solid lines) and minimum temperatures (dashed lines) with ground absorptivity for various locations.
Figure 3.11: Dependence of average temperatures with insulator cell emissivity for various locations.

Figure 3.12: Dependence of maximum temperatures (solid lines) and minimum temperatures (dashed lines) with insulator cell emissivity for various locations.
3.2.1.2 Parameter determination for obtaining an acceptable day/night temperature difference

At this point two problems arise:
• Some thermal inertia is needed to minimize the temperature difference between
day and night within a season (Fig. 3.13).

• The seasonal mean temperature for the cold season is far lower than the seasonal
mean temperature for the hot season (Fig. 3.14).

Each problem has been addressed separately. The solution for the first one is the ad-
dition of an accumulator layer in the ground which increases the thermal inertia, thus
reducing the day-night difference. The solution to the second issue involves some kind of
regulation mechanism in the greenhouse allowing to dissipate the extra heat generated
in the hottest seasons.

Increasing the thickness of the thermal intertia layer has little effect on the average
greenhouse temperature (Fig. 3.15) and the seasonal maximum and minimum temper-
atures (Fig. 3.16). The enhanced ground acts as a heat accumulator, releasing heat to
the greenhouse during the night (Fig. 3.17), however, its performance is not sufficient.
For this reason, a better accumulator must be found. Such an accumulator is water.

Water layer thickness has little influence on the average temperature of the greenhouse
(Fig. 3.18). Nevertheless, it helps adjust the average internal temperature. Water layer
thickness has major impact in the seasonal maximum and minimum temperatures and
day-night temperature difference (Figs. 3.19 and 3.20). For a thickness of more than 15
cm, Fig. 3.20 predicts an asymptotic behavior, i.e., by adding more thickness (thus more
water which in turn translates into more mass) no improvement is made. Nevertheless,
there is still a decrease of the seasonal maxima and minima. It must also be noted that
water is a more effective accumulator than the enhanced Martian ground, therefore,
water is used instead of the enhanced ground (in turn, it is also needed for the plants
life cycle as well). For these reasons, it has been proved that some sort of control is
needed for the greenhouse to operate within the crops’ temperature limits.
Figure 3.15: Dependence of average temperatures with ground layer thickness for various locations.

Figure 3.16: Dependence of maximum temperatures (solid lines) and minimum temperatures (dashed lines) with ground layer thickness for various locations.
Figure 3.17: Day-night temperature difference for Valles Marineris for each season as a function of the ground layer thickness.

Figure 3.18: Dependence of average temperatures with water layer thickness for various locations.
Chapter 3  Greenhouse Design and Results

3.2.2 Temperature active control

The last obtained results on Subsect. 3.2.1 indicate that active thermal control is needed for the greenhouse to operate inside the crops’ operating temperatures, otherwise, the day-night difference is too high for crops to live. Such control is based on having a double ground coating which is able to change from one side to another (Fig. 3.21). It
must be noted that the word active is not indicative of using energy to provide heat to
the greenhouse. Energy will be provided to a set of low power consuming actuators that
will change the sides of the insulator coating to provide thermal regulation. The first
side of the coating is set with high solar absorptivity and low thermal emissivity in order
to trap the heat in the ground and raise the temperature of the water fast. The other
side of the coating is set to have low solar absorptivity and high thermal emissivity in
order not to increase the ground temperature and help evacuate the extra heat. Table
3.4 shows the properties of the two coatings and the temperature thresholds in which
one coating changes to the other. Such temperature thresholds have been set to the op-
timal operation value for the crops of group $I$. So when the fluid temperature increases

<table>
<thead>
<tr>
<th>Coating</th>
<th>$\alpha$</th>
<th>$\varepsilon$</th>
<th>Threshold (in K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accumulating</td>
<td>0.95</td>
<td>0.30</td>
<td>293</td>
</tr>
<tr>
<td>Dissipating</td>
<td>0.25</td>
<td>0.80</td>
<td>288</td>
</tr>
</tbody>
</table>

Figure 3.21: Depiction of the active control coating with both sides, the accumulating
and the dissipating.

over 293 $K$, the ground coating changes to the dissipating side and helps evacuating the
extra accumulated heat. Then, when the fluid temperature decreases under 288 $K$, the
ground coating changes to the accumulating side and helps increasing the accumulated
heat.

The analysis for the active control is analogous to the discussion presented on Sub-
sect. 3.2.1. The two main control parameters are the insulator cell emissivity and the
water layer thickness. The values of these control parameters are chosen according to
the results presented on Subsect. 3.2.1. For the sake of simplicity, hereinafter the results are going to refer to a single location. Any other location will yield another set of parameters, however, the discussion will be analogous. The chosen location is Valles Marineris as this work considers it the most appropriate place in where to place the greenhouse. Design graphs for other locations are included in Appendix B. Fig. 3.22 clearly shows the need of an active control: when it is activated, seasonal temperatures remain comprised within the acceptable range while when there is no active control there is a huge difference between seasonal temperatures. Moreover, the results show that there is a combination of optimum values for the shell emissivity and water layer thickness that yield a suitable average temperature (Fig. 3.23), the lowest temperature difference (Fig. 3.25) and the lowest of the maximum temperatures and the highest of the minimum temperatures (Fig. 3.24). Such values are 0.3 and 15 cm, respectively. In fact, all the average temperatures are under the optimum zone for the crops (Fig. 3.23). Two considerations need to be taken:

- During the night a considerable amount of heat is lost through the insulator by radiative heat transfer.

- The amount of water needed for 15 cm is prohibitive. In a greenhouse of 2 m of diameter and 1 m wide by 0.15 m depth, the weight of water to carry to Mars would be of 300 kg.

**Figure 3.22:** Daily simulation comparing the performance without active control (solid lines) and with active control (dashed lines) for Valles Marineris.
Figure 3.23: Dependence of water layer thickness and insulator emissivity for Valles Marineris.

