Analysis of the Capability of C-Band Fully Polarimetric SAR Data to Monitor Snowed Environments

Author: Yu Zhan
Supervisor: Carlos López Martínez

Universitat Politècnica de Catalunya
Signal Theory and Communications Department

December 9, 2013
Acknowledgement

I would like to express my sincere gratitude to my advisor Prof. Carlos López for the continuous support of my study and research, for his patience, motivation, enthusiasm, and immense knowledge. His guidance helped me in all the time of research and writing of this Master thesis.
Abstract

This study focuses on snow cover monitoring based on C-Band fully Polarimetric Synthetic Aperture Radar (PolSAR) data. Due to the lack of polarimetric sensors, most of the previous studies have been based on single polarization data, supported by time or frequency diversities, resulting in low sensitivity to the presence of snow and its physical parameters below X-Band. As it is shown, the use of polarimetry may overcome the previous limitations allowing to differentiate between dry, wet and absence of snow, making it possible to introduce a snow monitoring methodology based on multi-temporal PolSAR data and the use of target decomposition theorems. In general, snow scattering at C-Band is due primarily to surface and volume scattering. The previous theses are supported by results based on a case study over the North-Eastern Pyrenees, Spain, monitored with the Radarsat-2 system.
## Contents

1 Introduction 9

2 Polarimetry SAR 11
   2.1 Introduction .................................................. 11
   2.2 Scattering Operators and Polarimetric Characterization ............. 15
      2.2.1 Coherency and Covariance Matrix in SAR systems ............. 15
   2.3 Polarimetric SAR Speckle Noise Filtering .......................... 17
   2.4 Review of Target Decomposition Theorems .......................... 20
      2.4.1 Model Based Decompositions ................................ 21
      2.4.2 Eigenvalues/Eigenvector Decompositions ..................... 28
         2.4.2.1 Roll Invariant Property .............................. 30
         2.4.2.2 Other Eigenvalue Based Parameters .................. 31
         2.4.2.3 $H/A/\alpha$ Classification Space .................... 32
         2.4.2.4 Unsupervised Classification Based on Scattering Mechanisms and the Wishart Classifier .......... 33
      2.4.3 Hybrid Decompositions .................................... 35
         2.4.3.1 Nonnegative Eigenvalue Decomposition with Predetermined Volume Scattering .................. 35
         2.4.3.2 Adaptive Volume Approach ............................. 37

3 Snow Remote Sensing using SAR : State of Art 41
   3.1 Introduction .................................................. 41
   3.2 Snow Remote Sensing .......................................... 42
   3.3 Snow Physical Modeling ....................................... 42
      3.3.1 Snow Water Equivalent .................................... 43
      3.3.2 Snow Interaction with Electromagnetic Waves ................ 44
         3.3.2.1 Wet Snow ............................................ 45
         3.3.2.2 Dry Snow ........................................... 45
      3.3.3 Snow Polarimetric Signature ................................ 46
   3.4 Snow Covered Area ............................................. 47
## CONTENTS

3.4.1 Wet Snow Mapping ........................................ 48  
3.4.2 Dry Snow Mapping ....................................... 48  
3.4.3 Wet+Dry Snow Mapping .................................. 49  
3.5 Non Coherent PolSAR Data Methods ....................... 49  
  3.5.1 Snow-Pack Thermal Resistance based Model for SWE estimation . 50  
  3.5.2 Multi-frequency SWE Estimation on Co-Polarization SAR Data . 50  
    3.5.2.1 Snow Density Estimation .......................... 51  
    3.5.2.2 Snow Depth Estimation ........................... 52  
  3.5.3 Backscattering Ratios Retrieval Model ................ 53  
    3.5.3.1 Estimation with Meteorological Model ............... 54  
3.6 Coherent PolSAR Data Methods ............................... 55  
  3.6.1 InSAR for Estimation of Changes in SWE .......... 55  
  3.6.2 Polarimetric Decomposition over Glacier Ice .......... 55  
3.7 Conclusions .................................................. 56  

4 Polarimetric Analysis ......................................... 59  
  4.1 Introduction ............................................... 59  
  4.2 PolSAR Data ................................................ 59  
  4.3 Area Description and Ground truth ....................... 61  
  4.4 Study of Diagonal Elements of Coherency and Covariance Matrix . 64  
  4.5 Study of Different Decomposition Theorems Applied to the Data . 65  
    4.5.1 Model Based Decomposition Study ................... 65  
      4.5.1.1 Freeman Three Component Decomposition .......... 66  
      4.5.1.2 Yamaguchi Four Component Decomposition ....... 67  
    4.5.2 Eigenvectors/Eigenvector Decomposition Study ....... 69  
      4.5.2.1 Eigenvales ........................................ 69  
      4.5.2.2 $\alpha$ angles of Different Eigenvectors ......... 69  
      4.5.2.3 One Component Analysis ........................... 70  
      4.5.2.4 Type of Coherency Matrix in Each Component ....... 70  
      4.5.2.5 $H/A/\alpha$ Interpretation ........................ 71  
      4.5.2.6 Type of Scattering Mechanism ..................... 71  
      4.5.2.7 Number of Scattering Mechanisms ................. 73  
    4.5.3 Hybrid Approach Analysis ............................. 74  
      4.5.3.1 Van Zyl NNE Decomposition ........................ 74  
      4.5.3.2 Study of Volume Adaptive Decomposition .......... 75  
  4.6 Dry Snow Covered Mapping ................................ 79  
    4.6.1 External Information ................................ 79  
    4.6.2 Mapping Algorithm ................................... 79  
  4.7 Conclusions ................................................ 82
## CONTENTS

5 Data Validation 85
  5.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 85
  5.2 SAR and MODIS Comparison . . . . . . . . . . . . . . . . . . . . . . . . . . 85
  5.3 Inverse Geocoding of Spots with Snow Characteristics . . . . . . . . . . . . 87

6 Decomposition via Convex Optimization 91
  6.1 Introduction . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 91
  6.2 Problem Formulation . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 91
     6.2.1 Nonconvex QCQP Solution . . . . . . . . . . . . . . . . . . . . . . . . 96
  6.3 Conclusions . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 97

7 Conclusions 99

Appendix A Tables 103
Chapter 1

Introduction

Snow cover and its parameters estimation play a important role in hydrology as well as in climate studies. The potential of synthetic aperture radar (SAR) systems to cover a large areas with fine spatial resolution has pushed the scientific community to use them to study natural targets, such as snow pack. Unlike the optic systems, the penetration capabilities of microwave radiation of SAR satellites represents a valuable tool in the enhancement of snow monitoring, proving almost weather independent images.

Several authors have reported snow studies with SAR imaging systems, using single polarization configuration plus some temporal or frequency diversity [29] [34] [42]. However, due to the presence of a strong speckle component in the natural environments like snow covers, the scattered waves are partially polarized, and single polarization measurements are not suitable to fully characterize the distributed media. Therefore, fully polarimetric information is needed to obtain higher statistics descriptor, able to extract expanded information of the target.

In this work, the capability of SAR fully polarimetric time series data at C-Band to monitor snowed environments is analyzed. The use of fully polarimetric data allow us to compute second order statistics as covariance and coherency matrices, which are more adequate to characterize distributed targets such as snowpack. Furthermore, target decomposition theorems can be applied to the processed data in order to study the target via different scattering mechanisms. As it will be shown, some decomposition parameters as polarimetric Entropy and $\bar{\sigma}$ can be a valuable key to differentiate between dry, wet and absence of snow, making it possible to introduce a snow monitoring methodology based on multi-temporal PolSAR data.

The motivation behind this work is to develop a snow monitoring service, which would target seasonal snow in northern hemisphere countries, and would be guaranteed in continuity by future’s CSA Radarsat Constellation Mission. The election of C-Band was
encouraged for its trade-off between the sensitivity to the snowpack, reported in previous studies [29] [42] and the accessibility of commercial fully polarimetric sensors.

The present work is organized as follows: Chapter 2 gives the basic background to understand the rest of the documents. Reader familiar with Polarimetric Synthetic Aperture Radar (SAR) systems and polarimetric target decompositions can skip it. Chapter 3 contains the state of art of snow quantitative and qualitative estimation using SAR systems. Chapter 4 describes the acquired data as well as the available ground truth measurements, accomplished with a full polarimetric analysis. In addition, a dry snow mapping algorithm is proposed. In Chapter 5, the proposed SAR snow map is validated against optic acquisition, and some hypothesis of snow analysis is corroborated by specific snow campaign measurements. Chapter 6 tries to explain the methodology used to convert the target decomposition problems into a convex optimization problems and finally Section 7, gathers some conclusions.
Chapter 2

Polarimetry SAR

2.1 Introduction

The study of geophysical medias as snow, rough surfaces, vegetation, ice, etc... is complicated because of their complex structure and composition. Consequently, using radar instruments for the purpose, the knowledge of the exact scattered electromagnetic field, when illuminated by an incident wave, is only possible if a complete description of the scene was available. This type of description is unattainable for practical applications. Hence, the alternative, is to describe them in an statistical form. Those targets are also named as distributed or partial scatteres.

Among the existing radar systems, the potential of Synthetic Aperture Radar (SAR) instruments to cover large areas with fine spatial resolution and the penetration capabilities of microwaves make it a valuable tool for the analysis of natural targets, while providing almost weather independent data. The imaging process of SAR system is laborious and complex, and it will not be detailed here, reader may refers to [1] [2]. Essentially, is an imaging radar mounted on a moving platform, and similar to a conventional radar, electromagnetic waves are sequentially transmitted and the backscattered echoes are collected by the radar antenna. In the case of SAR, the consecutive time of transmission/reception translates into different positions due to the platform movement. Therefore, an appropriate coherent combination of the received signals allows the construction of a virtual aperture that is much longer than the physical antenna length, implying thus an increase of spatial resolution. Nowadays, the existing SAR systems can map areas of tens of kilometers with spatial resolution of meters, for instance, a region of 25 $Km$ with resolution cell of 10 $m^2$ is translated into an image of 2500 x 2500 pixels, where the complex value of each pixel is described by the Equation (2.1)
\[ S_{qp}(x, r) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma_s(x', r') h(x - x', r - r') dx' dr', \quad (2.1) \]

where \( p \) and \( q \) indicate the transmitted and received polarization states respectively, \( x \) and \( r \) are the azimuth (typically height of the image) and range (typically width of the image) dimensions of the SAR image. \( \sigma_s(x, r) \) is the scene’s complex reflectivity, and \( h(x, r) \), the SAR system impulse response.

Assuming that the scattered wave from any distributed scatterer to be originated by a set of discrete sources, Equation (2.1) can be considered in its discrete form as follows

\[ S_{qp}(x, r) = \sum_{k=1}^{N} \sigma_s(x_k, r_k) h(x - x_k, r - r_k), \quad (2.2) \]

where the sub-index \( k \) refers to each particular discrete scatterer in the resolution cell, and \( N \) is the total number of scatterers embraced by the response of the SAR system \( h(x, r) \).

In the Figure 2.1 is shown a possible situation. Furthermore, for ease of interpretation of the underlying phenomenon, the Equation (2.2) can be rewritten by using

\[ \sigma_s(x_k, r_k) = \sqrt{\sigma_k} e^{j \theta_{sk}} \quad (2.3) \]

\[ h(x - x_k, r - r_k) = h_k e^{j \phi_k} \quad (2.4) \]

\[ \theta_{sk} = \theta'_{sk} + \phi_k, \quad (2.5) \]

and the complex description, where \( r \) now denotes amplitude and \( \theta \) phase.
\[ S_{qp}(r, \theta) = \text{Real}(S_{qp}) + j \text{Imag}(S_{qp}) = re^{j\theta} \]  

(2.6)

\[ re^{j\theta} = \sum_{k=1}^{N} A_k e^{j\theta s_k} \]  

(2.7)

\[ \text{Real}(S_{qp}) = \sum_{k=1}^{N} A_k \cos \theta s_k \]  

(2.8)

\[ \text{Imag}(S_{qp}) = \sum_{k=1}^{N} A_k \sin \theta s_k, \]  

(2.9)

where \( A_k = h_k \sqrt{\sigma_k} \). As observed in the Equation (2.7), the process to form a SAR image pixel consists of the complex coherent addition of the responses of each one of the discrete scatters, which are not accessible individually. And the sole available measure is the complex coherent addition itself. This coherent addition process receives the name of bi-dimensional random walk, see Figure 2.2.

![Figure 2.2: Complex coherent addition of scatterers inside the resolution cell.](image)

At this point, it is necessary to consider certain assumptions about the elementary scattered waves \( A_k e^{j\theta s_k} \) in order to derive an stochastic model for the observed SAR image \([3]\) \([4]\). For instance:

1. The amplitude \( A_k \) and the phase \( \theta s_k \) of the \( k_{th} \) scattered wave are statistically independent of each other and from the amplitudes and phases of all other elementary waves.

2. The phases of the elementary contributions \( \theta s_k \) are equal likely to lie anywhere in the primary interval \([−\pi, \pi]\).
Under these conditions, Equation (2.7) can be seen as an interference process, in which the interference itself is controlled by the phases \( \theta_{s_k} \). This interference can be constructive, as well as, destructive, the behavior is totally random. This aspect can be clearly identified in SAR images, as the amplitude, or the intensity of Equation (2.7) present a salt and pepper or grainy aspect. Such a phenomenon is called speckle.

When the number of discrete scatterers inside the resolution cell \( N \) is large, provided that \( A_k \cos \theta_{s_k} \) and \( A_k \sin \theta_{s_k} \) satisfy the Central Limit Theorem, the quantities \( \text{Real}(S_{qp}) \) and \( \text{Imag}(S_{qp}) \) are non-correlated normally distributed, that is, they follow a zero-mean, Gaussian probability density function (pdf). The parameters of this pdf, can be obtained on the basis of the discrete scatterers model. The mean values of \( \text{Real}(S_{qp}) \) and \( \text{Imag}(S_{qp}) \) are then

\[
E\{\text{Real}(S_{qp})\} = \sum_{k=1}^{N} E\{A_k \cos \theta_{s_k}\} = \sum_{k=1}^{N} E\{A_k\} E\{\cos \theta_{s_k}\} = 0
\]  

\[
E\{\text{Imag}(S_{qp})\} = \sum_{k=1}^{N} E\{A_k \sin \theta_{s_k}\} = \sum_{k=1}^{N} E\{A_k\} E\{\sin \theta_{s_k}\} = 0, \quad (2.11)
\]

where \( E\{\cdot\} \) express the ensemble average. Using the same arguments, the variances are obtained as

\[
E\{\text{Real}^2(S_{qp})\} = \sum_{k=1}^{N} E\{A_k^2\} E\{\cos^2 \theta_{s_k}\} = \frac{N}{2} E\{A_k^2\} \quad (2.12)
\]

\[
E\{\text{Imag}^2(S_{qp})\} = \sum_{k=1}^{N} E\{A_k^2\} E\{\sin^2 \theta_{s_k}\} = \frac{N}{2} E\{A_k^2\}.
\]  

Besides, the symmetry of the phase distribution of the discrete scatters produces

\[
E\{\text{Real}(S_{qp}) \text{Imag}(S_{qp})\} = \sum_{k=1}^{N} \sum_{l=1}^{N} E\{A_k A_l\} E\{\cos \theta_{s_k} \sin \theta_{s_l}\} = 0. \quad (2.14)
\]

However, as mentioned before, owing to the complexity of natural targets, the scattered wave has also a complex behavior, as it is a function of the incident wave properties, as well as, the scatterer features. Statistical characterization of single measurements present its limitations, specially in the case of distributed media, where the scattered wave is partially polarized. Therefore, the same spot should be measured with some diversity in order to obtain higher statistical moments. One possibility is to consider the change of the polarization state that a scatterer may induce to an incident field. Thus, by altering the wave polarization in transmission and reception, a full scattering matrix can be obtained. This scattering matrix allows to build up a powerful observation space sensitive to shape, orientation and dielectric properties of the scatterers and allows the development
of physical models for the identification and/or separation of scattering mechanisms occurring inside the same resolution cell. The SAR polarimetry (PolSAR) community has been developing specific models and techniques to study the mentioned matrix.

2.2 Scattering Operators and Polarimetric Characterization

The basic concept of SAR polarimetry is given by the 2x2 complex scattering matrix that describes the transformation of the two-dimensional transmitted (e.g., incidence) plane wave vector $\mathbf{E}^i$ into the received (e.g., scattered) wave vector $\mathbf{E}^s$ (two-dimensional in the far field of the scatterer). Thus, by altering horizontal and vertical polarization in transmission and reception, the form of scattering matrix is described by the Equation (2.15)

$$
\begin{bmatrix}
E_H^s \\
E_V^s
\end{bmatrix} = \frac{1}{r} \begin{bmatrix}
S_{HH} & S_{HV} \\
S_{VH} & S_{VV}
\end{bmatrix} 
\begin{bmatrix}
E_H^i \\
E_V^i
\end{bmatrix} e^{-jkr},
$$

(2.15)

where $H$ and $V$ indicate linear horizontal and vertical polarization states, respectively, $r$ is the distance between the scatterer and the receiving antenna and $k$ is the wavenumber of the illuminating field. The coefficient $\frac{1}{r}$ represents the attenuation between the scatterer and the receiving antenna, which is produced by the spherical nature of the scattered wave. On the other hand, the phase factor $-jkr$ represents the delay of the travel of the wave from the scatterer to the antenna.

It is worth to mention that the scattering matrix $S$ characterizes the target under observation for a fixed imaging geometry and frequency. In addition, the four elements must be measured at the same time, specially in those situations where the scatterer is not static or fixed. If they are not measured at the same time, the coherency between the elements may be lost, as the different elements may refer to a different scatterer. The best example of this situation appears when the ocean surface is measured. At a given frequency, the surface of the ocean is characterized by a coherent time that indicates the period of time in which the surface can be considered fixed. In this case, the scattering matrix elements should be measured in an interval lower that the coherence time of the ocean surface.

Once the scattering matrix is measured, higher statistic moments can be computed by spatial averaging, namely, covariance and coherency matrix are computed as second statistics.

2.2.1 Coherency and Covariance Matrix in SAR systems

To compute second order statistics, a polarimetric vector needs to be defined first, which is given by the Equation (2.16)

$$
k = \frac{1}{2} tr(S \Phi),
$$

(2.16)
where \( tr(\cdot) \) computes the trace of the given matrix, and \( \Phi \) is a complete set of 2x2 complex basis matrices that are constructed as an orthogonal set under Hermitian inner product. However, for a reciprocal target matrix, in the mono-static backscattering case, the reciprocity constrains the scattering matrix to be symmetrical, that is, \( S_{HV} = S_{VH} \). Thus, the 4-D polarimetric target vectors reduce to 3-D polarimetric target vectors. The are two orthogonal special sets [5]:

- **Pauli spin matrix basis set**, \( \{ \Phi_P \} \)

\[
\{ \Phi_P \} = \left\{ \sqrt{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \sqrt{2} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \sqrt{2} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right\}
\]  

(2.17)

and the corresponding 3-D Pauli feature vector becomes

\[
k_P = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{HH} + S_{VV} \\ S_{HV} - S_{VH} \\ 2S_{HV} \end{bmatrix}^T,
\]

(2.18)

and the coherency matrix is compute as

\[
T = \langle k_P k_P^H \rangle =
\frac{1}{2} \begin{bmatrix}
\langle |S_{HH} + S_{VV}|^2 \rangle & \langle (S_{HH} + S_{VV})(S_{HH} - S_{VV})^* \rangle & 2\langle (S_{HH} + S_{VV})S_{HV}^* \rangle \\
2\langle S_{HV}(S_{HH} + S_{VV})^* \rangle & \langle |S_{HH} - S_{VV}|^2 \rangle & 2\langle (S_{HH} - S_{VV})S_{HV}^* \rangle \\
2\langle S_{HV}^*(S_{HH} - S_{VV})^* \rangle & 4\langle |S_{HV}|^2 \rangle & \langle |S_{VV}|^2 \rangle 
\end{bmatrix}
\]

(2.19)

where \( \dagger \) is the complex conjugation and \( \langle \cdot \rangle \) means spatial ensemble average.

- **Lexicographic matrix basis set**, \( \{ \Phi_L \} \)

\[
\{ \Phi_L \} = \left\{ 2 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, 2\sqrt{2} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \sqrt{2} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}
\]

(2.20)

and the corresponding 3-D Lexicographic feature vector becomes

\[
k_L = \begin{bmatrix} S_{HH} & \sqrt{2}S_{HV} & S_{VV} \end{bmatrix}^T,
\]

(2.21)

and the covariance matrix is compute as

\[
C = \langle k_L k_L^H \rangle =
\begin{bmatrix}
\langle |S_{HH}|^2 \rangle & \sqrt{2} \langle S_{HH}S_{HV}^* \rangle & \langle S_{HH}S_{VV}^* \rangle \\
\sqrt{2} \langle S_{HV}S_{HH}^* \rangle & 2\langle |S_{HV}|^2 \rangle & \sqrt{2} \langle S_{HV}S_{VV}^* \rangle \\
\langle S_{VV}S_{HH}^* \rangle & \sqrt{2} \langle S_{VV}S_{HV}^* \rangle & \langle |S_{VV}|^2 \rangle 
\end{bmatrix}
\]

(2.22)

where \( \dagger \) is the complex conjugation and \( \langle \cdot \rangle \) means spatial ensemble average.
When the objective is to interpret physical phenomenon as snow scattering, the coherency matrix represents a more adequate choice, because of the directly identification of scattering mechanisms by its diagonal elements. For instance, \( \left| S_{HH} + S_{VV} \right|^2 \) is related to surface scattering, \( \left| S_{HH} - S_{VV} \right|^2 \) to double bounce mechanisms and \( \left| S_{HV} \right|^2 \) to volume component. However, once coherency matrix is computed, the covariance matrix can be easily obtained by applying a simple unitary transform \([5]\).

Notice that to compute both matrices, a spatial ensemble average is required. The aim of this filtering process is to reduce the impact of speckle noise mentioned in the previous section, which allows a better characterization of the studied target. Here an advanced technique, called Binary Partition trees, is used for the purpose, and it will be explained in the next section.

### 2.3 Polarimetric SAR Speckle Noise Filtering

As mentioned in the previous sections, due to the coherent nature of the target under study, SAR images are corrupted by the speckle noise. Consequently, to characterize its behavior, a filtering process should be carried out in order to estimate the second order statistics as the covariance or coherency matrices. Conventional speckle filtering techniques are pixel based, which performs a low pass filtering approach defining an homogeneous region around a given position. The main problem of those techniques are the mixture of non-homogeneous areas and the loss of spatial resolution. Thus, the image structure is not preserved.

Recently, a novel technique called Binary Partition Tree (BPT) was introduced by Alberto et. al. \([6]\) to filter the speckle noise of SAR images. BPT is a region-based and multi-scale image representation. It contains information of the image structure at different details levels within a tree. For instance, in the Figure 2.3 is shown a tree computed from a SAR image, where each node stands for a connected region of the image, the tree leaves represent single pixels and the tree edges describe the inclusion relationship between nodes. In this hierarchical representation, the nodes in higher levels represent larger regions as result of merging its two son nodes. Therefore, the root node symbolizes the whole image. Notice that between the leaves and the root there are a wide number of nodes representing regions of the image with different detail level. This multi-scale representation contains a lot of useful information about the image structure that can be used to filter the polarimetric SAR image.

The BPT technique can be mainly divided in two steps:

- Construction process: To construct the BPT representation from an image, an iterative algorithm is employed in a bottom-up approach. In the initial state, every pixel of the image becomes a one-pixel region. Then, at every step, the two most
similar regions are merged and this process is repeated until it reaches to the root of the tree, which contains the whole image. In order to apply this algorithm, two important concepts have to be defined first:

1. A region model: traditionally, under the complex Gaussian PolSAR model, the estimated covariance matrix $C$ is employed to measure the region polarimetric information

$$C = \langle k_L k_L^H \rangle = \frac{1}{n} \sum_{i=1}^{n} k_L i k_L^H$$ (2.23)

where $k_i$ represents the polarimetric target vector of the $i^{th}$ pixel and $n$ represents the region size in pixels.

Additionally, since during the BPT construction process regions of different sizes coexist, the region size information should be taken into account and it will be included in the region model.

2. A similarity measure on the region model space to compare two neighboring regions $d(X,Y)$. Here, the Geodesic dissimilarity $d_{sg}$ is considered. It is based on the positive definite matrix cone geometry [6], that is, the geometry of the region model space. $d_{sg}$ measures the distance over the geodesic path, instead of the euclidean path, that follow the curvature of the matrix cone space. A modified version is generated by adding a term depending on the region size, and shown in Equation (2.24)
\[ d_{sg}(X,Y) = \left\| \log(C_X^{-1/2}C_Y^{-1/2}) \right\|_F + \ln \left( \frac{2n_x n_y}{n_x + n_y} \right), \]  

(2.24)

where \( C_X \) and \( C_Y \) represent the estimated covariance matrices for regions \( X \) and \( Y \), respectively, \( \left\| \cdot \right\|_F \) is the Frobenius matrix norm, \( \log(\cdot) \) represents the matrix logarithm and \( \ln(\cdot) \) is the natural logarithm. Other dissimilarities as Wishart or Ward relative [6] can also be used to construct the structure, but the Geodesic distance is invariant to unitary transformations, which means that there are not differences if the coherency matrix is employed instead of covariance matrix as the measure of polarimetric information.

