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Títol

MOMENT – CURVATURE DIAGRAMS FOR LIGHT – WEIGHT SECTIONS

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Resum

MOMENT-CURVATURE DIAGRAMS FOR LIGHT-WEIGHT SECTIONS
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En construcció metàl·lica hi ha dues tipologies estructurals principals. La primera és la formada per seccions formades per làmines conformades en calent i ens és força coneguda. La segona, menys coneguda però d’importància creixent és la formada per seccions conformades en fred des de fletxos, acer laminat o barres planes d’acer extrudides. Aquesta última tipologia és coneguda com la dels acers conformatos en fred o seccions lleugerbes. La idea principal en aquest tipus de seccions és aconseguir capacitat portant a través de la forma de la secció enlloc d’augmentant el gruix de l’element. Donada la relativa facilitat de conformar l’acer en fred es pot produir una gran quantitat de seccions per ajustar-se a les necessitats de disseny.

Aquesta tesi s’ha concebut en el context d’un projecte de recerca europeu: “Seismic Design of Light-Gauge Steel Frame Buildings” en el que es van assajar a laboratori, sota càrregues sísmiques simulades, dos models a escala de marcs amb arriostaments en forma de X. Aquests dos marcs estaven formats per seccions conformades en fred de dimensions i gruixos habituels en construcció residencial, amb una alçada aproximadament quatre vegades inferior a l’habitual (Casafont, en preparació). Amb els assajos es van obtenir els diagrames força – desplaçament reals dels marcs. L’assaig consistia en imposar deformacions creixents als marcs, tant positives com negatives. Aquests desplaçaments imposats tenien un període creixent a cada cicle. Aquest tipus de càrrega simulà una ona sinusal, tipica càrrega per assajos a resposta sísmica (Paz, 2004).

L’objectiu d’aquesta tesi és analitzar les seccions conformades en fred presents a l’assaig estudiant el seu comportament davant l’abonyegament. A més, obtenir les seves corbes moment – curvatura sota diferents carregues d’esforç axial i així reconstruir la primera càrrega en el diagra de força – desplaçament per determinar fins a quin punt la perda de secció deguda a l’abonyegament influencia. Com a objectiu parcial necessitarem trobar el diagrames d’interacció entre el moment flector i l’eforç axial.

S’ha d’assenyalar que només es tindran en compte el moment flector i l’esforç axial en aquesta tesi. Per tant, l’esforç tallant serà negligit com ja és habitual en aquest tipus d’investigacions. Els efectes de l’esforç tallant són despreciables comparats amb els del moment flector i l’esforç axial. Cal dir també, que tots els càlculs i totes les comprovacions d’aquesta tesi es fan considerant l’Eurocode 3, especialment a les parts 1-1 i 1-3, que estan específicament dedicades a la construcció metàl·lica d’acers conformatos en fred.

Donat que les seccions conformades en fred són seccions del tipus 4, tenen tendència a presentar abonyegament i no entren el la brança plaústica. L’abonyegament és la clau per calcular acers conformatos en fred, ja que la seva aparició canvia les propietats de la secció estudiada per a cada càrrega que hi impossem. Els marcs arriostats assajats dissipen l’energia sota càrregues sísmiques a través de la plastificació de les diagonals en tracció. Per tant, els pastals i el dintell normalment es consideren elements no dissipatius del marc. Assumint que les seccions de tipus 4 no entren en la brança plaústica en compressió degut a l’aparició de l’abonyegament, però que si poden entrar-hi en tracció, intentarem tenir en compte la dissipació d’energia que els pastals i el dintell produceixen.

Algunes conclusions d’aquesta tesi són que les aproximacions de l’Eurocode 3 per tracció i flexió combinada poden ser força conservadores per les seccions assajades, assumint que poden entrar en la brança plaústica a tracció. A més, l’abonyegament té un efecte notori en el diagra de força – desplaçament però no ajuda a emular per complet el diagra real obtingut en els assajos (els models són massa rígids o mostren falta de rigidida per forces petites). Per acabar, les cartes tenen un impacte força important en els diagrames força – desplaçament. En la primera part de l’assaig les cartes actuen com a elements completament rígids, i a mesura que carreguem el marc arriostat van perdent aquesta rígida. Enlloc de mantenir propietats constants en la seva secció s’ha d’estudiar el seu comportament per determinar el seu comportament real. Aquest estudi del seu comportament condicionarà clarament els resultats d’aquesta tesi i ajudarà a modelar completament el comportament real del diagra de força – desplaçament que s’ha obtingut per assaig.
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List of symbols

Latin symbols

A  Axial force
A_{eff}  Effective area
A_g  Gross cross-section area
b  Web length
b_{e1}  Upper effective flange for section C-102-40-14-2
b_{e2}  Lower effective flange for section C-102-40-14-2
b_{eff}  Effective length for section's U-108-32-2 flanges
b_l  Flange loss for section C-102-40-14-2
b_s  Flange loss for section U-108-32-2
b_{g1}  Section's C-102-40-14-2 flange length
b_{g2}  Section's U-108-32-2 flange length
b_p  Web length accounting for rounded corners, midpoint of corner or bend
b_{p,c}  Flange length accounting for rounded corners, midpoint of corner or bend
b_{p,d}  Stiffener length accounting for rounded corners, midpoint of corner or bend
c  Stiffener length
C_g  Section's C-102-40-14-2 stiffener length
C_p  Stiffener length accounting for rounded corners, midpoint of corner or bend
D  Displacement
E  Young's modulus
\varepsilon_{Ny}  Shift of the centroidal axes
g_r  Distance between X and P
h  Yielding penetration
F  Force
f_u  Ultimate yield strength
f_{u,250}  Ultimate yield strength for steel S250
f_{u,350}  Ultimate yield strength for steel S350
f_y  Yield strength
f_{y,250}  Yield strength for steel S250
f_{y,350}  Yield strength for steel S350
f_{ya}  Average yield strength
h  Flange length
h_{a1}  Effective left part for section's C-102-40-14-2 web
h_{e2}  Effective right part for section's C-102-40-14-2 web
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>( h_{e21} )</td>
<td>Effective left part for section's U-108-32-2 web</td>
</tr>
<tr>
<td>( h_{e22} )</td>
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<td>( h_l )</td>
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<td>( h_g )</td>
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<td>( I_x )</td>
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</tr>
<tr>
<td>( I_{x,\text{eff}} )</td>
<td>Moment of inertia about the x-axis for the effective section</td>
</tr>
<tr>
<td>( I_y )</td>
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<td>( h_g )</td>
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</tr>
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<td>( h_{g2} )</td>
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</tr>
<tr>
<td>( h_p )</td>
<td>Flange length accounting for rounded corners, midpoint of corner or bend</td>
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<tr>
<td>( k )</td>
<td>Numerical coefficient that depends on the type of forming</td>
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<tr>
<td>( K_f )</td>
<td>Buckling factor</td>
</tr>
<tr>
<td>( L )</td>
<td>Applied force to the frame</td>
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<tr>
<td>( M )</td>
<td>Bending moment</td>
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<td>( n )</td>
<td>Number of 90° bends in the cross-section with an internal radius ( r \leq 5\cdot t )</td>
</tr>
<tr>
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<tr>
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<tr>
<td>( P )</td>
<td>Midpoint of corner</td>
</tr>
<tr>
<td>( R )</td>
<td>Radius</td>
</tr>
<tr>
<td>( r_m )</td>
<td>Radius plus half of the element's thickness</td>
</tr>
<tr>
<td>( s )</td>
<td>Net length of a section's element</td>
</tr>
<tr>
<td>( s_w )</td>
<td>Slant height</td>
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<td>Product of inertia in x-direction</td>
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<td>( t )</td>
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<td>( W )</td>
<td>Modulus of elasticity</td>
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<td>( X )</td>
<td>Intersection of midlines</td>
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<td>( X_{CG} )</td>
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Greek symbols

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<tr>
<td>ε</td>
<td>Strain</td>
</tr>
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<td>(\varepsilon_y)</td>
<td>Yielding strain</td>
</tr>
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<td>(\varepsilon_1)</td>
<td>Strain at the top fiber</td>
</tr>
<tr>
<td>(\varepsilon_2)</td>
<td>Strain at the lower fiber</td>
</tr>
<tr>
<td>(\Phi)</td>
<td>Angle between two plane elements</td>
</tr>
<tr>
<td>(\gamma M_0)</td>
<td>Partial safety factor</td>
</tr>
<tr>
<td>(\lambda_p)</td>
<td>Plate slenderness</td>
</tr>
<tr>
<td>(\lambda_{p,\text{red}})</td>
<td>Reduced plate slenderness</td>
</tr>
<tr>
<td>(\vartheta)</td>
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<td>(\rho)</td>
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<td>Stress</td>
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<td>(\sigma_E)</td>
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Chapter 1

Introduction

1.1 Cold-formed steel

In steel construction, there are two main families of structural members. One is the familiar group of hot-rolled shapes and members built up of plates. The other, less familiar but of growing importance, is composed of sections cold-formed from steel sheet, strip, plates, or flat bars in roll-forming machines or by press brake or bending brake operations. These are cold-formed steel structural members.

Although cold-formed steel sections are used in car bodies, railway coaches, various types of equipment, storage racks, grain bins, highway products, transmission towers, transmission poles, drainage facilities, and bridge construction, the discussions included herein are primarily limited to applications in building construction.