Figure 3.24: Dependence of water layer thickness and insulator emissivity with maximum temperatures (top) and minimum temperatures (bottom) for Valles Marineris.
Figure 3.25: Dependence of water layer thickness and insulator emissivity with the maximum day-night seasonal temperature difference for Valles Marineris.
Chapter 4

Conclusions and future work

The conclusions of this work are drawn in this chapter, as well as a discussion of the future developments is presented. The aim of this work was to perform a thermodynamic study and assess the feasibility of a greenhouse in Mars using the current insulation technology.

During the last 50 years, the insulation technologies have impressively evolved. Progress in harsh environment insulation has been made through the use of RHU and a uniquely built MLI structure (e.g., Spirit and Opportunity) to the use of the Martian atmosphere (mainly CO$_2$) as insulator (e.g., Curiosity). Such an approach is later taken in this work for the greenhouse shell. Insulation structures have also been improved with the implementation of new materials (e.g., aerogel) and more efficient structures. Such structures are comprised of various layers of different materials, each of them contributing by adding a property to the whole pack. Such structures and materials are currently being implemented and tested in space applications (e.g., Stardust). Moreover, up to the present date, many sketches of Martian greenhouses have been drawn by NASA in collaboration with other institutions.

The first part of this study has led to the thermodynamical design of a greenhouse able to operate in harsh environmental conditions. Several loops of iterations were performed on the design until the present model was found. Serious limitation was found in the first iteration aerogel walls due to the material’s interior radiation scattering. A model of radiation is not needed as the properties of the Martian atmosphere allow for multi-layer thermal insulation, therefore, a CO$_2$ based insulation, similar to the one used in Curiosity, is presented in this work. Not only is this method simpler but also allows easier in situ construction and probably yields a more reliable insulation. The present
literature does not provide any simplified free convection models for enclosed domes (i.e., a model for the enclosed air). Instead, a rather simple relation between the heat flux coming from the shell to the ground was found. Experimental correlations have been used in this work. Such correlations are found and studied in Earth-like conditions (i.e., pressure of 1 bar and gravity of 9.8 m/s²), however, Martian conditions are large different from Earth (i.e., pressure of 1 kPa and gravity of 3.71 m/s²). These facts result in the experimental correlations falling out of the tested ranges most of the times. For this reason, such correlations have been prolonged by continuity or have been saturated at the limit of their tested range. Results of this study show that Martian ground alone does not provide enough thermal inertia to withstand the temperature drop at night. For this reason, a thermal accumulator material is needed to increase the overall thermal inertia (i.e., water or the enhanced ground). The accumulator is followed by a layer of an insulator material (e.g. Aerogel) and a layer of Martian ground.

Results of simulating the greenhouse model have shown that greenhouse location must be carefully chosen. Martian climate behavior is different that on Earth thus leading to unexpected results (e.g., fall and spring seasons being hotter than summer or winter, this happens in the Ares Vallis location). Greenhouse location is tightly related with solar incidence, which effects on the overall achievable temperature. It has been shown that equatorial locations (e.g., Valles Marineris or Gale Crater) are feasible locations for a hypothetical Martian greenhouse. In particular, Valles Marineris is an interesting location as it is a valley of several kilometers deep where the atmospheric pressure is greater and the same valley can provide radiation shelter for personnel. On the other hand, it is a location difficult to reach. Results also show that with a low emissivity (e.g., around 0.05), fairly transparent walls and a high ground absorptivity (e.g. around 0.95) high temperatures can be reached on the surface of Mars (e.g., around 400 - 500 K). For this reason, one of the issues of this work has been how to cool the greenhouse rather than rising the temperature to an optimal value. It has been shown that such control enables the greenhouse to operate withing acceptable thermal limits during all the seasons. The analysis has also shown that there is a huge temperature difference between day and night (around 100 K), ergo a methodology to accumulate heat was devised. Such mechanism has been another issue of this work. From the two risen solutions, it has been shown that enhancing the Martian ground is an insufficient mechanism compared to the addition of water. Moreover, enhancing Martian ground requires knowing its properties. At the time of this work, Martian ground properties remain largely unknown. For this reason, and the fact that water is needed for the plants’ life cycle, water is used as a thermal accumulator. Such a solution has proved acceptable in terms of the temperature but it poses a major transportation problem: a layer of 15 cm of
water represents 300 kg of weight to be carried to Mars. Nevertheless, such amount could be reduced when taking into account water as a phase change material, resulting in the equivalent of 80 K of heat release or absorption. These facts will result in a thinner layer of water than the 15 cm expected. Such unknowns also raise interest in the active thermal regulation systems. This systems will allow the correct performance of the greenhouse in a wide range of values. Finally, the temperature requirements for crops of group I is too restrictive. As a matter of fact, some of the plants in group I (e.g., tomatoes) can withstand slightly negative temperatures (i.e., from minus 5 to minus 10 degrees centigrade), therefore, the temperature margin can be moderately relaxed.

Despite the uncertainties presented in the model and the improvements that need to be done, it seems clear that it is possible to obtain an appropriate temperature for the plants without any energy expense (i.e., heaters such as RHU that represent a biological threat), provided a proper design is made. Such a fact is illustrative of how the current technology can allow to save energy and shows that the acclimatization systems represent a huge energy expense that could be avoided.

As a final concluding remark, it is stressed that several works on the feasibility of a Martian greenhouse exist\textsuperscript{20} \textsuperscript{21}. Nonetheless, to the author’s knowledge, none provides an analysis of the climate (provided by MEMM) and thermal (provided by this work) aspects of a hypothetical greenhouse in Mars. This work is the culmination of a project to assess the thermal feasibility of a greenhouse in Mars. It gives an application to MEMM, therefore, it must not be taken separately.