It is worth to mention that this construction process depends only to the image and consequently it is application independent.

- Pruning process: For the speckle filtering application, the main objective is to obtain the biggest possible homogeneous regions of the image. The BPT and its multi-scale nature can be exploited for this purpose. For instance, an homogeneity-based tree pruning can be performed. Thus, a region homogeneity measure \( \phi \) has to be defined to be able to characterize a pruning process. In [6], is proposed the criterion based on the Frobenius matrix norm

\[ \phi_R(X) = \frac{1}{n_x} \sum_{i=1}^{n_x} \frac{||C_i - C_X||_F}{||C_X||_F^2}, \]  

(2.25)

where \( C_i \) is the estimated covariance matrix for the \( i^{th} \) pixel within the region \( X \). Notice that this measure depends on all the pixel values within the \( X \) region and its region model. Additionally, \( \phi_R \) is independent of the region size, since it is an average over all the region pixels, which is an important property of the homogeneity measure in order to define the region homogeneity independently of its size. The measure of Equation (2.25) can be seen as the mean information loss when modeling all the pixels within a region with its estimated covariance matrix. Finally, to determine if a region is homogeneous or not, a maximum value \( \delta_p \) for the homogeneity measure has to be defined, called pruning threshold. Therefore, the pruning process will consist to select the bigger regions \( X_i \) having \( \phi_R(X_i) < \delta_p \) from the tree. Normally, the \( \delta_p \) will be expressed in dB scale, corresponding to \( 10\log_{10}(\phi_R) \).

Furthermore, this BPT technique can be extended to estimate the temporal evolution of the scenario, for instance, detecting changes due to environmental phenomenons [7]. As in the previous explanation, devoted to process a single PolSAR image, the image pixels will be considered as the initial data elements, but in the extended version, a different neighborhood has to be defined over them, taking into account neighboring pixels in
space and time dimensions. This new connectivity allows the algorithm to seek changes of the homogenous regions through different time acquisitions. It is obvious that the images under study should be co-registered in order to have pixel to pixel correspondence, because measuring the time variation between two different regions does not make sense. In the Figure 2.4 is shown a example of space-time connectivity. For more details, refer to [?].

![BPT space-time connectivity](image_url)

**Figure 2.4: BPT space-time connectivity.**

### 2.4 Review of Target Decomposition Theorems

The extraction of physical parameters from observed backscattering of complex structures and their modeling are the objectives of the different decomposition theorems. In the PolSAR literature, many approaches have been proposed by several authors, and they can be classified mainly in three types [9]: those using an analysis of the covariance or coherency matrix, those based in the Mueller matrix and Stokes vector, and those employing coherent decomposition of the scattering matrix.

In the case of modeling natural scattering environments like snow cover, the observed scatterers can be considered as distributed targets. Therefore, the attention should be centered in the former type of decomposition theorems. Because, it is well known that scattering matrix does not offer a proper interpretation of the data due to the speckle noise, and only pure targets have a covariance or coherency matrix related to single equivalent scattering matrix. The case of the Muller matrix is neither interesting here, since all the considerations made in such matrix can be equally found in the coherency approach.

Within these covariance or coherency matrix decomposition methods, considering a speckle pre-filtering process, we can identify three branches: model based decompositions, eigenvector/eigenvalues analysis, and hybrid approaches. The last method tries to combine the previous two named techniques, taking the advantages and breaking the disadvantages.

In the rest of the section, coherency matrix is employed to explain the aforementioned
decomposition theorems, by the reason of an easier physical interpretation. All these
processes can be equally applied to the covariance matrix, because to pass one matrix
representation to another, only a linear transformation is needed. In addition, notice that
all the expressions consider mono-static satellite configuration.

2.4.1 Model Based Decompositions

Originally, Freeman et al. [11] proposed a model based decomposition theorem for forest
areas, where the dominant scatterer was set to be the volume contribution. Such approach
considers the coherency matrix as sum of three individual matrices: surface, double-bounce
and volume, giving a certain contribution to each of them. All the three components
have predefined forms, related to a specific scattering models. The suggested method
plus a few assumptions makes the decomposition really simple, resulting in to nothing
more than solving a linear equation system. However, the problem comes out when the
model does not match with the studied targets, which is the main drawback of model
based approaches. For example, the method is inappropriate for scenes where the surface
scattering is dominant due to the volume predominant assumption.

Let $\mathbf{T}$ be the observed coherency matrix after the speckle filtering process, then, it can
be written as

$$\mathbf{T} = f_v \mathbf{T}_v + f_s \mathbf{T}_s + f_d \mathbf{T}_d$$  (2.26)

where $\mathbf{T}$ is a hermitic matrix shown in Equation 2.19, the vector $\mathbf{k}_P$ is the 3-D Pauli
feature vector illustrated in Equation 2.18, and each of the individual matrices are set as
following:

- Surface scattering: a first order Bragg model is used, derived as a low frequency
  scattering approximation in the microwave region. The corresponding 2x2 scattering
  matrix $S_s$ is giving by the Equation (2.27)

$$S_s = \begin{bmatrix} R_h(\theta, \varepsilon_s) & 0 \\ 0 & R_v(\theta, \varepsilon_s) \end{bmatrix},$$  (2.27)

where the coefficients $R_h$ and $R_v$ are the horizontal and the vertical Bragg scattering
coefficients

$$R_v = \frac{(\varepsilon_s - 1)(\sin^2 \theta - \varepsilon_s (1 + \sin \theta))}{(\varepsilon_s \cos \theta + \sqrt{\varepsilon_s - \sin^2 \theta})^2},$$  (2.28)

$$R_h = \frac{\cos \theta - \sqrt{\varepsilon_s - \sin^2 \theta}}{\cos \theta + \sqrt{\varepsilon_s - \sin^2 \theta}}.$$  (2.29)
In consequence, the second order statistic after normalization is shown in the Equation (2.30)

$$T_s = \begin{bmatrix} 1 & \beta^* & 0 \\ \beta & |\beta|^2 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

(2.30)

the expression of $\beta$ is illustrated in the Equation (2.31)

$$\beta = \frac{R_h(\theta, \varepsilon_s) + R_v(\theta, \varepsilon_s)}{R_h(\theta, \varepsilon_s) - R_v(\theta, \varepsilon_s)}.$$  

(2.31)

- Double bounce scattering: is modeled by double reflection on smooth dielectric, leading the following form of scattering matrix

$$S_d = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -e^{i\phi} \end{bmatrix} \begin{bmatrix} R_{sh} & 0 & R_{th} & 0 \\ 0 & R_{sv} & 0 & R_{tv} \end{bmatrix} = \begin{bmatrix} R_{sh}R_{th} & 0 & -R_{sv}R_{tv}e^{i\phi} \end{bmatrix}. $$  

(2.32)

The horizontal and vertical Fresnel coefficients are $R_{sh}$ and $R_{sv}$ for the soil, and $R_{sv}$ and $R_{tv}$ for the trunk plane. These coefficients depend on their corresponding dielectric constant $\varepsilon_s$, $\varepsilon_t$, and the incidence angles $\theta_s$, $\theta_t$, with a particular relationship between angles: $\theta_t = \frac{\pi}{2} - \theta_s$. Equation (2.33) displays the definition of these parameters

$$R_{ih} = \frac{\cos \theta_i - \sqrt{\varepsilon_i - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{\varepsilon_i - \sin^2 \theta_i}},$$

(2.33)

$$R_{iv} = \frac{(\varepsilon_i - 1)(\sin^2 \theta_i - \varepsilon_i(1 + \sin \theta_i))}{(\varepsilon_i \cos \theta_i + \sqrt{\varepsilon_i - \sin^2 \theta_i})^2},$$

(2.34)

where $i \in \{t, s\}$. In addition, the phase $e^{i\phi}$ accounts for the case of a differential propagation phase introduced by vegetation layer. Figure 2.5 illustrates a possible situation.

Then, the coherency matrix after normalization takes the form

$$T_d = \begin{bmatrix} |\alpha|^2 & \alpha & 0 \\ \alpha^* & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

(2.35)

where the ratio $\alpha$ is given by

$$\alpha = \frac{R_{sh}R_{th} - R_{sv}R_{tv}e^{i\phi}}{R_{sh}R_{th} + R_{sv}R_{tv}e^{i\phi}}.$$  

(2.36)
2.4. REVIEW OF TARGET DECOMPOSITION THEOREMS

- Volumen scattering: it is assumed that the radar return comes from a cloud of randomly oriented, very thin, cylinder-like scatterers.

Supposing that the scattering matrix in the Equation (2.30) as the standard oriented one

\[ S = \begin{bmatrix} S_{HH} & 0 \\ 0 & S_{VV} \end{bmatrix}, \]  

therefore, the Equation (2.38) gives the expression of the rotated version, with the angle \( \phi \) from vertical polarization direction about the radar look direction

\[ \mathbf{S}_v(\phi) = \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} S = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} S_{HH} & 0 \\ 0 & S_{VV} \end{bmatrix} = \begin{bmatrix} S_{HH} \cos^2 \phi + S_{VV} \sin^2 \phi & (S_{VV} - S_{HH}) \cos \phi \sin \phi \\ (S_{VV} - S_{HH}) \cos \phi \sin \phi & S_{HH} \sin^2 \phi + S_{VV} \cos^2 \phi \end{bmatrix}. \]  

(2.38)

In addition, taking an uniform distribution of the angle \( \phi \), \( p(\phi) = \frac{1}{2\pi} \), the different elements of the \( \mathbf{T}_v \) matrix can be computed as

\[ T_{11} = \int T_{11}(\phi)p(\phi)d\phi = \frac{|S_{HH} + S_{VV}|^2}{2} \]  

(2.39)

\[ T_{22} = \int T_{22}(\phi)p(\phi)d\phi = \frac{|4a_1 + 1 - 4a_2||S_{HH} - S_{VV}|^2}{2} \]  

(2.40)

\[ T_{33} = \int T_{22}(\phi)p(\phi)d\phi = \frac{4a_3|S_{HH} - S_{VV}|^2}{2}, \]  

(2.41)

Figure 2.5: Double bounce scattering
with coefficients $a_i$ for $i = 1..3$ taking the form of

$$a_1 = \frac{1}{2\pi} \int_0^{2\pi} \cos(\phi)^4 d\phi = \frac{3}{8} \quad (2.42)$$

$$a_2 = \frac{1}{2\pi} \int_0^{2\pi} \cos(\phi)^2 d\phi = \frac{1}{2} \quad (2.43)$$

$$a_3 = \frac{1}{2\pi} \int_0^{2\pi} \cos(\phi)^2 \sin(\phi)^2 d\phi = \frac{1}{8} \quad (2.44)$$

Notice that only diagonal elements of the coherency matrix are shown, because all the rest are equal to zero.

One more step should be carried out in order to reach the final expression of the volume component, and it is to assume the cylinder scatterers as dipoles. There is no difference between vertical oriented $S_{v\text{-dipole}} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ or horizontal oriented $S_{h\text{-dipole}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, the resulting coherency matrix is

$$T_v = \frac{1}{4} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (2.45)$$

Once the three scattering components are presented, the remain step to complete the decomposition is to estimate the different parameters of the models. In order to have an easier interpretation, all the unknown parameters are grouped by following four equations

$$\frac{1}{2} \left| S_{HH} + S_{VV} \right|^2 = f_s + |\alpha|^2 f_d + \frac{f_v}{2} \quad (2.46)$$

$$\frac{1}{2} \left| S_{HH} - S_{VV} \right|^2 = |\beta|^2 f_s + f_d + \frac{f_v}{4} \quad (2.47)$$

$$\frac{1}{2} \langle (S_{HH} - S_{VV})(S_{HH} + S_{VV})^* \rangle = \beta f_s + \alpha^* f_d \quad (2.48)$$

$$2 \left| S_{HV} \right|^2 = \frac{f_v}{4}. \quad (2.49)$$

In this manner, volume contribution can be directly retrieved from $\langle |S_{HV}|^2 \rangle$, leaving an underdetermined inversion problem of three equations for four unknown parameters.

$$\frac{1}{2} \left| S_{HH} + S_{VV} \right|^2 - 4 \left| S_{HV} \right|^2 = |\alpha|^2 f_d + f_s \quad (2.50)$$
\[
\frac{1}{2} \langle |S_{HH} - S_{VV}|^2 \rangle - 2 \langle |S_{HV}|^2 \rangle = |\beta|^2 f_s + f_d 
\]
(2.51)

\[
\frac{1}{2} \langle (S_{HH} - S_{VV})(S_{HH} + S_{VV})^*\rangle = \alpha^* f_d + \beta f_s,
\]
(2.52)

to solve this system, an extra condition is added depending on the values of \(\langle |S_{HH} + S_{VV}|^2 \rangle\) and \(\langle |S_{HH} - S_{VV}|^2 \rangle\). If \(\langle |S_{HH} + S_{VV}|^2 \rangle > \langle |S_{HH} - S_{VV}|^2 \rangle\), then, surface scattering is dominant, and \(\alpha = 0\), otherwise, double bounce scattering is dominant and \(\beta = 0\).

Note that this condition is totally equivalent to the original condition used in Freeman’s paper, where the sign of \(\text{Re}(\langle S_{HH}S_{VV}^* \rangle)\) indicates the dominant scatterer, if \(\text{Re}(\langle S_{HH}S_{VV}^* \rangle) > 0\) means surface predominates over double bounce. To check their equivalence, the relation between the coherency and covariance matrix is used [5],

\[
\frac{1}{2} \langle |S_{HH} + S_{VV}|^2 \rangle = \frac{1}{2} \left[ \langle |S_{HH}|^2 \rangle + \langle |S_{VV}|^2 \rangle + 2 \text{Re}(\langle S_{HH}S_{VV}^* \rangle) \right] 
\]
(2.53)

\[
\frac{1}{2} \langle |S_{HH} - S_{VV}|^2 \rangle = \frac{1}{2} \left[ \langle |S_{HH}|^2 \rangle + \langle |S_{VV}|^2 \rangle - 2 \text{Re}(\langle S_{HH}S_{VV}^* \rangle) \right] 
\]

Therefore,

\[
\langle |S_{HH} + S_{VV}|^2 \rangle > \langle |S_{HH} - S_{VV}|^2 \rangle \implies \text{Re}(\langle S_{HH}S_{VV}^* \rangle) > 0.
\]
(2.54)

Finally, when all the contributions are acquired, the total power \(\text{Span}\) can be expressed as the sum of the power of each scattering mechanism. Starting with the definition shown in (2.55)

\[
\text{Span} = |k_p|^2 = \frac{1}{2} \left\langle |S_{HH} + S_{VV}|^2 \right\rangle + \frac{1}{2} \left\langle |S_{HH} - S_{VV}|^2 \right\rangle + 2 \left\langle |S_{HV}|^2 \right\rangle 
\]
(2.55)

then, substituting each elements of the coherency matrix for their model decomposition, \(\text{Span}\) can be rewritten as

\[
\text{Span} = P_s + P_d + P_v = f_s(1 + |\alpha|^2) + f_d(1 + |\beta|^2) + f_v.
\]
(2.56)

Where, \(P_s, P_d, P_v\) are power contributions of surface, double bounce and volume respectively.

As we have mentioned, the process is quite automatic and simple. However, when the studied scatterer does not correspond with the introduced pattern, serious problems come out:

1. Surface and double bounce scattering model: In principle, Freeman decomposition is an incoherent approach. However, except the volume model, the rest of components
are adopted from coherent scatterings with zeros cross-polarization. In consequence, the volume power tends to be overestimated, which may cause negative power of odd and even reflection when the volume is removed. Namely, aforementioned values like \( f_s \) and \( f_d \) could result negatives, which is physically unacceptable.

2. Volume scattering model: The volume scattering is the most difficult component to model. Freeman selects a random uniformly distributed dipoles to adjust it, but the model may not be proper for an specific wavelength or forest, leading also to a significant number of pixels with negative power.

3. Reflection symmetry: Is implicitly assumed for taking models that only five parameters of the coherency matrix is used instead of the completely nine values.

4. Orientation angle compensation effects: Very recently, the effects of orientation compensation on the scattering model based decomposition have been investigated by several researchers [12], [13]. Basically, for tilted surfaces, the polarization states is no longer parallel or normal to the surface, which may cause a possible deviation from the real backscattering coefficients.

Several authors have proposed possible solutions to deal with those problems:

- Yamaguchi et al. : Proposed a four component based decomposition to treat with problems 2 and 3 [14]. A fourth helix element is added to avoid the reflection symmetry assumption. This intent is to account for seven out of nine values of the coherency matrix instead of five. Equation (2.57) shows the decomposition scheme

\[
T = f_v T_v + f_s T_s + f_d T_d + f_c T_c,
\]  

where \( T_c \) can be right or left circular helix. The condition to choose one or another is annotated in the following equations

\[
If \quad \text{Im}(\langle (S_{HH}S_{HV}^*) + (S_{HV}S_{VV}^*) \rangle) > 0 \implies T_{c\text{RHC}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2}j \\ 0 & -\frac{1}{2}j & \frac{1}{2} \end{bmatrix} \tag{2.58}
\]

\[
If \quad \text{Im}(\langle (S_{HH}S_{HV}^*) + (S_{HV}S_{VV}^*) \rangle) < 0 \implies T_{c\text{LHC}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2}j \\ 0 & \frac{1}{2}j & \frac{1}{2} \end{bmatrix} \tag{2.59}
\]

the expression \( \langle (S_{HH}S_{HV}^*) + (S_{HV}S_{VV}^*) \rangle \) can be equally expressed as \( T_{13} + T_{23} + T_{13}^* - T_{23}^* \), where \( T_{ij} \) means the element of row number \( i \), column number \( j \) of the coherency matrix.
They also introduced a modification of the volume scattering matrix, suggesting a new distribution of dipoles orientation angle to complement with the original uniform approach: \( p(\phi) = \frac{1}{2} \sin \phi \). In this way, two different scattering component for vertical and horizontal oriented dipoles can be obtained:

\[
S_{\text{v-dipole}} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow T_{\text{v-dipole}}^\text{v} = \frac{1}{30} \begin{bmatrix} 15 & 5 & 0 \\ 5 & 7 & 0 \\ 0 & 0 & 8 \end{bmatrix}
\]

\[
S_{\text{v-dipole}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow T_{\text{v-dipole}}^\text{h} = \frac{1}{30} \begin{bmatrix} 15 & -5 & 0 \\ -5 & 7 & 0 \\ 0 & 0 & 8 \end{bmatrix}
\]

The choice of the specific kind of volume component yield in the ratio defined in the Equation (2.62)

\[
v_r = 10 \log \frac{\langle |S_{VV}|^2 \rangle}{\langle |S_{HH}|^2 \rangle}.
\]

If \( v_r > 2dB \), the horizontal oriented dipole with sinus distribution is taken, when \( v_r < -2dB \), the vertical oriented dipole with sinus distribution is picked. Otherwise, uniform approach as the case of Freeman is chosen. Both improvements help to moderate the presence of negative powers. More detail of the algorithm is presented in [14].

- I.Hajnsek et al [15]: Have introduced an extended Bragg surface model, called X-Bragg model, which accounts the cross polarization deficiency introduced in the former point

\[
T_X^\delta = \begin{bmatrix} 1 & \beta^* \text{sinc}(2\delta) & 0 \\ \beta \text{sinc}(2\delta) & \frac{1}{2} |\beta|^2 (1 + \text{sinc}(4\delta)) & 0 \\ 0 & 0 & \frac{1}{2} |\beta|^2 (1 - \text{sinc}(4\delta)) \end{bmatrix}
\]

The roughness effect is inserted by integrating a Bragg surface over a line of sight rotation angle parameterized by the width of \( \delta \) distribution. Consequently, this parameter controls the depolarization as well as the cross polarized power level. In the limit of \( \delta = 0 \), X-Bragg converges to the Bragg surface.

- Arii and van Zyl: treat more deeply the problem of negative pixels resulting from the model based decomposition, and proposed a hybrid non-negative decomposition, see more details in Section 2.4.3. Furthermore, they extended the algorithm to extract adaptively the volume component.
• Lee et al [12], [13] proposed to estimate the orientation angle from the data itself. Then, rotating the coherency matrix by the negative of those angles about the line of sight can mitigate the effect mentioned in point 4. Lee and Ainsworth [12] showed that after the compensation, the volume scattering power of model decomposition theorems is consistently reduced, while the double bound has increased, and the surface scattering power maintains relatively unchanged.

2.4.2 Eigenvalues/Eigenvector Decompositions

S. Cloude was the first to consider an eigenvalue/eigenvector decomposition of coherency matrix to analyze the scattering process of the observed data. Unlike the model based approaches, this technique is a mathematical method, which provides uniqueness besides the easy computation. This mathematical nature makes necessary an additional step in the decomposition in order to establish a physical interpretation of the eigenvalues and eigenvectors. However, no assumptions are needed to perform the decomposition.

The basic idea of the eigenvalue/eigenvector approach is similar to model based method: to represent the coherency matrix \( T \) as a sum of individual scattering mechanisms \( T_i \), related to a certain scattering matrix \( S_i \) [5]. The physical basis for choosing the target components is to establish statistical independence between the targets \( S_i \)

\[
T = U_3 \Sigma U_3^H ,
\]  

(2.64)

where \( \Sigma \) is a diagonal matrix, whose diagonal elements are eigenvalues of the matrix \( T \). And \( U_3 \) contains in columns the eigenvectors associated to these eigenvalues

\[
\Sigma = \begin{bmatrix}
\lambda_1 & 0 & 0 \\
0 & \lambda_2 & 0 \\
0 & 0 & \lambda_3 
\end{bmatrix} ,
\]  

(2.65)

\[
U_3 = \begin{bmatrix}
u_1 & u_2 & u_3
\end{bmatrix} .
\]  

(2.66)

Due to the Hermitian propriety of the coherency matrix, all the eigenvalues are positives values. Therefore, it is possible to sort them in a descent way, i.e., \( \lambda_1 > \lambda_2 > \lambda_3 > 0 \). This sorting process is implicitly assumed when the eigenvalue/eigenvector decomposition is considered. Notice also that, thanks to eigenvalues and eigenvectors computation, the expression of \( T \) can be rewritten as

\[
T = \sum_{i=1}^{3} \lambda_i u_i u_i^H = \sum_{i=1}^{3} \lambda_i T_i .
\]  

(2.67)

If only one eigenvalue is nonzero, then the coherency \( T \) matrix corresponds to a pure target and can be related to a single scattering matrix \( S \). In this case, the coherency \( T \)
matrix is \( \text{rank} = 1 \), and can be expressed as the outer product of a single target vector \( \mathbf{k}_1 \) with

\[
\mathbf{T} = \lambda_1 \mathbf{u}_1 \mathbf{u}_1^H = \mathbf{k}_1 \mathbf{k}_1^H.
\]  

(2.68)

Hence, the single nonzero eigenvalue \( \lambda_1 \) is equal to the Frobenius norm of the unit target vector \( \mathbf{u}_1 \) and corresponds to the Span of the associated scattering matrix. The corresponding target vector \( \mathbf{k}_1 \) may be then expressed as follows

\[
\mathbf{k}_1 = \sqrt{\lambda_1} \mathbf{u}_1 = e^{j\phi} \begin{bmatrix} \sqrt{2} A_0 \\ \sqrt{(B_0 + B)} e^{j\arctan \frac{D}{C}} \\ \sqrt{(B_0 - B)} e^{j\arctan \frac{D}{C}} \end{bmatrix}.
\]

(2.69)

Without using any ground truth measurements, this polarimetric parameterization of the target vector \( \mathbf{k}_1 \) involves the fit of a combination of three simple scattering mechanisms: surface scattering, dihedral scattering, and volume scattering, which are characterized from the three components (target generators) of the unit target vector \( \mathbf{u}_1 \):

- Surface scattering: \( A_0 \gg B_0 + B, B_0 - B \)
- Dihedral scattering: \( B_0 + B \gg A_0, B_0 - B \)
- Volume scattering: \( B_0 - B \gg A_0, B_0 + B \).

On the other hand, if all the eigenvalues are equal, the coherency \( \mathbf{T} \) matrix is composed of three orthogonal scattering mechanisms with equal amplitudes or probabilities. Then, the target is said to be “random”, with no correlated polarized structure at all. Between these two extremes, there exist cases of partial polarized targets where the coherency \( \mathbf{T} \) matrix has nonzero and non-equal eigenvalues. The analysis of its polarimetric properties requires a study of the eigenvalue distribution, as well as a characterization of each scattering mechanisms of the expansion represented by the different matrices \( \mathbf{T}_i \).

