Escola Tècnica Superior d'Enginyeria
de Telecomunicació de Barcelona
UNIVERSITAT POLITÈCNICA DE CATALUNYA

PROJECTE FINAL DE CARRERA

The impact of linear precoders on the capacity of a MIMO broadcast channel

Estudis: Enginyeria de Telecomunicació

Autor: Àlex Oliveras Martínez

Director: Niu, ZhiSheng

Any: 2013

"If you can't explain it simply, you don't understand it well enough."

Albert Einstein

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Resum

Escola Tècnica Superior d'Enginyeria de Telecomunicació de Barcelona
TSINGHUA UNIVERSITY, NiuLab

per Àlex Oliveras Martínez

L'escenari que s'estudia en aquest projecte es un sistema MIMO broadcast multiusuari. Això significa que l'estació base té un nombre significant d'antenes, varis usuaris amb una sola antena repartits per la cella, i només es considera l'enlla descendent. S'utilitza un precodificador per augmentar la capacitat del canal. S'estudien tres precodificadors diferents, els zero forcing, el matched filter i l'MMSE. La CSI se suposa incompleta. Això significa que alguns usuaris transmeten tota la CSI però d'altres només transmeten una part d'aquesta. En aquest escenari es realitzen varies simulacions i s'obté que quan el nombre d'usuaris amb CSI incompleta és suficientment alt els tres precodificadors es comporten semblant. També s'ensenya un resultat sorprenent quan el nombre d'antenes i usuaris s'augmenta. Es comparen les tècniques de suposar la CSI mancant i de dividir els usuaris amb CSI i sense. I s'obté que hi ha un llindar que fa una tècnica millor que l'altre. Aquest llindar depèn del nombre d'antenes, usuaris , SNR, entre d'altres. Finalment s'explica que quan la correlació del canal MIMO augmenta la capacitat decreix. I s'analitza una suposició de la CSI per explotar aquesta correlació.

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Resumen

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El escenario estudiado en este proyecto es un sistema MIMO broadcast multiusuario. Esto significa que la estación base tiene un gran numero de antenas, varios usuarios con una sola antena están repartidos por la cela, y solo se considera el enlace descendente. Se utiliza un precodificador para augmentar la capacidad del canal. Tres precodificadores lineales son analizados, zero forcing, matched filter y MMSE. La CSI se supone incompleta. Esto significa que algunos usuarios transmiten toda la CSI pero otros solo transmiten una parte de esta. En este escenario se realizan varias simulaciones y se obtiene que si el numero de usuarios con CSI incompleta es elevado, los tres precoders se comportan de forma similar. Se muestra un resultado sorprendente cuando el numero de antenas y usuarios se aumenta. Se compara la técnica de suponer la CSI y dividir los usuarios con y sin CSI, y se descubre que existe un lindar donde una técnica es mejor que la otra. Este lindar depende del numero de antenas, usuarios, SNR, entre otros. Finalmente se explica que cuando la correlación de un canal MIMO aumenta, su capacidad se reduce. Y una suposición para explotar esta correlación es analizada.

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Abstract

Escola Tècnica Superior d'Enginyeria de Telecomunicació de Barcelona
TSINGHUA UNIVERSITY, NiuLab

by Àlex Oliveras Martínez

The scenario studied in this project is a MIMO broadcast multiuser system. It means a base station with a high number of antennas, several users with only one antenna spread in the coverage area of this base station, and only considering the downlink. It is used a precodification to increase the capacity of the channel. Three linear precoders are studied, zero forcing, matched filter and MMSE. The CSI is suposed to be incomplete. It means that some users transmit all the CSI but some others just transmit a part of it. In this scenario is performed some simulations and its obtained that if the number of users with incomplete CSI is high the three precoders have similar performance. It is showed a surprising result when the number of antennas and users is increased. It is compared the technique of assuming the missing CSI and splitting the users with and without CSI and it is found that there is a threshold that makes one technique better than the other. This threshold depends on the numbers of antennas, users, SNR, etc. Finally it is explained that when the correlation of a MIMO channel increases its capacity decreases. And an assumption to exploit this correlation is analyzed.

Acknowledgements

I would like to express my deepest appreciation to all those who provided me the possibility to complete this report. A special gratitude I give to my final year project advisor, Dr. Zhisheng Niu, whose contribution in stimulating suggestions and encouragement, helped me to coordinate my project especially in finding ways to keep the research on.

Furthermore I would also like to acknowledge with much appreciation the crucial role of the students of NiuLab, who helped me so much. A special thanks goes to my team mate, Zhiyuan Jiang, who helped me discussing about some topics of my work and giving me his point of view. Thanks to his comments and advises. Last but not least, many thanks go to the advisor in my home university, Dra. Anna Umbert, who has invested her full effort in guiding me.

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Abbreviations

MIMO	**M**ultiple **I**nput **M**ultiple **O**utput
SISO	**S**ingle **I**nput **S**ingle **O**utput
SNR	**S**ignal to **N**oise **R**atio
LTE	**L**ong **T**erm **E**volution
IEEE	**I**nstitute of **E**lectrical and **E**lectronics **E**ngineers
CSI	**C**hannel **S**tate **I**nformation
MMSE	**M**inimum **M**ean **S**quered **E**rror
CSIT	**C**hannel **S**tate **I**nformation at the **T**ransmitter
LOS	**L**ine **O**f **S**ight
AWGN	**A**dditive **W**hite **G**aussian **N**oise
QoS	**Q**uality **O**f **S**ervice
LDPC	**L**ow **D**ensity **P**arity-check **C**odes
SINR	**S**ignal plus **I**nterference to **N**oise **R**atio
CSIR	**C**hannel **S**tate **I**nformation at the **R**eceiver
ML	**M**aximum **L**ikelihood
MAP	**M**aximum **A** posteriori **P**robability
WCDMA	**W**ideband **C**ode **D**ivision **M**ultiple **A**ccess

Symbols

M	number of transmitting antennas
N	number of receiving antennas
$\mathbf{G}$	channel matrix
g_{ij}	phase shift from transmitting antenna j to receiver user i
s	transmitted signal
y	received signal
n	AWGN noise
$\mathbf{R_{TX}}$	correlation matrix at the transmitter
r_{TX}	coefficient of spatial correlation at the transmitter
r_{RX}	coefficient of spatial correlation at the receiver
G_{cor}	channel matrix correlated
C	channel capacity
W	bandwidth
P	signal power
$\frac{N_0}{2}$	noise power spectral density
I	interference power
$\mathbf{W}$	precoding matrix
w_{ij}	multiplying weight of the information from antenna i to user j
P_n	power of information received for user n
I_n	interference to user n
β	weighting vale of the precoders
ρ	total power constrain

To my family

Chapter 1

Introduction

1.1 Context

Now a days mobile communications has experienced a huge progress. In part for the insertion of the smart phones in the society and in part due to the development of the technology. The users had become more demanding on the services, and it is a challenge to the companies to satisfy their demand. The only way to achieve this is through doing research for increasing the capacity of the communication systems. It has not been an easy way, because the old systems had reached their limits, so it has been designed new techniques. This techniques created unresolved problems to keep the research on.

One of the techniques to improve mobile communications is the MIMO systems. MIMO stands for multiple-input multiple-output, and refers to the use of multiple antennas at the transmitter and receiver in wireless systems. It was introduced by Foschini [1] and Telatar [2]. They predicted high spectral efficiency in MIMO systems with high scattering environments and traceable variations of the channel. The large spectral efficiency associated with MIMO channels are based on the premise that a rich scattering environment provides independent transmission paths from each transmitting antenna to each receiving antenna [3]. SISO systems used to exploit time and frequency resources, but MIMO systems also use the spatial dimension provided by the multiple antennas.

In old systems multipath scattering is seen as an impairment to the information transmission. However, in MIMO system, it can be seen as an opportunity to improve the capacity of the channel. This increase of the performance is due to several factors. First of all there is the array gain, that provides an increase of SNR by combining the received signals from several antennas. This improve the resistance in front of noise and interference. Also the spatial diversity gain increases the performance of the channel. It provides multiple path between the transmitter and the receiver and it makes the probability of deep fade decrease. Then there is the spatial multiplexing gain. It increases the capacity of the channel by transmitting multiple independent data streams. The receiver can separate the data streams if it is in a high scattering environment. Each stream has the capacity of a SISO system, so the capacity is increased by a factor of the number of streams. The last factor is the interference reduction. This is due to the MIMO systems can focus the signal energy towards the desired user minimizing the interference with other users.

To have an overview of the system in the figure 1.1 it can be seen a diagram of the basic building blocks of a MIMO communication system. The information bits to be transmitted are encoded and interleaved. This is used to reduce the error rate of the channel. The codeword is mapped to data symbols by the symbol mapper. This means that it is assigned a different electromagnetic wave pattern to each codeword to be transmitted. The data symbols are input to a space-time encoder that outputs spatial data streams. This add redundancy to provide reliability communication. The precoding maps the data streams to the transmitting antennas. The signal is propagated through the channel and arrives at the receiver. All the blocks in the receiver undo what has been done for the transmitter to obtain the original data. This project is focused on the precoder block and the channel.

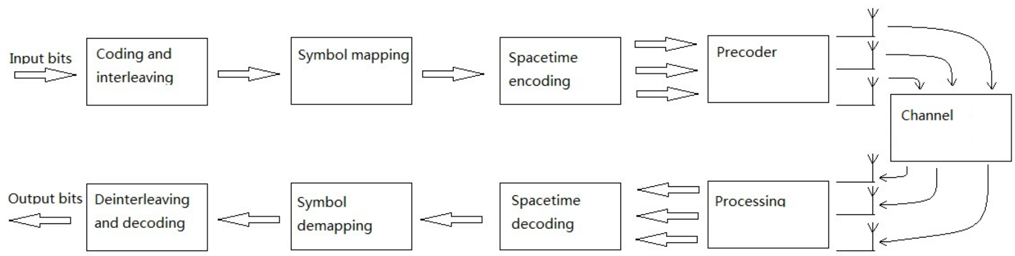

FIGURE 1.1: Block diagram of a MIMO system.