\section*{4.1 Future actions}

Despite trying to fulfill completeness, future actions are required to improve the model. Such actions are listed below:

- Water as a phase change material needs to be implemented as it will soften the maximum and minimum temperatures achievable in the greenhouse.

- The transitory effects in the greenhouse shell need to be taken into account to fully account for the temperature changes.

- A 3D geometry model is needed in order to provide accurate results. Such geometry can take the shape of a spherical dome or a semi-cylinder. With the addition of
a 3D geometry model, the view factors must be computed by solving a double integral. Such a computation has already been analyzed in this work (Appendix D). Moreover, a 3D model of radiation is needed in which the whole movement of the sun in the sky and shadowing must be analyzed. 3D geometry also accounts for 3D heat conduction analysis in to analyze the heat losses between wall and ground.

- A CFD analysis must be performed in both the insulator wall and the enclosed air in order to understand the free convection mechanism, thus performing a better evaluation of the heat transfer coefficients. The CFD model must be coupled with the boundary conditions of the conduction and radiation problem, as the conditions are dependent on each other. With the addition of CFD, no air correlations are going to be needed and the results will account for the exact studied case.

- Knowledge of the environmental conditions (e.g., dust, wind, micrometeorites) of the landing site is required to successfully analyze the feasibility of a Martian greenhouse. The presented model can be enhanced with CFD model of the outside environment of Mars to account for a better approximation on the convection due to Martian wind. Moreover, dust settlement and removal must be added to the model. Dust will set in the shell of the greenhouse, thus blocking the effective sunlight needed for the plants and the greenhouse to regulate its temperature.

- Crop growing and plants life cycle must be added to the whole model to further obtain a more accurate interior temperature.

- All the structural, energetic and life support considerations mentioned in the scope must be taken into account in the whole design.
Appendix A

Budget

In this section the budget of the *Study of advanced materials for thermal insulator in the inner Solar System* is presented. Man hours as well as hardware and software resources required to carry out this study are illustrated in Table A.1. Nine months were spent on the study, three of which on a part-time basis and the others at full time, totally accounting for 800 man hours. The price per hour worked is estimated in 15 €, then the total cost of the engineering work can be computed as 12000 €. Power consumption stands for 500 simulation hours required by the study. The electric bill adds up to 55 €. Hardware and software amortizations correspond to the partial cost of the tools and licenses used in the project; they sum up to a cost of 119 €. The total cost is of 12174 €.
## Appendix A: Budget

**Table A.1:** Budget of the *Study of advanced materials for thermal insulator in the inner Solar System.*

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<th>Cost (€)</th>
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<tr>
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<td>Report</td>
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<td>3000</td>
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<tr>
<td><strong>Total</strong></td>
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<td></td>
<td><strong>12174</strong></td>
</tr>
</tbody>
</table>
Appendix B

Location design graphs

The design plots for each location analyzed in this work are summarized in this Appendix. Some of the plots have already included in the body of the report, nevertheless, they are included here for the sake of completeness and to provide a quick design reference to the reader.

B.1 Ground emissivity ($\varepsilon_g$) graphs

![Graph showing temperature dependence on ground emissivity for various locations.](image)

**Figure B.1:** Dependence of average temperature with ground emissivity for various locations.
Appendix B. Location design graphs

Figure B.2: Dependence of maximum temperatures (solid lines) and minimum temperatures (dashed lines) with ground emissivity for various locations.

B.2 Ground absorptivity ($\alpha_g$) graphs

Figure B.3: Dependence of average temperature with ground absorptivity for various locations.
Appendix B. Location design graphs

B.3 Shell emissivity $\varepsilon$ graphs

**Figure B.4:** Dependence of maximum temperatures (solid lines) and minimum temperatures (dashed lines) with ground absorptivity for various locations.

**Figure B.5:** Dependence of average temperatures with insulator cell emissivity for various locations.
Appendix B. Location design graphs

Figure B.6: Dependence of maximum temperatures (solid lines) and minimum temperatures (dashed lines) with insulator cell emissivity for various locations.

B.3.1 Ares Vallis

Figure B.7: Day-night temperature difference for Ares Vallis for each season as a function of the insulator cell emissivity.
Appendix B. Location design graphs

B.3.2 Gale Crater

Figure B.8: Seasonal average temperature for Ares Vallis for each season as a function of the insulator cell emissivity.

Figure B.9: Day-night temperature difference for Gale Crater for each season as a function of the insulator cell emissivity.
Appendix B. Location design graphs

Figure B.10: Seasonal average temperature for Gale Crater for each season as a function of the insulator cell emissivity.

B.3.3 Gusev Crater

Figure B.11: Day-night temperature difference for Gusev Crater for each season as a function of the insulator cell emissivity.
B.3.4 Valles Marineris

Figure B.12: Seasonal average temperature for Gusev Crater for each season as a function of the insulator cell emissivity.

Figure B.13: Day-night temperature difference for Valles Marineris for each season as a function of the insulator cell emissivity.
Appendix B. Location design graphs

Figure B.14: Seasonal average temperature for Valles Marineris for each season as a function of the insulator cell emissivity.

B.4 Ground accumulator layer

Figure B.15: Dependence of average temperatures with ground layer thickness for various locations.
Figure B.16: Dependence of maximum temperatures (solid lines) and minimum temperatures (dashed lines) with ground layer thickness for various locations.

B.4.1 Ares Vallis

Figure B.17: Day-night temperature difference for Ares Vallis for each season as a function of the ground layer thickness.
B.4.2 Gale Crater

![Graph of Day-night temperature difference for Gale Crater for each season as a function of the ground layer thickness.](image)

**Figure B.18:** Day-night temperature difference for Gale Crater for each season as a function of the ground layer thickness.