Furthermore, collaborations between E.Pottier and S.Cloude \([5]\) have provided the definition of new basis invariant parameters to describe the different scatterers, which means that in such cases, angle compensation methods are not necessary anymore. However, to define these variable, a eigenvector parametrization is required. The authors adapted the one shown in Equation 2.70,

\[
\mathbf{U}_3 = \begin{bmatrix} \cos \alpha_1 e^{j\phi_1} & \cos \alpha_2 e^{j\phi_2} & \cos \alpha_3 e^{j\phi_3} \\ \sin \alpha_1 \cos \beta_1 e^{j(\delta_1 + \phi_1)} & \sin \alpha_2 \cos \beta_2 e^{j(\delta_2 + \phi_2)} & \sin \alpha_3 \cos \beta_3 e^{j(\delta_3 + \phi_3)} \\ \sin \alpha_1 \sin \beta_1 e^{j(\gamma_1 + \phi_1)} & \sin \alpha_2 \sin \beta_2 e^{j(\gamma_2 + \phi_2)} & \sin \alpha_3 \sin \beta_3 e^{j(\gamma_3 + \phi_3)} \end{bmatrix}.
\]

(2.70)
2.4.2.1 Roll Invariant Property

Is one of the most important properties in radar polarimetry. A parameter which is roll invariant means that the effect of rotation around the radar line of sight is nonexistent. For instance, considering the rotation of the coherency matrix as:

\[ T(\theta) = R(\theta)TR(\theta)^{-1}, \tag{2.71} \]

where \( R(\theta) \) is the unitary similarity rotation matrix given by

\[
R(\theta) = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos 2\theta & \sin 2\theta \\
0 & -\sin 2\theta & \cos 2\theta
\end{bmatrix}.
\tag{2.72}
\]

According to the eigenvalues/eigenvectors based decomposition approach, the rotated coherency matrix can be restructured as

\[ T(\theta) = R(\theta)U_3\Sigma U_3^{-1}R(\theta)^{-1} = U_{3}^{'}\Sigma U_{3}^{'}^{-1}. \tag{2.73} \]

Notice that in the Equation (2.73), the matrix \( \Sigma \) remains unchanged and the eigenvector matrix takes the new form of

\[
U_{3}^{'} = \begin{bmatrix}
\cos \alpha_{1}e^{j\phi_{1}'} & \cos \alpha_{2}e^{j\phi_{2}'} & \cos \alpha_{3}e^{j\phi_{3}'} \\
\sin \alpha_{1} \cos \beta_{1}e^{j(\delta_{1}+\phi_{1}')} & \sin \alpha_{2} \cos \beta_{2}e^{j(\delta_{2}+\phi_{2}')} & \sin \alpha_{3} \cos \beta_{3}e^{j(\delta_{3}+\phi_{3}')} \\
\sin \alpha_{1} \sin \beta_{1}e^{j(\gamma_{1}'+\phi_{1}')} & \sin \alpha_{2} \sin \beta_{2}e^{j(\gamma_{2}'+\phi_{2}')} & \sin \alpha_{3} \sin \beta_{3}e^{j(\gamma_{3}'+\phi_{3}')} \\
\end{bmatrix}. \tag{2.74}
\]

Where the new matrix \( U_{3}^{'} \) follows the same parameterization as the matrix \( U_{3} \), having angles \( \alpha_{1}, \alpha_{2} \) and \( \alpha_{3} \) invariant. Therefore, plus the three unchanged eigenvalues from the matrix \( \Sigma \), there are in total six parameters whose behavior keep unaffected by rotation around the radar line of sight: \( \alpha_{i} \) and \( \lambda_{i} \) \( \forall i \in \{i = 1..3\} \). Furthermore, taken them as the core ones, new roll invariant parameters can be defined:

- **Pseudo probabilities:** A statistical model of the scatterer is considered as a 3 symbol Bernoulli process:

\[ P_i = \frac{\lambda_i}{\sum_{i=1}^{3} \lambda_i}. \tag{2.75} \]

- **Polarimetry Entropy:** According to Von Neuman, Entropy is an efficient and suitable parameter to define the degree of statistical disorder of each distinct scatterer type within the ensemble. For instance, if only a pure target is present, \( H = 0 \), on the another extreme, when the target scattering is truly a random noise process, \( H \) became equal to 1:
2.4. REVIEW OF TARGET DECOMPOSITION THEOREMS

\[ H = - \sum_{i=1}^{3} P_i \log_3(P_i). \]  
\[ (2.76) \]

- Mean \( \bar{\alpha} \) angle: the use of this parameter relies to its useful range corresponding to a continuous change from surface scattering in the geometrical optics limit \((\bar{\alpha} = 0^\circ)\) through surface scattering under physical optics to Bragg surface model, encompassing dipole scattering or single scattering by a cloud of anisotropic particles \((\bar{\alpha} = 45^\circ)\), moving into double bounce scattering mechanisms between two dielectric surfaces and finally reaching dihedral scatterer from metallic surfaces \((\bar{\alpha} = 90^\circ)\):

\[ \bar{\alpha} = \sum_{i=1}^{3} P_i \alpha_i. \]  
\[ (2.77) \]

- Anisotropy: while the polarimetric Entropy \( H \) is a useful scalar descriptor of the randomness of the scattering problem, it is not a unique function of the eigenvalues ratios. Hence, another eigenvalue parameter defined as polarimetric Anisotropy can be introduced. Is specially useful for situations where \( H > 0.7 \), because lower entropies, the second and third eigenvalues are highly affected by noise, leading to A very noisy too:

\[ A = \frac{\lambda_2 - \lambda_3}{\lambda_2 + \lambda_3}. \]  
\[ (2.78) \]

2.4.2.2 Other Eigenvalue Based Parameters

Besides the Entropy and Anisotropy, others eigenvalue based parameters have been presented in the literature, which describe different aspects of the eigenvalue spectrum. Note that all these parameters are roll invariant because eigenvalues are doted with that property. It is worth to mention three of them:

- Polarization fraction: A complementary approach to the Entropy and Anisotropy parameterization is to remote the unpolarized portion of the radar return and the analyze the remaining polarized component. The percentage of the total power (Span) that remains completely unpolarized is equal to \( \frac{3\lambda_3}{Span} \), which follows the definition of the polarization fraction parameter:

\[ PF = 1 - \frac{3\lambda_3}{Span} = 1 - \frac{3\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3} \quad 0 \leq PF \leq 1. \]  
\[ (2.79) \]

- Radar vegetation index: The average return of a distributed target is, in general, partially polarized. Van Zyl [17] analyzed scattering from vegetated areas using a model of randomly oriented dielectric cylinders and showed that the second an third
eigenvalues are equal for this type of model. The radar vegetation index is thus given by:

\[ RVI = \frac{4\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3}, \quad 0 \leq RVI \leq \frac{4}{3}, \quad (2.80) \]

\( RVI \) is equal to \( \frac{4}{3} \) for thin cylinders and monotonically decreases to 0 for thick cylinders.

- Pedestal height: another way of measuring randomness in the scattering process is to measure the pedestal height in polarization signatures. It was shown by Durden et al. [18] that measuring the pedestal height is equivalent to measuring the ratio of the minimum eigenvalue to the maximum eigenvalue, in our case, the ratio between \( \lambda_3 \) and \( \lambda_1 \):

\[ PH = \frac{\min(\lambda_1, \lambda_2, \lambda_3)}{\min(\lambda_1, \lambda_2, \lambda_3)} = \frac{\lambda_3}{\lambda_1}, \quad 0 \leq PH \leq 1. \quad (2.81) \]

### 2.4.2.3 \( H/A/\pi \) Classification Space

Considering all the roll-invariant parameters defined in Section 2.4.2.1, Cloude and Pottier proposed an unsupervised classification scheme based in the use of the 2-D \( H/\pi \) plane [5], where all random scattering mechanisms can be represented. Such plane is illustrated in Figure 2.6.

As is shown, the \( H/\pi \) plane is subdivided into nine basic zones characteristic of classes of different scattering behavior. The location of the boundaries within the feasible combinations of \( H \) and \( \pi \) values is set based on the general properties of the scattering mechanisms. The own authors point out that there is some degree of arbitrariness on setting these boundaries [5], which are not dependent on a particular data set.

Inherent of the spatial averaging, the Entropy \( H \) may increase, and the number of distinguishable classes identifiable from polarimetric observations is then reduced. For example, the feasible region of the \( H/\pi \) plane is rapidly shrinking for high values of Entropy (\( H > 0.7 \)), where \( \pi \) parameter reaches the limit values of 60°. In order to discriminate new classes, the Anisotropy values is used, and a new extended 3-D \( H/A/\pi \) space [5] is defined in Figure 2.7.

Thus, it is possible to subdivide each plane of the \( H/A/\pi \) space into basic zone characteristics of classes of different scattering behavior. Furthermore, in order to improve the capability to distinguish different types of scattering process, the authors [5] proposed to use some combinations between Entropy and Anisotropy information:

- The \( (1-H)(1-A) \) image corresponds to the presence of a single dominant scattering process. Low Entropy and low Anisotropy with \( \lambda_2 \approx \lambda_3 \approx 0 \).
• The $H(1 - A)$ image characterizes a random scattering process. High Entropy and low Anisotropy with $\lambda_1 \approx \lambda_2 \approx \lambda_3$.

• The $HA$ image relates to the presence of two scattering mechanisms with the same probability. High Entropy and high Anisotropy with $\lambda_3 \approx 0$.

• The $(1 - H)A$ image corresponds to the presence of two scattering mechanisms with a dominant process and a second one with medium probability. Low to medium Entropy for the former two, and high Anisotropy with $\lambda_3 \approx 0$ for the last one.

2.4.2.4 Unsupervised Classification Based on Scattering Mechanisms and the Wishart Classifier

The classification using $H/\alpha$ plane may not be satisfactory in some cases, due to the fact that only partial polarimetric information from the coherency matrix is used, and that the $H/\alpha$ zone boundaries were preset somehow arbitrarily. For instance, clusters may be located near boundaries, and may not be confined in each individual zone. In addition, two or more clusters may fall in a zone. To deal with those problems, Lee et al. [16] proposed an algorithm, which is a combination of the unsupervised target decomposition classifier.
and the supervised Wishart classifier. The algorithm works in the following manner:

1. First, Cloude and Pottier unsupervised classification is applied over the data. The classification result is used to form training sets as input to the Wishart method.

2. From the initial classification map obtained in the previous point, the cluster center of coherency matrices, $V_i$, is computed for pixels in each zone as:

$$V_i = \frac{1}{n_i} \sum_{j=1}^{n_i} T_j$$

for all pixels in class $i$, \hspace{1cm} (2.82)

where $n_i$ is the number of pixels in class $i$.

3. Then, each pixel is then reclassified by applying the Wishart distance measure for the coherency matrix $(T)$:

$$d(T, V_m) = ln|V_m| + Tr(V_m^{-1}T).$$

4. The last classification result is used as the training set for same classification process employed in steps 2 and 3. The sorting process reiterates until the number of pixels switching classes becomes smaller than a predetermined number, or a termination criterion is met.
2.4.3 Hybrid Decompositions

Through the previous sections, the importance to have a suitable decomposition technique, which is physically interpretable and adaptable to an specific environment has been stated. Van Zyl et al. [20] [21] introduced a hybrid approach which combines the two previous methods: model and eigenvalue/eigenvector based decomposition. It tries to combine the two algorithms in order to extract their respective potential advantage. However, it is worth to mention that it is still a technique addressed to targets with the volume scattering as the dominant phenomenon.

The main idea is to first employ the typical model based decomposition to obtain the volume component, with the key point to always guarantee that the remaining matrix has not negative eigenvalues [21]. The volume model used in such approach can be either the conventional scattering matrix computed from a cloud of uniform dipoles or the adaptive scattering mechanisms introduced by Arii et al. [22]. Latter on, surface and double bounce components are gathered by using eigenvalue decomposition method. Depending on certain conditions, each of these mechanisms can be associated to one of the individual coherency matrices of eigenvalue decomposition.

2.4.3.1 Nonnegative Eigenvalue Decomposition with Predetermined Volume Scattering

Like the Freeman approach, this method considers the case of a medium with reflection symmetry. Consequently, the coherency matrix has the special form of

\[
\mathbf{T} = \begin{bmatrix}
\xi & \rho & 0 \\
\rho^* & \mu & 0 \\
0 & 0 & \nu
\end{bmatrix},
\]

(2.84)

where the parameters \(\xi, \rho, \mu\) and \(\nu\) all depend on the size, shape and electrical properties of the scatterers, as well as their statistical angular distribution. Considering the same symmetric form for the volume mechanism component, the volume extraction process can be modeled by the Equation (2.85)

\[
\mathbf{T}_{\text{res}} = \mathbf{T} - f_v \mathbf{T}_v = \begin{bmatrix}
\xi & \rho & 0 \\
\rho^* & \mu & 0 \\
0 & 0 & \nu
\end{bmatrix} - f_v \begin{bmatrix}
\xi_v & \rho_v & 0 \\
\rho_v^* & \mu_v & 0 \\
0 & 0 & \nu_v
\end{bmatrix}.
\]

(2.85)

Note that this equation is similar to the first step in the Freeman decomposition, where \(f_v\) is directly set as \(\frac{\xi}{\nu_v}\), being \(\nu_v = \frac{1}{4}\). However, here, the limit of \(f_v\) is set as the largest value that still ensures the non-negativity of all three eigenvalues of the remaining matrix \(\mathbf{T}_{\text{res}}\). To derive the general expressions limiting the value of \(f_v\), eigenvalues of the remaining matrix are computed and shown in Equation (2.86)
\[
\lambda_1 = \frac{1}{2}(t_1 + t_2) \quad (2.86)
\]
\[
\lambda_2 = \frac{1}{2}(t_1 - t_2)
\]
\[
\lambda_3 = \nu - f_v \nu_v
\]
\[
t_1 = \xi + \mu - f_v(\xi_v + \mu_v)
\]
\[
t_2 = \sqrt{(\xi - \mu - f_v(\xi_v - \mu_v))^2 + 4|\rho - f_v \rho_v|^2},
\]

notice that \(\lambda_1 \geq 0\), because \(\xi, \mu, f_v, \xi_v, \mu_v\) are positives by the definition, and since \(\lambda_1 \geq \lambda_2\), the maximum value of \(f_v\) is found when the smaller of \(\lambda_2\) or \(\lambda_3\) is equal to zero. Any value of \(f_v\) larger than this will cause at least one eigenvalue to be negative.

The case of \(\lambda_3 = 0\) is straightforward, and \(f_v = \frac{\nu}{\nu_v}\), which corresponds to the Freeman decomposition [11], while for \(\lambda_2 = 0\) we need to solve the Equation (2.87)

\[
t_1 = t_2 \implies (\xi - f_v \xi_v)(\mu - f_v \mu_v) = |\rho - f_v \rho_v|^2. \quad (2.87)
\]

The smaller of the two roots of the quadratic expression in the Equation (2.87) is

\[
f_v = \frac{1}{2(\xi_v \mu_v - |\rho_v|^2)^2} [t_3 - \sqrt{t_3^2 - 4(\xi_v \mu_v - |\rho_v|^2)(\xi \mu - |\rho|^2)}] \quad (2.88)
\]

\[
t_3 = (\xi \mu_v + \mu \xi_v) - \rho \mu_v^* - \rho^* \rho_v.
\]

Therefore, the resulting minimum \(f_v\) is given by

\[
f_{v_{\text{max}}} = \min \left\{ \frac{\nu}{\nu_v}, \frac{1}{2(\xi_v \mu_v - |\rho_v|^2)} [t_3 - \sqrt{t_3^2 - 4(\xi_v \mu_v - |\rho_v|^2)(\xi \mu - |\rho|^2)}] \right\}. \quad (2.89)
\]

The choice of \(T_v\) depends on the parameter defined in Equation (2.62), following the Yamaguchi’s criteria.

Once the extraction of volume component has been completed, the eigenvalue decomposition is performed to the remaining matrix \(T_{\text{res}}\). Considering it form as

\[
T_{\text{res}} = \begin{bmatrix}
\xi_{\text{res}} & \rho_{\text{res}} & 0 \\
\rho_{\text{res}}^* & \mu_{\text{res}} & 0 \\
0 & 0 & \nu_{\text{res}}
\end{bmatrix}, \quad (2.90)
\]

the computed eigenvalues are

\[
\lambda_1 = \frac{1}{2}(\xi_{\text{res}} + \mu_{\text{res}} + \sqrt{(\xi_{\text{res}} - \mu_{\text{res}})^2 + 4|\rho_{\text{res}}|^2}) \quad (2.91)
\]
\[
\lambda_2 = \frac{1}{2}(\xi_{\text{res}} + \mu_{\text{res}} - \sqrt{(\xi_{\text{res}} - \mu_{\text{res}})^2 + 4|\rho_{\text{res}}|^2})
\]
\[
\lambda_3 = \nu_{\text{res}}.
\]
and the corresponding eigenvector are given by

\[
\begin{align*}
\mathbf{e}_1 = & \begin{bmatrix} \Delta_1 \\ 1 \\ 0 \end{bmatrix} \\
\mathbf{e}_2 = & \begin{bmatrix} \Delta_2 \\ 1 \\ 0 \end{bmatrix} \\
\mathbf{e}_3 = & \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}
\end{align*}
\]

\[
\Delta_1 = \frac{(\xi_{res} - \mu_{res}) + \sqrt{(\xi_{res} - \mu_{res})^2 + 4|\rho|^2}}{2\rho^*}
\]

\[
\Delta_2 = \frac{(\xi_{res} - \mu_{res}) - \sqrt{(\xi_{res} - \mu_{res})^2 + 4|\rho|^2}}{2\rho^*}.
\]

Finally, depending on the values of \(\xi_{res}\) and \(\mu_{res}\), the \(\beta\) and \(\alpha\) values from surface and double bounce component defined in the Equation (2.30) and the Equation (2.35) respectively, can be computed as following

\[
\begin{align*}
\text{If } \xi_{res} \geq \mu_{res}, \text{ Surface is dominant} & \quad \begin{cases} 
\beta^* = \frac{1}{\Delta_1} \\
\alpha = \Delta_2
\end{cases} \\
\text{If } \xi_{res} < \mu_{res}, \text{ Double bounce is dominant} & \quad \begin{cases} 
\alpha = \Delta_1 \\
\beta^* = \frac{1}{\Delta_2}
\end{cases}
\end{align*}
\]

### 2.4.3.2 Adaptive Volume Approach

In the previous Subsection 2.4.3.1, a hybrid decomposition with a predefined volume component has been introduced. Here, the most general adaptive decomposition will be explained. However, before entering in details of the algorithm, a general characterization for polarimetric scattering from vegetation canopies needs to be considered first. Arii et al. [22] realized that the coherency matrix derived from vertically oriented cylinder can be expanded as the sum of three matrices. That is, considering the scattering matrix defined in the Equation (2.38) with \(S_{HH} = 0\) and \(S_{VV} = 1\)

\[
S_g(\phi) = \begin{bmatrix}
\sin^2 \phi & \cos \phi \sin \phi \\
\cos \phi \sin \phi & \cos^2 \phi
\end{bmatrix},
\]

such expression leads to form the coherency matrix as
\[
\mathbf{T}_{gv} = \frac{1}{2} \begin{bmatrix}
1 & 2 \sin^2 \phi - 1 & \sin 2\phi \\
2 \sin^2 \phi - 1 & \cos^2 2\phi & -\frac{\sin 4\phi}{2} \\
\sin 2\phi & -\frac{\sin 4\phi}{2} & \sin^2 2\phi
\end{bmatrix},
\] (2.96)

then, expanding the power of sinus and cosines, the matrix \(\mathbf{T}_{gv}\) can be expressed as sum of three matrices

\[
\mathbf{T}_{gv} = \mathbf{T}_\alpha + 2\mathbf{T}_\beta(2\phi) + \mathbf{T}_\gamma(4\phi),
\] (2.97)

with

\[
\mathbf{T}_\alpha = \frac{1}{4} \begin{bmatrix}
2 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix},
\] (2.98)

\[
\mathbf{T}_\beta = \frac{1}{4} \begin{bmatrix}
0 & -\cos(2\phi) & \sin(2\phi) \\
-\cos(2\phi) & 0 & 0 \\
\sin(2\phi) & 0 & 0
\end{bmatrix},
\] (2.99)

\[
\mathbf{T}_\gamma = \frac{1}{4} \begin{bmatrix}
0 & 0 & 0 \\
0 & \cos(4\phi) & -\sin(4\phi) \\
0 & -\sin(4\phi) & -\cos(4\phi)
\end{bmatrix}.
\] (2.100)

Thus, the coherency matrix describing the scattering from a cloud of randomly oriented thin cylinders is expressed as

\[
\mathbf{T}_v = \int_0^{2\pi} \mathbf{T}_{gv} p(\phi)d\phi,
\] (2.101)

where \(p(\phi)\) is the distribution of angle orientation of the cylinders. Therefore, the type of canopy scattering yields in the choice of the \(p(\phi)\). Arii et al. [22] proposed the \(n^{th}\) power cosine-squared distribution as a generalized distribution

\[
p(\phi, \phi_0, n) = \frac{\left|\{\cos^2(\phi - \phi_0)\}\right|^n}{\int_0^{2\pi} \left|\{\cos^2(\phi - \phi_0)\}\right|^n}.
\] (2.102)

Where the absolute value is used to avoid assigning negative probabilities. When \(n = 0\), this becomes exactly the same as the uniform distribution, and the final volume scattering mechanism is equal to the one used in Freeman’s approach. The randomness decreases monotonically as \(n\) increases, and with infinitely large \(n\), it becomes delta function. So, the generalized coherency matrix of a cloud of cylinder that are much thinner than the radar wavelength, \(ka \ll 1\) with \(a\) being the radius of the cylinder, is given by

\[
\mathbf{T}_v(\phi_0, n) = \frac{1}{An} \int_0^{2\pi} \mathbf{T}_{gv} \cos^{2n}(\phi - \phi_0)d\phi,
\] (2.103)
with \( n \) being an integer

\[
A_n = \int_0^{2\pi} \cos^{2n}(\phi - \phi_0) d\phi = \frac{\pi}{2^{2n-1}} \left( \frac{2n}{n} \right).
\] (2.104)

The final expression by computing the integral is shown in the Equation (2.105), development details can be located in [22]

\[
T_v(\phi_0, n) = T_\alpha + \frac{2n}{n + 1} T_\beta(2\phi_0) + \frac{n(n - 1)}{(n + 1)(n + 2)} T_\gamma(4\phi_0).
\] (2.105)

Note that the expression is only function of \( \phi_0 \) and \( n \), and the different cases appeared along the literature are only a particular examples

\[
T_v^{\text{uniform}} = T_v(\phi_0, 0) = T_\alpha,
\]

\[
T_v^{\text{vert, hor}} = T_v(\phi_0, 1) = T_\alpha + T_\beta(2\phi_0).
\] (2.106)

Also remark that, while (2.105) has been delivered assuming \( n \) integer, the researchers also verified numerically that the expression holds for no integer values of \( n \).

Once the general canopy model is given, the hybrid algorithm can described properly [23]. Figure 2.8 shows the scheme of the algorithm.

First of all, the number of iterations is set according to the amount of samples taken for the parameters \( \phi_0 \in [0, \pi] \) and \( n \in [0, \infty] \). In practice, there is little difference between distributions with values of \( n \) larger than 20. Thus, using Matlab notation, a possible sampling process is annotated in the following equations

\[
\phi_0 = 0 : \frac{\pi}{N_i - 1} : \pi
\] (2.107)

\[
n = 0 : \frac{20}{N_j - 1} : 20,
\] (2.108)

where the number of values of \( \phi_0 \) is equal to \( N_i \) of and there are \( N_j \) values of \( n \), resulting in a total of \( N_i \times N_j \) iterations. Then, \( f_v \) is computed for all the possible combination of those parameters using nonnegative eigenvalue approach. This method, introduced by Ariii et al. [21] states that, given a certain volume scattering model, \( f_v \) is the maxim extraction possible before the eigenvalues of coherency matrix after subtraction gets negative

\[
\text{All the eigenvalue}(T - f_v(i, j)T_v(\phi_0(i), n(j))) \geq 0,
\] (2.109)

unlike typical model based approaches, no symmetrical assumption is made here. As soon as \( f_v \) is obtained, the normal eigenvalue/eigenvector decomposition is applied to the remain matrix, producing also a residual matrix. The best pair of parameters are chosen with the criterion of lowest power in the residual matrix.
Figure 2.8: Adaptive model based algorithm
Chapter 3

Snow Remote Sensing using SAR: State of Art

3.1 Introduction

Mapping of snow and ice covered areas are important for many applications such as prediction of floods, snowmelt runoff modeling, water supply for irrigation and hydropower station, weather forecast and understudying climate change. The importance of the estimation of snow parameters such as the Snow Water Equivalent (SWE) in hydrology and climate studies has pushed the scientific community to explore the possibilities of the Earth Observation techniques for the snow quantitative estimation. Optical and near-Infrared (IR) remote sensing techniques are proved to be promising for snow cover mapping. However, in the presence of cloud cover and different weather conditions, these techniques fail in acquiring snow cover information. Microwave remote sensing has an advantage over optical and IR techniques due to its all weather capability, penetration through cloud and independence of sun illumination.

The aim of this chapter is summarize the basic knowledges necessaries to performance a snow mapping and its parameters estimation. Starting to introduce the snow physical properties, and how it interacts with the electromagnetic. Following by microwave remote sensing techniques of snow cover mapping and estimation of parameters based on coherent and incoherent SAR data. Finally, conclusions and possible future research topics will close this chapter.
3.2 Snow Remote Sensing

Remote sensing of snow can be classified depending on how the information is retrieval, for instance, the type of sensor employed in the acquisition process:

- Optic sensors: They offer good resolutions, but their nature only allow them to be used under very specific visibility conditions. For example, they can not work during the night or during inadequate meteorology conditions as heavy rain, strong wind, etc. Furthermore, at optical frequency range, the penetration effect is almost null.

- Microwave sensors: there are two different type of microwave sensors:
  - The first type is called passive microwave sensors, named also radiometers. They have good sensitivity to the snow parameters and are meteorology independent, but unfortunately, they provide very poor spatial resolution images.
  - Another type of microwave sensors are the active ones, denominated also SAR sensors. Unlike the optic configuration, they are meteorology independent and have an adequate trade-off between sensitivity and penetration. In addition, it offers much better spatial resolution than the passive ones.