The use of cold-formed steel members in building construction began in about the 1850s in both the United States and Great Britain. However, such steel members were not widely used in buildings until around 1940 (Yu 2001). Since 1940 cold-formed steel has been used in industrial and residential buildings. Nowadays it represents the 45% of the steel construction market. For instance, in Hawaii 50% of new home construction is made by cold-formed steel (Elhajj 2001).

Contrarily, in Spain steel structures market (of any type) is still small, since it only represents a 10% of construction. This is a percentage well below the one found in other countries such as the United Kingdom where steel construction can represent a 75% of the total construction, or the United States, where steel use is even more popular.

In general, cold-formed steel structural members provide the following advantages in building construction:

1. As compared with thicker hot-rolled shapes, cold-formed light members can be manufactured for relatively light loads and/or short spans.
2. Unusual sectional configurations can be produced economically by cold-forming operations, and consequently favorable strength-to-weight ratios can be obtained.
3. Load-carrying panels and decks can provide useful surfaces for floor, roof, and wall construction, and in other cases they can also provide enclosed cells for electrical and other conduits.

4. Load-carrying panels and decks not only withstand loads normal to their surfaces, but they can also act as shear diaphragms to resist force in their own planes if they are adequately interconnected to each other and to supporting members.

Compared with other materials such as timber and concrete the following qualities, the combination of which can result in cost savings, can be realized for cold-formed steel structural members:

1. Lightness
2. High strength and stiffness
3. Ease of prefabrication and mass production
4. Fast and easy erection and installation
5. Substantial elimination of delays due to weather
6. More accurate detailing
7. Nonshrinking and noncreeping at ambient temperatures
8. Termite-proof and rotproof
9. Uniform quality
10. Economy in transportation and handling
11. Noncombustibility
12. Recyclable material

1.2 Building technology

Cold-formed steel sections are made with high quality rolled steel. They are formed either by press-braking blanks sheared from sheets or coils (Figure 1.1a), or by roll forming the steel through a series of dies (Figure 1.2b).

![Press braking](image1)

![Roll forming](image2)

(a) Press braking  (b) Roll forming

Figure 1.1 Forming methods for cold-formed steel
The main idea for cold-formed steel is to achieve load carrying capacity by the shape of the section instead of the thickness of the element. Due to the relatively easy forming methods a wide variety of sections can be produced to adjust to design needs.

Houses are placed on a concrete foundation slab. Bottom tracks are fixed to the slabs by means of anchoring bolts. Then, panels conforming the outside and inside walls of the house can be placed easily. Due to their low weight they can be carried by workers without need of auxiliary means.

Walls are the resistant elements of the house. They are constructed with cold-formed steel studs and are covered with sheathing, usually gypsum wallboards. Mineral wool between boards is normally used as insulation.

Figure 1.2 shows some of the cold-formed sections generally used in structural framing. The usual shapes are channels (C-sections), Z-sections, angles, hat sections, I-sections, T-sections, and tubular members. Previous studies have indicated that the sigma section possesses several advantages such as high load-carrying capacity, smaller blank size, less weight, and larger torsional rigidity as compared with standard channels (Yu 2001). In general, the depth of cold-formed individual framing members ranges form 51 to 305 mm. The thickness of steel sheets or strip generally used in cold-formed steel structural members ranges from 0.4 mm to about 6.4 mm.

![Figure 1.2 Typical cold-formed steel sections.](image)

Due to the fact that the major function of this type of individual framing member is to carry load, structural strength and stiffness are the main considerations in design. Such sections can be used as primary framing members in buildings up to six stories in height (Yu 2001). In tall multi-storey buildings the main framing is typically of heavy hot-rolled shapes and the secondary elements may be of cold-formed steel members such as steel joists, decks, or panels. In this case the heavy hot-rolled steel shapes and the cold-formed steel sections supplement each other.
For all this, and much more that could be said about cold-formed steel we can conclude that this is a great alternative to other materials used in home construction. It is a deeply studied material with its own widely developed design manuals: AISI in the United States, Eurocode 3 in Europe, CIRSOC-301 in Argentina, and so on.

1.3 Seismic design

Analysis and design of these steel structures subjected to dynamic loads involves consideration of time-dependent inertial forces. The resistance to displacement exhibited by a structure may include forces which are a function of the displacement and velocity (Paz, 2004). As a consequence, the governing equations of motion of the dynamic system are generally nonlinear partial differential equations which are extremely difficult to solve in mathematical terms. Nevertheless, quite recent developments in the field of structural dynamics enable such analysis and design to be accomplished in a practical and efficient manner (Paz, 2004). The dynamic study of cold-formed steel is a deep and interesting field which deserves research and study to develop efficient solving models.

In seismic design the structure has to be ductile under large seismic loads in order to dissipate energy by means of plastic deformations. These requirements are fulfilled by incorporating X-braced frames made of steel flat straps.

The hysteretic behavior of shear walls is a key factor in seismic response of cold-formed steel structures: they are the main dissipative zones of the building and, consequently, their plastic behavior has to be well characterized in order to provide ductile designs. Since cold-formed steel members are C4 cross sections, they are prone to local buckling and do not get into plastic range. Thus, studs and tracks are non-dissipative members of the shear walls. In the case of X-braced frames the dissipation of energy comes from the yielding of the diagonal straps (Pastor, 2006).

All calculations and checks in this thesis will be done considering the Eurocode 3 specification, with special regards to part 1-1 and 1-3, which specifically attain cold-formed steel construction and design.
Chapter 2
Need and scope

2.1 Need of this thesis

This thesis is conceived as a complementary work to the doctoral thesis: Numerical modeling of the seismic behavior of cold-formed steel structures (Pastor, 2006). This doctoral thesis was developed in the UPC "Universitat Politècnica de Catalunya" under the advising of Antonio Rodríguez-Ferran.

In this doctoral thesis, two scale-model shear frames were tested in the laboratory under simulated seismic loads (Figure 2.1). These two identical frames were made of cold-formed steel sections of usual dimensions and thicknesses for home constructions, whose height is about for times shorter than the height of a conventional frame (Casafont, 2006). The scope was to model the seismic behavior of a typical light weight home structure and the cyclic response of X-braced frames by using the finite element analysis method (FEM) (Figure 2.2). The results obtained by the FEM analysis where to be calibrated and validated with the ones obtained in the laboratory; for this reason, it is important to be sure that no member of the frame would fail before its hysteretic response is recorded (Casafont, 2006).

Figure 2.1 Tested X-braced frame model.
Figure 2.2 Finite element bilinear model of the X-braced frames: (a) geometry of the frame; (b) horizontal force-displacement cyclic law.

Real force-displacement diagrams where obtained by testing. The test consisted in imposing crescent positive and negative deflections to the frames. This imposed displacements had an increasing period at each cycle. This type of loading simulates a sinusoidal wave (Figure 2.3), typical loading for seismic response tests (Paz, 2004).

Figure 2.3 Loading used in tests. Deflection-time diagram.

Test results showed a pinching in the hysteretic system (Figure 2.4), not quite explained by the x-braced frame finite element model.
Since the results where not exactly the ones expected two possibilities had to be further analyzed and studied:

a) Vertical supports contribution to stiffness in bracings with diagonals depends on behavior in front of local buckling in type 4 sections (Eurocode 3, 2004). For big horizontal deflections it is interesting to evaluate this influence in capacity and ductility for structures under seismic loads and actions.

b) Gusset plates may be out of scale and may not represent perfectly a real life situation. These plates were used as a bolted pinned connection element between the studs and the tracks and were calculated to disallow connection failure before section failure. The axial joints where also tested to determine their real behavior.

2.2 Scope of this thesis

The scope of this thesis is to deeply analyze the cold-formed sections involved in the test by studying their behavior in front of local buckling. Obtain their moment-curvature diagrams for different axial loads and rebuild the first loading step of the force – displacement diagram to determine to which extend section loss due to local buckling influences. As a partial objective we will need to find bending moment-axial load interaction diagrams.

It shall be remarked that only axial force and bending moment will be considered in this thesis. Thus shear force will be neglected as it is usual in this kind of investigations. Shear effects are quite small, compared to those from bending moment or axial force, and would considerably increase calculation efforts at all means.
X-braced frames basically dissipate energy under seismic loads due to yielding of the diagonal straps. Since cold-formed steel members are C4 cross sections, they are prone to local buckling and do not get into plastic range. Thus, studs and tracks are often considered non-dissipative members of the shear walls. Assuming that C4 cross sections do not get into plastic range in compression due to local buckling but are able to do so in tension we will try to account for the energy dissipation that the straps can produce.

With all this information, a proper gusset plate study and Natividad Pastor's finite element model a further study should be able to rebuild the load-deflection diagram accounting for the studs' contribution to stiffness and reproduce the whole force-displacement diagram including pinching.
Chapter 3
Test specimens

3.1 X-braced frames

Test specimens where two identical X-braced frames. The main components of the shear panels are two tracks, two studs and two diagonal straps (Figure 3.1). Tracks and studs are composed of U100 and C100 profiles. The diagonal bracings are two straps of 65 mm in width and 0.8 mm in thickness. Diagonals are connected to studs and tracks trough 210x140 mm gusset plates whose thickness is 1.5 mm. Ø6.3 mm self-drilling screws are used to connect the components of the frame (Casafont, 2006).

Figure 3.1 Shear frame tested.