B.4.3 Gusev Crater

![Graph of Day-night temperature difference for Gusev Crater for each season as a function of the ground layer thickness.](image)

**Figure B.19:** Day-night temperature difference for Gusev Crater for each season as a function of the ground layer thickness.
B.4.4 Valles Marineris

Figure B.20: Day-night temperature difference for Valles Marineris for each season as a function of the ground layer thickness.

B.5 Ground water layer

Figure B.21: Dependence of average temperatures with water layer thickness for various locations.
Figure B.22: Dependence of maximum temperatures (solid lines) and minimum temperatures (dashed lines) with water layer thickness for various locations.

B.5.1 Ares Vallis

Figure B.23: Day-night temperature difference for Ares Vallis for each season as a function of the water layer thickness.
B.5.2 Gale Crater

![Graph: Day-night temperature difference for Gale Crater for each season as a function of the water layer thickness.]

**Figure B.24:** Day-night temperature difference for Gale Crater for each season as a function of the water layer thickness.

B.5.3 Gusev Crater

![Graph: Day-night temperature difference for Gusev Crater for each season as a function of the water layer thickness.]

**Figure B.25:** Day-night temperature difference for Gusev Crater for each season as a function of the water layer thickness.
B.5.4 Valles Marineris

![Day-night temperature difference graph for Valles Marineris for each season as a function of the water layer thickness.](image)

**Figure B.26:** Day-night temperature difference for Valles Marineris for each season as a function of the water layer thickness.

B.6 Temperature active control

B.6.1 Ares Vallis

![Dependence of water layer thickness and insulator emissivity for Ares Vallis.](image)

**Figure B.27:** Dependence of water layer thickness and insulator emissivity for Ares Vallis.
Figure B.28: Dependence of water layer thickness and insulator emissivity with maximum temperatures (top) and minimum temperatures (bottom) for Ares Vallis.

Figure B.29: Dependence of water layer thickness and insulator emissivity with the maximum day-night seasonal temperature difference for Ares Vallis.
Appendix B

Location design graphs

B.6.2 Gale Crater

Figure B.30: Dependence of water layer thickness and insulator emissivity for Gale Crater.

Figure B.31: Dependence of water layer thickness and insulator emissivity with maximum temperatures (top) and minimum temperatures (bottom) for Gale Crater.
Appendix B. Location design graphs

B.6.3 Gusev Crater

Figure B.32: Dependence of water layer thickness and insulator emissivity with the maximum day-night seasonal temperature difference for Gale Crater.

Figure B.33: Dependence of water layer thickness and insulator emissivity for Gusev Crater.
Figure B.34: Dependence of water layer thickness and insulator emissivity with maximum temperatures (top) and minimum temperatures (bottom) for Gusev Crater.

Figure B.35: Dependence of water layer thickness and insulator emissivity with the maximum day-night seasonal temperature difference for Gusev Crater.
B.6.4 Valles Marineris

Figure B.36: Dependence of water layer thickness and insulator emissivity for Valles Marineris.

Figure B.37: Dependence of water layer thickness and insulator emissivity with maximum temperatures (top) and minimum temperatures (bottom) for Valles Marineris.
Figure B.38: Dependence of water layer thickness and insulator emissivity with the maximum day-night seasonal temperature difference for Valles Marineris.
Appendix C

Numerical algorithm for one-dimensional heat conduction

Conduction is the main mechanism of heat transfer in solids. In this work, a one-dimensional treatment for the heat transfer equation is assumed. Following the book of Patankar, a numerical approximation has been set up based on finite differences. The heat transfer equation for such a case is:

\[ \rho C \frac{dT}{dt} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + S, \]  
(C.1)

where \( \rho \) is the density of the material, \( C \) the heat capacity and \( S \) the source term. \( T \) stands for the temperature, \( t \) for time and \( x \) for the direction. For solids, the heat capacity at constant volume and the heat capacity at constant pressure have the same value. If the thermal conductivity is considered independent from temperature, Eq. C.1 can be rewritten as:

\[ \rho C \frac{dT}{dt} - k \frac{\partial^2 T}{\partial x^2} = S, \]  
(C.2)

being Eq. C.2 the basis for the numerical approximation.

C.1 Numerical approximation

A finite differences integration technique is used for numerically integrating Eq. C.2. The discrete version of Eq. C.2 is

\[ a_P T_P = a_E T_E + a_W T_W + b. \]  
(C.3)
Appendix C

One-Dimensional Heat Conduction Numeric Algorithm

111

\( a_P, a_E \) and \( a_W \) are the integration coefficients. \( T_P \) is the temperature at the current node whereas \( T_E \) and \( T_W \) are the temperatures of the nodes east and west of the current node, respectively. The source term \( b \) can be a function of temperature, position and time. Therefore, a linear approximation of the source term is made according to Eq. [C.4]

\[
 b = S_C(t, x) + S_P(t, x)T_P. \tag{C.4}
\]

A fully-implicit scheme is taken into account to maintain simplicity and ensure full convergence, even for large time intervals. The values of the coefficients for Eq. [C.3] are:

\[
a_E = \frac{k}{(\Delta x)_e}, \tag{C.5}
\]

\[
a_W = \frac{k}{(\Delta x)_w}, \tag{C.6}
\]

\[
a_0^P = \frac{\rho C\Delta x}{\Delta t}, \tag{C.7}
\]

\[
b = S_C\Delta x + a_0^P T_P, \tag{C.8}
\]

\[
a_P = a_E + a_W + a_0^P - S_P \Delta x. \tag{C.9}
\]

\( \Delta x \) refers to the spatial integration step and \( \Delta t \) refers to the temporal integration step. The subscripts indicate to which control volume each quantity belongs, with the use of upper case letters for the center and lower case letters for the face. The superscript 0 indicates that the variable takes the last instant value. Patankar presents four basic rules to ensure the stability and correct behaviour of the presented method:

1. Consistency at control-volume faces. In other words, the heat flux that leaves one control volume must be equal to the heat flux that enters the next control volume.