However, when the nature of the resulting information is considered, the classification becomes as follows:

- Qualitative estimation, which means binary information about the presence of snow or no. The process is called Snow Covered Area (SCA).

- Quantitative estimation, as the name says, in this group, the information recovered is quantitative, with certain magnitude. For instance, SWE retrieval is a good example.

The literature about snow remote sensing is therefore mostly focused on SAR based techniques given its proven appropriateness. The recent advances involve the multichannel SAR sensors, which exploit different diversity sources of information in order to have more than one channel of information about the observed targets. PolSAR techniques, which is going to be in the scope of this document, is included in this group, and it is based on the utilization of the polarization information of the electromagnetic waves in order to achieve the multichannel diversity.

3.3 Snow Physical Modeling

Snow is a mixture of three elements: air, ice and water. The percentage variation of these elements determine the kind of snow. Mainly, it can be classified in two classes: dry and wet snow. The difference between them can be described by following parameters:
• Liquid water content ($W$): Determine the dielectric properties and has strong relation with the temperature of the snow. Snow is considered dry if there is 0% of liquid water content. However, as the temperature raises, the amount of liquid water content increases, leading to the so called wet snow.

• Dielectric constant ($\varepsilon$): The complex dielectric constant is the weighted average of dielectric constants of snow components: air, ice and water

$$\varepsilon = \varepsilon' + i\varepsilon''.$$  

(3.1)

The real part of the dielectric constant $\varepsilon'$ is strongly dependent on the snow density, specially for dry snow. Whereas the imaginary part $\varepsilon''$ depends directly on the amount of liquid water content. Only a 0.5% increase of liquid water content can introduce changes of an order of magnitude of permittivity $\varepsilon''$. Therefore, the more water content, higher is the dielectric constant, which implies stronger absorption and less penetration.

• Snow density ($\rho_s$): snow density is defined as a ratio of snow mass over the water reference mass. It is the most important parameter influencing the dry snow backscattering power. For dry snow, the snow density in natural conditions yields: $0.2 < \rho_s < 0.5$ g/cm$^3$.

### 3.3.1 Snow Water Equivalent

Snow Water Equivalent is the amount of water contained within the snow-pack, equivalently to the depth of water that would cover the ground if the snow-pack was in a liquid state. Therefore, SWE estimation plays a important role in climate, hydrology and meteorology studies. It is worth to mention that the SWE can only be retrieved from dry snow-packs, and depending of the amount of SWE, dry snow cover can be further split in two more subclasses:

• Shallow snow cover: 5-15 cm of SWE.

• Deep snow cover: 20-64 cm of SWE.

The mathematical definition of SWE is shown in the Equation 3.2

$$SWE = d \cdot \rho_s \left[ \frac{Kg}{m^2} \right],$$  

(3.2)

where $d$ is snow depth in meter and $\rho_s$ is snow density in Kg per cubic meter. Thus, these two parameters are essential for the SWE estimation. However, they are generally not highly correlated, and must be considered as independent variable in the field surveys of
the spatial distribution of snow as well as in the study of snow’s microwave backscattering properties:

- Snow density can be estimated from the Looyenga’s semi-empiric dielectric formula

\[
\varepsilon_s = 1.0 + 1.5995\rho_s + 1.861\rho_s^3,
\]

where \(\varepsilon_s\) is the dielectric constant of the snow-pack.

- Snow depth is inversely proportional to the extinction coefficient and highly related to the snow size, which means that high frequencies estimation techniques show better performance, due to interaction magnification with low wavelength

\[
d = \frac{1}{k_e},
\]

where \(k_e = k_a + k_s\) is extinction coefficient, which is computed as the sum of the absorption coefficient plus the scattering coefficient.

As it has been pointed out before, the SWE is the most important measurement of snow-pack from the hydrological standpoint, but its acquisition is tedious and costly, since it consists of melting a known volume of snow in order to weight the water content. In field measurement campaigns, such test cannot be carried out in massive areas, and therefore SWE cannot be extensively studied for large scale applications that are the ones of scientific interest. From this requirement of mapping in higher spatial and frequency scales the SWE parameter, arises the interest of the remote sensing community in the estimation of this snow-pack parameter.

### 3.3.2 Snow Interaction with Electromagnetic Waves

From SAR point of view, the targets being imaged can be mainly distinguished in two groups: the point target and the distributed target. The point targets are those ones that are not affect by the speckle noise component, which is not the case of snow. Distributed target as snow can be interpreted as the sum of a set of point targets, and it should be analyzed by statistical techniques due to its randomness.

Leaving aside the speckle problem due to the distributed characteristic of the snow-pack. When the electromagnetic microwave interacts with the snow-pack, it produces different physical phonemes due to the snow heterogeneous composition. Hence, the total backscatter from snow covered ground at transmit and receive polarization, \(pq\), can be modeled by four contributions

\[
\sigma_{pq}^t = \sigma_{pq}^{as} + \sigma_{pq}^{uv} + \sigma_{pq}^{gv} + \sigma_{pq}^g,
\]
where $\sigma^{as}$ represents the scattering at the air/snow interface, $\sigma^v$ is the direct snow volume scattering term, $\sigma^{gv}$ are the contributions by the ground surface/snow volume and snow volume/ground surface interaction, and $\sigma^g$ is the backscatter at the snow/ground interface after traveling through the snow-pack. These backscattering measurements are affected by three set of parameters:

- Snow parameters: snow density, snow depth, particle size, stickiness, stratification, snow temperature.
- Sensor parameters: frequency, polarization, incidence angle.
- Under ground parameters: ground dielectric, roughness.

For example, small incidence angles makes surface scattering the dominating phenomenon, whereas for bigger incidence angles volume scattering became more important.

### 3.3.2.1 Wet Snow

Backscattering of wet snow can be modeled appropriately by sum of two components: air/snow surface backscattering and the volume scattering, neglecting the remain elements of the Equation (3.5) for their poor contribution. This behavior can be explained by the increase of liquid water content in wet snow, and hence, important lose of microwave penetration properties.

### 3.3.2.2 Dry Snow

The main backscattering contributions of dry snow varies from the wet snow. It can be extracted from the volume scattering and snow/ground backscattering. The reason is due to the heterogeneous property of dry snow, which can be seen as medium composed by ice particles with different sizes and microstructures. The omission of air/snow component comes from that dry snow has lower dielectric constant than air, then, the air/snow surface usually appears smooth to microwaves, leading to a really low reflection. At C-Band, the interaction between the snow particles and ground can be also considered negligible due to its poor contribution to the total backscattering.

Depending on the incidence wavelength, the dominant component varies from the volume contribution to the snow/ground component. For instance, for low frequencies, snow/ground backscattering has higher values, because, large wavelength does not interact with snow-pack, producing poor volume phenomenon. For this reason, retrieval of snow parameters at low frequencies is difficult because the similarities of backscattering of snow covered and snow-free zones. However, for high frequencies, the interaction with ice particles is higher, resulting in a large volume phenomenon. Consequently, higher frequencies is desired for retrieval of snow parameters.
In most of the cases, the snow/ground interface, also called soil surface or underlying surface, is addressed as Brag scattering or Small Perturbation Method (SPM), since it is considered that surface roughness is slight with regards to the incident wavelength.

The snow-pack volume backscattering is modeled as a Rayleigh scattering, resulting of a random or oriented cloud of particles whose scattered intensity in the far field is the addition of the individual intensities of each particle [24]. Thus, volume scattering in the snow-pack is mainly governed by the size of the ice crystals relative to the incident wavelength [25]: the bigger the snow particles with regards to the incident wavelength, the higher the interaction of the electromagnetic waves with the snow grains, and therefore the higher volume contribution. Therefore, as the incident wave frequency is reduced the volume scattering effect becomes less important. Another usual assumption in the snow-pack scattering response modeling is the Born approximation, which establishes that the scattering generated out of the interactions of the particles with the diffused radiation is negligible. Born approximation can be assumed since snow-pack medium fluctuation is very slight. Also, snow presents continuously distributed volume absorption and volume scattering, which can be described by the Radiative Transfer Method (RTM).

At this point of the document we can already point out a few interesting ideas. For instance, for quantitative snow remote sensing, snow-pack has to be dry and the volume has to be sensed in order to get the physical information. This means that microwaves signals have to penetrate the snow volume and at same time interact with the snow grains so that information about the snow-pack could be extracted from the scattered power. In conclusion, to achieve a proper snow parameter estimation, trade-off between the penetration capacity and the sensitivity to the volume of snow particles must be careful chosen. Then, for the rest of the document we will focus on dry snow, so, when we refer to snow without mentioning explicitly wet snow, it is always dry snow.

### 3.3.3 Snow Polarimetric Signature

Polarimetric signature of snow is an on-going research topic, and therefore, the outcome of each research work done contributes to a bit more light on the subject. From the point of view of the scattering matrix, snow-pack cannot be analyzed, since it consists of a distributed target subject to spatial and temporal variations, and therefore the received response is the averaged contribution of a myriad of single scatterers.

Snow-pack analysis requires a statistical approach, based on the second order moment of the scattering matrix. The polarimetric coherency and covariance matrices offer statistical information of the scattering Pauli and lexicographic vectors respectively, that contain the vectorized scattering matrix components. The elements of these two vectors can be physically interpreted according to the scattering mechanism that contributes to them. Specifically, the \( \langle |S_{HV}|^2 \rangle \) proportional component is sensible to volume scattering.
3.4 SNOW COVERED AREA

and the $\langle |S_{HH} - S_{VV}|^2 \rangle$ proportional to surface scattering. Therefore these components should have some information about snow volume and snow/ground interface respectively.

From [29] it can be concluded that at C-Band dry snow is almost transparent to the incident radar waves for any polarimetric configuration. However, it is proven that the backscatter is higher for snow free than for snow cover for both co-pol and cross-pol configurations. This might be due to the volume scattering that results in a power dispersion and therefore less power is back to the antennas. On the other hand, C-Band is not sensitive to re-frost layers of the snow-pack.

However, more recent publications [28] do find a reasonable sensitivity to the snow-pack volume at C-Band. This difference might be driven by the amount of liquid water content in the snow-pack, since the extinction coefficient at C-Band increases dramatically with the liquid water content, decreasing thus the backscattered power. Effort needs to be put on this issue in order to clarify the snow behavior at C-Band.

3.4 Snow Covered Area

Detecting the presence of snow cover, and of what kind is the first step before any snow quantitative estimation algorithm. In this section, different snow detection techniques proposed in the literature are reviewed.

Almost all the snow qualitative techniques are based on multi-temporal change detection. For instance, Mätler et al [29] used Network-Analyzer based on scatterometers at 5.3 and 35 Ghz to detect the impact in the backscattering coefficient of snow cover during a whole year. They concluded that the combination of measurements at C and Ka-Band, 40\degree degrees of incidence angle, permits a discrimination of various snow cover types. In particular, completely dry, shallow snow covers, deep snow covers, a wet layer on the surface of a dry snow cover, refrozen crusts on the surface of dry and wet snow covers. In addition, snow free test side can be discriminated from snow covers by observing the temporal evolution of backscattering coefficient at cross and co-polarization all over the year and by means of day and night measurements.

However, using only C-Band, the mapping of the wet snow is possible because of the considerable reduction of backscattering coefficient in contrast with snow free zones. Unlikely, there is only slight difference in backscattering coefficient between dry snow covers and snow free areas, usually undetectable. This difference is higher when does not exist soil frozen phenomenon in the winter, which makes suitable the discrimination between them.
3.4.1 Wet Snow Mapping

Based on the idea that the extinction coefficient of melting snow is very high, involving a considerable backscattering level reduction, Mätler, Nagler et al. [30] developed a specific algorithm to classify wet snow covers. The detection of wet snow is achieved by thresholding the target image, and the threshold is calculated between the image to be classified and a reference image revealing snow-free or dry snow conditions. They found the same threshold of $-3$ dB for identifying wet snow using C-Band $S_{HH}$ Radarsat SAR data as well as C-Band $S_{VV}$ ERS-SAR data. This technique is further improved by Eirik Malnes et al. [32], they suggest discriminate wet snow in different categories due to the different melting morphology and underlying surface response. For instance, wet snow in forest areas and on lakes must be treated differently from wet snow on open areas. Exploiting the work done by J.Pulliainen et al. [31] about compensation of forest canopy effects in the estimation of snow covered area, the sub-pixel classification with “soft” threshold function is introduced

$$F(\sigma) = 50 - 50 \tanh(a(\sigma + 3)), \quad (3.6)$$

where $F$ is the sigmoid function, $\sigma$ is the backscatter in dB and $a$ is a slope-parameter for the sigmoid function. The sub-pixel scheme is considering each pixel as the mixture of dry snow, wet snow and snow-free ground instead of binary assumption. Another idea replacing the binary decision of Nagler algorithm is introduced by Nicolas Longépé [47]. They use a nonlinear sigmoid activation function as the soft decision

$$F_a\left(\frac{\sigma^0_{ws}}{\sigma^0_{ref}}\right) = \left(1 + e\left(s\left(\frac{\sigma^0_{ws}}{\sigma^0_{ref}} + 3\right)\right)\right)^{-1}, \quad (3.7)$$

where $\sigma^0_{ws}$ is the backscattering of geometrically corrected winter snow and $\sigma^0_{ref}$ corresponds to snow-free reference image, and $s$ is a positive steepness factor.

3.4.2 Dry Snow Mapping

Thanks to the $H/A/\alpha$ polarimetric decomposition technique introduced by S.Cloud, E.Pottier and A.Martini et al. [33] [34] [36] presented a combination of multi-frequency and multi-temporal polarimetric algorithm to discriminate dry snow cover. They first separate the type of soil in surface or forest class using $H/A/\alpha$ at L-Band. Then, supervised Polariometric Contrast Variation Enhancement (PCVE) is applied to discriminate dry snow in surface class, and summer/winter $\alpha$ standard deviation is applied to classify snow over in forest areas. As using only seasonal polarimetric contrast optimization, is severely limited by the large variability of the underlying surface polarimetric response. Instead of merely optimizing a winter to summer polarimetric contrast over snow covered areas, the PCVE
3.5. NON COHERENT POLSAR DATA METHODS

The technique aims to determine the emitting/receiving polarization state, \((g_{\text{max}}, h_{\text{max}})\), for which the summer to winter contrast snow covered areas, represented by \(K_a\) in the Equation (3.8), is maximized, while maintaining the seasonal variation to low amplitude over snow-free zones, associated to \(K_b\) in the Equation (3.8)

\[
[g_{\text{max}}, h_{\text{max}}] = \arg \max_{(g,h)} \left( \frac{\rho_a}{\rho_b} \right) = \frac{1}{N_a} \sum_{h} h^T K_a^{\text{win}} \frac{1}{N_b} \sum_{h} h^T K_b^{\text{sum}}, \tag{3.8}
\]

where \(K_a\) and \(K_b\) are Kennaugh matrices sampled over a priori known snow covered and snow-free areas denoted by the subscripts \(a\) and \(b\), respectively. The subscripts \(\text{win}\) and \(\text{sum}\) indicates winter and summer situations respectively. Once the optimization algorithm has converged, the emitting/receiving polarization state, \((g_{\text{max}}, h_{\text{max}})\), shows a very low sensitivity to surface scattering response variations because of moisture or overlying short vegetation, and emphasizing the presence of snow. The response of the whole scene is then computed in the \((g_{\text{max}}, h_{\text{max}})\) polarization basis, and a comparison to a threshold is used to detect the presence of snow.

3.4.3 Wet+Dry Snow Mapping

Finally, Singh et al. [37] [38] developed the Radar Snow Index (RSI) algorithm to discriminate total (wet+dry) snow cover at L-Band without multi-temporal approach. It consists of thresholding the polarization fraction coefficient, introduced in the Equation (2.79). They demonstrated the easiness and robustness of this algorithm in comparison to \(H/A/\pi\) wishart supervised [26] classifier and 4-component decomposition [35] classifier, for solving the lack of training samples.

3.5 Non Coherent PolSAR Data Methods

In this section we will present the most relevant works of snow parameters estimation based on non coherent polarimetric SAR data. Non coherent polarimetric data means that the techniques are based on amplitude contrast or comparison, without the phase information. In other words, fully polarimetric processing techniques can not be applied.

Jiancheng Shi et al. [40] made different studies of polarimetric SAR response to dry snow, showing that backscattering coefficient and microwave brightness are sensitive to parameters describing snow microstructures. They point out that the relationship between the co-polarization channels \((S_{HH}, S_{VV})\) and snow water equivalence can be either positive or negative at X-Band. A positive correlation occurs when the scattering signal from the snowpack is greater than the signal from the ground, attenuated by the overlying snow. Besides the basic snow parameters as snow density and ice particle size, ice particle variation, snowpack stratification, and underlying conditions also have great impact
3.5.1 Snow-Pack Thermal Resistance based Model for SWE estimation

The work carried out by Monique Bernier et al. [41] aims to determine the snow water equivalent with multi-temporal C-Band SAR data. A semi-supervised method modeling the backscattering coefficient, snowpack and underlying soil physical parameters has been developed.

The work was focused on shallow snow, and the analysis has revealed that shallow snow cover with SWE less than 20 cm is undetectable. They propose to model the total backscattering as the sum of volume scattering and snow/ground surface backscattering, considering the second one as the dominant scatter. Since the dielectric constant of a given soil is determined by its temperature below the freezing point, the relation between backscattering power ratio \( \frac{(P_c)_{\text{ratio}(45)}}{(P_c)_{\text{snow}(45)}} \) and the SWE can be then explained by the thermal resistance \( R (°C/m^2s/J) \) of snow cover. Thus, having a few soil parameters and certain frozen conditions, soil is frozen over more than 5 cm, and \( R \) can be estimated by the following empirical equation

\[
R_{\text{estimated}} = e^{(P_c)_{\text{ratio}(45)} + 4.1}. \tag{3.9}
\]

Then, SWE is a linear function of \( R \)

\[
SWE = \rho_s K(\rho_s)R, \tag{3.10}
\]

where \( \rho_s \) is the snow density and \( \rho_s K(\rho_s) \) is the slope of the regression function, which can be estimated using some measured values at the site

\[
\rho_s K(\rho_s) = \frac{SWE_{\text{measured}}}{R_{\text{measured}}}. \tag{3.11}
\]

Another interesting point to mention is that depending on the type of soil texture and its temperature, there exist differences between the underlying surface of dry snow cover and snow-free. For instance, a temperature below the freezing point modifies the soil dielectric constant of clay or a loam, but does not alter the dielectric properties of a well drained rocky soil.

3.5.2 Multi-frequency SWE Estimation on Co-Polarization SAR Data

The research work done by Jiancheng Shi and Jeff Dozier [42] [43] separate the estimation of SWE in two parts, snow density estimation and snow depth estimation. First, they use low frequency L-Band, which is more sensitive to snow/ground backscattering, to retrieve snow density, underlying surface dielectric properties, and soil roughness. Afterwards,
using these recovered parameters they estimate the snow depth at C-Band, which is more sensitive to snow-pack volume scattering.

### 3.5.2.1 Snow Density Estimation

At long wavelength, namely 24 cm for L-Band, snow particle size has little effect on the backscattering signals from a dry snow cover. Therefore, volume scattering can be neglected, and the total backscattering can be expressed as

\[
\sigma_{pp}^t(k_0, \theta_i) = T_{pp}^2(\theta_i) \sigma_{pp}^g(k_1, \theta_r),
\]

where \( \sigma_{pp}^g(k_1, \theta_r) \) is the snow/ground backscattering given by the Integral Equation Method Model (IEM), \( T_{pp}^2 \) is the power transmission coefficient at air/snow interface, \( k_1 = k_0 \sqrt{\varepsilon_s} \) is the incident wave number at the snow/ground interface. \( \theta_i \) is the incidence angle at air/snow interface, which is related to the refractive angle \( \theta_r \) at the snow/ground interface by Snell’s law

\[
\sin^2 \theta_i = \varepsilon_s \sin^2 \theta_r.
\]

Note that the subsurface is treated as the surface without snow applying the change of parameters made by the snow-pack. For instance, the incidence angle is smaller due to the snow refraction, the wavelength is shortened for having snow dielectrically thicker than the air, and also power is reduced by the snow because of reflection and absorption phenomenons.

Based on the scattering mechanism described above, the relation between the \( S_{HH} \) and \( S_{VV} \) polarized backscattering can be use to estimate the snow density. The algorithm developed is a physical based regression model using the combination of co-polarization measurement, and simulated backscattering for the most bare surface condition computed by IEM model. The method, which aims to maximize the sensitivity of snow density to incidence angle and wavelength, minimizing the sensitivity to dielectric and roughness includes four set of equations

\[
10 \log_{10} \left( \frac{\sqrt{\sigma_{VV}}}{T_{VV}} + \frac{\sqrt{\sigma_{HH}}}{T_{HH}} \right) = a_d(\theta_r) + b_d(\theta_r)10 \log_{10} \left( \frac{\sqrt{\sigma_{VV}} \sigma_{VV}}{T_{VV} T_{HH}} \right) + c_d(\theta_i)10 \log_{10} \left( \frac{\sigma_{VV}^2}{T_{VV}^2} + \frac{\sigma_{HH}^2}{T_{HH}^2} \right)
\]

\[
a_vh(\theta_r) + b_vh(\theta_r)10 \log_{10} \frac{|\alpha_{HH}| |\alpha_{VV}|}{\sqrt{\sigma_{HH}^g \sigma_{VV}^g}} = 10 \log_{10} \left[ \frac{|\alpha_{HH}|^2 + |\alpha_{VV}|^2}{\sigma_{HH}^g + \sigma_{VV}^g} \right]
\]
\[
\sigma_{VV}^g = |\alpha_{VV}|^2 \left[ \frac{S_R}{a_{VV}(\theta_r) + b_{VV}(\theta_r)S_R} \right]
\] (3.16)

\[
\frac{\sigma_{HH}^g}{\sigma_{VV}^g} = \frac{|\alpha_{HH}|^2}{|\alpha_{VV}|^2} e^{a_r(\theta_r) + ks(\theta_r) + c_r(\theta_r))W},
\] (3.17)

where \(\sigma_{pp}^t\) and \(\sigma_{pp}^g\) are the total backscattering and the snow/ground surface backscattering coefficients. \(pp\) indicates polarization. \(\alpha_{pp}\) is the same as the small perturbation model and only depend on the incidence angle and dielectric contrast. \(S_R = (ks)^2W\) is the snow/ground surface roughness parameter. Where \(W\) is the Fourier transform of the power spectrum of the surface correlation function. Finally, \(a, b, c\) are the coefficients that depend only on the incidence angle, differ in each equation above, and are derived from nonlinear regression analyses. In the Equation (3.14) there is only one unknown, snow density, since both refractive angle and transitivity are depending on it. With estimated snow density from the Equation (3.14), the under ground dielectric constant can be estimated using the Equation (3.15). Furthermore, the Equation (3.16) and the Equation 3.17 can be used simultaneously to calculate the surface roughness parameter, root mean square (RMS) height.

The algorithm is successfully applied to soil or rock subsurfaces, but it cannot be applied where the subsurface is dominated by volume scattering.

### 3.5.2.2 Snow Depth Estimation

With estimated snow density, the underlying surface dielectric and roughness parameters from L-Band measurements, we can calculate the surface backscattering components at C-Band and X-Band. The inverse algorithm for estimation of snow depth and equivalent particle size is based on the first order backscattering model

\[
\sigma_{pp}^t = \sigma_{pp}^v + \sigma_{pp}^g e^{-\frac{2k_e d}{\cos \theta_r}}, \quad (3.18)
\]

where \(d\) is the snow depth and \(k_e\) is the snow volume extinction coefficient. \(\sigma_{pp}^v\) is the backscattering from the snow-pack and can be written as

\[
\sigma_{VV}^v = 0.75T_{VV}^2 \omega \cos \theta_r \left[ 1 - e^{-\frac{2k_e d}{\cos \theta_r}} \right] \left( 1 + R_{VV}^2 e^{-\frac{2k_e d}{\cos \theta_r}} \right) +
\]

\[
-2 \frac{2k_e d}{\cos \theta_r} R_{VV}^2 e^{-\frac{2k_e d}{\cos \theta_r}} \left( \sin^2 \theta_r - \cos \theta_r \right)^2 ] \quad (3.19)
\]

\[
\sigma_{HH}^v = 0.75T_{HH}^2 \omega \cos \theta_r \left[ 1 - e^{-\frac{2k_e d}{\cos \theta_r}} \right] \left( 1 + R_{HH}^2 e^{-\frac{2k_e d}{\cos \theta_r}} \right) +
\]

\[
-2 \frac{2k_e d}{\cos \theta_r} R_{HH}^2 e^{-\frac{2k_e d}{\cos \theta_r}} \left( \sin^2 \theta_r - \cos \theta_r \right)^2 \right], \quad (3.20)
\]
where $\omega$ is volume scattering albedo. $R_{pp}$ is reflectivity of snow-ground interface, its can be calculated from refractive angle and dielectric contrast. In the Equation (3.19) and the Equation (3.20), there are only two unknowns, snow equivalent particle size and snow depth. With C-Band $S_{VV}, S_{HH}$ backscattering measurements, two equations, two unknowns, the estimation of snow depth and snow equivalent particle size can be solved.