In the recent years there has been lots of publications about MIMO techniques to improve the capacity of the communications channels. And the achieved results can be seen in the implementation of this technologies in the real life systems. For example MIMO is used in the LTE/LTE-Advanced systems or in the IEEE 802.11n. But there is still a lot of work to do in this topic. One of the open topics in MIMO systems is the dealing with the CSI. It has been demonstrated that the use of CSI (at the receiver and/or the transmitter) is improving the performance of the system. But as the CSI is obtained at the receiver not always can be feedback to the transmitter. Some times it can be transmitted just a part of it, or just some users can transmit it, etc. There is still a lot of work to do on knowing the performance of the system in this scenarios.

1.2 Goals

The main goal of this project is to research on the performance, in terms of capacity, of a specific scenario in mobile communication systems. This scenario studied is a broadcast multiuser MIMO channel. In which the base station is the transmitter that has a lot of antennas, and the receivers are the users that each one has only one antenna. The project is focused in the different linear precoders, the zero forcing, the matched filter and the MMSE. In this scenario it will be analyzed the impact of several hypothesis for example the CSIT completeness and the number of transmitting antennas or receiving users, as well as the channel model or the SNR.

For all this scenarios it will be programmed a MATLAB code to perform the needed simulations and after that an analysis of this results will be performed to obtain the relationship between the above-mentioned parameters and the capacity of the system.

1.3 Structure

This project is structured in seven chapters. The first chapter is this introduction to the project and to the studied system. From chapter two to chapter six it is explained the necessary theory on communication engineering to understand the projects and the obtained results. It is also showed the state of the art in these topics. In the chapter two it is explained the channel used to transmit the information. In the chapter three it is discussed the evaluated parameter, the capacity. In the chapter four it is showed the importance of the CSI. In the chapter five it is explained the different precoders. And in the chapter six it is discussed about the assumptions of CSI used to transmit the information. Finally in the chapter seven it is explained the simulations performed, and the obtained results are showed and analyzed. At the end of the project, it can be find a conclusion.

Chapter 2

Channel

In this chapter it is explained the characteristics of the transmission channel and how this can affect the communication. It is also discussed two kind of channel models, uncorrelated and correlated, and how are this models obtained mathematically for the simulations.

2.1 Characteristics of the channel

The channel is the media between the transmitter and the receiver in a communication. In these systems there are a lot of channels that can be used to transmit information. Sometimes the channel is a wire, i.g., twisted pair, optical fiber. Other times it can be wireless, i.g., air, water. It is obvious that in mobile communications, like the system this project is focused on, the channel must be wireless to let the users move freely. It is of particular importance the characteristics of the channel since it will determine the design of the system.

In the scenario in which this project focuses, the transmission channel is the atmosphere. The waves coming from the base station moves through the air, until getting the user. This is not a static environment. Depending on the position of the receiver, it can have different properties. Even if the user is remaining in the same position for a long time, the behavior of the channel can change through the time. This is due to the channel being composed by moving objects, e.g., cars. So the path of the signal is varying over the time as the position of this objects. For this reason it is important to highlight the randomness of the channel.

The result of this randomness is the corruption of the transmitted signal. The randomness of the channel is the main reason to perform a Monte-Carlo simulation in this project. In each realization the channel has different properties, so the information that can be transmitted in a period of time changes. It is needed to do a high number of realizations to obtain the statistical properties of the channel and the throughput. And also in this way can be obtained the ergodic capacity of the channel. In chapter 3 it is explained in more details the capacity and the throughput.

There are several ways to the transmitted signal become corrupted. The most remarkable is the additive noise. It uses to be generated in the circuits of the system due to the temperature of the components. It becomes in a high random signal added to the signal that carries the information. It uses to be modeled by a white, Gaussian, zero mean random process. Its effects can be attenuated using the proper filters. There are other sources of noise that are external to the system like interference from other systems or other users in the same system. The signal is also degraded by the attenuation of the path. But in this project it is omitted such attenuation because it is affecting all the users of the MIMO system in the same way. If the position of the users is random, it can be assumed that all the users are at the same distance of the base station, in average. So the attenuation is not important for the goal of the project. The most important degradation of the signal for the project is the phase shift. Due to the multipath environment the signal of different paths arrive at the receiver with different phase shifts. Some times the signals can be added in-phase, so the received power is increased. But some times the signals can be added out-of-phase, and then the signals are canceled. It makes the channel random.

The antenna structure can make the channel become correlated. In the analyzed system the channel is the atmosphere. So the information is carried by electromagnetic waves. This waves are radiated by antennas. How to design this antennas and the physical principles that sustain this design will not be discussed in this project. However it is important to notice that in the studied system it is used an array of antennas at the transmitter side, i.e., base station, and only one antenna at the receiver side, i.e., user. The geometry of the antenna array is of capital importance to determine the correlation of the channel. For example, if the antennas are placed very near the correlation increases, if there is along distance between antennas the correlation decreases, but it is needed more space for the array. However, the scatters around the receiver can broke this correlation. The users are assumed to be far from each other and there is no correlation between them.

Note that it is taken into account a highly multi-path environment. In environments where there is no line-of-sight propagation (NLOS environments), scattering is the dominating propagation mechanism. It means that the transmitted signal is reflected in a huge amount of scatters around the receiver. The combination of the signal produced by this scatters can be constructive or destructive, depending on the phase shift of the signals. So a small variation of the objects in the path will perform a high variation in the received signal. It becomes in a highly random signal, i.e., Rayleigh. For this reason the correlation due to the antenna array pattern disappears (the scatters randomize the signals).

It is considered a multiuser system. The goal of these systems is to provide information to several users at the same time. To do this, some of the resources have to be distributed among the different users. There are several techniques to provide multiuser service. It can be time multiplexing, frequency multiplexing, coding multiplexing or spatial multiplexing. The idea of the time multiplexing is to transmit the information of each user in a different time slot. In this way during a period of time one user has all the resources of the channel and during an other period of time it has no resources. The frequency multiplexing distribute the bandwidth to the users. The capacity for each user is reduced, but they can be transmitting continuously. For the coding multiplexing, the users can be using all the resources of the channel during all the time, but the information is coded with a different code for each user. This codes makes the transmitted signals orthogonal and they can be isolated at the receiver.

Finally the spatial multiplexing is the studied in this project. The goal is to transmit a different signal for each antenna that modified by the channel and added to the signals of the other antennas the user can receive the desired information. As the path to each user is different, transmitting the same signals, different users can receive different information. The idea of this technique is to use the differences on the path for the different users to provide the desired information to each one. It supposes that the channel is known by the transmitter, see chapter 4. So the system knows the functions that modify the transmitted signal and the signal that each user have to receive. So solving an equations system it can be found which signal has to be transmitted by each antenna to let each user receive its information. Note that to be sure that the equations system has solution the number of transmitting antennas must be higher or equal to the number of users. See the section 2.2 for the mathematical model.

This work is focused on the spatial multiplexing, and the correlation of the signal. It is not taken into account the time delays or time correlation in this project. It is assumed that even if the signals are scattered around the user, all arrive at the receiver antenna at the same time. So, the user obtains a linear combination of the signal with the coefficients representing the shift phase. It is assumed that the channel properties remains constant during the transmission time of a signal. But the channel is completely different at the transmission of the next signal. It could be a way to keep the research in the future to study a time correlated channel.

2.2 Uncorrelated

The first part of the project is focused on a channel model with uncorrelated elements. It is the easiest way to generate the channel and to deal with it. But it is not a precise model of the reality. And also don't let make use of the correlation to assume the missing parts of the CSI. On the other hand, it is important for the MIMO systems to have an uncorrelated channel, because the spatial diversity is sustained by this hypothesis. If the path for all the users are independent, the transmitted signal is modified in a different way for each user. So even if the transmitted signal is the same, the received signals are different. This is the principle of the MIMO systems, to transmit a unique signal that all the users receive differently. If the channel is correlated the among of information that can be transmitted to different users decreases.

Now it will be explained the model of the channel. An stochastic MIMO channel is modeled as a matrix. Consider M transmitting antennas and N receiving users. The channel can be represented by the $N \times M$ matrix $\mathbf{G}$. Each component represents the phase shift in the path between the transmitted antenna (in the column) and the receive antenna or user (in the row).

$$\mathbf{G} = \begin{pmatrix} g_{11} & g_{12} & \cdots & g_{1m} \\ g_{21} & g_{22} & \cdots & g_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1} & g_{n2} & \cdots & g_{nm} \end{pmatrix}$$

where g_{ij} is the phase shift in the path from transmitter antenna j to receiver user i. It is assumed that the phase shift is zero-mean and unity variance, complex Gaussian distributed.

Let be the transmitted signal s, a vector of M components, each one for a different antenna. Each one of the components is the transmitted symbol of the corresponding antenna. And y is the received signal. It is a vector of longitude N, and each component correspond to the symbol received for a different user. The transmission equation is

$$y = G * s + n \tag{2.1}$$

where n is the AWGN of N components. In the simulations this parameter is not used because it is included in the SNR definition. So there is no need to establish a value for the noise power. The SNR for the typical scenarios can vary from 10 dB to 20 dB. In this project a lot of simulation has been performed with a 20 dB SNR.

The objective of the MIMO techniques is to transmit a unique signal in the base station and due to the channel properties each user receives its own information. So to do this it has to be solved the equation 2, knowing the received information for each user y and the channel characteristics, that it will be call CSI (see chapter 4). To be sure that this equation system can be solved it is needed that $M > N$,e.g., the number of unknowns greater than the number of equations.

Using this channel model is good in some cases. But in some others it is important to have a more real channel. For this reason there is an other part of the project that is focused in a correlated channel.

2.3 Correlated

There are many ways to generate a correlated channel, or in general a correlated matrix. Some of this ways can be really complex and it is very difficult to have a correlation as the reality, because in the real world each systems has a different behavior. As the goal of this project is not the correlation it self, it is used a simple correlation method described by A. Van Zelst and J. S. Hammerschmidt in [4].