2. Positive Coefficients. This rule is used as a verification that the written code is consistent.

3. Negative slope linearization of the source term. It basically indicates that the term \( S_P \) must be negative.

4. Sum of the neighbour coefficients. It indicates that the coefficient \( a_P \) must be the sum of the other coefficients. The used method automatically fulfills this rule.
C.2 Meshing

A non-uniform, multi-material, one-dimensional mesh is developed for the conduction solver. Such mesh consists in dividing a wall slab of a certain material with certain properties $k$, $\rho$ and $c_P$ into a certain number of control volumes. The mesh is node-centered, i.e., the nodes are located in the center of the control volumes. Different wall slabs can be specified to the algorithm, of different material and different number of nodes and longitudes. The total number of nodes is the sum of all the nodes of the slabs plus two, one for each boundary node. For a single slab of material, the nodes are spaced equally:

$$dx = \frac{x_f - x_0}{n - 1}, \quad (C.10)$$

where $n$ is the number of nodes. The boundary nodes’ spacing is half the computed spacing:

$$dx_1 = \frac{dx_1}{2}, \quad (C.11)$$
$$dx_{end} = \frac{dx_{end}}{2}. \quad (C.12)$$

The remaining spacing that has been cut, which is exactly $dx$, has to be equally added to the spacing of the interior nodes. The theoretical position of the nodes is computed recursively by:

$$x_i = x_{i-1} + dx_{i-1}, \quad (C.13)$$

and starting by setting $x_1 = x_0$. The subscript $i$ denotes an arbitrary node. Then, the position of the boundaries of the control volumes are computed as:

$$x_{vc_i} = x_i + \frac{dx_i}{2}, \quad (C.14)$$

where $x_{vc_i}$ denotes the boundary position of the control volume $i$. Such equations are repeated for as many slabs of material as defined. If multiple slabs are defined, the positions of the nodes need to be recomputed using the control volume positions by:

$$x_i = \frac{x_{vc_i} - x_{vc_{i-1}}}{2}. \quad (C.15)$$

The properties of the materials $k$, $\rho$ and $c_P$ of each node are also returned. The resulting mesh looks like shown in Fig. C.1.
C.2.1 Interface conductivity

A method to compute the interface conductivity for non-uniform meshes is also proposed. The resulting equation is derived from the heat flux and reads:

\[
    k_i = \left( \frac{1 - f_i}{k_P} + \frac{f_i}{k_I} \right)^{-1},
\]

where \( i \) stands for both \( e \) and \( w \). When the interface is located midway between \( I \) and \( P \), then \( f_i = 0.5 \) and the interface conductivity becomes the harmonic mean of \( I \) and \( P \):

\[
    k_i = \frac{2k_P k_I}{k_P + k_I}.
\]

C.3 Boundary conditions

Four types of boundary conditions are considered:

1. Given boundary temperature;
2. Given boundary heat flux;
3. Boundary heat flux specified via a heat transfer coefficient and the temperature of the surrounding fluid;
4. Radiation boundary conditions.
The derivations further presented are able to deal with the four types of boundary conditions. They are treated separately, thus a complex condition (i.e., radiation plus convection) includes the sum of both derivations in their coefficients.

C.3.1 Given boundary temperature

The condition for a fixed temperature in the boundary condition is written as:

\[ T(t, x = 0, L) = T^*, \]  

(C.18)

where \( T^* \) is a given temperature. In that case, no additional equations are required. The algorithm is arranged so that the solver always gives \( T^* \) as a solution for the boundary, which can be easily done by doing the following modifications to the coefficients:

\[ a_i = 1, \]  

(C.19)

\[ a_P = 0, \]  

(C.20)

\[ b = T^*. \]  

(C.21)

Where \( a_i \) is the adjacent node to the boundary, being either \( W \) or \( E \). The other boundary node, as it does not exist, has its coefficient set to zero.

C.3.2 Given boundary heat flux

The condition for a given boundary flux \( \dot{q}^* \) is written as:

\[ \frac{\partial T}{\partial x}_{|x=0,L} = \dot{q}^*. \]  

(C.22)

Eq. [C.22] is numerically integrated in the half boundary control volume shown in Fig. ??.

The input heat flux is given by

\[ q_B = \dot{q}^*. \]  

(C.23)
The effect on this derivation is the addition of a parameter in the source term $b$. The coefficients in such case are:

\[
a_i = \frac{k_i}{(\delta x)_i},
\]
\[
a_P = a_i + a_P^0 - S_P \Delta x,
\]
\[
b = S_C \Delta x + q_B + a_P^0 T_P^0,
\]

where $a_i$ is the adjacent node to the boundary, being either $W$ or $E$. The other boundary node, as it does not exist, has its coefficient set to zero.

### C.3.3 Given boundary flux via heat transfer coefficient

The condition for a given boundary flux via a convection coefficient $h$ can be written as:

\[
\frac{\partial T}{\partial x}|_{x=0,L} = -h (T - T_\infty),
\]

where $T$ is the temperature at the surface and $T_\infty$ is the temperature of the fluid. Eq. [C.27] is also numerically integrated in the half boundary control volume (Fig. ??) where the input heat flux is given by:

\[
q_B = h(T_f - T_B),
\]

where $T_f$ is the fluid temperature and $T_B$ the temperature of the boundary node. Once integrated according the procedures shown by Patankar, the obtained coefficients are:

\[
a_i = \frac{k_i}{(\delta x)_i},
\]
\[
a_P = a_i + a_P^0 - S_P \Delta x + h,
\]
\[
b = S_C \Delta x + h T_f + a_P^0 T_P^0.
\]

$a_i$ is the adjacent node to the boundary, being either $W$ or $E$. The other boundary node, as it does not exist, has its coefficient set to zero. In addition to Patankar’s derivation, in this deduction the transient term has been included in the boundary.