### 3.5.3 Backscattering Ratios Retrieval Model

Gulab Singh et al. [44] [45] developed an algorithm for retrieval of snow density from co-polarization polarimetric SAR data at C-Band using volume and snow/ground interface scattering models. Under assumption that snow surface has no significant air/snow interface scattering contribution and the angle of refraction remains constant in dry snow pack. Total backscattering from the dry snow pack can be defined as

$$\sigma_{pp}(k_0, \theta_i) = \sigma_{pp}^v(k_1, \theta_r) + \sigma_{pp}^g(k_1, \theta_r),$$  \hspace{1cm} (3.21)

where $k_1 = k_0 \sqrt{\varepsilon_s}$ is the incident wave number at the snow/ground interface. $\theta_i$ is the incidence angle at air/snow interface, which is related to the refractive angle $\theta_r$ at the snow/ground interface by Snell’s law, Equation (4.5). $\sigma_{pp}^g(k_1, \theta_r)$ is the snow/ground backscattering given by the Integral Equation Method Model

$$\sigma_{pp}^g(k_1, \theta_r) = \sigma_{s}^{pp} T_{pp}^2 \cdot \text{LossFactor},$$  \hspace{1cm} (3.22)

$T_{pp}^2$ is the power transmission coefficient at air/snow interface, $\sigma_{pp}^s$ and LossFactor are defined by IEM. And the volume backscattering contribution is a first order model

$$\sigma_{pp}^v = \frac{3}{4} \omega T_{pp}^2 \cdot \text{LossFactor},$$  \hspace{1cm} (3.23)

where $\omega$ is the volume scattering albedo.

Given these models, Singh et al. have defined a set of ratios $D_T, D_{TV}, D_{TH}$, which is used to produce the following equation that only depends on dielectric constant of snow and the incidence angle

$$\sigma_{AP}^{T}(vvhh) = \frac{\sigma_{HH}^{I} (|T_{VV}|^2 |T_{HH}|)}{(|T_{VV}|^2 |T_{HH}|) + |\alpha_{VV}|^2 + |\alpha_{HH}|^2 + 1},$$  \hspace{1cm} (3.24)

where

$$\sigma_{AP}^{I}(vvhh) = \sqrt{\sigma_{VV}^{I} \sigma_{HH}^{I}} = \sigma_{AP}^{I}(vvhh) + \sigma_{AP}^{g}(vvhh)$$  \hspace{1cm} (3.25)

$$\alpha_{HH} = \frac{\cos \theta_i - \sqrt{\varepsilon_s - \sin \theta_i^2}}{\cos \theta_i + \sqrt{\varepsilon_s - \sin \theta_i^2}}$$  \hspace{1cm} (3.26)
\[ \alpha_{VV} = (\varepsilon_s - 1) \frac{\sin \theta_i^2 - \varepsilon_s(1 + \sin \theta_i^2)}{\varepsilon_s \cos \theta_i + \sqrt{\varepsilon_s - \sin \theta_i^2}} \]  

(3.27)

\[ T_{HH} = \frac{2\sqrt{\varepsilon_s - \sin \theta_i^2}}{\cos \theta_i + \sqrt{\varepsilon_s - \sin \theta_i^2}} \]  

(3.28)

\[ T_{VV} = \frac{2\sqrt{\varepsilon_s - \sin \theta_i^2}}{\varepsilon_s \cos \theta_i + \sqrt{\varepsilon_s - \sin \theta_i^2}} \]  

(3.29)

From the Equation (3.24) one can estimate the dielectric constant, given the incidence angle, and then, extract snow density from the Looyenga’s semi-empirical formula (Equation (4.6)).

3.5.3.1 Estimation with Meteorological Model

Unlike the single-layer electromagnetic approach employed in previous methods, Nicolas Longépé et al. [47] unitize a multilayer meteorological snow model to performance qualitative and quantitative snow assessment based on the combination of dual-polarization at C-Band. The algorithm works as the follows:

- First of all, snow cover profiles are estimated by means of the SAFRAN/Crocus meteorological chain developed by “Météo France” [48]. For the computation of these profiles, all the relevant meteorological parameters affecting the evolution of snowpack structure are needed, for instance, precipitation, solar radiation, etc.

- The second step is to acquire the Crocus snowpack numerical model using the SAFRAN outputs. It simulates the physical processes inside the snowpack and its structure, with up to 50 different layers.

- Soil parameters such as moisture and RMS surface height also are desired, and they are estimated with a summer snow-free image using Oh’s retrieval approach [49].

- Then, total backscattering, as the sum of air/snow, volume, snow/ground backscattering contribution are simulated by all the precious information.

- At end, simulated VV polarized intensities \( \sigma_{VV,\text{sim}}^0 \) and \( \sigma_{VV,\text{meas}}^0 \) from SAR data at C-Band are compared, and the difference is minimized to obtain the best snow profile. Thus, snow parameters can be easily extracted from it.

The main drawback of this model is the neediness of an \textit{in situ}, continuous, wide meteorological information.
3.6 Coherent PolSAR Data Methods

PolSAR data is normally referring to coherent polarimetric data, which means that both the amplitude and phase information of all the components of the scattering matrix are available. Having the complete amplitude and the phase information allows the polarimetric characterization of the target and therefore the application of polarimetric data processing such as polarimetric decomposition. In the following subsections the most relevant works of snow monitoring using PolSAR data are summarized.

3.6.1 InSAR for Estimation of Changes in SWE

Tore Guneriussen, et al. [51] first proposed to employ interferometric SAR technique to estimate changes in SWE, which is further refined by Helmut Rott et al. [52]. The phase difference image is obtained by a complex multiplication of the corresponding pixels of two repeat-pass images [52]

\[ \Phi(m, n) = \frac{4\pi}{\lambda_i} \Delta R(m, n) + \Phi_s + \Phi_n. \]  
(3.30)

From the term \( \Phi_s \), which is the two way propagation phase difference in free air and snow. The relation between this term and the change of SWE can be derived

\[ \Delta \Phi_s = \frac{4\pi}{\lambda_i} 0.87 \frac{\Delta d_s \rho_s}{\Delta SWE}. \]  
(3.31)

Where \( \lambda_i \) is the incidence wavelength, the \( \Delta d_s \) and \( \rho_s \) are the variation of snow depth and snow density respectively.

This method is only suitable when there is significant change in snow properties between two repeat-pass images, and having an initial acquisition of snow parameters. Nevertheless, their studies in PolInSAR [50] domain at L-Band shows how the variation of the interferometric phase with polarization indicates that the phase centers at different polarization are located within the snow layer at different heights. Which validates that the volume scattering component of the snow layer and supports fully the interpretation of the multi-baseline observation, useful for future snow parameter retrieval algorithms.

3.6.2 Polarimetric Decomposition over Glacier Ice

The ideas proposed by Jayanti J.Sharma et al. in [53] about decomposition over glacier ice are interesting to analyze. Because, snow and ice have considerable similarities by the scattering process point of view, therefore, such ideas could be applied or adapted in snow characterization.

In [53] the authors suggest, a new model-based polarimetric decomposition approach to describe an ice medium, assuming winter or early spring condition without melt ice. This
decomposition is applied to the covariance matrix defined in the Equation (2.22). After the decomposition process, the covariance matrix is divided as the sum of four components

\[
C^{HV} = C^{HV}_{\text{surface}} + C^{HV}_{\text{volume}} + C^{HV}_{\text{sasrugi}} + C_{\text{noise}},
\]

as the subindices indicate, \(C^{HV}_{\text{ground}}\) is the surface scattering (equivalent to air/snow in case of snow) component, for instance, at L-Band is postulated to originate from the snow-ice interface and to conform to the first-order small perturbation method for a slightly rough surface. \(C^{HV}_{\text{volume}}\) is the volume scatterer, which in this research work is either studied as an oriented or random volume, it incorporates polarimetric-dependent transmissivities as well as differential extinction and refractivity effects. \(C^{HV}_{\text{sasrugi}}\) is the contribution to explain temporal variations in the co-polar ratio in the absence of melt ice. Finally, \(C_{\text{noise}}\) is the additive noise component.

The model has even evaluated and validated on simple forward-modeling simulations and was applied to L- and P-Band polarimetric airborne data over two glacier test sites with varying near-surface structure. Decomposition results conform to known melt characteristics. As all the model-based methods, the greatest limitation of the decomposition process lies in the model assumptions, which could not be satisfied in specific environments. The most critical being as following:

- Presumption of a random volume which does not account for differential extinction and refractivity. These characteristics are required to reconstruct some of the properties of the observed data, including copolar phases and their behavior with incidence angle.
- Assumption of reflection symmetry in the decomposition of the experimental data. No precise criteria have been derived to determine when cocross covariance elements become significant.
- Assumption of a dipole shape for all particles (no variable shape, multishape distributions, shape dependence on depth, etc., are considered).
- Terrain slope is assumed to be constant over the averaging window, which may not always be applicable to alpine glaciers with steep terrain. In such a scenario, surface range and azimuth slopes must be separated from sastrugi and volume orientations, which is nontrivial.

### 3.7 Conclusions

Currently, satellite borne SAR radars are mainly available at L-Band and C-Band frequencies. Other frequencies such as X-Band or Ku-Band, which are more sensitive to
snow parameters retrieval algorithms still remains inaccessible. However, requirement of applications based on the existing technology is demanded. This section will present the conclusion of the existing techniques at C-Band and propose a possibles future research alternatives.

Almost all the authors describe the backscattering of dry snow-pack as the sum of two contributions, underlying surface plus volume scattering, being the first component the dominant one. This assumption seems to be adequate for the underlying surfaces where the volume scattering is not the dominant phenomenon. The problem is how to separate these two components and characterize each one. Cause of the good penetration capabilities at C-Band, and poor interaction with snow volume, extraction of the interesting volume part is not simple. Different authors use multi-temporal, multi-frequencial approach plus simulated surface to characterize the underlying surface in order to extract the interesting contribution from the total one. In order to better model the volume scatterer phenomenon, Nicolas Longépé, et al. further added meteorological models to define a complex snow pack model instead of the single layer assumed by the rest of researches. Other aspects that have been shown is for example, the importance of the incidence angle due to its impact in backscattering, the need to distinguish clearly between wet snow and dry snow, because of drastically change in snow dielectric when the transition from one to other occurs.

Model based decomposition using single layer snow-pack characterization, assuming the snowpack as random oriented cloud of ice particles seems to be a first interesting research step. Nevertheless, further distinguish with phase information multi-layer snow-pack, considering each layer as an oriented cloud of particles should be more adequate approach.
Chapter 4

Polarimetric Analysis

4.1 Introduction

In the previous chapters, the background of PolSAR and basic knowledges of snow monitoring were introduced. In this one, the acquired data and the available ground truth for snow monitoring are presented, followed by a complete polarimetric analysis of the second order statistics and the result of applying different decomposition techniques. In addition, a dry snow mapping algorithm is proposed as a result of the performed case study.

4.2 PolSAR Data

Three Radarsat-2 images were acquired on 17\textsuperscript{th} February 2011, 13\textsuperscript{th} March 2011 and 15\textsuperscript{th} October 2011, centered in the Pla de Beret area, North-Easter Pyrenees mountains, Spain. The covered field is approximately 25 km x 25 km, with the SAR system operating at ascending Fine Quad Pol FQ28 mode. Thus, the data are fully polarimetric, with an incidence angle of 46.6\textdegree and a pixel spatial resolution of around 8x8 m\textsuperscript{2}. The three images were co-registered and filtered afterwards via BPT technique, performing Geodesic distance with the pruning threshold set equal to $-3dB$. The Pauli images after co-registration and filtering process are presented in Figure 4.1.

Furthermore, a BPT change detection algorithm is applied to the three obtained images. Figure 4.2 shows the output of the algorithm. This image presents the polarimetric distance among temporal acquisitions in such a way that large distances imply large changes of the polarimetric behavior of the data.
Figure 4.1: RGB (Double bounce: red, volume: green, surface: blue) PolSAR images acquired in 2011 (a) 17th February, (b) 13th March, (c) 15th October.

Figure 4.2: Polarimetric distance among temporal acquisitions. Large distances imply a change in the polarimetric behavior of the data.
4.3 Area Description and Ground truth

In order to achieve an extensive statistical analysis and adequate characterization of the snowpack, specific areas were identified within the limits of the image. The idea was trying to find homogeneous distributed target areas from which, significant polarimetric backscatter information concerning snow type and presence can be extracted. Finally, six different areas have been selected for the analysis purpose: two alpine meadows reasonably flat at an altitude of 1900 to 2100 m, two forested areas with opposed slopes, another alpine meadow at 2300 m, and finally an urban zone. The different characteristic of the test areas chosen is as follows [8]:

1. Pla de Beret (main test site)
   - Geo-coordinates: longitude 0.966° and latitude 42.732°
   - Altitude: 1914 m
   - Area: 60 x 300 px (trapezoid)
   - Description: Alpine meadow
   - Comments: Quite flat, so that local angle of incidence is 46.6° in average. There is a little water stream vertically crossing the meadow that may appear in the SAR image.

2. Bosc Er Anherar
   - Geo-coordinates: longitude 0.980° and latitude 42.744°
   - Altitude: 2080 m
   - Area: 40 x 100 px
   - Description: Alpine forest in the north-est corner of Pla de Beret
   - Comments: A very steep slope facing the sensor azimuth trajectory, therefore the local incidence angle has been estimated to be 22.6° [8].

3. Bosc de Parros
   - Geo-coordinates: longitude 0.968° and latitude 42.475°
   - Altitude: 1973
   - Area: 40 x 100 px
   - Description: Alpine forest in the north-west corner of Pla de Beret
   - Comments: A steep slope facing opposite to the sensor azimuth trajectory, therefore the local incidence angle has been estimated to be 68.4° [8].

4. Bonaigua - Pla de Moredo
5. Pla de Moredo

- Geo-coordinates: longitude 0.899° and latitude 42.751°
- Altitude: 1959 m
- Area: 70x60px
- Description: Alpine meadow
- Comments: Quite flat, so that local angle of incidence is 46.6° in average.

6. Naut Aran

- Geo-coordinates: longitude 0.903° and latitude 42.709°
- Altitude: 1333 m
- Area: 60x50px
- Description: Urban zone (village)
- Comments: Quite flat, so that local angle of incidence is 46.6° in average.

With the objective of providing an easy area identification for the rest of the document, the principle characteristics of the six test areas are summarized in the Table 4.1.

Additionally, snow surveys were carried out on 16th - 17th February 2011 and 16th - 18th March 2011, considering 33 equally spaced measurements covering the Pla de Beret area [54], a region of 1800 m length and 300 m width, see Figure 4.3. The main results extracted from the campaign is summarized in Table 4.2. And in the Table A.1, Table A.2 are shown the specific measured positions and the retrieved snow physical parameters.

To complete the ground-truth data, daily temperatures were obtained from a meteorological automatic station in the Pla de Beret area, illustrated in Figure. 4.4. Contrasting the evolution of the temperature and the samples collected from the snow campaign, the dry snow condition is assumed for the image acquired on 17th February 2011, as the maximum temperature is below 0°C. In the frame of 13th March 2011, with a temperature of about 2°C, the wet snow hypothesis is assumed, which may be easily identified in the Pauli image in Figure 4.1b, where surface scattering dominates in the wet snow regions due to the decrease of backscatter power, represented by the blue color. Finally, a Shuttle Radar Topography Mission (SRTM) digital elevation model was employed and projected in the Radarsat-2 geometry, shown in Figure 4.5.
4.3. AREA DESCRIPTION AND GROUND TRUTH

Figure 4.3: Pla de Beret North looking November 2011.

Figure 4.4: Temperature evolution of the Pla de Beret. (— : average temperature, — — : minimum temperature, — · — : maximum temperature).
Table 4.1: Test areas.

<table>
<thead>
<tr>
<th>Area</th>
<th>Name</th>
<th>Height</th>
<th>Incidence Angle</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Pla de Beret</td>
<td>1914 m</td>
<td>46.6°</td>
<td>Alpine meadow</td>
</tr>
<tr>
<td>2</td>
<td>Bosc Er Anherar</td>
<td>2080 m</td>
<td>22.6°</td>
<td>Alpine forest</td>
</tr>
<tr>
<td>3</td>
<td>Bosc Parros</td>
<td>1973 m</td>
<td>68.4°</td>
<td>Alpine forest</td>
</tr>
<tr>
<td>4</td>
<td>Bonaigua</td>
<td>2310 m</td>
<td>46.6°</td>
<td>Alpine meadow</td>
</tr>
<tr>
<td>5</td>
<td>Pla de Moredo</td>
<td>1959 m</td>
<td>46.6°</td>
<td>Alpine meadow</td>
</tr>
<tr>
<td>6</td>
<td>Naut Aran</td>
<td>1333 m</td>
<td>46.6°</td>
<td>Urban zone</td>
</tr>
</tbody>
</table>

Table 4.2: Ground campaigns summary.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>February 2011</th>
<th>March 2011</th>
<th>∆ Feb-Mar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dryness</td>
<td>Dry snow</td>
<td>Wet snow</td>
<td>liq.H₂O</td>
</tr>
<tr>
<td>Density</td>
<td>236 Kg/m³</td>
<td>317 Kg/m³</td>
<td>34%</td>
</tr>
<tr>
<td>Depth</td>
<td>32.24 cm</td>
<td>62 cm</td>
<td>100%</td>
</tr>
</tbody>
</table>

4.4 Study of Diagonal Elements of Coherency and Covariance Matrix

To start the analysis, the diagonal elements of the coherency matrix, as well as the covariance matrix, in all the test areas are considered. The mean power values are shown in Table A.3. Where Span is defined as

\[
Span = \left< |S_{HH}|^2 \right> + 2 \left< |S_{HV}|^2 \right> + \left< |S_{VV}|^2 \right>,
\]

(4.1)

and ∆(x, y) (dB) means the difference in dB between the x and y acquisition. Notice that an abuse of language is used, as the acquisition in February is assumed to be the dry snow image, March acquisition as the wet snow and October acquisition as the snow free reference.

The change of the backscatters due to the presence of wet snow can be easily identified in the alpine meadow areas 1 and 5, where all the diagonal elements of covariance matrix suffer a reduction of more than 3 dB [29]. However, this phenomenon is not appreciable in the alpine meadow 4 and the two forest regions. The reason could be as follows, for forest areas the snow fall down from the trees once it gets wet and do not highly contribute to the surface scattering. While in the meadow area 4, due its location, much higher than the area 1 and 4, where lower temperature could still maintains the snow in dry conditions.
4.5 Study of Different Decomposition Theorems Applied to the Data

As indicated in the Section 2.4, polarimetric decomposition techniques are widely used to characterize distributed target, as well as their physical parameter retrieval. In this section, existing techniques in the PolSAR literature are implemented and applied to the acquired data, with the objective to assess the possibility of achieving a snow quantitative and qualitative estimation.

4.5.1 Model Based Decomposition Study

First of all, model based decomposition theorems are applied to these three images.
4.5.1.1 Freeman Three Component Decomposition

Freeman method is applied the data, and different scattering contributions in \( dB \) are obtained from the decomposition technique and combined as an RGB image. Namely, surface scattering \( P_s(dB) = 10\log(f_s(1 + |\alpha|^2)) \) is set as a blue component, double bounce scattering \( P_d(dB) = 10\log(f_s(1 + |\beta|^2)) \) is fixed as a red, and volume scattering \( P_v(dB) = 10\log(f_v) \) is assigned as a green element. The gathered results are shown in Figure 4.6.

Figure 4.6: Freeman decomposition images in \( dB \), (a) 17\(^{th}\) February, (b) 13\(^{th}\) March, (c) 15\(^{th}\) October.

It is not difficult to perceive that, from the dry snow situation (17\(^{th}\) February 2011) to wet snow scenario (13\(^{th}\) March 2011) there is a considerable increase of surface scattering. Such phenomenon is due to the apparition of wet snow zones, intensifying then the forward scattering effect. Nevertheless, snow free situation (15\(^{th}\) October 2011) does not seem to have significant changes compared to the dry snow circumstance. In other hand, the existence of black zones in all the three images is consequence of shadowing effect of SAR imaging. In order to analyze the situation more precisely, the proportion of each scattering to the total \( Span \)

\[
Pr_s = \frac{f_s(1 + |\beta|^2)}{f_s(1 + |\beta|^2) + f_d(1 + |\alpha|^2) + f_v}
\]

\[
Pr_d = \frac{f_d(1 + |\alpha|^2)}{f_s(1 + |\beta|^2) + f_d(1 + |\alpha|^2) + f_v}
\]

\[
Pr_v = \frac{3f_v}{8} \frac{3f_v}{8}
\]

is computed and illustrated in Figure 4.7, the same assignation of RGB to double bounce, volume and surface is employed.
4.5. STUDY OF DIFFERENT DECOMPOSITION THEOREMS APPLIED TO THE DATA

Basically, similar conclusions can be extracted for the transition from dry snow to wet snow. However, unlike the representation in dB, here a clear increase of volume scattering is observed from dry snow to snow free image. Thus, the hypothesis of the smoothing effect of snow over random surfaces is first stated. Notice also that double bounce scattering barely contributes in all the three images, leading us to think that maybe a model composed by two scattering elements is sufficient to achieve a final inversion model for snow parameter retrieval.

In order to test the decomposition method with some ground truth information, detail studies are effectuated for each area defined in Table 4.1. The average result from the decomposition is shown in Table A.4. Where ∆Comp(dB) means the difference in dB between the contribution of scattering mechanism of row above and the actual one.

Table A.4 indicates that the dominant component in quasi all the area and snow situations is the volume contribution, which is not realistic. For instance, area 4 and area 5 are quite flat zones with low vegetation, the existence of scenes where surface scattering contribution is less than half of the volume scattering should not appear. Another inconsistency is the area 6, a city area should be clearly dominated by double bound scattering. The reason of such overestimation of volume scattering has already been pointed out in the Section 2.4, leading us dicard this decomposition for inappropriateness.

4.5.1.2 Yamaguchi Four Component Decomposition

After checking the unfeasibility of the Freeman method, Yamaguchi technique is employed to analyze the data set. Like in the previous approach, double bounce, volume and surface scattering in dB from the decomposition are combined as an RGB image. The fourth component is not taken in account due to its poor physical information. The computed images are shown in the Figure 4.8.

Essentially, similar conclusions as Freeman decomposition for snow transition can be
reached: surface scattering increases from dry snow to wet snow, and less random behavior is present from snow free to dry snow. The difference yields in the single image observation, where in Freeman approach images are quite dominated by volume scattering, here the surface contribution starts to gather importance. As it has been done with the previous technique, to better visualize this effect, proportion of each scattering power to the total Span is computed and illustrated in Figure 4.9.

As expected, compared to Freeman method, surface scattering is more emphasized, specially in the dry snow scenario. To enter to more detail discussion, the situation of each particular test area is summarized in Table A.5, the definition of $\Delta$Comp(dB) is the same as in the Freeman’s case.

Unlike the Freeman technique, the dominant scattering of each zone seems to be more reasonable. For example, in the urban area, surface and double bounce become more prominent than volume contribution. Area 4, which was a flat area dominated by volume,
4.5. STUDY OF DIFFERENT DECOMPOSITION THEOREMS APPLIED TO THE DATA

Pass to be controlled by surface scattering, while area 2 and 3 maintain their random behavior. Regardless, area 5 still has unexpected response, quite flat zone governed by volume contribution in all the snow situations.

After all the analysis done through the two model based decomposition theorems, the most important problem of having an inadequate adaptation of the method to the specific scenario has been pointed out, namely, trying to fit a volume dominant decomposition model to situations where the surface mechanism predominates (see Section 4.5.2). Angle compensation could have improved the results slightly, but it still unsatisfactory for our purpose.

4.5.2 Eigenvalues/Eigenvector Decomposition Study

In this subsection, eigenvector based decomposition is applied to the data. Roll invariant parameters and some combination of them are analyzed to obtain information of the scene, which would contribute to structure a proper decomposition method in the future. The interpretation of different results is only centered in the six previously introduced areas, due to not meaningful reading from the whole image.

4.5.2.1 Eigenvalues

First, the analysis starts with different eigenvalues from the mathematical decomposition. Table A.6 shows the average values of each area. The mean power in $dB$ and percentage to total $\lambda_1 + \lambda_2 + \lambda_3$ are computed, as also the temporal evolution through the subtraction between snow conditions. $\Delta(a,b)$ means component $a$ minus component $b$, in this table, contributions are targets to effectuate the difference, not the values in $dB$.

Checking the table in detail, few conclusions can be extracted. For instance, the first eigenvalue represent almost two third of the total in all the studied zones. While the contribution of the third eigenvalue rarely reaches to fifteen per cent, having only around five in urban areas, case opposite to forest regions where get nearly to the local maximum (among all the analyzed areas). This aspect is not surprising, since $\lambda_3$ measures somehow the unpolarized phenomenon, and the un cities are man-made structures, where this phenomenon is poorly induced. When we analyze the temporal evolution of eigenvalues, it shows that from snow free to dry snow situation, the contribution of the first eigenvalue increases, decreasing the presence of second and third eigenvalue. Similar conditions happen for the transition from snow free scenario to wet snow, which may indicate the association of the $\lambda_1$ to surface scattering and its magnification (see Section 4.5.2.2).

4.5.2.2 $\alpha$ angles of Different Eigenvectors

Following the study of the eigenvalues, roll invariant angles from the parameterization of the eigenvectors shall be studied. Particularly, the alpha angles $\alpha_1, \alpha_2, \alpha_3$, which exhibit
information about the type of scattering mechanism, are the objectives. Average and variance values for each area are summarized in the Table A.7.