This correlation method is described as a single coefficient spatial correlation model for MIMO channels. The main idea is that the correlation is due to the proximity of the antennas so if the antennas are equidistant, the correlation is proportional for all the antennas. It is created a correlation matrix

$$\mathbf{R_{TX}} = \begin{pmatrix} 1 & r_{TX} & r_{TX}^4 & \dots & r_{TX}^{(M-1)^2} \\ r_{TX} & 1 & r_{TX} & \dots & \vdots \\ r_{TX}^4 & r_{TX} & 1 & \dots & r_{TX}^4 \\ \vdots & \vdots & \vdots & \ddots & r_{TX} \\ r_{TX}^{(M-1)^2} & \dots & r_{TX}^4 & r_{TX} & 1 \end{pmatrix}$$

This matrix is defined for the antenna array at the transmitter side, but it can be also defined a correlation matrix in the same way for the receiver side.

In this matrix it can be seen that the first antenna is completely correlated with itself, is highly correlated with the second antenna. It is less correlated with the third, etc. It is used only the parameter r_{TX} to define the correlation.

$$r_{TX} \approx exp(-23\Lambda^2 d^2) \tag{2.2}$$

where d is the distance between the antennas (in wavelength) and Λ is the angular spread. The correlation parameter can go from 0, there is no correlation, to 1 it is completely correlated. To choose the proper value for the parameter r_{TX} is quite difficult. In this project it has been performed simulations for several values and it has been done an analysis for all of them.

It can be established different values for the correlation at the transmitter antennas and the receiver antennas. But the scenario studied is a MIMO multiuser broadcast channel. So the transmitter is the base station and the receiver is the user. As the users are distributed among the cell, there is no correlation between them. So the parameter of the correlation for the receiver antennas is 0 for all the simulations.

To obtain the correlated channel it is used an uncorrelated channel and the correlation matrices $\mathbf{R_{TX}}$ and $\mathbf{R_{RX}}$. The uncorrelated channel can be the same defined in the section 2.2. In order to obtain the correlated matrix for the channel is used the next equation,

$$G_{cor} = R_{RX}^{1/2} G R_{TX}^{1/2} \tag{2.3}$$

It will be seen in the chapter 6 that the correlation matrix can be used to obtain the the missing values of the CSI matrix.

Chapter 3

Capacity

In this chapter it is discussed the difference between capacity and throughput. And which parts of the project are focused on each one. It is also explained the ergodic capacity that is important in the random channels. Finally it is explained the equation of the capacity by Shannon and the equation used to calculate the throughput of each realization in the simulations.

3.1 Throughput vs. capacity

The throughput is a measurement of the performance of a channel. It depends on the amount of information that can be transmitted through this channel. Some times it is also called rate or, more precisely, information rate. As the channel properties are variable, in each realization the throughput is different. It is not the same to transmit an electromagnetic wave between very near antennas in a sunny day, than between very far antennas in the middle of a storm. In the first case the amount of information that can be transmitted in a certain period of time is much more large than in the second case. The properties of the channel are different so the throughput is also different. As the throughput is changing, the performance of the communications systems use to be measured in terms of the capacity, that is a fixed parameter for each channel.

The capacity is the maximum throughput that can achieve a channel. In a more formal way, it is the supreme of the rates that permit a reliable transmission. Reliable means that it is possible to transmit information at this rate, with the averaged error probability tending to zero as the block length increases [5]. It can be expressed in this equation:

$$C = \max_{p(x)} I(X;Y) \tag{3.1}$$

The capacity is the maximum amount of information that can be transmitted between a source (X) and a receiver (Y), over all channel input probability density functions $p(x)$. The units of the capacity are bits/s but some times (and also in this project) it can be normalized for the bandwidth used. So its units becomes bits/s/Hz.

The communication companies need to have a specific value of the the amount of information that can be transmitted with their system. The same channel has a lot of different throughput but it has just only one capacity. This is used to compare different systems, or to offer a QoS to the clients of this companies. The capacity is a theoretical limit that can be used to know how good is a communications system. Sometimes it is also a goal for the scientists working in this field. They know that the capacity of the channel is the best it can perform, so they try to find a way to achieve this capacity. Now a days there are some studies using LDPC codes that had achieved data rates very near to the capacity of the channel. Other researchers are working in new channels with higher capacities.

3.2 Ergodic capacity

The capacity that has been explained in the previous section is a theoretical view of the capacity. But there are other channel models that can have a different description of the capacity. It doesn't mean that one is correct and the other not. These are just different points of view of the same thing. If we think that each realization of a channel is, in fact, a channel by it self, as the channel is changing over the time the capacity can be seen as a random variable. And as such, it can be described with the same parameters as the random variables. For example it has a mean and a variance. The ergodic capacity can be described as the mean of the capacity random variable. Remember that ergodic means that a long number of samples of the channel realizations has a distribution similar to the statistical distribution of the channel.

It can be considered two scenarios. One is that the channel variations are much faster than the duration of the transmission. It implies that a single transmission experiences all the possible channels. In this case the time averages during the transmission are equal to the statistical averages over all the channel's properties. So the capacity of this system can be approximated by the ergodic capacity. As the channel considered in this work is varying slowly over the time don't fits the first type of channels. In this case it can be considered that the channel remains constant during the transmission, so each transmission has a different information rate. So the capacity can be modeled as a random variable [5]. For this reason in the simulations it can be obtained the cdf or pdf of the capacity, as well as the averaged capacity.

In the results of the simulations it will be shown an averaged capacity of a specified number of realizations. If the number of realizations is large enough the result is very similar to the ergodic capacity. Because the ergodic capacity is the statistical mean. So it is considered an average of infinite number of realizations. It is very important to perform a high number of realizations, because if several systems are compared, the averaged capacity should be as similar as possible, to make a fair comparison.

3.3 Shannon equation

The first person to talk about the channel capacity was Claude E. Shannon. On 1948 his work on telegraphic communication lead him to establish this parameter of the performance of a channel.

> The Shannon's second Theorem says: *"Reliable communication over a discrete memoryless channel is possible if the communication rate R satisfies $R < C$, where C is the channel capacity. At rates higher than capacity, reliable communication is impossible."*

This is one of the most important theorems in communications theory. Since it was announced has established the goal of the majority of studies in this topic. Every day the communications systems becomes more close to the channel capacity. But there is still a lot of work to do.

The Shannon-Hartley theorem says that the capacity of an ideal band-limited, power-limited, additive white Gaussian noise channel is given by

$$C = Wlog(1 + \frac{P}{N_0 W}) \tag{3.2}$$

where W denotes the bandwidth, P denotes the signal power, and $N_0/2$ is the noise power spectral density. The capacity C in this case is given in bits/sec if the logarithm is in base 2 or in nats/sec if it is used the natural logarithm.

If it is analyzed this equation it is easy to see that the bandwidth has an important role. On one hand the larger the bandwidth, the larger the capacity. This explains the big fights between communications operators to obtain more bandwidth. It can be said that the bandwidth is a limited resource and it has to be shared. So they try to obtain as much bandwidth as possible to increase their capacity offers to the users. On the other hand, the larger the bandwidth, the more noise power will be introduced in the system. If it is considered white noise, its power is directly proportional to the bandwidth.

There is a performance parameter that is called spectral-efficiency, that measures the capacity of a channel given a specific bandwidth. It is important to achieve a high spectral-efficiency to use as much as possible a limited resource. If several communications companies have the same bandwidth, the one with a system with better spectral-efficiency, has more capacity to offer to their users. Now a days there are some studies that conclude that it is not possible to have spectral-efficiency and energy-efficiency at the same time [6]. So as the energy is becoming a limited resource of the earth, the communications will tend to be more energy-efficiency and less spectral-efficiency. And this is a important decision for the communications companies to become more respectful of the earth resources.

Another parameter that is important to take into account is the SNR. It expresses the ratio between the signal power and the noise power. In the equation 3.2 it can be seen as $SNR = \frac{P}{N_0 W}$. The interpretation is that if the signal power is much larger than the noise power, the shape of the transmitted symbol is not distorted by he noise, so it is easier to obtain the transmitted information, and the transmission is more reliable. Otherwise, if the signal power and the noise power are equivalent, the received signal is very distorted by the noise and the communication becomes slower.

As explained in the previous section, the capacity can be normalized for the bandwidth. In such case and using the definition of SNR showed in the previous paragraph, the capacity equation 3.2 becomes

$$C = log(1 + SNR)[bits/sec/Hz] \tag{3.3}$$

This is a very basic channel model for SISO systems. But if the channel model changes the equation has to be adapted to the new channel conditions. The model used in this project is MIMO. So the capacity will be computed in a variation of (3.3). For time-varying MIMO channels there are multiple Shannon-theoretic capacity definitions.

3.4 Capacity of MIMO Channels

The equation for the channel capacity could seem very simple, but it has to be adapted for each system. The idea is always the same, to perform the logarithm of the relation between the signal power and the noise power. But there are some specific things in each system that has to be taken into account.

The first thing that is different in the MIMO system studied in this project is that there are other users in the communication. The channel model is a MIMO broadcast multiuser system. So the communication is performed at the same time to a lot of users. And the information is transmitted from the base station, to the users. And it is used the spatial diversity. So the base station has to transmit all the information for all the users at the same time. The result is that the information from one user can be received for other user and it becomes an interference. The information for all the users get mixed with the information for the receiving user and due to this, it is more difficult to obtain the correct information.

As the channel resources (time, frequency) are the same for all the users it is very difficult to eliminate these interferences. It can be seen as a noise added to the original signal. This has not the same properties as the noise described in the capacity equation in the previous section 3.3, but it is affecting the capacity in the same way. So it can be added to the equation as an additive noise. For this scenario instead of using the SNR, it will be used the $SINR = \frac{P}{I+N}$. Where it is added the power of the interference signal to the noise.

It is important to notice that in the equation 3.3, if the power is increased the capacity is increased. But in the scenario studied, if all users increase their power, the interference power (I) also increases, so the capacity of other users is reduced.

The second thing that is different in the MIMO system studied in this project is the notation. As the communication is performed from lots of antennas to lots of users at the same time, it is used a matrix notation to express all the transmissions. As it is shown previously, the channel is modeled as a matrix G and the communications equation is $y = G * s + n$.

As it will be explained in the chapter 5, the signal is precodified before the transmission. And the output power of the transmitting antennas is determined by the precoding matrix W. Where the columns are the information for each user, and the rows are each transmitting antenna.

$$\mathbf{W} = \begin{pmatrix} w_{11} & w_{12} & \dots & w_{1n} \\ w_{21} & w_{22} & \dots & w_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{m1} & w_{m2} & \dots & w_{mn} \end{pmatrix}$$

where w_{ij} is the multiplying weight of the information for the user j transmitted by the antenna i.