Several measures are taken to be sure that the dissipative yielding of the straps occurs (Casafont, 2006):

1. The diagonal straps are thin and narrow. On the contrary, the cold-formed profiles chosen for tracks and studs have high load bearing capacity to prevent their premature failure.
2. For the same reason, the steel grade of the diagonals is lower than the steel grade of the other members of the frame.
3. A steel of low grade and ductile is used in diagonal straps, thus giving them more dissipation capacity.
Figure 3.2 Original design plot for section C-102-40-14-2 (out of scale)
Figure 3.3 Original design plot for section U-108-39-2 (out of scale)
Figure 3.4 Building scheme 1 (out of scale)
Figure 3.5 Building scheme 2. Joint detail. Gusset plate (out of scale)
3.2 Material characterization

According to Eurocode 3, the average yield strength can be found with equation 3.1 unless adequate testing is conducted.

\[ f_{yu} = \min\left( f_{yul}, \frac{f_u + f_{yb}}{2} \right) \]  

(3.1)

Where:

- \( f_{yul} = f_{yb} + (f_u - f_{yb}) \frac{k \pi t^2}{A_s} \)
- \( k \) is a numerical coefficient that depends on the type of forming. In this case: \( k = 7 \) because we are dealing with a rolled section (Eurocode 3, 2004).
- \( n \) is the number of 90° bends in the cross-section with an internal radius \( r \leq 5t \) (Eurocode 3, 2004). In this case \( n = 6 \).

In our case there will be no need to use equation 3.1 because cold-formed steel was tested by using a usual traction test, where gradual deflections are imposed to the test specimen and stress levels are measured to build a stress-strain diagram (Ortiz, 1997). Test specimens were two diagonals. The reason is that 4 more diagonals than necessary for the test were bought and delivered (Table 3.4). The rest of the elements could not be characterized because there where the exact amount necessary for the test.

<table>
<thead>
<tr>
<th>Data obtained: 17/02/2005</th>
<th>Section</th>
<th>Thickness (mm)</th>
<th>Length (mm)</th>
<th>Steel type</th>
<th>Number of elements for each frame</th>
<th>Total number of units delivered</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stud</td>
<td>C-102-40-14</td>
<td>2</td>
<td>600</td>
<td>S350</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Stud</td>
<td>U-108-39</td>
<td>2</td>
<td>562</td>
<td>S350</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>High Track</td>
<td>C-102-40-14</td>
<td>2</td>
<td>1079</td>
<td>S350</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>High Track</td>
<td>U-108-39</td>
<td>2</td>
<td>1079</td>
<td>S350</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Low Track</td>
<td>C-102-40-14</td>
<td>2</td>
<td>998</td>
<td>S350</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Low Track</td>
<td>U-108-39</td>
<td>2</td>
<td>1079</td>
<td>S350</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Gusset</td>
<td>Plate</td>
<td>1,5</td>
<td>-</td>
<td>S350</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>Diagonal</td>
<td>Plate 215mmx140mm</td>
<td>1,5</td>
<td>-</td>
<td>S350</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>Plate 1040mmx65mm</td>
<td>0,80</td>
<td>-</td>
<td><strong>S250</strong></td>
<td>4</td>
<td>12</td>
</tr>
</tbody>
</table>

Table 3.4 Delivered material characteristics

As we can see in table 3.4, marked in bold letters, diagonals are made of steel S250 whereas all the rest of the elements are made of steel S350. This means that the material characterization was only made for one of the two steels used.

Stress-strain diagrams were obtained by testing steel S250 (Figure 3.5). In order to characterize steel 350 what has been done is a homothetic diagram of the one
obtained for steel S250 (Figure 3.6, equation 3.2). This assumption should not include any major error in the further calculations.

\[ \sigma_{S350} = \frac{350}{250} \sigma_{S250} \]  

(3.2)

Figure 3.6 Stress-strain diagram for steel S250 & S350

For cold formed steel, as well as in many other ductile materials such as aluminum, the value of the yielding stress is determined by drawing a parallel line to the elastic part of the diagram starting at a unitary longitudinal elongation of 0.2% (Ortiz, 1997). The reason for this procedure is that we cannot find an elongation increase at constant stress since the elongation increases until the ultimate stress is reached.

Thus we can find the yielding stress, \( f_y \), and ultimate stresses, \( f_u \), for steel S250 (Table 3.5). These properties will be proportionally incremented for steel S350 for all further calculations in this document as indicated in equations 3.3 and 3.4.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>( f_u ) (N/mm(^2))</th>
<th>( f_y ) (N/mm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diagonal 1</td>
<td>365</td>
<td>222</td>
</tr>
<tr>
<td>Diagonal 2</td>
<td>365</td>
<td>226</td>
</tr>
<tr>
<td>Diagonal 3</td>
<td>367</td>
<td>225</td>
</tr>
<tr>
<td>Diagonal 4</td>
<td>365</td>
<td>223</td>
</tr>
<tr>
<td>Mean value (N/mm(^2))</td>
<td>365.5</td>
<td>224</td>
</tr>
<tr>
<td>Standard deviation (N/mm(^2))</td>
<td>1.0</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Table 3.5 Steel S250 properties from testing.
\[ f_{y,5350} = \frac{350}{250} f_{y,5250} \]  
(3.3)
\[ f_{u,5350} = \frac{350}{250} f_{u,5250} \]  
(3.4)

The yield stress results to be lower than the nominal value: \( f_y = 224 \text{ N/mm}^2 \). This is good for the experimental investigation because the yielding of the strap will occur earlier than planned. The measured ultimate stress of the steel is also different from the nominal value: \( f_u = 365 \text{ N/mm}^2 \). In this case, the experimental value is higher, which is also a good result because this increases even more the \( f_u/f_y \) ratio and, as a consequence, the ductility of the diagonal straps (Casafont, 2006).

### 3.3 Stress displacement curve fitting

We will need to fit this diagram and find an analytical equation in order to be able to find strain values for each stress level for a given combination of bending moment and axial force in our studied section. This will be necessary later on in order to determine moment-curvature diagrams.

The usual approach for a stress-strain diagram in steel structures is to take a bilinear model (Figure 3.7).

![Figure 3.7 Usual bilinear approach for the stress-strain diagram.](image)

In this report we will fit the first part of the diagram to an analytical equation. Once we reach the yielding point we will assume a linear constant approach as usual. This means that once we reach the yielding point we will always have \( \sigma = f_y \) and \( \varepsilon = \varepsilon_y \). Before the yielding point we shall use the equation in figure 3.8.
Figure 3.8 Stress-strain diagram curve fitting.

3.4 Test procedure

Figs. 3.9 and 3.10 show the test setup. It can be seen that the specimen is assembled to the testing frame in horizontal position. Loading in the horizontal direction is chosen because a smaller force is required to yield the strap (the strap component of the horizontal force is higher than when testing the specimen in the vertical position).

![Test setup diagram](image-url)

Figure 3.9 Test setup (Casafont, 2006)
A 100 kN hydraulic cylinder is used to apply the load. This load is measured by means of a load cell placed at the end of the cylinder. Two measurements of displacement are also taken: the displacement of the cylinder and the displacement of the upper track of the specimen, which is measured by means of an external displacement transducer. All the measured data is stored in a computer.

Tests are displacement controlled. The displacement input is shown in Fig. 2.3. There are five loading cycles with an increasing value of displacement amplitude that ranges from ±15 mm to ±75 mm. The displacement law is chosen so that yielding of the diagonal straps occurs from the first cycle of the test. The maximum displacement is limited to the maximum allowable displacement amplitude of the hydraulic cylinder, 160 mm. The loading rate is constant for all the cycles: 0.2 mm/s. Measured data, i.e., force and displacement, are read at the same rate.
Chapter 4
Cold-formed steel section calculation

Cold-formed steel has several unique properties which require consideration. The main difference with hot-rolled steel is that cold-formed steel is prone to local buckling in compression which produces loss of part of the section properties at certain stress levels (area and moment of inertia are reduced, the center of gravity's position changes, etc.). This means, amongst other things, that cold-formed steel does not get into plastic range in compression. Herein we will assume that cold-formed steel can go into plastic range in tension.

In fact, structural engineers often forget about plasticity since Serviceability Limit States are too restrictive to enter the plastic range. But if we are worried about an Ultimate Limit State under seismic loads, and we must account for all the energy dissipation that the structure can give, plastic analysis becomes essential.

The compound section that must be studied is a type 4 section according to Eurocode 3, or a slender section according to American Institute of Steel Construction (AISC, 2005). This means that we cannot always use a linear analysis with constant section properties. In fact, the non-linear approach that must be followed for type 4 sections assumes that section properties are not the same for all stress distributions. Tension and compression are considered separately as follows:

- When the section is tensioned we can use linear analysis and the usual elasto-plastic approach. Failure in tension happens when the whole section plastifies, and therefore reaches the material's limit (yielding).

- In compression we must account for the effective section loss as local buckling reaches each element of the section. Once the buckling stress is achieved in each element the effective section is reduced (Figure 4.1). As we increase the compression level further reduction appears (lower area and inertia, different position of the mass center, etc.). Section properties are reduced, but there is no section failure until the maximum compressive stress that the material can stand (yielding stress) strikes the extreme fiber of the section.
A compressive axial force is considered positive; therefore a tensile axial force shall be negative.

As it is well known in structures design bending moment and axial force studied separately generate stresses perpendicular to the section's plane. Due to the superposition principle when they act together they will only produce this kind of stresses (Canet, 2000).