Examining the obtained table, is not difficult to notice that the first component seems to have different scattering mechanisms respect to the second and third element, which are very similar. The computed values of $\alpha_1$ present low values in all the non-urban areas, indicating a possible surface dominant scatterer, even in the two forest regions. However, in the last two mentioned areas the variance is quite higher, so more accurate studies should be taken before making any conclusions. As we have mentioned, roughly, the values $\alpha_2$ and $\alpha_3$ are taking similar forms, pointing to a possible double bounce scattering. Such situation does not match with the expectations, maybe both components should be analyzed together in order to get extra information. Because the individual coherency matrix from the product of two eigenvector $T_i = u_i u_i^H$ has rank equal to one, and either purely surface (not Bragg) or volume can not be modeled by this type of matrix.

4.5.2.3 One Component Analysis

The analysis of eigenvalues has revealed that the first eigenvalue predominates over the rest, reaching even in some cases over two thirds of the total power. Then, the full coherency matrix can be approximately estimated by only the contribution of the this first component. In other words, the main scattering mechanism can be well modeled if the rest of the contributions are omitted. Following this idea, the parameters employed to distinguish among different scattering mechanisms are computed. Specifically, those introduced in the previous review Section 2.4.2: $A_0, B_0 + B, B_0 - B$.

The computed values are shown in the Table A.8, where $i - j$ means the value of row number $i$ minus the value of row number $j$.

It is shown that surface condition ($A_0 \gg B_0 + B, B_0 - B$) is hold for all non-urban areas. Besides, double bounce condition is satisfied by urban zone for dry and wet snow conditions. We did not expect to have surface as dominant scatterer even for forest regions, the reason is may due to having a low wavelength, which can produce some diffraction effect. Further studies about the type of forest and the size of their branches should be taken before reach any conclusion.

4.5.2.4 Type of Coherency Matrix in Each Component

After all the studies effectuated using eigenvalues and eigenvectors. It is worth to check the specific form of the coherency matrix component: $T_1 = u_1 u_1^H$. In the Tables A.9 and A.10 are shown the average values of each studied area. For instance, for every single matrix component, the mean real and imaginary parts are plotted separated.

As it is expected, for all the non-urban zones, the first scattering matrix component $T_1$ shows surface component behavior, taking roughly the form of the Equation (2.30).
Furthermore, the approximation to the ideal model is better in flat zones than the forest regions, which is quite logic. In contrast, the second and third components $T_2$, $T_3$ can not generally be matched with any canonical form of coherency matrix, only in some cases, the second contribution exhibits slightly double bounce behavior, introduced in the Equation (2.35).

4.5.2.5 $H/A/\alpha$ Interpretation

In order to contrast with the previous studies [8], $H$ and $\alpha$ values computed after BPT based filtering process are presented in the Table A.11, and a graphical comparison with the achievement by 7x7 box car filter is shown in Figure 4.10. As we can see, $H$ was slightly overestimated and the $\alpha$ angle underestimated. However, the average values of different areas still remains in the same region of the $H/\alpha$ plane.

Five remarks should be discussed here:

- Dry snow decreases the underlying media randomness, see Figure 4.10, by the reduction of $H$ in all the natural areas due to the increase of the surface scattering contribution.

- The scattering mechanism seems to be governed by surface scattering, as random surface, in meadow areas.

- Forest area 2 appears in the random surface region due to the low incidence angle, around 22.6°.

- The different wet snow behavior of meadow area 4, respect to the rest of flat areas, is due to its higher location, where temperature difference could still keep the snow in dry condition when others have passed to wet. This singularity was already pointed out in the Section 4.4.

- The reason to have similar snow free and wet snow situations in the forest areas is probably due to the snow falling process from the trees when its get wet, which matches with the study of diagonals elements in the Section 4.4.

4.5.2.6 Type of Scattering Mechanism

In the Section 2.4.2.3, we have mentioned that Anisotropy became a proper complementary discriminator when Entropy reaches values over 0.7, since the scenario fits into the situation, this parameter is added to perform $H/A/\alpha$ unsupervised classification based on scattering mechanisms and wishart classifier (Section 2.4.2.4). The obtained images displaying the dominant class are shown in Figure 4.11, like previous assignations, the
same combination of color is used to identify different mechanisms: red, green and blue are attributed to double bounce, volume and surface respectively.

Clearly, surface scattering predominates in most of the regions, while volume mechanism controls certain areas. Another interesting observation is the progressive increase of pixels having volume as a dominant scatterer from dry snow to wet snow, and finally reaching the free snow situation. Again, dry snows play an important function of smoothing random scatterers. The phenomenon of having the wet snow scenario more volume pixels than dry situation may be due to the falling process of snow from the trees when it gets wet, as explained in the previous subsection. However, it seems to contradict the results captured from the model decomposition techniques, where in wet snow situations, the surface scattering is enhanced. It may due to that the contribution of surface to the total Span is intensified in pixels where the dominant scatterer is still surface, as here only binary assessment is done, this phenomenon is undetectable.
All the previous studies show a clear dominance of surface scattering in the majority of the defined areas. Thus, for future assessment of a proper decomposition technique for snow environments, the next step should be to determine the number of scattering mechanisms necessary to model the scenario. For this purpose, different combinations between Entropy and Anisotropy can be used to distinguish the type of scattering process.

- \((1-H)(1-A)\) vs \(H(1-A)\) image: As it can be checked in the Figure 4.12, in general, almost all the test areas present low values of the \((1-H)(1-A)\) combination, and large values of the \(H(1-A)\) component, which means that the chance to have a single dominant scattering process is small, opposite to the probabilities of belonging to a random scattering process. In terms of the temporal transition, snow free situations seem to have more random behavior because of a considerable increase of \(H(1-A)\) values respect to the wet and dry snow scenarios. In other hand, dry snow environments seem to augment the chance of having single dominant scatterer, because the values \((1-H)(1-A)\) became higher with the snow presence, which is quite logic because of its smoothing effect over random scattering mechanisms.

![Figure 4.12](image)

Figure 4.12: \((1-H)(1-A)\) vs \(H(1-A)\), d: dry snow, w: wet snow, f: snow free.

- \(HA\) vs \((1-H)A\) image: The possibilities to have two scattering mechanisms either with same probability expressed by \(HA\) values or one dominant over another specified by \(H(1-A)\) values, are close to zero, having the last option larger chances. Therefore, the idea of having only two components in the decomposition model need to be further studied.
Through the study of different combinations between Entropy and Anisotropy, we find out that the scenario to characterize is not completely random, but tends to it, which means that three component decomposition theorem is needed to model the situation properly. However, here, we are only considering the environment constructed by a sum of three pure targets, \( T = \sum \lambda_i T_i \), because all the individual scattering components \( T_i \) are \( \text{rank} = 1 \) coherency matrices. Then, the possibilities to have more complex mechanisms and their composition to form scenarios, are not considered.

### 4.5.3 Hybrid Approach Analysis

#### 4.5.3.1 Van Zyl NNE Decomposition

Van Zyl non negative eigenvalues (NNE) decomposition is applied over the three snow images, the different scattering contributions in \( dB \) combined as RGB image is shown in the Figure 4.14. The same color assignation is used: red for double bounce, green for volume, and blue for surface. Without examining the images in detail, the obtained results see to be really similar to the achievement done with Yamaguchi decomposition. However, if the contribution of each scattering power to total Span is checked (Figure 4.15). We notice an considerable increase of surface scattering in all the three images, even surpassing the volume scattering, and becoming the dominant component in dry and wet snow situation. This phenomenon is in ageement with the studies done using eigenvalue/eigenvector decomposition. However, there is still an amount of the double bounce (red) contribution that is still unexpected according to the emitted short wavelength (around 5 centimeters).
4.5. STUDY OF DIFFERENT DECOMPOSITION THEOREMS APPLIED TO THE DATA

Figure 4.14: Van Zyl non negative eigenvalues decomposition images, (a) 17th February, (b) 13th March, (c) 15th October.

More details can be found in the Table A.12, where the average of each contribution is computed for the different named areas. Unlike the purely model based decomposition, this method seems to have better performance and can be adapted to a specific scenario. For instance, according to the table, surface become dominant in all the flat meadow areas, and volume plus surface is alternating in forest regions depending on the snow situation. Furthermore, in the urban area, the dry snow situation is dominated by double bounce scattering, all consistent with the expectations.

Figure 4.15: Van Zyl non negative eigenvalues decomposition, the proportion of each scattering to the total power is combined in RGB image, (a) 17th February, (b) 13th March, (c) 15th October.

4.5.3.2 Study of Volume Adaptive Decomposition

The last target decomposition to analyze is the adaptive approach proposed by Arii et al. [23], described in Section 2.4.3.2. The values of $n$ and $\theta$ have been sampled using the Equation 2.107 and the Equation 2.108, with $N_i = N_j = 30$. The results in dB is almost identical to the non negative eigenvalue approach and it will be omitted. Higher difference
can be appreciated in the contribution of each scattering power to the total Span, which is plotted in Figure 4.16.

Figure 4.16: Volume adaptive decomposition, the proportion of each scattering to the total power is combined in RGB image, (a) 17\textsuperscript{th} February, (b) 13\textsuperscript{th} March, (c) 15\textsuperscript{th} October.

As it may be observed, unlike the NNE decomposition, double bounce contribution badly appears in all the three images, having the surface and volume mechanisms spreading the amount of scattering power. Also, the decomposition seems to not have the volume and surface overestimation problem as the purely model and eigenvalue based decompositions techniques.

To get in deeper analysis, the amount of scattering contributions of the six tested areas were computed and shown in Table A.13. In terms of the dominant scatterer, all the test zones seems to be under expectation: all the meadow areas are dominated by surface scattering independent of the three snow situations, the urban area alters the dominant mechanism between double bounce and surface according to the snow scenario, and the forest regions almost dominated by volume scatterer. Particularities as the volume dominance of forest areas in wet snow situation can be easily explained if the snow falling process from the trees when its get wet is considered true. Also, the meadow area 5 dominated by volume scattering is may due to the kind of vegetation present, which have been already pointed out in the previous $H/\alpha$ analysis.

Furthermore, two novel parameters called scattering Entropy $H_s$ (Equation (4.3)) and scattering Anisotropy $A_s$ (Equation (4.4)), are considered for an easier interpretation of the variation of the scattering components through the change of snow enviroment. They are based on the definition of Entropy and Anisotropy, introduced in the Section 2.4.2.

$$H_s = -Pr_s \log(Pr_s) - Pr_d \log(Pr_d) - Pr_v \log(Pr_v)$$

$$A_s = \frac{|Pr_d - Pr_v|}{Pr_d + Pr_v},$$

where $Pr_s$, $Pr_d$ and $Pr_v$ are defined in the Equation(4.2), and the values in percentage
4.5. STUDY OF DIFFERENT DECOMPOSITION THEOREMS APPLIED TO THE DATA

Figure 4.17: Variation of scattering components for the three snow conditions, d: dry snow, w: wet snow, f: snow free.

of each test area are shown in Table A.13. The mean values of $H_s$ and $A_s$ for the six analyzed areas are illustrated in Figure 4.17.

Figure 4.17 shows that the scattering Anisotropy, defined as the difference between the volume and double bounce components does not suffer large alterations through the three snow conditions, while the scattering Entropy does. This means that the redistribution of scattering components in the three snow states only occurs between the surface and the volume components. Regarding the weight of the double bounce component in the six areas in Table A.13, it seems to be negligible for snow studies at C-Band. The $\pi$ valued obtained in the previous section, far from 90$^0$ confirms also that double-bounce scattering mechanism may be considered negligible.

If the parameters $H_s$ and $A_s$ plays the same function as the Entropy and Anisotropy values. We can also call them Entropy and Anisotropy of the decomposition. And like we have done with the $H$ and $A$, different combinations of those parameters are evaluated in order to compare the differences between eigenvalue based decomposition and adaptive volume approach.

- $(1 - H_s)(1 - A_s)$ vs $H_s(1 - A_s)$ image: Like the previous $H/A$ approach (Section 4.5.2.7), single dominant scattering process seems to be improbable due to the low values of $(1 - H_s)(1 - A_s)$. However, the $H_s(1 - A_s)$ axis has different performance, for instance, unlike the previous result, forest regions 2 and meadow area 5 can be easily identified as more random process than the rest of the areas, for having higher $H_s(1 - A_s)$ values with similar $(1 - H_s)(1 - A_s)$ levels.
Figure 4.18: $(1 - H_s)(1 - A_s)$ vs $H_s(1 - A_s)$, d: dry snow, w: wet snow, f: snow free.

- $H_s A_s$ vs $(1 - H_s)A_s$ image: Compared to the $H/A$ analyzed in the Section 4.5.2.7, here, the change of having two scattering mechanisms with same probability, corresponding to $H_s A_s$ values, is higher. Maintaining almost the possibilities to have two scattering with one predominant, specially for the forest areas, which means that it is corroborating the fact of possessing more complex scattering mechanisms to model the target will reduce the number of component in the decomposition theorem. For instance, in the eigenvalue based decomposition, volume scattering could not be modeled for limitations of the technique.

Figure 4.19: $H_s A_s$ vs $(1 - H_s)A_s$, d: dry snow, w: wet snow, f: snow free.

Is not worth to mention that even for different decompositions, the ensemble information seems to offer approximately similar results, which may correspond to the real environment behavior. For instance, eigenvalues decomposition and adaptive approach are different decomposition techniques, but both show up the importance of surface scattering in all the test areas. The adaptive approach further show the possibility of having only two complex scattering mechanisms with different probabilities. Therefore, a proper
decomposition technique considering only two components, surface and volume, could be a step to follow for quantitative parameters estimation at C-Band.

4.6 Dry Snow Covered Mapping

As mentioned in Section 3.3.1, identifying dry snow cover is essential for subsequent application of inversion algorithms. Because, the final desired SWE parameter can only be extracted from dry snow situations. Thus, summarizing the conclusions extracted from the previous extensive polarimetric analysis, a simple algorithm combining $H/\pi$ parameters and backscattering coefficients with some external information is proposed to generate the dry snow map.

4.6.1 External Information

In order to achieve an accurate mapping of snow cover, external informations are employed. Specifically, digital elevation model, temperature information and results from BPT change detection algorithm:

- Digital elevation model is computed using interferometry SAR technique in the geometry of the acquisition by inverse imaging. The result is shown in Figure 4.5.

- Temperature datas are provided by automatic meteorological station at Pla de Beret. In Figure 4.4 is shown a month acquisition including the transition from dry snow scenery to wet snow situation.

- BPT change detection algorithm is applied taking into account all the three acquired images. In Figure 4.2 is shown the output of the algorithm. Is interesting to observe that higher places implies larger changes, maybe due to the lower temperatures of the scene, which makes the accumulation of snow easier.

4.6.2 Mapping Algorithm

The polarimetric analysis carried out in the previous subsections showed that the $H/\pi$ values are sensitive to the presence of dry snow, and the $\langle |S_{HH}|^2 \rangle$ backscatter coefficient is the element among the covariance and coherency components that suffers the largest change due to wet snow. Also notice that the average temperature of main Pla de beret area has been maintained almost under $0^\circC$ until the second acquisition in March, Figure 4.4. Furthermore, the snow cover was found deeper in the March frame (Table 4.2), which means that there was snow accumulation between the February and March image. Thus, the assumption that snow covers between the two acquisitions have not suffered large alterations was made. Thus, transition between snow situations should be governed by the following diagram:
And the proposed Algorithm to estimate the snow cover in February is shown in Table 4.3.

<table>
<thead>
<tr>
<th>Algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1</strong>) If $H_{\text{dry}} &lt; \text{thre}<em>H H</em>{\text{free}}$ and $\pi_{\text{dry}} &lt; \text{thre}<em>\alpha \pi</em>{\text{free}}$ ⇒ possible snow pixels, we call it subset snowDF.</td>
</tr>
</tbody>
</table>
| **Step 2**) a) The pixels from snowDF subset that does not fulfill $\left< |S_{HH}|^2 \right>_\text{wet} < \left< |S_{HH}|^2 \right>_\text{dry} - \text{thre}_{\text{snow}} dB$ is discarded as snow pixels, except zones higher than thre$_h$ m.  
  
  b) If pixels not include in snowDF subset satisfy $\left< |S_{HH}|^2 \right>_\text{wet} < \left< |S_{HH}|^2 \right>_\text{dry} - \text{thre}_{\text{snow}} dB$, then, they are added to the snow group ⇒ the new snow subset result from the previous two steps is called snowDFW. |
| **Step 3**) Same methodology as step 2 is applied to the group snowDFW changing the condition by $\left< |S_{HH}|^2 \right>_\text{free} > \left< |S_{HH}|^2 \right>_\text{wet} + \text{thre}_{\text{snow}} dB$. |
| **Step 4**) Final mask is applied to the obtained snow pixels by thresholding the full covariance matrix change detection with thre$_{ch}$. |

The different thresholds are defined as:

- $H/\pi$: thre$_H$ and thre$_\alpha$ are set as 0.938 and 0.914, results from the previous work [8].

- Backscattering: thre$_{\text{snow}}$ is fix to 3, knowing that according to state of art of snow remote sensing, wet snow backscattering is considerably reduced comparing to dry snow and snow free situations. The statement is also corroborated with our acquired data and ground truth.

- Height: Wet snow condition was set thanks to the monitoring of temperature at Pla de Beret, about 2°C at 1914m. Such condition does not hold anymore for much higher altitudes, for instance, assuming that temperatures normally decreases 0.6 per each 100 m, then, altitudes larger than 2247 meters have temperatures below 0°C, condition necessary to have dry snow. We have fixed the threshold to thre$_h$ = 2250 m.

- Changes: The last mask threshold of change detection is set according to the changes produced in each area defined in the Section 4.3, the average values is summarized in the Table 4.4.
Table 4.4: Average change values of six test areas.

<table>
<thead>
<tr>
<th>Area</th>
<th>Polarimetric distance between matrices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area 1</td>
<td>2.99</td>
</tr>
<tr>
<td>Area 2</td>
<td>1.18</td>
</tr>
<tr>
<td>Area 3</td>
<td>1.74</td>
</tr>
<tr>
<td>Area 4</td>
<td>2.61</td>
</tr>
<tr>
<td>Area 5</td>
<td>3.58</td>
</tr>
<tr>
<td>Area 6</td>
<td>1.64</td>
</tr>
</tbody>
</table>

The lowest value of change is produced at forest area 2, but considering its low incidence angle, we have chosen a mean value between the two lowest change magnitudes, setting the $thre_{ch}$ to 1.5.

The results from the Algorithm is illustrated in the Figure 4.20. The dry snow map seems quite homogenous in the majority part of the image, however, there are still noise like features around the valley parts (see Figure 4.5). Thus, a filtering process is employed to reduce the noise phenomenon in the snow map, particularly, image reconstruction technique using morphological erode operation is applied. The achieved result is shown in the Figure 4.21.

Figure 4.20: Snow covered mapping.
4.7 Conclusions

Through all the studies done using different decomposition algorithms, it is clear that there is need to use both temporal and full-polarimetry information in order to complement the low sensibility of C-Band to snow parameter retrieval processes. Note that we have focused more in the transition between different acquisitions rather than analyzing images separately. Furthermore, new coherency or covariance matrix models need to be defined for specific snow environments, plus an intelligent decomposition technique that is able to identify the dominant scattering mechanism and proceed with decomposition accordingly. The components involved in this new decomposition should be based on the combination of the surface and volume scattering mechanisms.

Once a proper decomposition is defined, the retrieval of the surface model can be computed in different snow situations, and the change of incidence angle from snow free scenarios to dry snow environments could be a proper candidate to estimate snow parameters, the situation is shown in Figure 4.22.
Notice that the values of $\beta$ defined in the Equation (2.31) will take different forms for the two different snow situations. For instance, considering $\beta_{\text{free}} = f(\theta_i, \varepsilon_{\text{soil}})$ as the standard incidence case, in the dry snow situation, $\beta_{\text{dry}} = f(\theta_r, \varepsilon_{\text{soil}})$, because the dry snow layer is almost transparent to the wavelength, producing only a change of the incidence angle on the ground. If we are able to estimate both $\beta$ values from the decomposition theorem and assuming that the dielectric constant of the soil has not suffered large alterations, then, we can formulate a system of two equation with two unknowns, $\varepsilon_{\text{soil}}$ and $\theta_r$, because $\theta_i$ is one of the known parameter of satellite. We also know that both incidence angles, $\theta_i$ and $\theta_r$ are related with snow dielectric by Snell’s law

$$\sin^2 \theta_i = \varepsilon_s \sin^2 \theta_r,$$  \hspace{1cm} \text{(4.5)}

with the $\varepsilon_s$ recovered, snow density $\rho_s$ can be easily computed from the Equation

$$\varepsilon_s = 1.0 + 1.5995\rho_s + 1.861\rho_s^3.$$ \hspace{1cm} \text{(4.6)}

However, to obtain the final desired SWE, further studies to estimate snow depth should be effectuated.
Chapter 5

Data Validation

5.1 Introduction

In the Section 4.6, a snow cover detection algorithm has been proposed to achieve snow maps over Pyrenees’ mountainous areas. In this section, the effectiveness of the proposed algorithm is tested by comparing the results against optical acquisitions. Specifically, the Moderate Resolution Imaging Spectroradiometer (MODIS) system is chosen for the purpose, because of its facility to provide snow covered areas. The MODIS image with multiple bands was obtained on 17th February 2011. Taken in nadir geometry, with a spatial resolution of 500 m. Therefore, ground truth of snow cover can be recovered, by computing the Normalized Difference Snow Index (NDSI) from the optic images. The NDSI parameter indicates the existence of snow when its value exceeds 0.4.

Moreover, in order to validate the conclusions extracted from the study of snow behavior over homogenous areas carried out in Section 4.5, indicating that snow covers tend to decrease the randomness of scattering mechanism. The around eight ground-truth points with full snow physical parameters have been inversely geocoded to the satellite coordinates. In this way, polarimetric characterization of snow can be performed in more accurate way.

5.2 SAR and MODIS Comparison

Due to the different acquisition geometries between SAR and MODIS systems, before any comparison, geometrical and resolution mismatches should be solved. The methodology to follow is summarized in five points:

- Undoing the co-registration step of SAR image.
Figure 5.1: SAR and MODIS images projected to the ground geometry, (a) SAR image (b) MODIS image.

Figure 5.2: Comparison between (a) SAR and (b) MODIS snow maps after ground projection and interpolation.

- Project both SAR and MODIS images to the ground geometry, see Figure 5.1.
- Overlap both image, and mask out the zone of not interest.
- Interpolate the MODIS image in order to reach resolution of SAR image.
- Obtain the MODIS snow map by thresholding the NDSI parameter.

The obtained results after the described steps is shown in Figure 5.2. By checking pixel to pixel correspondence, around a 70% was found. Taking into account the edges maladjustment between both systems, and the interpolation errors caused by the poor resolution, it seems quite satisfactory.

Furthermore, an in-depth analysis of this agreement is presented in Figure 5.3, where the SAR snow map is compared against the MODIS map as a function of height, for six height ranges. *Dry snow detection* and *snow free detection* represent the cases in which
both systems detect snow or absence of snow, respectively. False alarm considers the cases where the SAR system detects snow whereas the MODIS one does not and detection losses accounts for pixels where the SAR system does not detect snow and the MODIS one does. For a height below 1600 m, both systems detect the absence of snow with a rate above 90%. As it would be expected, the snow free detection decreases and the dry snow detection increases with height. In the range from 1600 m to 2100 m, the misclassification obtained with SAR compared to MODIS data is maximum as the detection loses and the false alarm are maximum. Finally, for heights above 2100 m, both systems agree on the detection of the snow cover with a rate above 60%. Nevertheless, the false alarm rate is low and the detection losses are moderated. The later could be due to the differences in spatial resolution, and the layover and shadowing effects affecting the SAR data.

5.3 Inverse Geocoding of Spots with Snow Characteristics

Besides the snow cover map, the polarimetric response characterization due to snow density is further studied. The eight ground points, D-2, D-4, D-8, D-13, D-15, D-19, D-20 and D-25 (Table A.1), from the February snow campaign, with information of snow density and snow depth, were inversely geocoded to satellite coordinates. In this way, more accurate polarimetric studies can be carried out directly in the image coordinates. The distribution of points during the campaign is shown in the Figure 5.4, and the localization of spots in the image space is illustrated in the Figure 5.5.

The corresponding $H$ and $\bar{H}$ values are in Figure. 5.6, where $D-i$ states for the dry snow points on 17th February frame, and $F-i$ are the same points in the snow free image on 15th
Figure 5.4: Distribution of snow points in the Pla de Beret test site.

Figure 5.5: Distribution of snow points in SAR coordinates, a) Pauli image marked with area of acquisition, (b) the considered snow points.

October acquisition. As observed, the points present the same tendency previously stated. The randomness decreases with the presence of dry snow. The sole conflicting point is $d_5$, where $\bar{\alpha}$ increases differently from the rest. This behavior may due to the incidence angle, which is the lowest. Additionally, no direct dependency of $H$ and $\bar{\alpha}$ parameters to snow physical parameters is observed.