### C.3.4 Radiation boundary conditions

Radiation boundaries depend on the wavelength absorption of the material. However, two major wavelengths are considered: the shortwave or visible and the longwave or IR. They follow a different treatment, i.e., a body emits radiation in the IR but not in
the visible. The visible radiation treatment is analogue to that of a given heat flux as described by Eq. (C.22) but setting:

$$\dot{q}^* = \alpha \dot{q}_{vis},$$

where \(\alpha\) is the visible wavelength absorption coefficient of the surface of the material.

The IR radiation, however, does not have such a direct treatment, i.e., a body emits thermal radiation according to its temperature at the fourth power. Therefore, the boundary condition is written slightly different:

$$\frac{\partial T}{\partial x}|_{x=0,L} = \varepsilon (\dot{q}_{IR} - \sigma T_B^4),$$

where \(\varepsilon\) is the thermal emissivity of the surface of the material and \(\dot{q}_{IR}\) is an incoming thermal flux. Kirchoff’s law states that for a gray body (which is a hypothesis of this derivation) the absorptivity at a given wavelength is equal to the emissivity at that same wavelength. Eq. (C.34) presents a major complication: it is no longer linear. Therefore, the boundary condition must be linearly approximated. Such an approximation takes the form of:

$$\frac{\partial T}{\partial x}|_{x=0,L} = \varepsilon (\dot{q}_{IR} - \sigma T_B^3 T^*_B),$$

where \(T_B^*\) is the last known boundary temperature. The coefficients are:

$$a_i = \frac{k_i}{(\delta x)_i},$$

$$a_P = a_i + a_P^0 - S_P \Delta x + \varepsilon T^*_B,$$

$$b = S_C \Delta x + \varepsilon \dot{q}_{IR} + a_P^0 T_B^0.$$

\(a_i\) is the adjacent node to the boundary, being either \(W\) or \(E\). The other boundary node, as it does not exist, has its coefficient set to zero.

### C.4 Solver

Thomas algorithm, also called TDMA (Tri Diagonal Matrix Algorithm), is a fast and direct algorithm to solve such sparse systems of equations. The full derivation will not be presented in this work, as it can be found in Patankar’s book. The main advantage of this algorithm with respect to a standard Gaussian elimination or a Gauss-Seidel algorithm is that with a two sweeps of the mesh, one forwards and one backwards, the temperature field can be known. Let us then see a summary of the algorithm:
1. Computing $P_1$ and $Q_1$ from:

$$P_1 = \frac{aE_1}{aP_1},$$

$$Q_1 = \frac{b_1 + aW_1 T_0}{aP_1};$$  \hfill (C.38)  \hfill (C.39)

2. Using the recurrence

$$P_i = \frac{aE_i}{aP_i - aW_i P_{i-1}},$$

$$Q_i = \frac{b_i + aW_i Q_{i-1}}{aP_i - aW_i P_{i-1}},$$  \hfill (C.40)  \hfill (C.41)

to obtain $P_i$ and $Q_i$ for $i = 2, 3, \ldots, N$;

3. Setting $T_N = Q_N$;

4. Use

$$T_i = P_i T_{i+1} + Q_i,$$  \hfill (C.42)

to compute the temperatures for $i = N - 1, N - 2, \ldots, 3, 2, 1$.

### C.5 Solution algorithm

The following steps need to be taken to solve the conduction problem for a single time step:

1. Suppose $T^*$;

2. Make $T = T^*$;

3. Compute any material properties and/or radiation using $T$;

4. Compute the coefficients $a_e, a_w, a_p, b$;

5. Solve the system of equation;

6. Is $T - T^* < \epsilon$? where $\epsilon$ is an arbitrary small value, say $10^{-6}$.

If the answer is yes proceed to next time step, else make $T^* = T$ and back to 2. This approach is needed when the problem is not linear, i.e., the material properties depend on the temperature $T$ or the boundary conditions force any non-linearity.
C.6 Validation

A simple methodology to verify the solution is to perform two energy balances: one in the interior of the material and another in the boundaries. Such balances check that the solution is physically consistent; however, they cannot check the accuracy nor the convergence of the solution.

C.6.1 Energy balance in the interior of the material

The energy balance in the interior of the material is performed by taking a control volume that includes the inside nodes and excludes the boundary nodes (Fig. C.3). To generalize the algorithm, the walls take the name of left (L) and right (R). Then, the energy balance states that the difference between the heat flux that enters the control volume $\dot{q}_L$ and the heat flux that exits the control volume $\dot{q}_R$ must be equal to the generated $\dot{q}_{gen}$ and accumulated heat $\dot{q}_{ac}$ in such control volume, i.e.,

$$\dot{q}_L - \dot{q}_R = \dot{q}_{gen} + \dot{q}_{ac}. \quad (C.43)$$

The conduction heat fluxes are computed by recalling the conduction Fourier law:

$$\dot{q}_{cond} = -k \frac{dT}{dx}. \quad (C.44)$$
Then the left and right conduction heat fluxes can be written as:

\[ \dot{q}_L = k \frac{T_2 - T_1}{x_2 - x_1}, \quad (C.45) \]

\[ \dot{q}_R = k \frac{T_{n-1} - T_n}{x_n - x_{n-1}}, \quad (C.46) \]

The accumulated heat is due to the transitory part is written as:

\[ \dot{q}_{ac} = \int_{c.v.} \rho c_p \frac{dT}{dt} dx, \quad (C.47) \]

which can be numerically integrated as

\[ \dot{q}_{ac} = \rho c_p \sum_{i=2}^{n-1} \frac{T_i - T_i^0}{\Delta t} \Delta x_i. \quad (C.48) \]