So to obtain the power of the signal received for a specific user it is done as,

$$P_n = |GW_{nn}|^2 \tag{3.4}$$

Multiplying the matrices G and W it can be obtained an other matrix A. A is a $N \times N$ matrix where a_{ij} is the signal energy of the information going to user j and arriving at user i. So to obtain the information energy that has been successfully transmitted to the user n it has to be seen the value a_{nn} of the matrix.

Then adding the other values of the row n it can be obtained the interference energy coming from other users to user n. It can be computed as,

$$I_n = \sum_{i \neq n} |GW_{ni}|^2 \tag{3.5}$$

If it is considered an scenario with a normalized noise power. Because it has been explained that the important parameter is the relation between the signal power and the noise power. Then the SINR can be obtained as,

$$SINR = \frac{|GW_{nn}|^2}{1 + \sum_{i \neq n} |GW_{ni}|^2} \tag{3.6}$$

In [7] it can be seen that the same method is used to calculate the SINR.

Then it can be used the equation 3.3 to obtain the capacity of the studied channel.

$$C = log(1 + \frac{|GW_{nn}|^2}{1 + \sum_{i \neq n} |GW_{ni}|^2}) \tag{3.7}$$

Chapter 4

Channel State Information

In this chapter it is explained why it is so important to have a good estimation of the channel and how it is obtained the CSI. And also how is this CSI used to improve the transmissions of the MIMO systems.

4.1 The importance of the CSI

In the previous chapters it has been explained the nature of the channels used for mobile communications. One of the most remarkable properties is its randomness. And it is important to know that it makes the communication much more difficult. To don't know how the information will be modified during a transmission, and also that this modification is different in different periods of time, gives to the received signal a certain randomness. There exist some way to obtain the information of the behavior of the channel, and this information can be used to improve the information transmission. The instantaneous channel gains are called channel state information (CSI). If the receiver and the transmitter can know the CSI they can adapt the transmitted signal to the channel to increase its performance.

As it has been explained in the chapter 2, in the studied system the channel is modeled as a matrix G. So the CSI is also a matrix with the same size as G. If the CSI is known or used at the receiver it can be called CSIR, otherwise, if it is known or used at the transmitter, it is used to be called CSIT. Depending on the system resources and design, it can be several kinds of CSI. All this situations lead the system to have different performances. And is the work of researcher to analyze and discover how this affects the capacity or other parameters of the communication. It is very important also to know in which channels can be applied each kind of CSI model.

Perfect CSIT or CSIR means that the transmitter or the receiver knows all the components of G instantaneously and without errors. This is an ideal scenario where the performance of the system can be increased very much. This is in fact, impossible to achieve because the transmission should be instantaneously, the precision of the electronics should be infinite, and some other hypothesis that in the reality are impossible. But there are more realistic approaches. For example it can be considered that the CSI is incomplete. It means that only a part of the channel is known. It can be due to some user not knowing its channel. Then a whole row of G is missing. Or due to some resources constrains in the system that don't let know an specific amount of information of the channel. The imperfect CSI is also a very common problem. It can be originated because an error on the obtainment of the CSI or a quantification error. In the next section 4.2 it is explained how the CSI is obtained. But even without knowing this, it is obvious that the process can have some errors. And also is easy to think that the information from the channel gain has an analog value, but as the studied system is digital, this information has to be discretized and there is an approximation error.

Another kind of CSI is the non instantaneous. This is a very usual problem in the CSI. Because the CSI has to be transmitted from the receiver to the transmitter and the time used for this transmission makes the information to be transmitted not instantaneously, i.e., the channel has changed when the CSI arrives at the transmitter side. It can be analyzed in terms of the coherence time of the channel. If the channel varies very fast the CSI will be outdated also very fast. So there will be a big difference between the CSI used and the real channel properties. On the other hand if the coherence time is very large, the channel will remain similar during a long period of time. So the CSI will be up to date longer. To solve this problem, the CSI can be obtained more often, in this way the information will be updated. If it is obtained the CSI more often, it needs to be used more time resources. And this resources can not be used for transmitting information. So there is a trade-off between the obtainment rate of the CSI and the time resources wasted (not used for user data).

The same reasoning can be done for the coherence bandwidth. There is a trade-off between how many frequencies are used to obtain the CSI and the frequency resources not used for sending user information. The more information is get from the performance of the channel the faster you can transmit the user information, but you have less resources to transmit it. As the CSI can not be obtained for all the frequencies used, it will be interpolated. Some samples of the channel will be obtained and the rest of the channel will be decided by an interpolating filter. It can be also analogized in terms of the coherence bandwidth of the channel. If the channel has similar behavior over all frequencies, or the bandwidth of the system is small enough, then it is not need a deeply knowledge of the channel. With a small number of samples it is easy to obtain the remaining of the channel. But otherwise, if the coherence bandwidth is small, the channel varies a lot over different frequencies and it is needed more frequency resources to have a good estimation of the channel.

In a complete CSI scenario, the users transmit all the CSI to the transmitter. But the studied scenario is an incomplete CSI scenario, so there are some users that do not transmit all the information of its channel. For some practical reason it has been considered that each user at least transmit one path of its CSI. The reason for this is explained in chapter 6. It is a model that fit the reality, because to send the CSI from the receiver to the transmitter is used a reverse link and some times this link is slow or with small capacity. Then the information that can be sent is limited and some times it is not possible to feedback all the channel information but just one path or a few. To consider the worst case, the simulations had been performed with only one path for such a user. The reason to do that is because the zero forcing precoder has to invert the matrix of the channel, and if there is one user that transmit no CSI and this is assumed using a linear combination of other path, the CSIT matrix become rank deficient and can not be inverted. In addition to this, it is used a kind of assumption that uses the spatial correlation between the antennas to perform the assumption, so it need at least one value of the channel for the users that has incomplete CSI. This scenario is not very strange because the transmission of the CSI from the receiver to the transmitter is done by a reverse link. Some times this reverse link has severe limitation on the bandwidth so it is difficult to transmit all the values of the CSI. But is no difficult to transmit just one value of the channel's paths. An example of the CSIT could be like this

$$\mathbf{G} = \begin{pmatrix} g_{11} & g_{12} & g_{13} & g_{14} & g_{15} & g_{16} \\ g_{21} & g_{22} & g_{23} & g_{24} & g_{25} & g_{26} \\ \emptyset & \emptyset & g_{33} & \emptyset & \emptyset & \emptyset \\ g_{41} & g_{42} & g_{43} & g_{44} & g_{45} & g_{46} \\ \emptyset & \emptyset & \emptyset & \emptyset & \emptyset & g_{56} \end{pmatrix}$$

In this example it can be seen that the users 1, 2 and 4 transmit all the CSI but users 3 and 5 just transmit the information only for the path from antenna 3 and 6 respectively.

4.2 How the CSI is obtained

To obtain the CSI,it is required to know the behavior of the channel. The objective is to know how the channel modify a certain transmitted signal. If this information is known at the transmitter the signal can be shaped in a way that when being corrupted by the channel the result is the proper signal for the receiver. On the other hand, if this information is known at the receiver, it can calculate which signal has been sent that at being modified by the channel the result is the received signal. In order to do this, the easiest way is to send a known signal by the receiver and analyze how this signal has been modified. If the receiver know which is the transmitted signal, can compare it with the received signal and find the differences. Using this technique the receiver can know the behavior of the channel. The signals transmitted are called pilots.

The properties of the channel are obviously depending on the frequency. The different wavelength will interact in a different way with the objects forming the transmission path. And also the phase shift depends on the wave length. So what for a specified wave length the signals from two different paths can be added destructively, in a different wave length this addition can become constructive. The conclusion is that the behavior of the channel can not be described using a single parameter, but it need a different parameter for each frequency. And also it is deduced that it will be necessary to transmit one pilot for each frequency. As the resources occupied by the pilot signals can not be used to transmit any information, it is not possible to send one pilot for each frequency. There is a trade-off between the number of pilots and the information sent. If a lot of pilots are sent, the knowledge of the channel is better and it can be performed a better communication. But a lot of frequency resources are used by the pilots and very few information can be sent to the user.

Only some pilots are sent for each channel, so only a few frequencies behavior can be determined. The other frequencies behavior is obtained through interpolation filters. There is a trade-off between how much is it known the channel and how many resources are free to sent information. If the channel is smooth, i.e., has a large coherence bandwidth, the information obtained through the interpolation is better than if the channel has strong variations over the frequencies. In the first case it can be assumed a communication with a very narrow bandwidth. If this bandwidth is smaller than the coherence bandwidth the communication channel has very few frequency dependence. The second case can be a system with a large bandwidth. So the channel has strong variations in this bandwidth.

The transmitter can not use this techniques to acquire the CSI. So it has to use an indirect way. Receiving this information from the receiver through a reverse channel (closed-loop method), or using the hypothesis of the reciprocity of the channel (open-loop method). If it decides to receive the information from the receiver, the main problem is the time that it takes. As explained before, if the coherence time of the channel is small, during the feedback of the information the channel can change, so the CSI is no more up to date. An other problem is the resources used to transmit information. It can be that the link from the receiver to the transmitter is no good or is very used, then it just can be sent a part or none of the CSI. On the other hand if it is used the reciprocity of the channel, is assumed that the properties of the channel from the transmitter A to receiver B is identical to the transpose of the channel from B to A. This model assumes that both transmissions are held in the same time, frequency and antenna positions. In the reality this is not practical, even if the link is full-duplex the forward and reverse transmissions can not share the same frequency.

In the present project, it has been used an specific CSIT. As the zero forcing precoder need to invert the matrix, if the matrix is rank deficient this inversion can not be performed. So if one user don't transmit any path of its channel and the assumption of the CSI is a linear combination of the other users information the CSIT can become in a rank deficient matrix. Also there is some kind of CSI assumptions that uses the correlation between the channel's paths for one user. So at least it needs to have one path for each user to perform the correlation and obtain the other paths.