It worth to mention that the different threshold of the snow mapping algorithm was tested for these dry snow points, for instance, $thre_H$ and $thre_{\alpha}$, and both of them satisfies the conditions.
5.3. INVERSE GEOCODING OF SPOTS WITH SNOW CHARACTERISTICS

Figure 5.6: Points with snow properties: height [m], snow depth [cm], snow density [Kg/m$^3$] and SWE [Kg/m$^2$].
Chapter 6

Decomposition via Convex Optimization

6.1 Introduction

Many decomposition theorems have been proposed in literature, see Section 2.4. However, none of them is designed for snow application. Moreover, the existing methods always propose solutions based on suppositions or iterative methods that do not guarantee an optimum solution. As the definition of snow models for scattering phenomenon requires snow physical knowledge that is out of our area of expertise, we are going to focus on the problem of providing a methodological step to estimate the polarimetric decomposition parameters optimally. Convex optimization is a powerful tool that is able to find optimal solutions within a set of equation, so let’s see if it can be applied to target decompositions.

6.2 Problem Formulation

Back to the definition of the decomposition theorems, the objective behind is to seek the best parameters that minimizes the difference between a given coherency matrix $T$ and the sum of proposed scattering models $f_s T_s + f_d T_d + f_v T_v$, where subindex $s$ is the surface component, $d$ refers to the double bounce scattering mechanism, and $v$ states for the volume one. In addition, a few restrictions should be taken in account, for instance, the three contribution parameters $f_s, f_d, f_v$, have to be real and positive, because of being physical parameters. Or that all the scattering matrices should be hermitian semidefinite, due to the definition of the coherency matrix. Therefore, the convex problem can be summarized as following
\[
\min \ \text{dif}(T, f_s T_s + f_d T_d + f_v T_v)
\]
\[
\text{subject to}
\]
\[
f_s \geq 0
\]
\[
f_d \geq 0
\]
\[
f_v \geq 0
\]
\[
T_s \succeq 0
\]
\[
T_d \succeq 0
\]
\[
T_v \succeq 0,
\]
\[
(6.1)
\]

where \( \succeq 0 \) states for the positive semidefinite positive. Notice that normally for definition, the three matrices \( T_i \), modeling surface, double bounce and volume are already hermitian semidefinite. Consequently, the problem is reduced to

\[
\min \ \text{dif}(T, f_s T_s + f_d T_d + f_v T_v)
\]
\[
\text{subject to}
\]
\[
f_s \geq 0
\]
\[
f_d \geq 0
\]
\[
f_v \geq 0
\]
\[
(6.2)
\]

The main drawback of this formulation is the quantification of the differences between two matrices, which has not an obvious answer. A point to start is to compare component to component, and minimize their difference. Therefore, as example, the simplest Freeman decomposition (Section 2.4.1) has been chosen to follow the rest of the mathematical approach. Substituting the specific matrix models and by checking component to component equalities, all the unknown parameters can be grouped by the following four equations

\[
\frac{1}{2} \langle |S_{HH} + S_{VV}|^2 \rangle = |\alpha|^2 f_d + f_s + \frac{f_v}{2}
\]
\[
(6.3)
\]
\[
\frac{1}{2} \langle |S_{HH} - S_{VV}|^2 \rangle = |\beta|^2 f_s + f_d + \frac{f_v}{4}
\]
\[
(6.4)
\]
\[
\frac{1}{2} \langle (S_{HH} - S_{VV})(S_{HH} + S_{VV})^* \rangle = \alpha^* f_d + \beta f_s
\]
\[
(6.5)
\]
\[
2 \langle |S_{HV}|^2 \rangle = \frac{f_v}{4}.
\]
\[
(6.6)
\]
In this manner, volume contribution can be directly retrieved from $\langle |S_{HV}|^2 \rangle$, leaving an underdetermined inversion problem of three equations and four unknown parameters

$$\frac{1}{2} \langle |S_{HH} + S_{VV}|^2 \rangle - 4 \langle |S_{HV}|^2 \rangle = |\alpha|^2 f_d + f_s$$  \hspace{1cm} (6.7)

$$\frac{1}{2} \langle |S_{HH} - S_{VV}|^2 \rangle - 2 \langle |S_{HV}|^2 \rangle = |\beta|^2 f_s + f_d$$  \hspace{1cm} (6.8)

$$\frac{1}{2} \langle (S_{HH} - S_{VV})(S_{HH} + S_{VV})^* \rangle = \alpha^* f_d + \beta f_s.$$  \hspace{1cm} (6.9)

In order to deal only with real numbers, the last equation has been split in two, having

$$\text{Real}(\frac{1}{2} \langle (S_{HH} - S_{VV})(S_{HH} + S_{VV})^* \rangle) = \text{real}(\alpha^*) f_d + \text{real}(\beta) f_s$$  \hspace{1cm} (6.10)

$$\text{Imag}(\frac{1}{2} \langle (S_{HH} - S_{VV})(S_{HH} + S_{VV})^* \rangle) = \text{imag}(\alpha^*) f_d + \text{imag}(\beta) f_s.$$  \hspace{1cm} (6.11)

Hence, the original problem has been converted into an underdetermined system of four equation and five unknowns. Notice that until now the decomposition follows the same steps of the original algorithm. However, from here is where the original method starts to make suppositions, which will not be the case of convex formulation.

After a few iterations, trying several combinations, the following change of variable is employed to simply the formulation

$$x_1 = |\alpha|^2 f_d$$  \hspace{1cm} (6.12)

$$x_2 = f_s$$  \hspace{1cm} (6.13)

$$x_3 = |\beta|^2 f_s$$  \hspace{1cm} (6.14)

$$x_4 = f_d$$  \hspace{1cm} (6.15)

$$x_5 = \text{real}(\alpha^*) f_d$$  \hspace{1cm} (6.16)

$$x_6 = \text{real}(\beta) f_s$$  \hspace{1cm} (6.17)

$$x_7 = \text{imag}(\alpha^*) f_d$$  \hspace{1cm} (6.18)
\[ x_8 = \text{imag}(\beta) f_s. \]  

(6.19)

Then, the problem can be seen as a system of linear equation with quadratic constrains. Having \( x = \begin{bmatrix} x_1 & x_2 & \ldots & x_8 \end{bmatrix}^T \), the minimization problem becomes

\[
\begin{align*}
\min & \quad \text{dif}(Ax, b) \\
\text{subject to} & \quad h_i(x) = 0 \text{ for } i = 1, 2 \\
& \quad g_i(x) \leq 0 \text{ for } i = 1..4,
\end{align*}
\]

(6.20)

where

\[
A = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1
\end{bmatrix},
\]

(6.21)

and the new constrains functions are

\[
\begin{align*}
h_1(x) &= x_5^2 + x_7^2 - x_1x_4 \\
h_2(x) &= x_6^2 + x_8^2 - x_2x_3 \\
g_i(x) &= -x_i \text{ for } i = 1..4.
\end{align*}
\]

(6.22) \quad (6.23) \quad (6.24)

The advantage of this new definition is that the distance between two vectors can be easily measured by the norm of the difference. Therefore, the final proposal of the optimization problem is

\[
\begin{align*}
\min & \quad \|Ax - b\|_2^2 \\
\text{subject to} & \quad h_i(x) = 0 \text{ for } i = 1, 2 \\
& \quad g_i(x) \leq 0 \text{ for } i = 1..4,
\end{align*}
\]

(6.25)

where \( \| \cdot \|_2 \) represents norm operation with base 2.
Actually, the cost function, as well as the equality constraints are quadratic functions, because they can be expressed as

\[ ||Ax - b||^2_2 = (Ax - b)^T(Ax - b) = \]
\[ = x^TA^TAx - 2b^TAx + b^Tb = \]
\[ = x^TP_0x + q_0^Tx + r_0 \]  

(6.26)

\[ h_1(x) = x^TP_1x \]  

(6.27)

\[ h_2(x) = x^TP_2x, \]  

(6.28)

where \( P_0 = A^TA \), \( q_0^T = -2b^TA \), \( r_0 = b^Tb \), and

\[
P_1 = \begin{bmatrix} 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]  

(6.29)

\[
P_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}. \]  

(6.30)

It is worth to mention that \( P_0 \) is positive semidefinite, which implies that the cost function is convex, so there is a minimum to the problem to find.

With the new notations, the problem can be rewritten as
\[
\min x^T P_0 x + q_0^T x + r_0 \\
\text{subject to}\\
x^T P_i x = 0 \text{ for } i = 1, 2\\
g_i(x) \leq 0 \text{ for } i = 1..4.
\] (6.31)

It results a typical quadratically constrained quadratic program (QCQP). However, as the constrains \( P_i \) for \( i = 1, 2 \) has negative and positive eigenvalues, they are not positive semidefinite neither negative semidefinite, leading to the whole problem become a non-convex approach.

### 6.2.1 Nonconvex QCQP Solution

As the formulated problem is not completely convex, the best choice is to find a lower bound through its dual. By definition, the dual problem always fulfills the convex condition, in consequence, it can be easily introduced to the existing cvx programs [81] and be solved.

First of all, the Lagrange function should be computed

\[
\mathcal{L}(x, \lambda, \mu) = x^T P_0 x + q_0^T x + r_0 + \sum_{i=1}^{2} \lambda_i x^T P_i x - \sum_{i=1}^{4} \mu_i x_i = \\
= x^T P x + q^T x + r_0,
\] (6.32)

where \( P = P_0 + \sum_{i=1}^{2} \lambda_i P_i \), \( q^T = q_0^T - \mu^T D \), being

\[
D = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}.
\] (6.33)

Hence, the dual function is calculated as

\[
g(\lambda, \mu) = \min_x \mathcal{L}(x, \lambda, \mu).
\] (6.34)

To evaluate the minimum of the Equation 6.34, the gradient of the Lagrange function is computed

\[
\frac{\partial \mathcal{L}(x, \lambda, \mu)}{\partial x^T} = \frac{\partial x^T P x + x^T q + r_0}{\partial x^T} = 2P x + q.
\] (6.35)

equaling the gradient to zero, the expression of \( x \) that minimizes the Lagrange function can be calculated
6.3. CONCLUSIONS

\[
\frac{\partial L(x, \lambda, \mu)}{\partial x^T} = 0 \implies x = -\frac{1}{2} P^{-1} q \quad \text{if } P \succeq 0. \tag{6.36}
\]

Finally, by substituting the expression of the \( x \) into the Lagrange function, the form of the dual function shows up

\[
g(\lambda, \mu) = \begin{cases} 
  r_0 - \frac{1}{4} q^T P^{-1} q & \text{if } P \succeq 0 \\
  -\infty & \text{otherwise.}
\end{cases} \tag{6.37}
\]

Once the dual function is calculated, using the Schur complement \([81]\), a semidefinite programming can be achieved to solve the dual problem

\[
\begin{align*}
\max & \quad \gamma + r_0 \\
\text{subject to} & \\
\begin{bmatrix} P & q \\ q^T & -\gamma \end{bmatrix} & \succeq 0 \\
\mu_i & \geq 0 \quad \text{for } i = 1..4.
\end{align*} \tag{6.38}
\]

This last expression can be easily introduced to the Matlab software package provided by \([81]\) to find the solution.

6.3 Conclusions

The need of providing optimal estimation of the target decomposition parameters, once the different scattering models are defined, is essential for the adequate characterization of the studied media. The methods in the literature tends make suppositions based on observations that are not accurate enough or iterative methods that do not guarantee the optimal solution. Here, we have provided a possible solution using Convex Optimization tool, describing the methodological steps to follow. Despite the example was done using the simplest case, it can be easily extended to more general coherency matrix models.
Chapter 7

Conclusions

This study has addressed snow monitoring based on C-Band, multi-temporal PolSAR data. As shown, snow scattering at C-Band is primary due to surface scattering and secondary to volume scattering. C-Band PolSAR data present sensitivity to the different snow conditions, that is, dry, wet and absence of snow, as the results on real Radarsat-2 data point out to the hypothesis that the presence of snow increases the contribution of surface respect to volume scattering, leading to a lower Entropy and $\bar{\pi}$. The largest contribution of surface scattering to the total scattering would suggest the possibility that the sensitivity to the snow condition is related with the surface type. Nevertheless, due to penetration at C-Band, volume scattering can not be neglected, as well as to determine the contribution of the snow/soil interface.

Despite the results with Radarsat-2 PolSAR data allow to extract the previous conclusions, one must be aware that they correspond to a case-study. On the one hand, the generalization of the previous observations should take into account incidence angle dependency. On the other hand, more complex snow scenarios should be considered, specially a deeper snowpack, and determining the impact of other snow parameters as layering or snow roughness, as it is observed from the data. Finally, the type of forest may also affect the double bounce contribution, which means the need to have wider dataset and ground truth information.

For snow qualitative estimation, dry snow mapping was achieved by using the dependency of multi-time C-Band PolSAR data on the snow conditions, together with height, temperature and changes information. The comparison against MODIS data shows the feasibility of the proposed approach, but with a finer spatial resolution than the optical approach. Nevertheless, for SAR data, a combination of data taken at different incidence angles or acquired at ascending and descending passes is necessary to overcome the limitations due to layover and shadowing.
The snow quantitative estimation may be possible if a proper scattering matrix model for snowed environments is defined, plus an adequate target decomposition theorem to estimate its parameters. The two components involved in this new decomposition should be based on the combination of the surface and volume scattering mechanisms. We have shown that the parameters estimation step can be carried out using convex optimization processes.

Concerning future lines of work, they should be addressed to define surface and volume matrix scattering models for snowed environments, considering various complex snow scenarios. In addition, the convex optimization approach should be extended to estimate the parameters of these new models, which should be studied and validated for widespread of data and ground truth.

Finally, it is worth to indicate an important limitation that this study had to face during all its development. As indicated, the study of snow parameters retrieval should be considered in the frame of multi time acquisition. In this case, the thesis had only three Radarsat-2 images, so a more in-depth analysis of this problem should need a more dense time series of SAR images during one year with incidence angle diversity and different complex snow scenarios, to be able to extract more solid conclusions. In addition, the two images that were acquired during the winter had the problem that the snow cover was very limited in the area of study, making complex the extraction of snow parameters.
Appendices
Table A.1: Snow survey on 16<sup>th</sup> – 17<sup>th</sup> February 2011.

<table>
<thead>
<tr>
<th>Reference</th>
<th>UTM Coord. X (m)</th>
<th>UTM Coord. Y (m)</th>
<th>Altitude (m)</th>
<th>Snow depth (cm)</th>
<th>Density (Kg/m&lt;sup&gt;3&lt;/sup&gt;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D-1</td>
<td>333473</td>
<td>4732817</td>
<td>1843</td>
<td>33</td>
<td>-</td>
</tr>
<tr>
<td>D-2</td>
<td>333080</td>
<td>4733053</td>
<td>1838</td>
<td>21</td>
<td>233</td>
</tr>
<tr>
<td>D-3</td>
<td>333725</td>
<td>4733137</td>
<td>1839</td>
<td>20</td>
<td>-</td>
</tr>
<tr>
<td>D-4</td>
<td>333725</td>
<td>47331209</td>
<td>1830</td>
<td>23</td>
<td>327</td>
</tr>
<tr>
<td>D-5</td>
<td>333792</td>
<td>4733307</td>
<td>1835</td>
<td>20</td>
<td>-</td>
</tr>
<tr>
<td>D-6</td>
<td>333846</td>
<td>4733408</td>
<td>1831</td>
<td>30</td>
<td>-</td>
</tr>
<tr>
<td>D-7</td>
<td>333938</td>
<td>4733456</td>
<td>1837</td>
<td>42</td>
<td>-</td>
</tr>
<tr>
<td>D-8</td>
<td>334047</td>
<td>4733596</td>
<td>1826</td>
<td>38</td>
<td>258</td>
</tr>
<tr>
<td>D-9</td>
<td>334089</td>
<td>4733712</td>
<td>1818</td>
<td>33</td>
<td>-</td>
</tr>
<tr>
<td>D-10</td>
<td>334157</td>
<td>4733774</td>
<td>1812</td>
<td>30</td>
<td>-</td>
</tr>
<tr>
<td>D-11</td>
<td>334208</td>
<td>4733885</td>
<td>1813</td>
<td>18</td>
<td>-</td>
</tr>
<tr>
<td>D-12</td>
<td>334322</td>
<td>4733977</td>
<td>1809</td>
<td>19</td>
<td>-</td>
</tr>
<tr>
<td>D-13</td>
<td>334375</td>
<td>4733984</td>
<td>1820</td>
<td>30</td>
<td>184</td>
</tr>
<tr>
<td>D-14</td>
<td>334450</td>
<td>4734246</td>
<td>1803</td>
<td>40</td>
<td>-</td>
</tr>
<tr>
<td>D-15</td>
<td>334542</td>
<td>4734358</td>
<td>1799</td>
<td>40</td>
<td>321</td>
</tr>
<tr>
<td>D-16</td>
<td>334417</td>
<td>4734390</td>
<td>1782</td>
<td>30</td>
<td>-</td>
</tr>
<tr>
<td>D-17</td>
<td>334340</td>
<td>4734992</td>
<td>1779</td>
<td>25</td>
<td>-</td>
</tr>
<tr>
<td>D-18</td>
<td>334212</td>
<td>4734376</td>
<td>1801</td>
<td>22</td>
<td>-</td>
</tr>
<tr>
<td>D-19</td>
<td>334148</td>
<td>4734269</td>
<td>1812</td>
<td>18</td>
<td>204</td>
</tr>
<tr>
<td>D-20</td>
<td>334080</td>
<td>4734171</td>
<td>1831</td>
<td>50</td>
<td>184</td>
</tr>
<tr>
<td>D-21</td>
<td>333955</td>
<td>4734008</td>
<td>1828</td>
<td>13</td>
<td>-</td>
</tr>
<tr>
<td>D-22</td>
<td>333877</td>
<td>4733915</td>
<td>1826</td>
<td>15</td>
<td>-</td>
</tr>
<tr>
<td>D-23</td>
<td>333778</td>
<td>4733852</td>
<td>1832</td>
<td>15</td>
<td>-</td>
</tr>
<tr>
<td>D-24</td>
<td>333658</td>
<td>4733680</td>
<td>1847</td>
<td>13</td>
<td>-</td>
</tr>
<tr>
<td>D-25</td>
<td>333502</td>
<td>4733502</td>
<td>1842</td>
<td>34</td>
<td>180</td>
</tr>
<tr>
<td>D-26</td>
<td>333451</td>
<td>4733377</td>
<td>1848</td>
<td>13</td>
<td>-</td>
</tr>
<tr>
<td>D-27</td>
<td>333391</td>
<td>4733071</td>
<td>1836</td>
<td>28</td>
<td>-</td>
</tr>
<tr>
<td>D-28</td>
<td>332849</td>
<td>4732061</td>
<td>1852</td>
<td>40</td>
<td>-</td>
</tr>
<tr>
<td>D-29</td>
<td>332866</td>
<td>4732038</td>
<td>1853</td>
<td>38</td>
<td>-</td>
</tr>
<tr>
<td>D-30</td>
<td>332594</td>
<td>4731737</td>
<td>1869</td>
<td>40</td>
<td>-</td>
</tr>
<tr>
<td>D-31</td>
<td>332643</td>
<td>4731701</td>
<td>1864</td>
<td>25</td>
<td>-</td>
</tr>
<tr>
<td>D-32</td>
<td>332393</td>
<td>4731452</td>
<td>1866</td>
<td>9</td>
<td>-</td>
</tr>
<tr>
<td>D-33</td>
<td>332453</td>
<td>4731096</td>
<td>1870</td>
<td>40</td>
<td>-</td>
</tr>
</tbody>
</table>
Table A.2: Snow survey on 16th – 18th Mars 2011.

<table>
<thead>
<tr>
<th>Reference</th>
<th>UTM Coord. X (m)</th>
<th>UTM Coord. Y (m)</th>
<th>Altitude (m)</th>
<th>Snow depth (cm)</th>
<th>Density (Kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W-1</td>
<td>333458</td>
<td>4732778</td>
<td>1842 m</td>
<td>60</td>
<td>388</td>
</tr>
<tr>
<td>W-2</td>
<td>333670</td>
<td>4733037</td>
<td>1842 m</td>
<td>50</td>
<td>465</td>
</tr>
<tr>
<td>W-3</td>
<td>333735</td>
<td>4733131</td>
<td>1837 m</td>
<td>72</td>
<td>357</td>
</tr>
<tr>
<td>W-4</td>
<td>333825</td>
<td>4733234</td>
<td>1841 m</td>
<td>55</td>
<td>312</td>
</tr>
<tr>
<td>W-5</td>
<td>333876</td>
<td>4733298</td>
<td>1841 m</td>
<td>65</td>
<td>-</td>
</tr>
<tr>
<td>W-6</td>
<td>334015</td>
<td>4733540</td>
<td>1833 m</td>
<td>76</td>
<td>387</td>
</tr>
<tr>
<td>W-7</td>
<td>334047</td>
<td>4733604</td>
<td>1829 m</td>
<td>85</td>
<td>-</td>
</tr>
<tr>
<td>W-8</td>
<td>334126</td>
<td>4733712</td>
<td>1827 m</td>
<td>60</td>
<td>-</td>
</tr>
<tr>
<td>W-9</td>
<td>334158</td>
<td>4733821</td>
<td>1818 m</td>
<td>55</td>
<td>312</td>
</tr>
<tr>
<td>W-10</td>
<td>334210</td>
<td>4733912</td>
<td>1813 m</td>
<td>65</td>
<td>358</td>
</tr>
<tr>
<td>W-11</td>
<td>334321</td>
<td>4733985</td>
<td>1814 m</td>
<td>45</td>
<td>-</td>
</tr>
<tr>
<td>W-12</td>
<td>334369</td>
<td>4734022</td>
<td>1824 m</td>
<td>35</td>
<td>280</td>
</tr>
<tr>
<td>W-13</td>
<td>334458</td>
<td>4734254</td>
<td>1802 m</td>
<td>60</td>
<td>184</td>
</tr>
<tr>
<td>W-14</td>
<td>334522</td>
<td>4734397</td>
<td>1806 m</td>
<td>45</td>
<td>190</td>
</tr>
<tr>
<td>W-15</td>
<td>334562</td>
<td>4734523</td>
<td>1801 m</td>
<td>50</td>
<td>321</td>
</tr>
<tr>
<td>W-16</td>
<td>334575</td>
<td>4734715</td>
<td>1794 m</td>
<td>20</td>
<td>245</td>
</tr>
<tr>
<td>W-17</td>
<td>334450</td>
<td>4734377</td>
<td>1788 m</td>
<td>55</td>
<td>323</td>
</tr>
<tr>
<td>W-18</td>
<td>334421</td>
<td>4734377</td>
<td>1785 m</td>
<td>75</td>
<td>-</td>
</tr>
<tr>
<td>W-19</td>
<td>334358</td>
<td>4734400</td>
<td>1782 m</td>
<td>60</td>
<td>367</td>
</tr>
<tr>
<td>W-20</td>
<td>334219</td>
<td>4734369</td>
<td>1808 m</td>
<td>46</td>
<td>184</td>
</tr>
<tr>
<td>W-21</td>
<td>334134</td>
<td>4734263</td>
<td>1815 m</td>
<td>50</td>
<td>269</td>
</tr>
<tr>
<td>W-22</td>
<td>334047</td>
<td>4734135</td>
<td>1818 m</td>
<td>40</td>
<td>-</td>
</tr>
<tr>
<td>W-23</td>
<td>333954</td>
<td>4734000</td>
<td>1825 m</td>
<td>30</td>
<td>-</td>
</tr>
<tr>
<td>W-24</td>
<td>333850</td>
<td>4733885</td>
<td>1833 m</td>
<td>55</td>
<td>245</td>
</tr>
<tr>
<td>W-25</td>
<td>333745</td>
<td>4733772</td>
<td>1840 m</td>
<td>50</td>
<td>-</td>
</tr>
<tr>
<td>W-26</td>
<td>333666</td>
<td>4733680</td>
<td>1846 m</td>
<td>47</td>
<td>287</td>
</tr>
<tr>
<td>W-27</td>
<td>333519</td>
<td>4733489</td>
<td>1849 m</td>
<td>50</td>
<td>-</td>
</tr>
<tr>
<td>W-28</td>
<td>333465</td>
<td>4733554</td>
<td>1852 m</td>
<td>60</td>
<td>-</td>
</tr>
<tr>
<td>W-29</td>
<td>333343</td>
<td>4733070</td>
<td>1853 m</td>
<td>56</td>
<td>-</td>
</tr>
<tr>
<td>W-30</td>
<td>332841</td>
<td>4732068</td>
<td>1866 m</td>
<td>90</td>
<td>-</td>
</tr>
<tr>
<td>W-31</td>
<td>332808</td>
<td>4732069</td>
<td>1869 m</td>
<td>60</td>
<td>-</td>
</tr>
<tr>
<td>W-32</td>
<td>332887</td>
<td>4732040</td>
<td>1855 m</td>
<td>81</td>
<td>-</td>
</tr>
<tr>
<td>W-33</td>
<td>332915</td>
<td>4732029</td>
<td>1851 m</td>
<td>95</td>
<td>-</td>
</tr>
</tbody>
</table>
Table A.3: Diagonal elements of the correlation and coherence matrix.


Table A.4: Freeman decomposition for the six test areas.

<table>
<thead>
<tr>
<th>Area</th>
<th>Dry snow</th>
<th>Wet snow</th>
<th>Snow free</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Contribution(dB)</td>
<td>Contribution(%)</td>
<td>∆Comp(dB)</td>
</tr>
<tr>
<td></td>
<td>Surface</td>
<td>Volume</td>
<td>Double bounce</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>57.46</td>
<td>60.80</td>
<td>49.06</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>67.74</td>
<td>68.89</td>
<td>51.23</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>61.53</td>
<td>62.48</td>
<td>49.34</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>59.33</td>
<td>59.33</td>
<td>49.49</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>68.99</td>
<td>68.99</td>
<td>68.21</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


Table A.5: Yamaguchi decomposition for the six test areas.