The generated heat is due to the internal heat sources of the material and it corresponds to their integration in the control volume, hence,

\[ \dot{q}_{gen} = \int_{c.v.} b dx = \sum_{i=2}^{n-1} S_{C_i} + S_{P_i} T_i. \quad (C.49) \]

### C.6.2 Energy balance in the boundaries

An energy balance in the boundaries is performed in order to check the consistence of the boundary condition through the whole algorithm. The control volume taken involves only the boundary node (Fig. C.4). By applying the first principle of thermodynamics

![Figure C.4: Control volume and heat fluxes of the energy balance in the boundary](image)

in such control volume:

\[ \rho \frac{dU}{dt} A dx = (\dot{q}_{cond} - \dot{q}_{gen} + \dot{q}_i^{rad} - \dot{q}_i^{rad} - \dot{q}_i^{rad} - \dot{q}_{conv}) A, \quad (C.50) \]
where $A$ is the area of the boundary and the superscripts in the radiation flux $i$ stands for incident, $e$ stands for emitted and $r$ stands for reflected. For a solid, $dU = dh$ and $dh = c_p dT$, so the left hand side of Eq. (C.50) becomes:

$$\rho \frac{dU}{dt} dx = \rho c_p \frac{dT}{dt} dx,$$

(C.51)

which corresponds to the accumulated heat. By integrated as shown in Eq. (C.48)

$$\rho c_p \frac{dT}{dt} dx = \rho c_p \frac{T_i - T_i^0}{\Delta t} \Delta x_i.$$  

(C.52)

The generated heat is integrated similarly to Eq. (C.49)

$$\dot{q}_{\text{gen}} = S_C + S_P T_i.$$  

(C.53)

The conduction heat is integrated using Fourier law in Eq. (C.44)

$$\dot{q}_{\text{cond}} = k \frac{T_2 - T_1}{x_2 - x_1}.$$  

(C.54)

The convective heat that leaves the wall is:

$$\dot{q}_{\text{conv}} = h(T_w - T_f).$$  

(C.55)

The radiative fluxes read:

$$\dot{q}_{\text{rad}}^i = \alpha \dot{q}_{\text{vis}} + \varepsilon \dot{q}_{\text{IR}},$$

(C.56)

$$\dot{q}_{\text{rad}}^r = (1 - \alpha) \dot{q}_{\text{vis}} + (1 - \varepsilon) \dot{q}_{\text{IR}},$$

(C.57)

$$\dot{q}_{\text{rad}}^e = \varepsilon \sigma T_w^4.$$  

(C.58)

By putting everything together:

$$\rho c_p \frac{T_i - T_i^0}{\Delta t} \Delta x_i = k \frac{T_2 - T_1}{x_2 - x_1} - S_C + S_P T_i + \dot{q}_{\text{rad}}^i - \dot{q}_{\text{rad}}^r - \dot{q}_{\text{rad}}^e - h(T_w - T_f).$$

(C.59)

### C.7 Method of Manufactured Solutions (MMS)

Roache\cite{Roache} proposes a methodology for performing code verification from known benchmark solutions. Such methodology is called Method of Manufactured Solutions, in short MMS. MMS provides a general procedure for generating an analytic solution for code accuracy verification. The method consist of picking first the example solution $U(t, x)$ before specifying the governing equations. The example solution must satisfy the problem’s boundary conditions for which it has been defined. Then, the solution $U(t, x)$ is
introduced to the governing equations to find the source term $b$. The source term then feeds the numerical procedure which returns the numerical solution for the problem which, by construction, should be equal to the selected example solution $U(t, x)$. For the transient heat problem, two meshes shall be analyzed. First of all, the mesh domain $x$ shall be analyzed by setting $dt \to \infty$. Such condition will make the transient term disappear from the equation. For this case the picked solution is:

$$U(t, x) = \sin 1 \sin x - \sin^2 x,$$  \hspace{1cm} (C.60)

with the boundary conditions:

$$U(0, x) = 0,$$ \hspace{1cm} (C.61)

$$U(t, 0) = 0,$$ \hspace{1cm} (C.62)

$$U(t, 1) = 0.$$ \hspace{1cm} (C.63)

Then,

$$\frac{dU}{dt} = 0,$$ \hspace{1cm} (C.64)

$$\frac{dU}{dx} = \sin 1 \cos x - 2 \sin x \cos x,$$ \hspace{1cm} (C.65)

$$\frac{dU}{dx} = - \sin 1 \sin x - 2 \cos 2x.$$ \hspace{1cm} (C.66)

Therefore, the source term that needs to be put in the numerical solver is:

$$S = k \sin 1 \sin x + 2k \cos 2x.$$ \hspace{1cm} (C.67)

To analyze the temporal mesh, a known working value of $dx$ will be picked. For this case, the selected solution is:

$$U(t, x) = x(1 - x)t,$$ \hspace{1cm} (C.68)

with the boundary conditions:

$$U(0, x) = 0,$$ \hspace{1cm} (C.69)

$$U(t, 0) = 0,$$ \hspace{1cm} (C.70)

$$U(t, 1) = 0.$$ \hspace{1cm} (C.71)
Then,
\[
\frac{dU}{dt} = x(1 - x), \quad (C.72)
\]
\[
\frac{dU}{dx} = (1 - 2x)t, \quad (C.73)
\]
\[
\frac{dU}{dx} = -2t. \quad (C.74)
\]
Therefore, the source term that needs to be put in the numerical solver is:
\[
S = \rho c p x (1 - x) + 2kt. \quad (C.75)
\]