4.3 Using the CSI to improve the transmission

As the channel used for mobile communications use to be frequency selective. It means that its performance depends on the frequency in a random way. To transmit information through a random channel is not easy. As you do not know how your information will be modified by the channel, it is difficult for the receiver to obtain the transmitted information. It means that the capacity of the channels is random and unknown. This is not convenient for the communication, because do not let it to decide which amount of information transmit in a reliable way. The use of coding has been demonstrated to increase the channel capacity. But it needs to know the characteristics of the channel. There are two ways to increase the capacity of a channel using the CSI. One is to pre-code the signal at the transmitter side, and the other way is to equalize the signal at the receiver side. Both can be down at the same time, increasing the performance of the channel.

The idea at the receiver is to process the received signal to make it more similar to what was transmitted. It is demonstrated that the optimum demodulator can be realized as a filter matched to the channel, followed by a sampler operating at the symbol rate and a subsequent processing algorithm for estimating the information sequence from the sample values. This is called the maximum-likelihood (ML), if the symbol at the transmitter are considered equiprobable, or maximum a posteriori probability (MAP) if not.

To improve the transmission using the CSI at the transmitter, is done by pre-coding the signal. The goal of this project is to analyze the performance of the pre-coders. The objective of using pre-codification is to allocate the information energy to the antennas in a way that maximize the transmitted information. For example the precoder can minimize the interference between the users and increase the SINR and the capacity of the channel. Or use the better paths to transmit more information than the deep fading paths.

Chapter 5

Precoders

In this chapter it is discussed three different linear precoders, the zero forcing, the matched filter and the MMSE. There is an explanation about non-linear precoding and linear precoding. Finally it is also explained how this precoders improve the communication using the CSI and how the equation for this precoders is obtained.

5.1 Properties

The precoders is the last block that processes the signal before the transmission. A linear precoder acts as a combination of a multimode beamformer and an input shaping matrix, matching each side to the channel and to the input signal structure, respectively. As it has been shown in the chapter 1 the shape of the signal is completely determined before this block, as well as the bit codification and all the signal processing needed before the transmission. So the goal of this precoders is to adapt the transmitted signal to the channel through which it will be transmitted. In this project the scenario taken into account is a MIMO system, so the transmitter has several antennas. The precoder is what decides how the signals are distributed among this antennas and the power that the signal has in each antenna. Note that the scenario is a MIMO multiuser, so there are as much information beams as users. To exploit the spatial diversity all the information for an specific user is sent through all the antennas. So the most important value that has to determine the precoder is the weighting of each signal stream. Here weighting is referred as the gain and the phase of the signal for a specific user in each antenna.

Using the techniques explained in the chapter 4, the precoder can have the information of the channel properties with the CSIT. Using this information it can adapt the transmission to the channel to increase the capacity of the system. The precoder will adjust the transmission to the channel that it knows. If the information that is using the precoder don't corresponds to the real channel, this increment of the capacity can be less important than if it is using a perfect and complete information of the channel.

Another point of view of the precoder is that it works as a beam forming. It is well known that an array of antennas can be used to create a directional beam or even decide the direction of this beam changing the phase of the transmitted signal. This is not the goal of this project, so it will not be discussed here. But it is useful to think of the precoders as a way to create a directive beam pointing each user information stream to the specific user. In this way the energy is used much more efficiently than in old systems. This old systems needed to send the information omnidirectionally because they couldn't know were was the receiver so they needed to have coverage over all the cell. In MIMO systems the information can be sent in the direction of the user, so there is no need to waste energy in other directions. It can be said that MIMO systems is a way of green communications.

Precoding is also helping to reduce the inter-users interference. Because in old systems all the users could receive the information sent to other users and it was a big source of interference. In fact, the signal with the information for one user was spread all around the cell to obtain the maximum coverage area. As a result of this, all the antennas placed inside the cell where receiving this signal as an interference. And also antennas outside the cell received interference from this user. But using this precoders the energy is focused on the desired user and the other users hardly receive any interference. So the output power at the receiver can be maximized. The precoders can also be designed to maximize the SNR.

To design the precoders it can be used different methods. There are some constraints that are important to incorporate at the designing equations and the goals of the precoder are also important. An important constraint to use as an hypothesis of the design is the power used. The SNR parameter explained in the chapter 3, increases when the power increases. So if there is no limitation in the power used, and the goal of the precoder is maximize the SNR, the output of the precoder will be infinite power. Obviously this is not feasible, so it must be some power constrains involved in the design of the precoder. This power constrains can be a total power transmitted, or a per-antenna power constrain. If it is used the first method, all the power is distributed among the antennas in the best possible way. But some times it can involve one antenna transmitting a very high power and an other one without transmitting anything. As very often the antennas has their own power amplifiers and their own energy sources, it seams a better option to consider a specific power constrain for each antenna. In this way there is a maximum power that can be used for a single antenna but it can use less power than the allowed. The design of the precoder also has to think on which parameter of the communication has to be optimized. For example, some precoders can reduce the interference produced between different users, while other precoders can increase the SNR.

5.2 Linear precoding

The precoding is a transformation of the signal streams to be transmitted. And such a transformation can be done lineally or non-lineally. The importance of it is based on the complexity of the software that performs the precoding. In many cases it is not feasible to implement a non-linear precoder and the solution is to use a sub-optimal linear one. This is because now a days the mobile devices are becoming smaller and are used for a lot of purposes, so it means the the computational power used for the communication is limited. There is also a big constrain in the battery power of the devices. Because the increase of the battery use is bigger than the increase of the battery storage. For all this reasons it can not be implemented a non-linear precoder at the user device.

The complexity of the precoders is not an important issue in small bandwidths, as for example in WCDMA (5MHz). However, if the bandwidth is increased, for example in LTE (20MHz), the complexity of straightforward high-performance equalization starts to become a serious issue [8]. As MIMO techniques are used in large bandwidth systems like LTE in this project are used linear sub-optimal precoders. In this way, the performance of the system is reduced, but the complexity of the receiver terminals and in general of the whole system is feasible.

In this project are studied three linear precoders. Let it be zero forcing, matched filter and MMSE. Each of this precoders has different goals and different performance of the system. This will be explained in posterior sections.

5.3 Non-linear precoding

Non-linear precoders can achieve an optimization of the design parameters for the precoders. Some examples of non-linear precoders are the dirty paper coding of M. Costa [9]. Dirty coding is a technique that precancels the interference without power penalty and without requiring the receiver to know the interference. Other examples of non-linear precoding can be Tomlinson-Harashima precoding [10] [11] and the vector perturbation technique [12].

Al this precoders has a huge computational complexity and are difficult to implement in real systems, like MIMO systems. So it won't be explained in this project. It will be focused on the linear precoders.

5.4 Zero forcing

The zero forcing precoder has attracted a lot of attention in the recent years, because it has a good trade-off between performance and simplicity. It is known for having good results in high SNR environments [13]. The main characteristic of the zero forcing precoder is that it inverts the channel matrix. It means that the channel matrix has to be full rank. This is very important when there is incomplete CSI. If a user do not transmit any information of its channels, a whole row of the CSI matrix have to be fulfilled. If it is done using a linear combination of the others users information (using the other rows of the matrix), the CSI matrix become a non full rang matrix and it can not be inverted. In this project it will be assumed that all the users transmit at least one value of the CSI, so the CSI matrix will be full rank.

As the number of transmitting antennas use to be different than the number of receiving antennas, the channel matrix use to be non-squared. It means that it is no possible to perform the inverse of the matrix and the zero forcing precoder has to use some pseudo-inverse. There are several ways to perform a matrix pseudo-inversion. And to use the proper one to perform the zero forcing codification is very important to achieve the desired results. As can be seen in [7], in the total power constraint design, the use of pseudo-inverses obtains an optimal result. But if the design is done with per-antenna power constrains, then the use of pseudo-inverses is not necessarily optimal and is needed a more detailed study of the particular case.

In [14] it can be seen the procedures to obtain the equations of the linear precoders studied in this project. In the following sections it is summarized this equations.

The equation that has to be optimized to find the zero forcing precoder is to minimize the output of the transmission, subject to the signal received having no interference from other users. Then it is applied the total power constrain ρ.

$$W = \arg\min_{W} E(||Wx||^2) \tag{5.1}$$

Subject to

$$GW = I \tag{5.2}$$

The solution is

$$W = \beta G^H (GG^H)^{-1} \tag{5.3}$$

and applying the power constrain

$$\beta^2 tr(W'W'^H) \leq \rho \tag{5.4}$$

it is obtained

$$\beta = \sqrt{\frac{\rho}{tr(W'W'^H)}} \tag{5.5}$$

W is the precoder. The input signal to the block is x and the output is s. G is the channel matrix and G^H is the transposed conjugated of G. W' is W without the factor β.

5.5 Matched filter

The precoder matched filter was introduced by R. Esmailzadeh and M. Nakagawa in [15]. They called it a Pre-RAKE coder because they used the idea of the RAKE decoder to code the signal previously to the transmission. The idea of a matched filter is to correlate a known signal with an unknown signal. It maximizes the power of the desired signal at the respective receiver. The matched filter is derived by maximizing the ratio between the power of the desired signal portion in the received signal and the signal power under the transmit power constraint.

The equation to be optimized to obtain the matched filter is

$$W = \arg\max_{W} \frac{E(||x^H \tilde{y}||^2)}{E(||n||^2)} \tag{5.6}$$

The solution to this equation is

$$W = \beta G^H \tag{5.7}$$

Then it can be applied the total power constrain

$$E(||Wx||^2) \leq \rho \tag{5.8}$$

Solving this equation it can be found

$$\beta = \sqrt{\frac{\rho}{tr(G^H G)}} \tag{5.9}$$

$\tilde{y}$ is the received signal without noise.

5.6 MMSE

The goal of this precoders is to minimize the mean of the squared error. The error here is considered the difference between the signal that wants to be transmitted and the received signal. It can also be called the Wiener filter [16].