In elasticity, provided that the behavior is linear, an axial force $N$ applied to a section (area $A$ and elastic section modulus $W$) generates a stress level given by equation 4.6; and a bending moment $M$ given by equation 4.7. According to the superposition principle, the stress level produced by the two forces acting together will be given by equation 4.8 (Timoshenko, 1976). This concept is very clearly presented in figure 4.7.

$$\sigma_1 = \frac{N}{A} \quad (4.6)$$
$$\sigma_2 = \frac{M}{W} \quad (4.7)$$
$$\sigma = \sigma_1 + \sigma_2 = \frac{N}{A} + \frac{M}{W} \quad (4.8)$$

Figure 4.7 Stress distribution in elasticity for combined axial force and bending moment (Canet, 2000).
Once we get into the plastic range we cannot find a stress distribution from given axial forces and bending moments. The inverse procedure must be followed, first we define a stress distribution (Figure 4.8) and then we calculate the bending moment and the axial force by setting equilibrium equations (equations 4.9 and 4.10).

\[ N = \sum F_i \]  
\[ M = \sum F_i d_i \]  

Figure 4.8 Stress distribution in plasticity.

Although this may seem an easy procedure it can get quite tedious to determine the forces Fi depending on the complexity of the section and the number of elements it has.

4.3 Local buckling

Since thickness of cold-formed steel members is very small, they are classified by Eurocode 3 as C4 cross-sections under compression loads; this means that they are likely to present local buckling (Yu, 1991). Figure 4.9 shows the local buckling mode obtained at the laboratory for a compressed cold-formed steel member.

Figure 4.9 Local buckling mode of a compressed cold-formed steel member.
Local buckling reduces a section's capacity to carry load because the buckled part of the section stops helping to carry load. This means that a buckled element reduces its resistant area. This reduction can be calculated by following Eurocode 3 procedures as explained in the following paragraphs.

The elastic critical plate buckling stress according to Eurocode 3 is given by equation 4.11.

$$\sigma_{\text{buckling}} = K_{\sigma} \cdot \sigma_E$$  \hspace{1cm} (4.11)

$\sigma_E$ is Euler's buckling stress given by equation 4.12 and $K_{\sigma}$ can be obtained by calculating the stress ratio, $\psi$, and using figures 4.10 and 4.11 depending on the type of element for which we want to calculate the critical stress, internal or outstanding compression elements.

$$\sigma_E = \frac{\pi^2 \cdot E \cdot t^2}{12(1 - \nu^2) \cdot b_p}$$  \hspace{1cm} (4.12)

If the stress level in the element, $\sigma$ (calculated according to section 4.2), is lower than $\sigma_{\text{buckling}}$, no section reduction shall be expected. In the other hand, if it is higher or equal we will have to calculate the plate's loss and the new effective properties of the section.

<table>
<thead>
<tr>
<th>Stress distribution (compression positive)</th>
<th>Effective width $b_{\text{eff}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_1$</td>
<td>$\psi = 1$: $b_{\text{eff}} = \rho \overline{b}$ $b_{\text{eff}} = 0.5 b_{\text{eff}}$, $b_{\text{eff}} = 0.5 b_{\text{eff}}$</td>
</tr>
<tr>
<td></td>
<td>$1 &gt; \psi &gt; 0$: $b_{\text{eff}} = \rho \overline{b}$ $b_{\text{eff}} = \frac{2}{5 - \psi} b_{\text{eff}}$, $b_{\text{eff}} = b_{\text{eff}} - b_{\text{eff}}$</td>
</tr>
<tr>
<td></td>
<td>$\psi &lt; 0$: $b_{\text{eff}} = \rho \overline{b} = \rho \overline{b} / (1 - \psi)$ $b_{\text{eff}} = 0.4 b_{\text{eff}}$, $b_{\text{eff}} = 0.6 b_{\text{eff}}$</td>
</tr>
</tbody>
</table>

$\psi = \sigma / \sigma_1$

<table>
<thead>
<tr>
<th>$\psi$</th>
<th>$1 &gt; \psi &gt; 0$</th>
<th>$0 &gt; \psi &gt; -1$</th>
<th>$-1 &gt; \psi &gt; -3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td>$1$</td>
<td>$7.81$</td>
<td>$7.81 - 6.29\psi + 9.78\psi^2$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>$4.9$</td>
<td>$2.39$</td>
<td>$5.98 (1 - \psi)^2$</td>
</tr>
</tbody>
</table>

Figure 4.10 Internal compression elements (Eurocode 3, 2004).
Effective widths are calculated by using figures 4.10 and 4.11. But in order to use the equations given in these figures we must find the reduction factor, \( \rho \). This is not a trivial operation and several parameters must be calculated in advance. First of all we must find the parameter \( \varepsilon \) by using equation 4.13, then the plate’s slenderness, \( \lambda_p \), (equation 4.14) and the reduced plate slenderness, \( \lambda_{p,\text{red}} \) (equation 4.15).

\[
\varepsilon = \frac{235}{\sqrt{f_y[N/mm^2]}} \tag{4.13}
\]

\[
\lambda_p = \frac{b_p}{\sqrt{\sigma_{\text{buckling}}}} = \frac{b_p}{t} \sqrt{\frac{28.4\varepsilon\cdot\sqrt{K_\sigma}}{f_y}} \tag{4.14}
\]

\[
\lambda_{p,\text{red}} = \lambda_p \cdot \sqrt{\frac{\sigma_{\gamma M_0}}{f_y}} \tag{4.15}
\]

Finally we can calculate the reduction factor, \( \rho \), with equation 4.16 for doubly supported compression elements and equation 4.17 for outstand compression elements. Once we have found this reduction factor we can calculate the element’s effective width with figures 4.10 and 4.11. By doing this for each element of the section we can calculate new section properties for any stress distribution in elasticity (same properties as section 4.2.1 but with the new values).
\[
\rho = \frac{1 - 0.055(3 + \psi)}{\frac{\lambda_{p,\text{red}}}{\lambda_{p,\text{red}}} + 0.18\frac{\lambda_p - \lambda_{p,\text{red}}}{\lambda_p - 0.6}} \leq 1
\] (4.16)

\[
\rho = \frac{1 - 0.188}{\frac{\lambda_{p,\text{red}}}{\lambda_{p,\text{red}}} + 0.18\frac{\lambda_p - \lambda_{p,\text{red}}}{\lambda_p - 0.6}} \leq 1
\] (4.17)

The above procedure is conservative and requires an iterative calculation in which stress ratio, \( \psi \), is calculated at each step from the stresses calculated on the effective cross-section defined at the end of the previous step.

4.4 Axial force - Bending moment interaction diagram

An axial force \( (A) \) – bending moment \( (M) \) interaction diagram gives us the combinations of these parameters above which we will have structural failure. Therefore, as long as we stay below these values our structure will be able to carry the given loads.

The tested section has a different behaviour for positive and negative moment for it is not symmetric. It also has a different behaviour for tensile and compressive stresses for being a cold-formed section. In addition we will allow plasticity in tension remaining in the elastic range for compression.

Figure 4.12 is a conceptual interaction diagram. It shows how the stress distribution varies in each part of the diagram. As it is only a conceptual diagram, once we find the real one it may slightly differ.
4.4.1 Diagram in elasticity

In elasticity we must find a combination of $A - M$ values which effective stresses stay in the elastic range and produce yielding at the extreme compressed fiber of the section.

Figure 4.13 shows the flowchart for this procedure. This is a quite tedious procedure and for ease of comprehension I have added a numerical example for given bending moment and axial force in annex A.

Summarizing we define a stress distribution from an arbitrary combination of bending moment and axial load. These stress levels change our section's properties due to local buckling by lowering its mass center and decreasing its area and elastic modulus. According to equation 4.8 this creates new effective stresses of higher value. We must find effective stresses that produce yielding at the extreme compressed fiber.

By repeating this procedure several times we will find an axial force – bending moment interaction diagram which will exactly tell us the maximum possible combinations of these forces that our section can undertake.
Figure 4.13 Flowchart for calculation of the A - M interaction diagram.

In chapter 5 these diagrams and the concepts developed in this chapter will help us find the moment-curvature diagrams, for they will let us know the maximum moment resistance for a determined axial load.

Tables 4.2 and 4.4 show the results obtained by following the procedure shown in figure 4.13.
### Table 4.3 Calculated values for the interaction diagram in elasticity.