<table>
<thead>
<tr>
<th>Area</th>
<th>Dry snow</th>
<th>Wet snow</th>
<th>Snow free</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Surface</th>
<th>Contribution(dB)</th>
<th>Contribution(%)</th>
<th>∆Comp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Double bounce</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Helix</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPAN</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Area</th>
<th>Contribution(dB)</th>
<th>Contribution(%)</th>
<th>∆Comp</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Surface</th>
<th>Contribution(dB)</th>
<th>Contribution(%)</th>
<th>∆Comp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Double bounce</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Helix</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPAN</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Area</th>
<th>Contribution(dB)</th>
<th>Contribution(%)</th>
<th>∆Comp</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Surface</th>
<th>Contribution(dB)</th>
<th>Contribution(%)</th>
<th>∆Comp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Double bounce</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Helix</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPAN</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Area</th>
<th>Contribution(dB)</th>
<th>Contribution(%)</th>
<th>∆Comp</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Surface</th>
<th>Contribution(dB)</th>
<th>Contribution(%)</th>
<th>∆Comp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Double bounce</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Helix</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SPAN</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table A.6: Average eigenvalues for the six test areas.

<table>
<thead>
<tr>
<th>Area 1</th>
<th></th>
<th></th>
<th></th>
<th>Area 2</th>
<th></th>
<th></th>
<th></th>
<th>Area 3</th>
<th></th>
<th></th>
<th></th>
<th>Area 4</th>
<th></th>
<th></th>
<th></th>
<th>Area 5</th>
<th></th>
<th></th>
<th></th>
<th>Area 6</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dry snow</td>
<td></td>
<td></td>
<td></td>
<td>Wet snow</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Snow free</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Area</td>
<td>Average(dB)</td>
<td>Average(%)</td>
<td>∆(Dry,Free)</td>
<td>Area</td>
<td>Average(dB)</td>
<td>Average(%)</td>
<td>∆(Dry,Free)</td>
<td>Area</td>
<td>Average(dB)</td>
<td>Average(%)</td>
<td>∆(Dry,Free)</td>
<td>Area</td>
<td>Average(dB)</td>
<td>Average(%)</td>
<td>∆(Dry,Free)</td>
<td>Area</td>
<td>Average(dB)</td>
<td>Average(%)</td>
<td>∆(Dry,Free)</td>
<td>Area</td>
<td>Average(dB)</td>
<td>Average(%)</td>
</tr>
<tr>
<td>L1</td>
<td></td>
<td>59.04</td>
<td>72.50</td>
<td>7.22</td>
<td></td>
<td>54.21</td>
<td>71.88</td>
<td>0.61</td>
<td></td>
<td>58.61</td>
<td>65.37</td>
<td>-6.51</td>
<td></td>
<td>54.09</td>
<td>25.86</td>
<td>2.44</td>
<td></td>
<td>55.78</td>
<td>33.56</td>
<td>4.05</td>
<td></td>
<td>56.74</td>
<td>4.71</td>
</tr>
<tr>
<td>L2</td>
<td></td>
<td>72.07</td>
<td>17.52</td>
<td>-5.51</td>
<td></td>
<td>48.84</td>
<td>16.09</td>
<td>-1.19</td>
<td></td>
<td>67.01</td>
<td>18.63</td>
<td>-0.66</td>
<td></td>
<td>47.82</td>
<td>15.93</td>
<td>-2.03</td>
<td></td>
<td>67.67</td>
<td>18.32</td>
<td>-0.35</td>
<td></td>
<td>66.47</td>
<td>18.45</td>
</tr>
<tr>
<td>L3</td>
<td></td>
<td>70.43</td>
<td>9.58</td>
<td>-3.58</td>
<td></td>
<td>45.43</td>
<td>9.52</td>
<td>0.47</td>
<td></td>
<td>51.78</td>
<td>33.56</td>
<td>4.05</td>
<td></td>
<td>45.56</td>
<td>33.56</td>
<td>0.00</td>
<td></td>
<td>51.60</td>
<td>33.56</td>
<td>0.04</td>
<td></td>
<td>52.60</td>
<td>33.56</td>
</tr>
<tr>
<td>SPAN</td>
<td></td>
<td>60.44</td>
<td>100.00</td>
<td></td>
<td></td>
<td>50.65</td>
<td></td>
<td></td>
<td></td>
<td>60.45</td>
<td></td>
<td></td>
<td></td>
<td>60.45</td>
<td></td>
<td></td>
<td></td>
<td>60.45</td>
<td></td>
<td></td>
<td></td>
<td>60.45</td>
<td></td>
</tr>
</tbody>
</table>

Area 1
L1 67.97 69.37 8.28 L1 67.80 62.45 6.92 L1 67.33 61.09 -1.36 L1 67.33 61.09 -1.36
L2 62.21 18.39 -4.55 L2 63.35 22.41 -4.02 L2 63.07 22.93 0.52 L2 63.07 22.93 0.52
L3 55.69 12.84 -1.95 L3 54.53 14.41 -2.90 L3 54.81 14.79 0.84 L3 54.81 14.79 0.84
SPAN 69.56 100.00 SPAN 69.84 SPAN 69.47 SPAN 69.47

Area 2
L1 60.69 64.73 5.46 L1 60.72 59.85 4.88 L1 60.84 59.26 -0.58 L1 60.84 59.26 -0.58
L2 56.99 22.43 -3.51 L2 57.05 25.77 -3.31 L2 57.25 25.95 0.90 L2 57.25 25.95 0.90
L3 53.66 12.84 -3.73 L3 54.53 14.41 -2.90 L3 54.81 14.79 0.84 L3 54.81 14.79 0.84
SPAN 62.58 100.00 SPAN 62.95 SPAN 63.11 SPAN 63.11

Area 3
L1 59.04 72.50 7.22 L1 54.21 71.88 0.61 L1 58.61 65.37 -6.51 L1 56.61 65.37 -6.51
L2 72.07 17.52 -5.51 L2 48.84 16.09 -1.19 L2 67.01 18.63 -0.66 L2 67.01 18.63 -0.66
L3 70.43 9.58 -3.58 L3 45.43 9.52 0.47 L3 51.78 33.56 4.05 L3 51.78 33.56 4.05
SPAN 60.44 100.00 SPAN 50.65 SPAN 60.45 SPAN 60.45

Area 4
L1 59.04 72.50 7.22 L1 54.21 71.88 0.61 L1 58.61 65.37 -6.51 L1 56.61 65.37 -6.51
L2 72.07 17.52 -5.51 L2 48.84 16.09 -1.19 L2 67.01 18.63 -0.66 L2 67.01 18.63 -0.66
L3 70.43 9.58 -3.58 L3 45.43 9.52 0.47 L3 51.78 33.56 4.05 L3 51.78 33.56 4.05
SPAN 60.44 100.00 SPAN 50.65 SPAN 60.45 SPAN 60.45

Area 5
L1 59.04 72.50 7.22 L1 54.21 71.88 0.61 L1 58.61 65.37 -6.51 L1 56.61 65.37 -6.51
L2 72.07 17.52 -5.51 L2 48.84 16.09 -1.19 L2 67.01 18.63 -0.66 L2 67.01 18.63 -0.66
L3 70.43 9.58 -3.58 L3 45.43 9.52 0.47 L3 51.78 33.56 4.05 L3 51.78 33.56 4.05
SPAN 60.44 100.00 SPAN 50.65 SPAN 60.45 SPAN 60.45

Area 6
L1 72.16 78.40 1.38 L1 71.61 77.63 0.77 L1 70.86 76.88 3.64 L1 70.86 76.88 3.64
L2 65.65 17.46 -0.89 L2 65.25 17.56 -0.30 L2 64.07 18.45 0.64 L2 64.07 18.45 0.64
L3 59.40 4.14 -6.57 L3 59.18 4.43 -2.97 L3 58.74 4.71 0.28 L3 58.74 4.71 0.28
SPAN 73.63 100.00 SPAN 72.71 SPAN 72.01 SPAN 72.01
Table A.7: Average and variance of α angles for the six test areas.

<table>
<thead>
<tr>
<th>Area</th>
<th>Average (°)</th>
<th>Variance (°)</th>
<th>Average (°)</th>
<th>Variance (°)</th>
<th>Average (°)</th>
<th>Variance (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8.43</td>
<td>15.37</td>
<td>5.66</td>
<td>14.52</td>
<td>10.27</td>
<td>32.95</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>82.46</td>
<td>14.56</td>
<td>85.39</td>
<td>13.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>81.38</td>
<td>30.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>87.02</td>
<td>5.61</td>
<td>87.36</td>
<td>4.82</td>
<td>85.41</td>
<td>11.22</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td>87.02</td>
<td>102.25</td>
<td>25.05</td>
<td>167.73</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>24.61</td>
<td>181.48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>9.89</td>
<td>34.48</td>
<td>10.39</td>
<td>26.07</td>
<td>7.18</td>
<td>25.37</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td>81.98</td>
<td>28.05</td>
<td>80.44</td>
<td>25.94</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>84.99</td>
<td>21.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>19.04</td>
<td>102.25</td>
<td>25.05</td>
<td>167.73</td>
<td>24.61</td>
<td>181.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>9.89</td>
<td>34.48</td>
<td>10.39</td>
<td>26.07</td>
<td>7.18</td>
<td>25.37</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td>81.98</td>
<td>28.05</td>
<td>80.44</td>
<td>25.94</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>84.99</td>
<td>21.33</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>19.04</td>
<td>102.25</td>
<td>25.05</td>
<td>167.73</td>
<td>24.61</td>
<td>181.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>19.04</td>
<td>102.25</td>
<td>25.05</td>
<td>167.73</td>
<td>24.61</td>
<td>181.48</td>
</tr>
</tbody>
</table>
Table A.8: Scattering mechanism identification for the six test areas.

<table>
<thead>
<tr>
<th>Area</th>
<th>Dry snow</th>
<th>Wet snow</th>
<th>Snow free</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average(dB) 1-2,2-1,1-1</td>
<td>Average(dB) 1-2,2-1,1-1</td>
<td>Average(dB) 1-2,2-1,1-1</td>
</tr>
<tr>
<td></td>
<td>1) A0</td>
<td>2) B0+B</td>
<td>3) B0-B</td>
</tr>
<tr>
<td>Area 1</td>
<td>111.817</td>
<td>110.759</td>
<td>110.759</td>
</tr>
<tr>
<td></td>
<td>27.07</td>
<td>23.70</td>
<td>23.70</td>
</tr>
<tr>
<td>Area 2</td>
<td>129.679</td>
<td>84.746</td>
<td>73.783</td>
</tr>
<tr>
<td></td>
<td>31.90</td>
<td>10.96</td>
<td>38.03</td>
</tr>
<tr>
<td>Area 3</td>
<td>114.178</td>
<td>73.783</td>
<td>99.976</td>
</tr>
<tr>
<td></td>
<td>15.57</td>
<td>38.03</td>
<td>29.70</td>
</tr>
<tr>
<td>Area 4</td>
<td>121.785</td>
<td>98.612</td>
<td>96.041</td>
</tr>
<tr>
<td></td>
<td>25.88</td>
<td>2.57</td>
<td>18.14</td>
</tr>
<tr>
<td>Area 5</td>
<td>112.972</td>
<td>95.001</td>
<td>95.346</td>
</tr>
<tr>
<td></td>
<td>19.35</td>
<td>0.35</td>
<td>26.24</td>
</tr>
<tr>
<td>Area 6</td>
<td>132.454</td>
<td>93.622</td>
<td>95.770</td>
</tr>
<tr>
<td></td>
<td>-4.48</td>
<td>7.85</td>
<td>27.20</td>
</tr>
</tbody>
</table>
Table A.9: Coherency matrix $T$ from the eigen-decomposition areas 1 to 3.

<table>
<thead>
<tr>
<th></th>
<th>Area 1</th>
<th>Area 2</th>
<th>Area 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dry</strong> snow</td>
<td>0.975</td>
<td>0.021</td>
<td>0.004</td>
</tr>
<tr>
<td>Wet snow</td>
<td>0.000</td>
<td>1.000</td>
<td>0.014</td>
</tr>
<tr>
<td>Snow Free</td>
<td>0.059</td>
<td>0.054</td>
<td>0.569</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Area 1</th>
<th>Area 2</th>
<th>Area 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dry</strong> snow</td>
<td>0.021</td>
<td>0.017</td>
<td>0.041</td>
</tr>
<tr>
<td>Wet snow</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Snow Free</td>
<td>0.059</td>
<td>0.054</td>
<td>0.569</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Area 1</th>
<th>Area 2</th>
<th>Area 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dry</strong> snow</td>
<td>0.004</td>
<td>0.008</td>
<td>0.003</td>
</tr>
<tr>
<td>Wet snow</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Snow Free</td>
<td>0.059</td>
<td>0.054</td>
<td>0.569</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Area 1</th>
<th>Area 2</th>
<th>Area 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dry</strong> snow</td>
<td>0.004</td>
<td>0.008</td>
<td>0.003</td>
</tr>
<tr>
<td>Wet snow</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Snow Free</td>
<td>0.059</td>
<td>0.054</td>
<td>0.569</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Area 1</th>
<th>Area 2</th>
<th>Area 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dry</strong> snow</td>
<td>0.004</td>
<td>0.008</td>
<td>0.003</td>
</tr>
<tr>
<td>Wet snow</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Snow Free</td>
<td>0.059</td>
<td>0.054</td>
<td>0.569</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Area 1</th>
<th>Area 2</th>
<th>Area 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dry</strong> snow</td>
<td>0.004</td>
<td>0.008</td>
<td>0.003</td>
</tr>
<tr>
<td>Wet snow</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Snow Free</td>
<td>0.059</td>
<td>0.054</td>
<td>0.569</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Area 1</th>
<th>Area 2</th>
<th>Area 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dry</strong> snow</td>
<td>0.004</td>
<td>0.008</td>
<td>0.003</td>
</tr>
<tr>
<td>Wet snow</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Snow Free</td>
<td>0.059</td>
<td>0.054</td>
<td>0.569</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Area 1</th>
<th>Area 2</th>
<th>Area 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dry</strong> snow</td>
<td>0.004</td>
<td>0.008</td>
<td>0.003</td>
</tr>
<tr>
<td>Wet snow</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Snow Free</td>
<td>0.059</td>
<td>0.054</td>
<td>0.569</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Area 1</th>
<th>Area 2</th>
<th>Area 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dry</strong> snow</td>
<td>0.004</td>
<td>0.008</td>
<td>0.003</td>
</tr>
<tr>
<td>Wet snow</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Snow Free</td>
<td>0.059</td>
<td>0.054</td>
<td>0.569</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Area 1</th>
<th>Area 2</th>
<th>Area 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dry</strong> snow</td>
<td>0.004</td>
<td>0.008</td>
<td>0.003</td>
</tr>
<tr>
<td>Wet snow</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Snow Free</td>
<td>0.059</td>
<td>0.054</td>
<td>0.569</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Area 1</th>
<th>Area 2</th>
<th>Area 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dry</strong> snow</td>
<td>0.004</td>
<td>0.008</td>
<td>0.003</td>
</tr>
<tr>
<td>Wet snow</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Snow Free</td>
<td>0.059</td>
<td>0.054</td>
<td>0.569</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Area 1</th>
<th>Area 2</th>
<th>Area 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dry</strong> snow</td>
<td>0.004</td>
<td>0.008</td>
<td>0.003</td>
</tr>
<tr>
<td>Wet snow</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Snow Free</td>
<td>0.059</td>
<td>0.054</td>
<td>0.569</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Area 1</th>
<th>Area 2</th>
<th>Area 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dry</strong> snow</td>
<td>0.004</td>
<td>0.008</td>
<td>0.003</td>
</tr>
<tr>
<td>Wet snow</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Snow Free</td>
<td>0.059</td>
<td>0.054</td>
<td>0.569</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Area 1</th>
<th>Area 2</th>
<th>Area 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dry</strong> snow</td>
<td>0.004</td>
<td>0.008</td>
<td>0.003</td>
</tr>
<tr>
<td>Wet snow</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Snow Free</td>
<td>0.059</td>
<td>0.054</td>
<td>0.569</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Area 1</th>
<th>Area 2</th>
<th>Area 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dry</strong> snow</td>
<td>0.004</td>
<td>0.008</td>
<td>0.003</td>
</tr>
<tr>
<td>Wet snow</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Snow Free</td>
<td>0.059</td>
<td>0.054</td>
<td>0.569</td>
</tr>
<tr>
<td>Areas</td>
<td>Dry snow</td>
<td>Wet snow</td>
<td>Snow Free</td>
</tr>
<tr>
<td>-------</td>
<td>----------</td>
<td>----------</td>
<td>-----------</td>
</tr>
<tr>
<td>T1</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>T2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>T3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>T4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>T5</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>T6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A.10: Coherency matrix \( T_i \) from the eigen-decomposition areas 4 to 6.
Table A.11: \( \alpha \) for the six test areas.

<table>
<thead>
<tr>
<th>Area</th>
<th>Average Variance ∆(Dry,Free)</th>
<th>Average Variance ∆(Dry,Wet)</th>
<th>Average Variance ∆(Free,Wet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>H 0.70</td>
<td>0.00</td>
<td>-0.11</td>
</tr>
<tr>
<td></td>
<td>( \alpha ) (°)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>29.61</td>
<td>12.21</td>
<td>-6.77</td>
</tr>
<tr>
<td></td>
<td>Anisot</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.27</td>
<td>0.01</td>
<td>0.06</td>
</tr>
<tr>
<td>2</td>
<td>H 0.72</td>
<td>0.01</td>
<td>-0.02</td>
</tr>
<tr>
<td></td>
<td>( \alpha ) (°)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>29.57</td>
<td>22.98</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>Anisot</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.32</td>
<td>0.01</td>
<td>-0.05</td>
</tr>
<tr>
<td>3</td>
<td>H 0.81</td>
<td>0.00</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>( \alpha ) (°)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>36.38</td>
<td>16.64</td>
<td>6.80</td>
</tr>
<tr>
<td></td>
<td>Anisot</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.27</td>
<td>0.01</td>
<td>0.06</td>
</tr>
<tr>
<td>4</td>
<td>H 0.75</td>
<td>0.01</td>
<td>-0.09</td>
</tr>
<tr>
<td></td>
<td>( \alpha ) (°)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>32.13</td>
<td>35.69</td>
<td>-6.97</td>
</tr>
<tr>
<td></td>
<td>Anisot</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>5</td>
<td>H 0.84</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>( \alpha ) (°)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>39.87</td>
<td>32.53</td>
<td>-5.01</td>
</tr>
<tr>
<td></td>
<td>Anisot</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.28</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>6</td>
<td>H 0.67</td>
<td>0.01</td>
<td>-0.12</td>
</tr>
<tr>
<td></td>
<td>( \alpha ) (°)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>28.50</td>
<td>33.71</td>
<td>-5.19</td>
</tr>
<tr>
<td></td>
<td>Anisot</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.22</td>
<td>0.01</td>
<td>0.06</td>
</tr>
</tbody>
</table>
### Table A.12: Van Zyl decomposition for the six test areas.

<table>
<thead>
<tr>
<th>Area 1</th>
<th>Dry snow</th>
<th>Wet snow</th>
<th>Snow free</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Contribution(dB)</td>
<td>Contribution(%)</td>
<td>∆Comp</td>
</tr>
<tr>
<td></td>
<td>Surface</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface</td>
<td>58.32</td>
<td>64.93</td>
<td>4.77</td>
</tr>
<tr>
<td>Volume</td>
<td>53.75</td>
<td>22.68</td>
<td>4.57</td>
</tr>
<tr>
<td>Double bounce</td>
<td>51.12</td>
<td>12.39</td>
<td>2.63</td>
</tr>
<tr>
<td>SPAN</td>
<td>60.19</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Area 2</th>
<th>Dry snow</th>
<th>Wet snow</th>
<th>Snow free</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Contribution(dB)</td>
<td>Contribution(%)</td>
<td>∆Comp</td>
</tr>
<tr>
<td></td>
<td>Surface</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface</td>
<td>66.75</td>
<td>57.06</td>
<td>0.58</td>
</tr>
<tr>
<td>Volume</td>
<td>64.69</td>
<td>35.33</td>
<td>2.68</td>
</tr>
<tr>
<td>Double bounce</td>
<td>57.89</td>
<td>10.77</td>
<td>2.68</td>
</tr>
<tr>
<td>SPAN</td>
<td>69.19</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Area 3</th>
<th>Dry snow</th>
<th>Wet snow</th>
<th>Snow free</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Contribution(dB)</td>
<td>Contribution(%)</td>
<td>∆Comp</td>
</tr>
<tr>
<td></td>
<td>Surface</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface</td>
<td>58.65</td>
<td>46.95</td>
<td>0.75</td>
</tr>
<tr>
<td>Volume</td>
<td>56.87</td>
<td>38.84</td>
<td>0.78</td>
</tr>
<tr>
<td>Double bounce</td>
<td>52.53</td>
<td>10.97</td>
<td>5.84</td>
</tr>
<tr>
<td>SPAN</td>
<td>62.13</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Area 4</th>
<th>Dry snow</th>
<th>Wet snow</th>
<th>Snow free</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Contribution(dB)</td>
<td>Contribution(%)</td>
<td>∆Comp</td>
</tr>
<tr>
<td></td>
<td>Surface</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface</td>
<td>63.20</td>
<td>60.27</td>
<td>3.77</td>
</tr>
<tr>
<td>Volume</td>
<td>54.42</td>
<td>27.22</td>
<td>7.16</td>
</tr>
<tr>
<td>Double bounce</td>
<td>52.53</td>
<td>9.97</td>
<td>5.84</td>
</tr>
<tr>
<td>SPAN</td>
<td>61.12</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Area 5</th>
<th>Dry snow</th>
<th>Wet snow</th>
<th>Snow free</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Contribution(dB)</td>
<td>Contribution(%)</td>
<td>∆Comp</td>
</tr>
<tr>
<td></td>
<td>Surface</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface</td>
<td>58.63</td>
<td>55.34</td>
<td>2.87</td>
</tr>
<tr>
<td>Volume</td>
<td>56.37</td>
<td>32.15</td>
<td>2.28</td>
</tr>
<tr>
<td>Double bounce</td>
<td>52.17</td>
<td>12.54</td>
<td>4.10</td>
</tr>
<tr>
<td>SPAN</td>
<td>61.20</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Area 6</th>
<th>Dry snow</th>
<th>Wet snow</th>
<th>Snow free</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Contribution(dB)</td>
<td>Contribution(%)</td>
<td>∆Comp</td>
</tr>
<tr>
<td></td>
<td>Surface</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Volume</td>
<td>64.71</td>
<td>35.46</td>
<td>4.37</td>
</tr>
<tr>
<td>Double bounce</td>
<td>69.08</td>
<td>12.39</td>
<td>2.63</td>
</tr>
<tr>
<td>SPAN</td>
<td>72.95</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
Table A.13: Adaptive decomposition for the six test areas.

<table>
<thead>
<tr>
<th>Area</th>
<th>Surface Contribution (dB)</th>
<th>Surface Contribution (%)</th>
<th>Volume Contribution (dB)</th>
<th>Volume Contribution (%)</th>
<th>Double B Contribution (dB)</th>
<th>Double B Contribution (%)</th>
<th>SPAN Contribution (dB)</th>
<th>SPAN Contribution (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>59.10</td>
<td>63.05</td>
<td>56.38</td>
<td>33.71</td>
<td>46.21</td>
<td>3.24</td>
<td>61.11</td>
<td>100.00</td>
</tr>
<tr>
<td>2</td>
<td>67.96</td>
<td>60.03</td>
<td>67.07</td>
<td>48.32</td>
<td>55.27</td>
<td>3.23</td>
<td>70.18</td>
<td>100.00</td>
</tr>
<tr>
<td>3</td>
<td>59.90</td>
<td>48.69</td>
<td>60.27</td>
<td>49.79</td>
<td>52.36</td>
<td>8.57</td>
<td>63.03</td>
<td>100.00</td>
</tr>
<tr>
<td>4</td>
<td>64.50</td>
<td>67.08</td>
<td>60.82</td>
<td>28.79</td>
<td>52.38</td>
<td>4.13</td>
<td>66.23</td>
<td>100.00</td>
</tr>
<tr>
<td>5</td>
<td>59.59</td>
<td>56.77</td>
<td>57.99</td>
<td>47.36</td>
<td>50.11</td>
<td>6.40</td>
<td>62.05</td>
<td>100.00</td>
</tr>
<tr>
<td>6</td>
<td>74.37</td>
<td>60.57</td>
<td>71.08</td>
<td>49.22</td>
<td>71.16</td>
<td>51.79</td>
<td>76.55</td>
<td>100.00</td>
</tr>
</tbody>
</table>
Bibliography


[38] G.Venkataraman, Gulab Singh and Y.Yamaguchi, *Fully Polarimetric ALOS PALSAR Data Application for Snow and Ice Studies*, IEEE, IGARSS 2010


[40] Jiancheng Shi, Jeff Dozier and Helmut Rott, *Modeling and Observation of Polarimetric SAR Response to Dry Snow*, University of California and Institute for University of Innsbruck.