The equation to be optimized to obtain the MMSE precoder is

$$W = \arg\min_{W} E(||x - \beta^{-1}\tilde{y}||^2) \tag{5.10}$$

And it is subject to the power constrain

$$E(||Wx||^2) \leq \rho \tag{5.11}$$

The solution is

$$W = \beta F^{-1} G^H \tag{5.12}$$

$$F = (G^H G + \frac{N}{\rho} I) \tag{5.13}$$

$$\beta = \sqrt{\frac{\rho}{tr(F^{-2} G^H G)}} \tag{5.14}$$

The equation shown for the three linear precoders are used in the simulation code to obtain the capacity of the channel.

Chapter 6

CSI assumptions

In this chapter it is studied how to estimate the missing values of an incomplete data, i.e., incomplete CSI. This estimations are called assumptions some times during this project. And it is analyzed the loss of capacity due to this missing data. Finally it is explained all the assumptions studied in this project.

6.1 The need of assumptions

In the previous chapters it has been explained the importance of the CSI to improve the performance of the channel used to transmit information. And this is of a capital importance in MIMO systems because uses the channel properties to increase the capacity. So if there is any error in the CSI this is expressed in a reduction of the channel capacity. This errors can be due to a temporal delay, an imprecision obtaining the channel information, etc. And when this information is delivered to the transmitter to be used at the precoder this errors are unknown and the precoding is not as good as it should be.

In other situations the problem can be worse. Sometimes the CSI can be incomplete. It means that there are some parts of the channel matrix that are unknown. So the precoder can no be used because do not have all the information of the channel. This can happens because the CSI is obtained at the receiver as it is explained in the chapter 4, so it has to be sent back to the transmitter to perform the precodification. So the result is that a huge amount of resources are needed to feedback this information. And also it is needed a reverse link from the receiver to the transmitter. In general this is not a problem, because most of the communications links are full-duplex. But it can be that this reverse link is used for other purposes so the information of the channel can not be sent through this link.

This scenario does not mean that the transmitter do not have any information of the channel. It can be that some users can transmit their information and some others can not. Or even that an user can transmit just a part of the CSI because of throughput constrains in the reverse link. For this reason in this project it is studied the performance of the channel when using incomplete CSIT.

When the precoder has this incomplete CSIT, it is a matrix with some values known and other values unknown, it can do two things. The first one is to not perform any precoding. It means that the MIMO techniques can not be used and instead it is performed an "old" transmission using other techniques. The second thing that can be done is to obtain the missing values of the matrix through other ways, different from the reverse link of the receiver. It can be said that the missing values of the CSI are assumed. The information to obtain this values can come from several sources. For example if the channel has some correlation and the transmitter knows this correlations, it can be used the known values of the CSI to obtain the unknown values, because there is a correlation between them. In the case that there is no correlation in the channel, the information can be obtained with statistical properties, or assuming that there is no noise in the channel.

6.2 Channel without spatial correlation

There are several channel models that can be used in a communication system. A model is just a representation of the reality so it can be adjusted to the necessities of the system. The more close to the reality the more complex it becomes. A very simple model is to think that there is no correlation in the transmission paths. This model makes the design of the system more easy, but is no very close to the reality. This is the first channel model used in the simulation of this project.

In order to fulfill the missing CSI there are two groups of assumptions. On one hand, there are the assumptions that do not use any previous information. These are the simplest assumptions, because do not involve any training of the system or any transmission of information. This assumptions can be applied to all the systems the same way and do not change. This assumptions also do no change during the communication. In this case, the precoder uses the programmed algorithm to obtain the complete CSI from the incomplete matrix. But this simple assumptions are not very good because are not close to the real channel. So it does not help very much to adapt the transmission to the channel.

On the other hand, there are the assumptions that use some known information. The information used for this precoders, can be some correlation between the channel paths or between the same path, e.g., spatial correlation, time correlation. This assumptions are more complex because are using some information specific of the system where are implemented. This information has to be provided for the operator or the system manager, or obtained automatically by the system. So it involves more complexity. This kind of assumptions are more close to the reality so it means that its performance is much better.

In this project has been used two assumptions of the first group, no previous information required. To obtain this assumptions it can not be used any previous information so the algorithm do not need any extra data. The assumptions used are not based in any mathematical result. It has been obtained out of the intuition. And after it, as been performed some simulations and the results has been analyzed.

One of this assumptions is to assume a perfect channel. It can be called ones assumption because it fills the gaps in the CSI matrix with ones. It is easy to see that this assumption does not use any known information, and its algorithm is very simple. But this assumption is far from the reality because it does not contemplate the noise or the destructive phase shift of the signal. Another assumption is to use the mean of the paths of each antenna to get the gain of the path between this antenna and the incomplete CSI user. If all the users except one transmit the information of the path coming from this antenna there are a lot of values to average, if not there are less values. As the channel is considered Gaussian with zero mean, if a lot of values are used to assume the missing value, the result tends to be zero. It means that no information can be transmitted through this path, so the capacity is reduced. So this assumption is really bad.

In the second group, it has been programmed two assumptions, one for the uncorrelated channel model and the other for a spatial correlated channel. The first assumption of this group is to generate the missing CSI with random numbers with the same distribution of the channel. The information that is using this assumption is the statistical properties of the channel. this statistical properties has to be known for the specific channel where the communication takes place and has to be programmed in each system individually. A more complex algorithm could obtain the properties of the channel using the previous information of the CSIT. As the results of this assumption will be random, the performance of this assumption also will be random.

6.3 Channel with spatial correlation

As it has been explained before, the channel can be modeled as a correlated channel. This approach is more close to the reality than the previous one. But is more difficult to implement. In the chapter 2 it is explained how to generate a correlated channel. In this assumption it is used the information of the correlation to obtain the missing values if the channel.

In this project it is easy to obtain the information of the correlation of the channel. As the channel is generated by an algorithm, an the correlation matrix is also generated by another algorithm, it can be used this information to produce the assumption. In a real system it would be more difficult to obtain this information, because the correlation of the channel is determined by the position of the antennas and the objects around the base station. So there is no correlation matrix known at the transmitter. The solution could be to analyze each system and obtain this parameter or to design an algorithm to obtain this parameter by itself. For example, using the information of the CSI.

The specific configuration of the system can modify the properties of the channel as well as the properties of the correlation. In the studied scenario, the transmitting antennas are close, so there is a correlation between them, but the users are spread in the cell, so there is no correlation between them. The result obtained is that all the path for the same user are correlated, but the paths for different users are uncorrelated. If a user do not transmit his CSI, there is no correlation with the other users to be exploited. So the project is focused in the situation where a user transmit, at least the information of one of the paths of the channel. In this way, there is a correlation between this and the missing information. And this correlation can be exploited to get a better communication. This can be seen in the figure 6.1

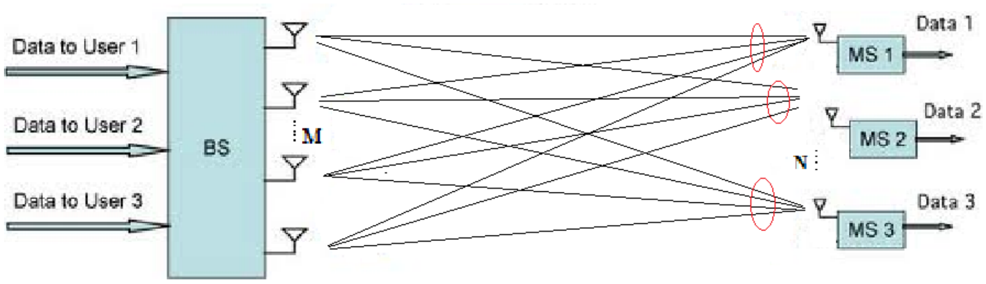

FIGURE 6.1: Channel correlation in the transmitting side but no correlation at the receiving side.

To perform the assumption it is multiplied the CSIT with the correlation matrix. This is an example of the result.

$$G * R_{TX} = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ \emptyset & g_{22} & \emptyset \end{pmatrix} \begin{pmatrix} 1 & r_{TX} & r_{TX}^4 \\ r_{TX} & 1 & r_{TX} \\ r_{TX}^4 & r_{TX} & 1 \end{pmatrix}$$
$$= \begin{pmatrix} g_{11} + g_{12}r_{TX} + g_{13}r_{TX}^4 & g_{11}r_{TX} + g_{12} + g_{13}r_{TX} & g_{11}r_{TX}^4 + g_{12}r_{TX} + g_{13} \\ \emptyset + g_{22}r_{TX} + \emptyset r_{TX}^4 & \emptyset r_{TX} + g_{22} + \emptyset r_{TX} & \emptyset r_{TX}^4 + g_{22}r_{TX} + \emptyset \end{pmatrix}$$

This system has 2 users and 3 antennas. The second user just transmit the path from the second antenna. To assume the missing path from the first antenna it is used the value $g_{22}r_{TX}$. To assume the path from the third antenna to the second user it is used the other obtained value $g_{22}r_{TX}$.

The spatial correlation has some drawbacks, as it has been shown. It needs at least one channel path information. But has some good things, like it don't need any memory. Because the correlation is done instantaneously. So with the parts of the CSI that the precoder receives, it can obtain at the same moment the missing information. A part from the spatial correlation there are other kinds of correlation that can be used. One option that could be to assume the CSI is to use the time correlation of the channel. This is a more useful information to be used because the precoder can obtain it from the CSI matrix that receives every time. Each channel path is correlated with the previous values of itself. So its a more reliable information and does not need any condition like that each user has to transmit at least one path. So it is more appropriate for a real scenario. In this project the channel is generated randomly for each realization so there is no time correlation to be exploited. But it could be a good way to keep the research in the future.

Chapter 7

Simulation

The simulations of this project has been programmed and executed with Matlab. The code has been written in several steps during the time the author has been working on the project, and after each part being finished the corresponding simulations has been performed. The code added in subsequent steps do not change the results of previous simulations. The simulation code can be seen in A.

In the previous chapters it has been explained the background knowledge necessary to understand the simulations and the posterior analysis. In this chapter it will be explained the parameters of each simulation and the obtained results, but not the knowledge already explained.

7.1 Structure of the algorithm

In the appendix A it can be seen the Matlab code programmed to simulate the studied system. In this section it is explained in a general way. It is shown the most important ideas and algorithms used and not the programming it self. For more detailed information please refer to the appendix A.

The code is structured in two main parts. The first is intended to calculate the sum capacity of the system, or other parameters, for example the SINR, the capacity for each user, etc. The second part is the one that plots the results, using the information obtained in the first part.