C.7.1 Validating the code

The error \(E\) between the numerical and exact solution is computed in order to validate the numerical approximation made in the code:
\[
E = f(\Delta) - f^{\text{exact}}, \quad (C.76)
\]
where \(f(\Delta)\) stands for the numerical solution and \(f^{\text{exact}}\) stands for the analytic one. For a well behaved problem, such as the finite differences method, the following relation holds:
\[
E = C \Delta^p + \varepsilon (\Delta^2). \quad (C.77)
\]
As it can be seen, by refining the mesh the approximation must improve. That is, the logarithm of the Error versus the logarithm of the mesh refinement must be a linear relation of order \(p\). Therefore this method is valid to assess how much refined must the mesh be. Another method to check the proper behavior of the code is to compute the order of approximation \(p\). By taking two different mesh refinements \(\Delta_1\) and \(\Delta_2\) with their respective errors \(E_1\) and \(E_2\):
\[
f_1(\Delta_1) - f_1^{\text{exact}} = C \Delta_1^p + \varepsilon (\Delta_1^2), \quad (C.78)
\]
\[
f_2(\Delta_2) - f_2^{\text{exact}} = C \Delta_2^p + \varepsilon (\Delta_2^2). \quad (C.79)
\]
By dividing Eqs. (C.78) and (C.79)
\[
\frac{f_1(\Delta_1) - f_1^{\text{exact}}}{f_2(\Delta_2) - f_2^{\text{exact}}} = \frac{C \Delta_1^p}{C \Delta_2^p}, \quad (C.80)
\]
and then taking logarithms:
\[
\log \frac{f_1(\Delta_1) - f_1^{\text{exact}}}{f_2(\Delta_2) - f_2^{\text{exact}}} = p \log \frac{\Delta_1}{\Delta_2}. \quad (C.81)
\]
Then, the order of approximation \( p \) can be computed as:

\[
p = \frac{\log \frac{f_1(\Delta_1) - f_1^{exact}}{f_2(\Delta_2) - f_2^{exact}}}{\log \frac{\Delta_1}{\Delta_2}}.
\]  
(C.82)

### C.7.2 Results from MMS validation

A MMS run is done by both the mesh in spacing \( dx \) and the mesh in time \( dt \). The error is plotted versus the mesh refining to assess the quality of the mesh and obtain the order of approximation. The logarithm of the error for the spacing mesh is linear with the logarithm of the mesh refinement (Fig. C.5). The slope of the curve is 2 as expected from an approximation of second order of the second derivative of the heat equation. It has been verified that the code runs smoothly for steady state cases. There is a linear dependence of the logarithm of the error with the logarithm of the time step refinement (Fig. C.6). The slope of the curve obtained is 1 as expected from a first order approximation of the derivative of the temperature. Therefore, it has been verified that the code runs smoothly for transient cases.
Figure C.6: MMS error plot for the time mesh. Order of approximation $p$ of 1.00.
Appendix D

View factors computation

The radiation heat transfer between surfaces depends on the orientation of the surfaces relative to each other and on their radiation properties and temperatures. To account for the effects of the orientation, a new parameter, the view factor, is defined. The view factor $F_{ij}$ of surface $i$ towards surface $j$ is a purely geometric quantity and is independent from the surface properties and temperature and it is defined as:

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos g_i \cos g_j}{\pi r^2} dA_j dA_i,$$  \hspace{1cm} (D.1)

where $A$ indicates the area of each surface, $g$ the irradiosity and $r$ the distance between the surfaces. The problem with the view factors is that the computation time increases at the square power of the number of surfaces involved. Let us consider a situation involving $N$ surfaces. Since each one may potentially interact with every other surface, a $N^2$ problem quickly arises. It is important to note the following properties derived from Eq. (D.1):

- Reciprocity theorem: for any pair of surfaces $A_i$ and $A_j$

  $$A_i F_{ij} = A_j F_{ji};$$ \hspace{1cm} (D.2)

- For a closed shape of $N$ surfaces

  $$\sum_{k=1}^{N} F_{ij} = 1.$$ \hspace{1cm} (D.3)
For the two-dimensional case, the determination of the view factors is rather simple. However, for the general case, there is no other solution than integrating either analytically or numerically Eq. D.1. A more detailed derivation of analytic view factors can be found in the *Heat Transfer Handbook* by Bejan.

### D.1 Crossed strings method

The crossed strings method is an algorithm to determine the view factors in bi-dimensional enclosures with constant cross section:

$$F_{ij} = \frac{\sum \text{diagonals} - \sum \text{sides}}{2A_i}.$$  \hspace{1cm} (D.4)

### D.2 Double line integration

Mazumder and Ravishankar present a method to compute the view factors using double line integration (2LI). Such method is based on the application of Stoke’s theorem to Eq. D.1 to obtain:

$$F_{ij} = \frac{1}{2\pi A_i} \int_{C_i} \int_{C_j} \ln r \, d\vec{v}_j \, d\vec{v}_i.$$ \hspace{1cm} (D.5)

A Matlab code is obtained and modified in order to compute Eq. D.5.

#### D.2.1 Singularity of Meeting Edges

Eq. D.5 presents a singularity when \( r = 0 \). Such a situation arises when the polygons have some meeting edge. In this case, a really accurate solution is to obtain the analytic solution by performing the limit of the 2LI integral, as proposed by Mazmuder and Ravishankar:

$$F_{ij}|\text{shared edge} = \frac{L^2}{2} \left(3 - \ln L^2 \right),$$ \hspace{1cm} (D.6)

where \( L \) is the length of the common edge.

### D.3 Validation of the view factor computation

Mazumder and Ravishankar also compare the results obtained by 2LI in various configurations with their analytic solutions. Such results have been used in the present work.
to validate the obtained code. The computed view factors (Table D.1) highly approximate the results obtained by Mazmuder and Ravishankar, therefore the obtained code is successfully validated.

Table D.1: Validation results of the view factor for various geometries.

<table>
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<th>Case</th>
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<th>Computed View Factor</th>
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Bibliography