<table>
<thead>
<tr>
<th>Positive moment and compression</th>
<th>Negative moment and compression</th>
</tr>
</thead>
<tbody>
<tr>
<td>M (N·m)</td>
<td>A (N)</td>
</tr>
<tr>
<td>0</td>
<td>199700</td>
</tr>
<tr>
<td>229</td>
<td>183300</td>
</tr>
<tr>
<td>424</td>
<td>169600</td>
</tr>
<tr>
<td>623</td>
<td>155800</td>
</tr>
<tr>
<td>706</td>
<td>150000</td>
</tr>
<tr>
<td>902</td>
<td>135200</td>
</tr>
<tr>
<td>1148.5</td>
<td>114850</td>
</tr>
<tr>
<td>1332</td>
<td>100000</td>
</tr>
<tr>
<td>1377</td>
<td>96390</td>
</tr>
<tr>
<td>1927</td>
<td>57810</td>
</tr>
<tr>
<td>2031.5</td>
<td>50000</td>
</tr>
<tr>
<td>2528</td>
<td>12640</td>
</tr>
<tr>
<td>2660</td>
<td>2660</td>
</tr>
<tr>
<td>2695</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>199700</td>
</tr>
<tr>
<td></td>
<td>-233.8</td>
</tr>
<tr>
<td></td>
<td>-439.8</td>
</tr>
<tr>
<td></td>
<td>-657.4</td>
</tr>
<tr>
<td></td>
<td>-899</td>
</tr>
<tr>
<td></td>
<td>-969</td>
</tr>
<tr>
<td></td>
<td>-1263</td>
</tr>
<tr>
<td></td>
<td>-1515</td>
</tr>
<tr>
<td></td>
<td>-1584</td>
</tr>
<tr>
<td></td>
<td>-2088</td>
</tr>
<tr>
<td></td>
<td>-2260</td>
</tr>
<tr>
<td></td>
<td>-2751</td>
</tr>
<tr>
<td></td>
<td>-2898</td>
</tr>
</tbody>
</table>

### Table 4.4 Calculated values for the interaction diagram in elasticity.

<table>
<thead>
<tr>
<th>Positive moment and tension</th>
</tr>
</thead>
<tbody>
<tr>
<td>M (M·m)</td>
</tr>
<tr>
<td>A (N)</td>
</tr>
<tr>
<td>2695</td>
</tr>
<tr>
<td>2800</td>
</tr>
<tr>
<td>2890</td>
</tr>
</tbody>
</table>

### 4.4.2 Diagram in plasticity

As exposed in section 4.2 we cannot find a stress distribution starting with given axial load and bending moment. This means that we must follow a different procedure. In this part of the diagram the effective compressive stress will be constantly the yielding stress while the tensioned part of the section plastifies.

When the studied section approaches plasticity in the tensioned part but still continues in the elastic range calculations show that it looses a constant width in the webs and has no loss from stiffeners or flanges. This means that we know the section loss for all points in the diagram that involve plastic analysis. Therefore local buckling has a constant, determined value in plasticity for our section (Table 4.5).

<table>
<thead>
<tr>
<th>Negative moment</th>
<th>C-102-40-14-2 web</th>
<th>Section U-108-32-2 web</th>
</tr>
</thead>
<tbody>
<tr>
<td>A = 1.97 m² (No loss)</td>
<td>I = 0.0066 cm⁴ (No loss)</td>
<td>A = 1.66 cm²</td>
</tr>
<tr>
<td>Positive moment</td>
<td>A = 1.63 cm²</td>
<td>I = 0.0054 cm⁴</td>
</tr>
<tr>
<td>A = 2.09 cm² (No loss)</td>
<td>I = 0.0069 cm⁴ (No loss)</td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.5 Constant values for effective area and local moment of inertia in plasticity.**
All we will have to do is follow section 4.2 accounting for this constant section loss, set the equilibrium equations and find the different combinations of bending moment and axial force that the section can stand (table 4.6).

<table>
<thead>
<tr>
<th>Bending Moment (N.m)</th>
<th>Axial Force (N)</th>
<th>Bending Moment (N.m)</th>
<th>Axial Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2920</td>
<td>0</td>
<td>2950</td>
<td>-21200</td>
</tr>
<tr>
<td>-2986</td>
<td>-10030</td>
<td>3001</td>
<td>-31480</td>
</tr>
<tr>
<td>-3020</td>
<td>-15300</td>
<td>2994</td>
<td>-41760</td>
</tr>
<tr>
<td>-3040</td>
<td>-25960</td>
<td>2990</td>
<td>-49480</td>
</tr>
<tr>
<td>-3047</td>
<td>-36790</td>
<td>2971</td>
<td>-57240</td>
</tr>
<tr>
<td>-3020</td>
<td>-47880</td>
<td>2921</td>
<td>-67670</td>
</tr>
<tr>
<td>-2947</td>
<td>-59400</td>
<td>2853</td>
<td>-76900</td>
</tr>
<tr>
<td>-2829</td>
<td>-71690</td>
<td>2770</td>
<td>-88850</td>
</tr>
<tr>
<td>-2754</td>
<td>-78300</td>
<td>2430</td>
<td>-115100</td>
</tr>
<tr>
<td>-2661</td>
<td>-85440</td>
<td>2056</td>
<td>-142400</td>
</tr>
<tr>
<td>-2546</td>
<td>-93420</td>
<td>1528</td>
<td>-172700</td>
</tr>
<tr>
<td>-2398</td>
<td>-102900</td>
<td>658</td>
<td>-209500</td>
</tr>
<tr>
<td>-2190</td>
<td>-115300</td>
<td>0</td>
<td>-235300</td>
</tr>
<tr>
<td>-1845</td>
<td>-134200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1637</td>
<td>-145600</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-1323</td>
<td>-163900</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-987</td>
<td>-182300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-825</td>
<td>-201600</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-317</td>
<td>-217400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>-235300</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.6 Calculated values for the interaction diagram in plasticity.

4.4.3 Interaction diagram

In figure 4.14 we can see represented the values calculated in sections 4.4.1 and 4.4.2. As it is only a conceptual diagram there's no need to fit the curves to have a numerical equation of the results because it would not give us further information.

As a reminder, this diagram is only valid for the cross-section studied in this report. No extrapolation for other cross-sections shall be made.
4.5 Results analysis

Eurocode 3 specifications give specific approaches for both: combined tension and bending and combined compression and bending.

Cross-sections subjected to combined axial tension $N_{Ed}$ and bending moment $M_{y,Ed}$ should satisfy the criterion given by equation 4.18 (Eurocode 3, 2004). If equation 4.19 is true then the criterion given by equation 4.20 should also be satisfied.

$$\frac{N_{Ed}}{N_{1,Ed}} + \frac{M_{y,Ed}}{M_{y,Ed,tm}} \leq 1$$  \hspace{1cm} (4.18)
$$M_{cy,Rd,com} \leq M_{cy,Rd,ten}$$  \hspace{1cm} (4.19) \\
$$\frac{M_{y,Ed}}{M_{cy,Rd,ten}} \frac{N_{Ed}}{N_{c,Rd}} \leq 1$$  \hspace{1cm} (4.20) \\

Where:

- $N_{c,Rd}$ is the design resistance of a cross-section for uniform tension.
- $M_{cy,Rd,ten}$ is the design moment resistance of a cross-section for maximum tensile stress if subject only to moment.

Cross-sections subjected to combined axial compression $N_{Ed}$ and bending moment $M_{y,Ed}$ should satisfy the criterion given by equation 4.21 (Eurocode 3, 2004). In this case, if equation 4.22 is true, equation 4.23 should also be satisfied.

$$\frac{N_{Ed}}{N_{c,Rd}} + \frac{M_{y,Ed} + \Delta M_{y,Ed}}{M_{cy,Rd,com}} \leq 1$$  \hspace{1cm} (4.21) \\
$$M_{cy,Rd,ten} \leq M_{cy,Rd,com}$$  \hspace{1cm} (4.22) \\
$$\frac{M_{y,Ed} + \Delta M_{y,Ed}}{M_{cy,Rd,ten}} - \frac{N_{Ed}}{N_{c,Rd}} \leq 1$$  \hspace{1cm} (4.23) \\

Where:

- $N_{c,Rd}$ is the design resistance of a cross-section for uniform compression (calculated with the effective area).
- $M_{cy,Rd,com}$ is the design moment resistance of a cross-section for maximum compressive stress if subject only to moment.
- $\Delta M_{y,Ed}$ is an additional moment due to shifts of the centroidal axes. This additional moment can be calculated by using equation 4.24.

$$\Delta M_{y,Ed} = N_{Ed} e_{Ny}$$  \hspace{1cm} (4.24) \\

- $e_{Ny}$ is the shift of the centroidal axes (Figure 4.15).
Equations 4.18, 4.20, 4.21 and 4.23 are linear equations. Figure 4.16 illustrates the results obtained in this thesis compared to those specified by Eurocode 3. We can see that for the studied section these equations fit quite well for combined axial compression and bending moment, in fact, they give us a fast, easy and relatively accurate estimation of our results.

![Normalized Bending Moment - Axial Force Interaction Diagram](image)

**Figure 4.16 Normalized interaction diagram**

It must also be noticed that for the studied section Eurocode 3 equation's in tension are very conservative, giving much lower values than those calculated before in this chapter. The reason for this noticeable difference is that Eurocode 3 does not account for plasticity in tension. It is the renowned plastic reserve. Points that lay between our calculated curve and the line given by Eurocode 3 will represent combinations of axial load and bending moment that our section can resist in exchange of large deflections, but never section fracture.
Chapter 5

Moment-Curvature diagrams

5.1 Definition of curvature

Curvature ($\psi$) is a parameter that indicates the rotation angle of a section in a strain diagram (Salmon, 1998). Figure 5.1 clearly illustrates this concept and equation 5.1 gives the geometric approach to calculate curvature.