In the calculation part, first of all there are the parameters definitions. It is where the programmer can define the system to be simulated. These parameters can be the number of antennas, the SNR of the channel, number of realizations, etc. All this parameters are placed all together at the beginning of the simulation to make it more easy for the user to see the whole system simulated. All this parameters remain constant during all the realizations.

It is used a matrix to establish which user has complete CSI and which has not. The rows of the matrix contain the users, and the columns contain the paths from each antenna. The users with incomplete CSI as well as the only path information transmitted are chosen randomly to be sure that this parameter don't modify the results. This matrix is used in several parts of the code to know in which position are the users with incomplete CSI and which path information are transmitting.

Some parts of the code are used to perform the simulation varying the SNR. This parts are commented by default and only are uncommented when this simulation is performed.

Then there are three loops. The first is for the realizations. For each realization it is established a random channel (correlated or uncorrelated), so after all the realization it is obtained the performance of different channels and some statistical results can be obtained.

Between each realization there is another loop for the kind of technique used: complete CSI, assumption or dividing users. Then it is calculated the CSI. For the first technique it is equal to the channel, for the second technique it is calculated depending on the assumption chosen by the simulation, and for the third technique it is deleted the part of the channel for the users with incomplete CSI. In this loop it is also calculated the output of the three precoders using the CSI matrix.

Inside this second loop there is the third one. Which computes the capacity for each user with the output of the precoders and the channel matrix. The equation used to obtain the capacity of the channel is described in the chapter Throughput 3.

There is an algorithm that is worth to mention. It is the way to calculate the capacity for the technique of dividing the users. In this algorithm first of all the rows of the channel matrix corresponding to the users with incomplete CSI are removed. Then it is computed the capacity for the remaining users and this capacity is stored in the vector of capacities. But as some users has been removed, the user does not correspond to the position of its capacity in the vector. Note that if there are N users with complete CSI and U users with incomplete CSI, the capacities vector will have its first N positions fulfilled and the last U positions equal to 0. Then it is computed the capacity for all the users with incomplete CSI. Finally there is a loop from the last user to the first, and for each user if the matrix that says if it has complete or incomplete CSI, says that it has complete CSI it is assigned the last capacity stored in the capacity vector. If the matrix says the user has incomplete CSI it is assigned the previous calculated capacity for this case.

Finally, after calculating the capacity for each user it is calculated the sum capacity. It is done inside the loop for each kind of technique used.

The flux diagram of the calculus of the capacity can be seen in the figure 7.1. First it is generated a random channel, then it is deleted the information to have incomplete CSI. If it is needed it is done the assumption on this CSI. Then it is obtained the CSI assumed. With this information is performed the precoding. At the end, using the output of the precoder and the real channel can be computed the throughput of the system.

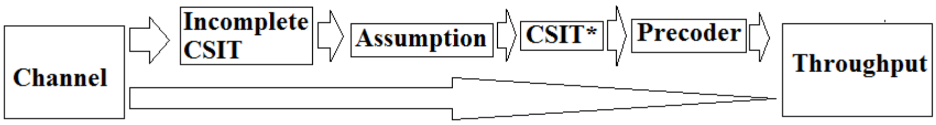

FIGURE 7.1: Flux diagram to calculate the capacity.

The other main part in the code is the plotting part. This part is divided in the code to perform each kind of plotting. By default all the code is commented and it is only uncommented the specific plot that is needed. The results to be plotted are obtained through a matrix generated by the calculating section of the code. This is a 3D matrix that one axis is for the users, an other one for the technique used, and the last for the realization. It uses specific functions provided by Matlab to perform the representation of the results.

7.2 Plotting

The results of a simulation can be seen in several ways. To choose the proper one, is fundamental to get right conclusions or to express the desired information. In this simulations has been programmed different plotting ways depending on which simulations was performed. The code for performing each simulation is written in the same file as the other code. All the simulation's code is commented and it is just uncommented the one needed in that moment.

One kind of plots is the cdf of the channel capacity. As the channel characteristics are random, the capacity is also random. So, it has some statistical properties. Plotting a cdf of the capacity helps to show this properties. For example in this plots can be seen the probability of the capacity to be under a certain threshold. It is useful for the companies to know the QoS that they can offer. This kind of plots was programmed at the initial part of the project because it was thought to be interesting to compare the statistical properties of the results. And also thinking in a more general way as a good manner to show information to the communications companies. After some time this plots did not give good results. In fact the interesting information was the averaged capacity. And also to compare different scenarios (different users capacity or different precoders) in the same plot, and it was not possible with the cdf plots. In the cdf section 7.3.2 it will be shown some examples of this plots but very few conclusions will be derived of it. It also can be plotted the pdf of the capacity.

Another kind of plots is the bars plot. It is useful to compare two parameters at the same time. The parameters are placed in the axis x and y and the capacity in the axis z. In this way it is shown how the capacity varies in both parameters. For example the axis x can be each one of the users, and the axis y can be the complete or incomplete CSIT.

7.3 Results

In this section it is showed the plots of the performed simulations and the results are analyzed.

7.3.1 SNR sweeping

The first result obtained with the simulation is the capacity of the channel for each one of the precoders while sweeping the SNR. It shows a result that is already known, but it is used to understand the performance of the precoders in further simulations. It is explained the result for the complete CSI scenario and the incomplete CSI using the three assumptions explained in chapter 6. For each kind of assumption it has been performed a different simulation, so the channel properties should different. But as the number of realizations is high enough, this properties tend to be equal. And the results can be compared.

The simulations has been done with 8 transmitting antennas, 4 users and 1 user with incomplete CSI. The total number of realizations for each simulation is 10000. And sweeping the SNR from -20 dB to +20 dB. It is plotted the sum capacity for all the users including the user with incomplete CSI.

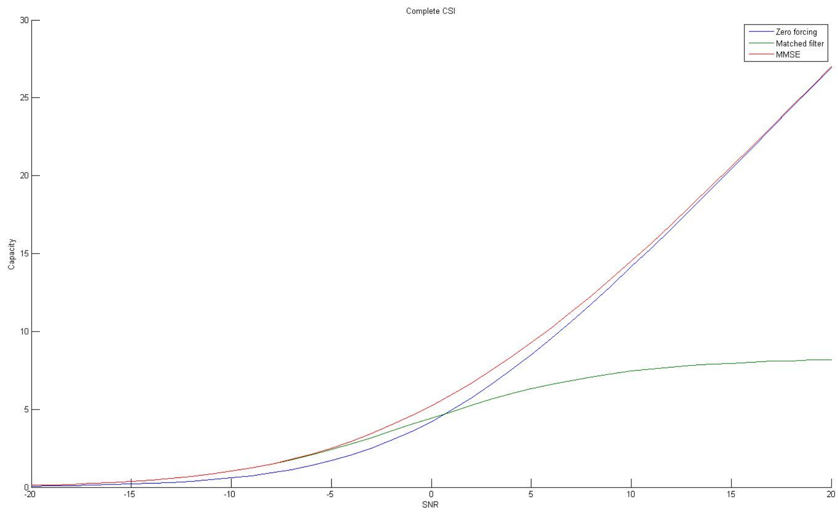

Figure 7.2: Capacity for different SNR. Complete CSI.

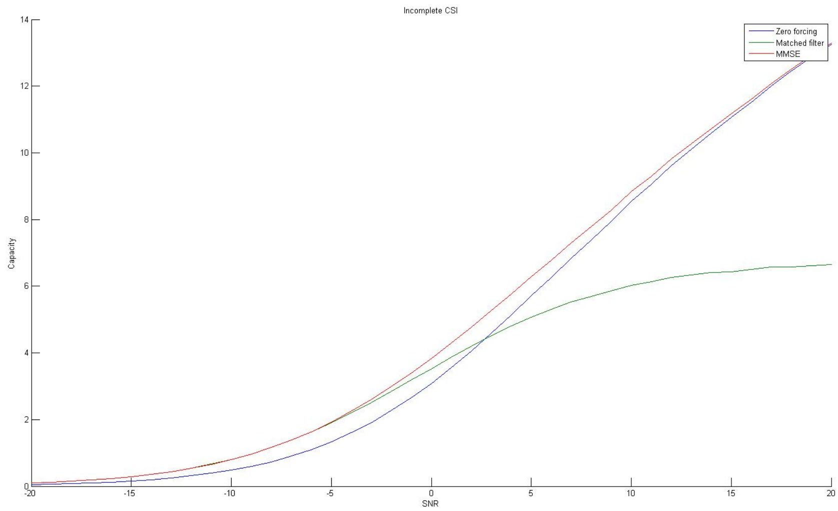

Figure 7.3: Capacity for different SNR. Incomplete CSI, with ones assumption.

A very obvious result obtained is that the capacity improves as the SNR is higher. This is a straight forward result from the Shannon's capacity formula [17].

The second thing that is obtained from this simulations is that the MMSE precoder has a better behavior in all the SNR. And for low SNR the matched filter is asymptotically the same as the MMSE, while for high SNR the zero forcing is asymptotically the same as the MMSE. Seeing this result, somebody can think that the MMSE is the best precoder and the others are not used. But it is not true because the computational complexity is very important in this kind of systems. So if it can be known in advance the SNR of a system and there are two precoders behaving the same, it will be used the simplest one.

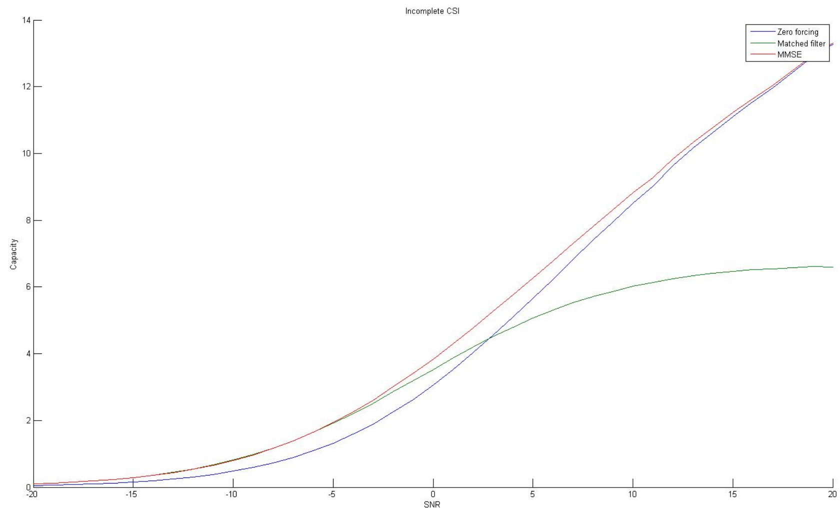

FIGURE 7.4: Capacity for different SNR. Incomplete CSI, with random assumption.