Figure 5.1 (a) Generic cross-section; (b) Stress diagram with yielding penetration, $h$; (c) Strain diagram with curvature, $\psi$.

$$\theta = \frac{\varepsilon_1 - \varepsilon_2}{b} \quad (5.1)$$

5.2 Need of moment – curvature diagrams

Moment – curvature diagrams help us find deflections not only in the elastic range, but also in plasticity until section failure. Usually design manuals or software packages have analytical expressions to determine deflections under any given loads but they only work while we stay in the elastic range, respecting the small deflections theory. Moment-curvature diagrams are a part of the large deflections theory.
5.3 Finding moment – curvature diagrams

Equation 5.2 is the elastic line differential equation under a given bending moment (Ortiz, 1997). When small deflections can be assumed we can ignore the term \( y'^2 \) for it is much smaller than 1. In this case the approximate elastic line equation is given by equation 5.3. Expression from which we can define curvature \((\theta)\) for small deflections (equation 5.4).

\[
\frac{y''}{(1 + y'^2)^{\frac{3}{2}}} = \frac{M}{E \cdot I}
\]

(5.2)

\[E \cdot I \cdot y'' = M\]

(5.3)

\[\theta = \frac{y''}{E \cdot I}\]

(5.4)

The problem is that equations 5.3 and 5.4 are only valid while we are in the elastic range. Once our stress level causes plasticity appearance a different approach must be followed. In this cases will have to find a moment – curvature diagram.

According to equation 5.2 if we are able to find an analytical equation \( M = f(\theta) \) for our section under any given loads we will only have to integrate it to determine the sections deflection, \( y \).

In the study of our section we have further complications added. We must not only account for a bending moment, \( M \), but also an axial load, \( A \). Thus, the analytical equation that we must find must depend on \( A \) (equation 5.5).

\[M = f(\theta, A)\]

(5.5)

The diagram developed in this report will be a series of parametric moment – curvature curves with constant values of \( A \). To find each one of these parametric curves we must first, select a constant value for the axial load applied; second, calculate the section’s loss due to local buckling (Chapter 4); third calculate the effective stresses and determine their associated strains; and finally, find the values of curvature (equation 5.1).
5.4 Moment – curvature diagrams

Figures 5.2 to 5.5 are the moment-curvature diagrams found for our cross-section. Numerical equations for these diagrams are also given in each figure.

![Positive bending moment and axial compression graph](image)

**Figure 5.2** Moment-curvature diagram for positive bending moment and axial compression.

![Positive bending moment and axial tension graph](image)

**Figure 5.3** Moment-curvature diagram for positive bending moment and axial tension.
Chapter 5. Moment-Curvature diagrams

Figure 5.4 Moment-curvature diagram for negative bending moment and axial compression.

Figure 5.4 Moment-curvature diagram for negative bending moment and axial tension.
Chapter 6

Force – Displacement diagrams

Now that we have deeply studied the test specimen's sections it is the time to see to what extent accounting for local buckling in the studs and the top track helps reproduce the real loading part of the force-displacement diagram obtained by testing.

6.1 Structural modeling

To do so, two different structures have been modeled using the software package ED-Tridim (CIMNE, 1997, version 1.0). This software tool uses structural matrix analysis and simplifies a structure to a series of nodes, beams and loads. Properties such as area, moment of inertia and elastic modulus are given to the beams and restraints are imposed to each node as desired (fixed node, pinned node, rigid node, etc.). Loads may be applied to both, beams and nodes. The outcome results of this software are deflections, stress levels and bending moments at each node of the structure.

ED-Tridim does not account for plasticity, thus we must check at each step the stresses at the diagonals to check if they are or are not in the plastic range. Once diagonals enter the plastic range we must substitute the beams in the model for constant forces (F) that correspond to the maximum stress level in the diagonals (Figure 6.1). This must be done for both models. F can be easily calculated as the diagonal's yielding stress multiplied by their cross-section's area, resulting a force of 23296 N.

![Figure 6.1 (a) Diagonal not plastified; (b) Plastified diagonal](image)
Diagonals will be modeled as if they acted only under axial load. Only the diagonal in tension will be of interest because overall buckling will quickly neglect the other diagonal's effect, as it is very slender (Figure 6.2).

![Figure 6.2 Quick buckling of the compressed diagonal (Casafont, 2006).](image)

The imposed loading force (L) is applied in the center of the top track as a horizontal point load. Local buckling is expected to appear in the upper and lower corners as in the tested model (Figure 6.3).

![Figure 6.3 (a) Local buckling of the stud flanges; (b) Local damage in lower corner joints (Casafont, 2006).](image)

The calculation procedure is to impose a loading force L and calculate the stress levels at each element. If these stresses are high enough to reduce the section's properties in whole or in part, the model is changed and the new
sectional properties are added. As stated before, local buckling shall be expected but luckily stress levels will not be so high to expect plasticity in tension. This means that we will not have the need to integrate moment-curvature diagrams in any case.

6.1.1 Model 1

The first structure (Model 1) has been modeled as a very rigid structure. The length of the studs has been reduced as if gusset plates where completely rigid and the top track has been modeled with a very high moment of inertia. The reason for giving such a high moment of inertia to the top track is that during the test the machine that imposed deflections to the structure was attached to it and conferred it great rigidity (Figure 3.1). The studs are fixed at the bottom and the lower node of the diagonal has a pinned connection.

Figure 6.4 shows the geometry entered in ED-Tridim for Model 1. Black dots are nodes. Notice that several nodes have been added to the studs. These nodes are separated 30mm from each other. This is where reductions due to local buckling are expected. If local buckling appears, section properties for these element will be recalculated and changed.

![Figure 6.4 Model 1](image)

6.1.2 Model 2

The second structure (Model 2) has been modeled as if gusset plates only conferred a partial rigidity to the structure and the top track is given the section’s properties without incrementing them to account for the testing machine attached to it. The studs are fixed at the bottom. This is conceptually less rigid than Model 1.

Figure 6.5 shows the geometry entered in ED-Tridim for Model 2. Here black dots also represent nodes and there are also several of them to model the gusset plate and to account for possible appearance of local buckling.
reductions. Here we may expect reductions in the top track too since it is not as rigid as in Model 1.

![Figure 6.5 Model 2](image)

Figure 6.5 Model 2

Figure 6.6 is the approach followed for gusset plates. The main idea is that as we get farther from the fixed node (1) rigidity from the gusset plate proportionally vanishes with distance. Thus, at the last point we will have the same section properties as the studied section by itself.

Figure 6.7a is the section introduced for the beam between nodes 1-2, as we can see it has bigger area and moment of inertia than the compound section studied. These properties are reduced for beams 2-3 to 6-7 proportionally with distance. The beam between node 7 and the next gusset plates has the original compound section properties.

Figure 6.7b indicates the section introduced for the beam between nodes 1-2’. These properties are also reduced with distance and the beam between node 6’ and the next gusset plate has the original section properties.

![Figure 6.6 Gusset plate model](image)
6.2 Force – displacement diagrams

The process followed is to reproduce the loading process as a series of steps in time. Starting from a null force at each step the applied force is increased by 5000 N. Then the stress levels at each node are checked and effective cross-section properties reduced according to this stresses (Chapter 4). Diagonal plasticity is also checked. Then structure is recalculated with the effective properties to find the deflections. The maximum force applied is 40000 N. Force-displacement diagrams for both models can be seen in figure 6.8.
6.2 Results analysis

The main reason for developing force-displacement diagrams was checking to what extent local buckling explains test results. Figure 6.9 is a comparison between the force-displacement diagram obtained by testing and those calculated from computer models (earlier in this chapter). Model 1 is intended to be a rigid structure and adapts quite well to the steep slope of the first part of the real diagram, however, it is so rigid that the loss of linearity due to local buckling does not correctly explain the second part of the real curve. On the other hand, we have model 2 which does not adapt well in the beginning, showing much larger deflections than expected, but it explains a lot better than model 1 the second part of the diagram.

![Force-Displacement Diagram](image)

**Figure 6.9 Force-displacement diagrams**

In fact model 1 assumes that gusset plates are so rigid that they shall never allow bending whereas model 2 assumes that gusset plates do not have constant cross-section properties and that as we move farther from the fixed nodes the structure becomes less rigid allowing for bending.

Section loss due to local buckling in the studs and the top track clearly produces a change in the diagram's slope but does not seem to be the main characteristic that controls results.

The reason why accounting for local buckling and plasticity in the diagonal will not emulate the real force-displacement diagram is clear: we are
dealing with cold-formed steel in all of our sections; therefore we must expect local buckling not only in the studs and the top track, but also in gusset plates. We cannot forget that gusset plates are also cold-formed and are under compressive forces.

This means that in the beginning gusset plates are completely rigid and as we load the X-braced frame they loose their rigidity. In spite of dealing with constant section properties deep study must be carried to determine gusset plate's real behavior. This further study of their behavior will clearly condition this report's results and help to completely model the real force-displacement diagram obtained by testing.
Chapter 7

Conclusions

This thesis dealt with a cold-formed X-braced frame tested in the laboratory under simulated seismic loads. A deep study of the tested cross-sections has been carried out accounting for local buckling in compression and assuming that the section could undergo plasticity in tension. According to this section analysis computer models of the X-braced frame have been calculated through matrix analysis using the computer software ED-Tridim to reproduce the force-displacement diagram obtained from testing.

The main conclusions that can be extracted from this thesis are:

1. Eurocode 3 specific approaches for combined tension and bending can be very conservative for the tested cross-section assuming that it can enter the plastic range in tension. Plastic reserve is quite significant. However Eurocode 3 equations for combined compression and bending are quite accurate for this section.

2. The tested compound cross-section is now completely characterized. Moment-curvature diagrams for it have been developed in chapter 5.

3. Local buckling has a noticeable effect on the force-displacement curve although it does not help to fully emulate the real diagram obtained from testing. Models are either too rigid or show lack of rigidity for small forces.

4. Gusset plates have a very important impact on force-displacement diagrams. In the first part of the test gusset plates are completely rigid and as we load the X-braced frame they loose their rigidity. In spite of dealing with constant section properties deep study must be carried to determine gusset plate's real behavior. This further study of their behavior will clearly condition this report's results and help to completely model the real force-displacement diagram obtained by testing.

5. If all the elements of our X-braced frames were fully characterized under local buckling in compression a usual design software package (such as ED-Tridim) can be used to determine the structural behavior.
Bibliographic references


CIRSOC 303 (1991), "Estructuras livianas de acero" El INTI-CIRSOC, ERREPAR


Elhajj, N. (2001). "Designing Homes Using Cold-Formed Steel Framing", NAHB Research Center, Upper Marlboro, MD, USA.


Annex A
Calculation in elasticity

A.1 Geometrical data

Geometrical data and the gross cross-sectional properties are given in chapter 4. Once local buckling begins we will have a loss of effective section. Figure A.1 show the notation that has been used to calculate these losses and find the new section properties.

Figure A.1 Effective and ineffective portions of the nominal cross-section.

A.2 Calculation procedure

Herein we will follow the flowchart presented in figure 4.13. First of all we must choose a combination of axial force (A) and bending moment (M). Our initial assumption is $M = 623N\cdot m$ and $A = 1558800N$. The resulting stress distribution is conceptually drawn in figure A2 and given numerically by equations A1 and A2.
Figure A.2 Stress distribution.

\[ \sigma_1 = \frac{A}{A_e} + \frac{M}{I_x} y_{up} = 271 \frac{N}{mm^2} \]  \hspace{1cm} (A.1)

\[ \sigma_2 = \frac{A}{A_e} - \frac{M}{I_x} y_{down} = 148.6 \frac{N}{mm^2} \]  \hspace{1cm} (A.2)

Where \( y_{up} \) and \( y_{down} \) are the distances of the upper and lower extreme fibers from the center of rotation.

A.2.1 Section loss

Now we must find each element's section loss according to these stress levels.

A.2.1.1 Section C-102-40-14-2. Calculation of the flange's effective width

According to figure A.2 in this element we have the following stress levels: \( \sigma_{f11} = 270.3 \frac{N}{mm^2} \) and \( \sigma_{f12} = 159.4 \frac{N}{mm^2} \). Now we can find the plate's slenderness with equation A.3.

\[ \psi = \frac{\sigma_{f12}}{\sigma_{f11}} = 0.59 \]  \hspace{1cm} (A.3)

The buckling factor can be calculated with equation A.4 (Erocode 3, 2004)

\[ K'_{\sigma} = \frac{8.2}{1.05 + \psi} = 5 \]  \hspace{1cm} (A.4)
Local buckling begins to appear when the plate slenderness ($\lambda_p$) is 0.673. Equation A.5 lets us find the tension level that would cause buckling appearance.

$$\sigma_{buckling} = \frac{235\cdot(28.4 \times \lambda_p)^2 \cdot K_\sigma}{b_p^2} = 1286 \frac{N}{mm^2} > \sigma_{f11}$$  \hspace{1cm} A.5

Section’s C-102-40-14-2 flanges are not expected to have any reduction due to local buckling in this case. Their effective properties will be their nominal properties. The reason is that the stress level that causes buckling is higher than the stress in the flange.

**A.2.1.2 Section C-102-40-14-2. Calculation of the web’s effective width**

The stress level at this section’s web will be: $\sigma_w = 268.1 \frac{N}{mm^2}$. In this case we have a uniform stress distribution: $\psi = 1$, which leads us to a buckling factor: $K_\sigma = 4$ for this internal compression element (Eurocode 3, 2004).

Again, local buckling will appear when the plate slenderness ($\lambda_p$) reaches a value of 0.673, which makes us able to find the tension level that will cause the appearance of buckling in this element (equation A.6).

$$\sigma_{buckling} = \frac{235\cdot(28.4 \times \lambda_p)^2 \cdot K_\sigma}{h_p^2} = 141.5 \frac{N}{mm^2} < \sigma_w$$  \hspace{1cm} A.6

The stress level in the web is higher than the buckling stress. Thus, we will have a reduction in our section properties at this compression level. In order to find the effective section we must calculate the real plate slenderness and the reduced plate slenderness (equations A.7 and A.8).

$$\frac{h_p}{\lambda_p} = \frac{f_y}{235 \sqrt{f_y} K_\sigma} = 1.002$$ \hspace{1cm} A.7

$$\lambda_p,\text{reduced} = \lambda_p \sqrt{\frac{\sigma_w \chi_w}{f_y}} = 0.98$$ \hspace{1cm} A.8

The reduction factor can be calculated with equation A.9.

$$\rho = \frac{1 - \frac{0.055 \cdot (3 + \psi)}{\lambda_p,\text{reduced}}}{\lambda_p,\text{reduced} + 0.18 \cdot \frac{\lambda_p - \lambda_p,\text{reduced}}{\lambda_p - 0.6}} = 0.82 < 1$$  \hspace{1cm} A.9

Finally, the effective portions are given by equations A.10 to A.12.
\[ h_{h1} = 0.5 \cdot 0.2 \cdot h_p = 0.04m \]  
\[ h_{\epsilon 2} = h_{\epsilon 1} = 0.04m \]  
\[ h_i = h_p - h_{\epsilon 1} - h_{\epsilon 2} = 0.018m \]

**A.2.1.3 Section C-102-40-14-2. Calculation of the stiffener's effective width**

The stress level will be \( \sigma_{f13} = 157.3 \frac{N}{mm^2} \). Again we must find the plate slenderness. In this case we have a uniform stress distribution: \( \psi = 1 \), which leads us to a buckling factor: \( K_\sigma = 0.43 \) for this outstanding compression element (Eurocode 3, 2004).

As usual, local buckling begins to appear when the plate slenderness \( (\lambda_p) \) is 0.673, which makes us able to find the tension level that will cause the appearance of buckling in this element (equation A.13).

\[
\sigma_{\text{buckling}} = \frac{235 \cdot (28.4 + \lambda_p)^2 \cdot K_\sigma}{c_p^2} = 981.1 \frac{N}{mm^2} > \sigma_{f13}
\]

Section's C-102-40-14-2 stiffeners are not expected to have any reduction due to local buckling in this case. Their effective properties will be their nominal properties.

**A.2.1.4 Section U-108-38-2. Calculation of the flange's effective width**

AS a first step, we must find the stresses at the extremes of the element (FigureA.2). These stresses are: \( \sigma_{f21} = 262.3 \frac{N}{mm^2} \) and \( \sigma_{f22} = 153.6 \frac{N}{mm^2} \). The plate's slenderness is given by equation A.14.

\[ \psi = \frac{\sigma_{f22}}{\sigma_{f21}} = 0.586 \]

The buckling factor can be calculated with equation A.15 (Eurocode 3, 2004).

\[ K_\sigma = 0.57 - 0.21\psi + 0.07\psi^2 = 0.471 \]

Local buckling begins to appear when the plate slenderness is 0.673. Let's find the tension level that would cause buckling appearance (equation A.16).
\[ \sigma_{\text{buckling}} = \frac{235 - (28.4 t \lambda_p)^2 \cdot K_\sigma}{b_p^2} = 116.5 \frac{N}{mm^2} < \sigma_{f21} \]  

A.16

Again, section reduction shall be expected due to local buckling. The plate slenderness and reduced plate slenderness are given by equations A.17 and A.18.

\[ \lambda_p = \frac{b_{p2}}{28.4 \sqrt{\frac{235}{f_y} - K_\sigma}} = 1.104 \]  

A.17

\[ \lambda_{p,\text{reduced}} = \frac{\sigma_{f21} \cdot Y_{M0}}{f_y} = 1.01 \]  

A.18

The reduction factor can now be easily calculated with equation A.19.

\[ \rho = 1 - \frac{0.188}{\lambda_{p,\text{reduced}}} + 0.18 \frac{\lambda_p - \lambda_{p,\text{reduced}}}{\lambda_p - 0.6} = 0.802 < 1 \]  

A.19

The effective portions are given in equations A.20 and A.21.

\[ b_{eff} = \rho \left( b_{p2} - NA \right) = 0.03m \]  

A.20

\[ b_{2t} = b_{p2} - b_{eff} - b_{2t} = 0.0074m \]  

A.21

A.2.1.5 Section U-108-38-2. Calculation of the web’s effective width

The stress level at this section’s web is \( \sigma_{w2} = 151.5 \frac{N}{mm^2} \). Once again, we have a uniform stress distribution: \( \psi = 1 \), which leads us to a buckling factor: \( K_\sigma = 4 \) for this internal compression element (Eurocode 3, 2004).

Local buckling begins to appear when the plate slenderness (\( \lambda_p \)) is 0.673, which makes us able to find the tension level that will cause the appearance of buckling in this element (equation A.22).

\[ \sigma_{\text{buckling}} = \frac{235 - (28.4 t \lambda_p)^2 \cdot K_\sigma}{h_{p2}^2} = 125.7 \frac{N}{mm^2} < \sigma_{w2} \]  

A.22

We will have a reduction in our section properties at this compression level. In order to find the effective section we must calculate the real plate slenderness (equation A.23) and the reduced plate slenderness (equation A.24).
\[
\lambda_p = \frac{h_p}{28.4 \cdot \frac{235}{f_y} \cdot K_o} = 1.063 \quad \text{A.23}
\]

\[
\lambda_{p, \text{reduced}} = \lambda_p \sqrt{\frac{\sigma_w \gamma_{M0}}{f_y}} = 0.739 \quad \text{A.24}
\]

The reduction factor can be calculated with equation A.25.

\[
\rho = 1 - \frac{0.055(3 + \psi)}{1 - \frac{\lambda_{p, \text{reduced}}}{\lambda_{p, \text{reduced}}} + 0.18 \frac{\lambda_p - \lambda_{p, \text{reduced}}}{\lambda_p - 0.6}} = 0.939 < 1 \quad \text{A.25}
\]

The effective portions are given in equations A.26 to A.28.

\[
h_{z1} = 0.5 \cdot \rho \cdot h_p = 0.049\text{m} \quad \text{A.26}
\]

\[
h_{z2} = h_{z1} = 0.049\text{m} \quad \text{A.27}
\]

\[
h_i = h_p - h_{z1} - h_{z2} = 0.0064\text{m} \quad \text{A.28}
\]

### A.2.1 Effective cross-section properties

Now we must set a co-ordinate system (Figure A.3) to determine each effective element’s section co-ordinates (Table A.1).

![Figure A.3 Co-ordinate system and numbering of the effective cross-section elements.](image-url)
<table>
<thead>
<tr>
<th>Element</th>
<th>( x_{\text{start},i} ) (m)</th>
<th>( y_{\text{start},i} ) (m)</th>
<th>( x_{\text{end},i} ) (m)</th>
<th>( y_{\text{end},i} ) (m)</th>
<th>( x_{\text{CG},i} ) (m)</th>
<th>( y_{\text{CG},i} ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.053</td>
<td>-0.053</td>
<td>7.322 \times 10^{-4}</td>
<td>0.031</td>
<td>-0.053</td>
<td>0.016</td>
</tr>
<tr>
<td>2</td>
<td>-0.052</td>
<td>-0.0032</td>
<td>0</td>
<td>0</td>
<td>-0.028</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.0032</td>
<td>0.052</td>
<td>0</td>
<td>0</td>
<td>0.028</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.053</td>
<td>0.053</td>
<td>7.322 \times 10^{-4}</td>
<td>0.031</td>
<td>0.053</td>
<td>0.016</td>
</tr>
<tr>
<td>5</td>
<td>-0.05</td>
<td>-0.05</td>
<td>2.732 \times 10^{-3}</td>
<td>0.039</td>
<td>-0.05</td>
<td>0.021</td>
</tr>
<tr>
<td>6</td>
<td>-0.037</td>
<td>-0.049</td>
<td>0.002</td>
<td>0.002</td>
<td>-0.043</td>
<td>0.002</td>
</tr>
<tr>
<td>7</td>
<td>0.037</td>
<td>0.049</td>
<td>0.002</td>
<td>0.002</td>
<td>0.043</td>
<td>0.002</td>
</tr>
<tr>
<td>8</td>
<td>0.05</td>
<td>0.05</td>
<td>2.732 \times 10^{-3}</td>
<td>0.030</td>
<td>0.05</td>
<td>0.021</td>
</tr>
<tr>
<td>9</td>
<td>0.0085</td>
<td>0.049</td>
<td>0.04</td>
<td>0.04</td>
<td>0.029</td>
<td>0.04</td>
</tr>
<tr>
<td>10</td>
<td>-0.049</td>
<td>-0.0085</td>
<td>0.04</td>
<td>0.04</td>
<td>-0.029</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Table A.1 Co-ordinates of the effective cross-section.

With this co-ordinates determined it is easy to find the section properties for each element (Table A.2).

<table>
<thead>
<tr>
<th>Element</th>
<th>( s_i ) (m)</th>
<th>( t_i ) (m)</th>
<th>( A_i ) (m²)</th>
<th>( S_{xli} ) (m³)</th>
<th>( l_{xi} ) (m)</th>
<th>( S_{yl} ) (m³)</th>
<th>( l_{yi} ) (m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.03</td>
<td>0.002</td>
<td>5.98 \times 10^{-5}</td>
<td>9.38 \times 10^{-7}</td>
<td>4.46 \times 10^{-9}</td>
<td>-3.17 \times 10^{-6}</td>
<td>1.99 \times 10^{-11}</td>
</tr>
<tr>
<td>2</td>
<td>0.049</td>
<td>0.002</td>
<td>9.81 \times 10^{-6}</td>
<td>0</td>
<td>6.97 \times 10^{-11}</td>
<td>-2.72 \times 10^{-6}</td>
<td>1.90 \times 10^{-7}</td>
</tr>
<tr>
<td>3</td>
<td>0.049</td>
<td>0.002</td>
<td>9.81 \times 10^{-6}</td>
<td>0</td>
<td>6.97 \times 10^{-8}</td>
<td>2.72 \times 10^{-6}</td>
<td>1.90 \times 10^{-7}</td>
</tr>
<tr>
<td>4</td>
<td>0.03</td>
<td>0.002</td>
<td>5.98 \times 10^{-5}</td>
<td>9.38 \times 10^{-7}</td>
<td>4.46 \times 10^{-9}</td>
<td>3.17 \times 10^{-6}</td>
<td>1.99 \times 10^{-11}</td>
</tr>
<tr>
<td>5</td>
<td>0.037</td>
<td>0.002</td>
<td>7.31 \times 10^{-5}</td>
<td>1.53 \times 10^{-6}</td>
<td>8.13 \times 10^{-9}</td>
<td>-3.65 \times 10^{-6}</td>
<td>2.44 \times 10^{-11}</td>
</tr>
<tr>
<td>6</td>
<td>0.012</td>
<td>0.002</td>
<td>2.45 \times 10^{-6}</td>
<td>4.91 \times 10^{-8}</td>
<td>8.18 \times 10^{-12}</td>
<td>-1.06 \times 10^{-6}</td>
<td>3.08 \times 10^{-10}</td>
</tr>
<tr>
<td>7</td>
<td>0.012</td>
<td>0.002</td>
<td>2.45 \times 10^{-6}</td>
<td>4.91 \times 10^{-8}</td>
<td>8.18 \times 10^{-12}</td>
<td>1.06 \times 10^{-6}</td>
<td>3.08 \times 10^{-10}</td>
</tr>
<tr>
<td>8</td>
<td>0.037</td>
<td>0.002</td>
<td>7.31 \times 10^{-5}</td>
<td>1.52 \times 10^{-6}</td>
<td>8.13 \times 10^{-9}</td>
<td>3.65 \times 10^{-6}</td>
<td>2.44 \times 10^{-11}</td>
</tr>
<tr>
<td>9</td>
<td>0.04</td>
<td>0.002</td>
<td>8.08 \times 10^{-6}</td>
<td>3.23 \times 10^{-6}</td>
<td>2.69 \times 10^{-11}</td>
<td>2.35 \times 10^{-6}</td>
<td>1.10 \times 10^{-8}</td>
</tr>
<tr>
<td>10</td>
<td>0.04</td>
<td>0.002</td>
<td>8.08 \times 10^{-6}</td>
<td>3.23 \times 10^{-6}</td>
<td>2.69 \times 10^{-11}</td>
<td>-2.35 \times 10^{-6}</td>
<td>1.10 \times 10^{-8}</td>
</tr>
</tbody>
</table>

Table A.2 Element section properties.

Finally, the section properties for the effective cross-section are:

- **Effective area:**
  \[
  A_{\text{eff}} = \sum_{i=1}^{10} A_i = 6.73 \times 10^{-4} \text{m}^2
  \]  
  A.29

- **Mass center in x-direction:**
  \[
  x_{\text{CG,eff}} = \frac{\sum_{i=1}^{10} S_{xli,eff}}{A_{\text{eff}}} = 0 \text{m}
  \]  
  A.30

- **Mass center in y-direction:**
  \[
  y_{\text{CG,eff}} = \frac{\sum_{i=1}^{10} S_{yl,eff}}{A_{\text{eff}}} = 0.017 \text{m}
  \]  
  A.31
- Moment of inertia about \( x \):

\[
I_{x,\text{eff}} = \sum_{i=1}^{g} \left( I_{yi} + A_{i} \left( y_{ci,i} - y_{co} \right)^2 \right) = 1.81 \times 10^{-7} \, m^4
\]  
\[\text{A.32}\]

- Moment of inertia about \( y \):

\[
I_{y,\text{eff}} = \sum_{i=1}^{g} \left( I_{yi} + A_{i} \left( x_{ci,i} - x_{co} \right)^2 \right) = 4.04 \times 10^{-7} \, m^4
\]  
\[\text{A.33}\]

### A.2.3 Effective Stresses

Now we must find the new stress distribution (equations A.34 and A.35). Sectional properties have changed due to local buckling. As a reaction the stresses are redistributed in the section producing the effective stresses.

\[
\sigma_{1,\text{eff}} = \frac{A}{A_{\text{eff}}} + \frac{M}{I_{x,\text{eff}}} y_{up} = 313.6 \frac{N}{mm^2}
\]  
\[\text{A.34}\]

\[
\sigma_{2,\text{eff}} = \frac{A}{A_{\text{eff}}} - \frac{M}{I_{x,\text{eff}}} y_{down} = 169.3 \frac{N}{mm^2}
\]  
\[\text{A.35}\]

As we can see we have found a combination of \( M \) and \( A \) that pertains to the interaction diagram. Here the final iteration has been presented, but logically, several iterations must be carried before finding these values.