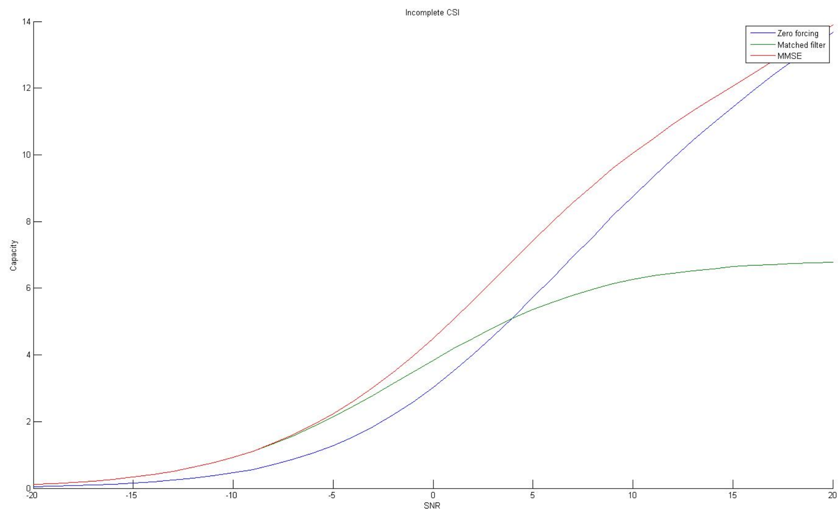

FIGURE 7.5: Capacity for different SNR. Incomplete CSI, with mean assumption.

In this way it can be reduced the cost of the device, it can use less battery, the design is easier, etc.

An other result that is easy to get from the figures is that with incomplete CSI the capacity is reduced to the half of the complete CSI scenario. The reduction of the capacity depends on the number of antennas, users with complete and incomplete CSI, etc. But it is clear that the complete CSI is the best scenario, and the others will always reduce the capacity.

Now it can be explored what happens if in such scenario the number of user with incomplete CSI increase. In the figure 7.6 it has been performed the same simulation with 3 users with incomplete CSI and the capacity is reduced significantly. It can also be seen that the performance of the three precoders tends to be the same. It means that what made the zero forcing and MMSE precoders better than the matched filter was having the CSI of a lot of users, but when this CSI becomes incomplete this precoders loose its advantage.

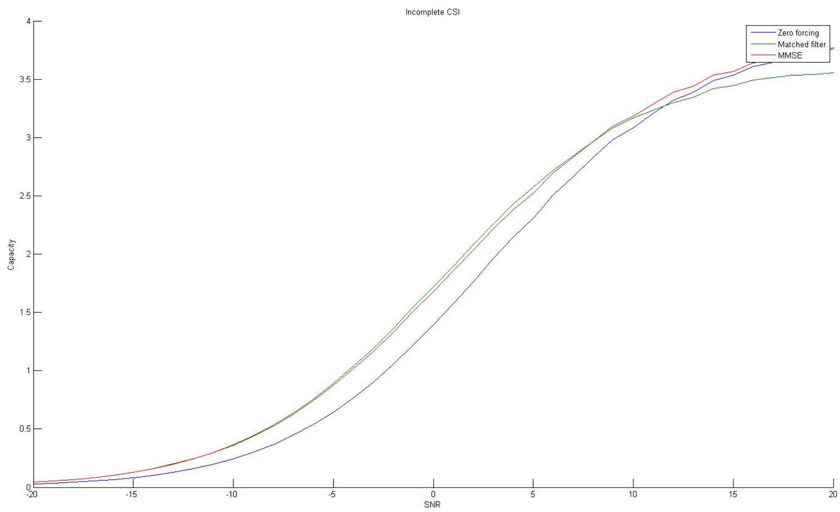

FIGURE 7.6: Capacity for different SNR. Incomplete CSI, with rand assumption and 3 incomplete CSI users.

The same simulation can be performed increasing the number of antennas and users but keeping the same proportion. The parameters of the simulation are, 80 transmitting antennas, 40 users, 10 of which has incomplete CSI. The results are shown for SNR from -20 dB to 30 dB. The result of this simulation is unexpected, and can not be explained by the author of the project. For the case that there is complete CSI the obtained capacity of the system has the same behavior that in the previous simulations. That is a well known result that does not need any explanation. But the result for the incomplete CSI, and using the means assumption to make it complete is not following the same behavior.

What it is seen is that the MMSE precoder increases the capacity of the system while the SNR is increasing, but it get a maximum value and then it decreases tending to a certain value in the limit. For the other precoders the function of the capacity is always crescent but is also tending to a certain value in the infinite limit. The three precoders tends the capacity to the same value. It will be studied more deeply in a future work.

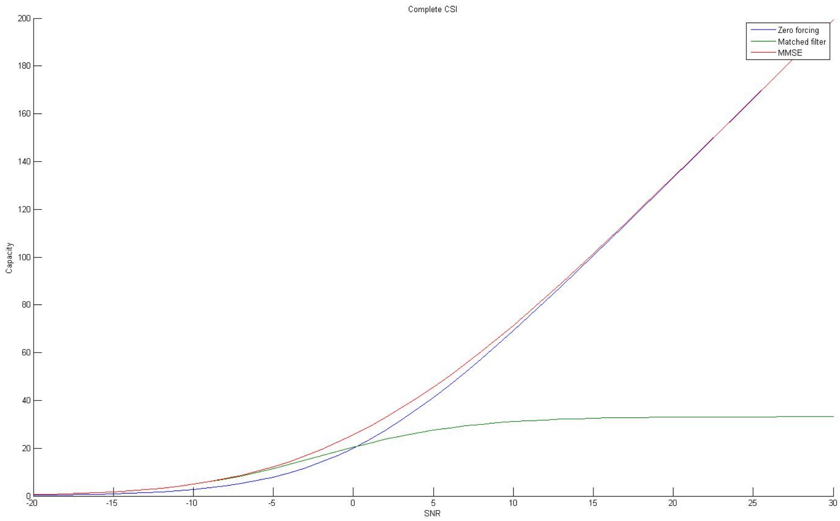

FIGURE 7.7: Capacity for different SNR. Complete CSI, with mean assumption and 3 incomplete CSI users.

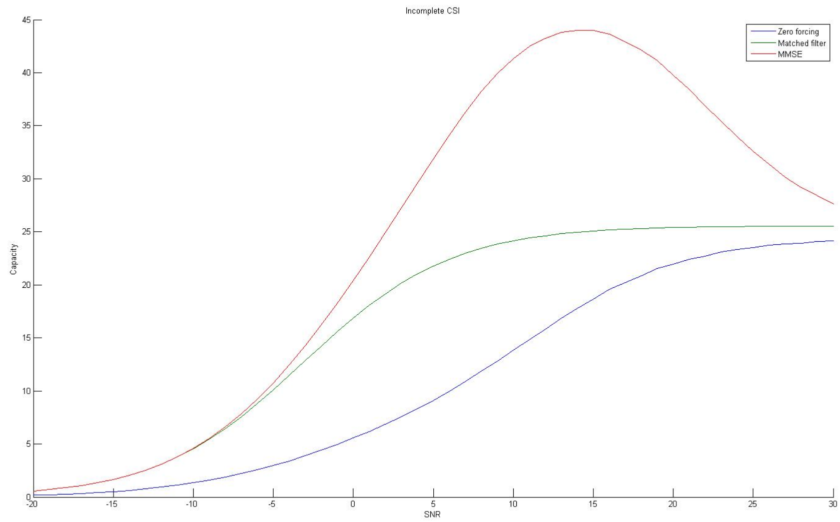

FIGURE 7.8: Capacity for different SNR. Incomplete CSI, with mean assumption and 3 incomplete CSI users.

7.3.2 cdf plot

This simulation has been performed with an SNR of 20 dB that is considered a typical SNR of the analyzed scenarios. 8 antennas at the base station, 4 users, and 1 of this users do not transmit all the CSI. The type of assumption is ones and it has been performed 10000 realizations.

It is shown the result for each one of the users. User 1, 3 and 4 has the same result. User 2 is different. So user 2 is the one that do not transmit all the CSI.

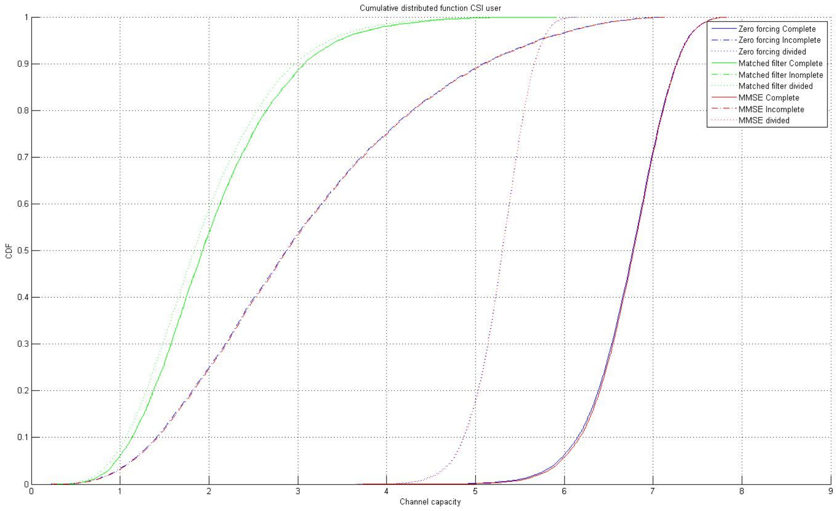

Figure 7.9: cdf of the capacity. User 1 for 3 precoders and complete CSI.

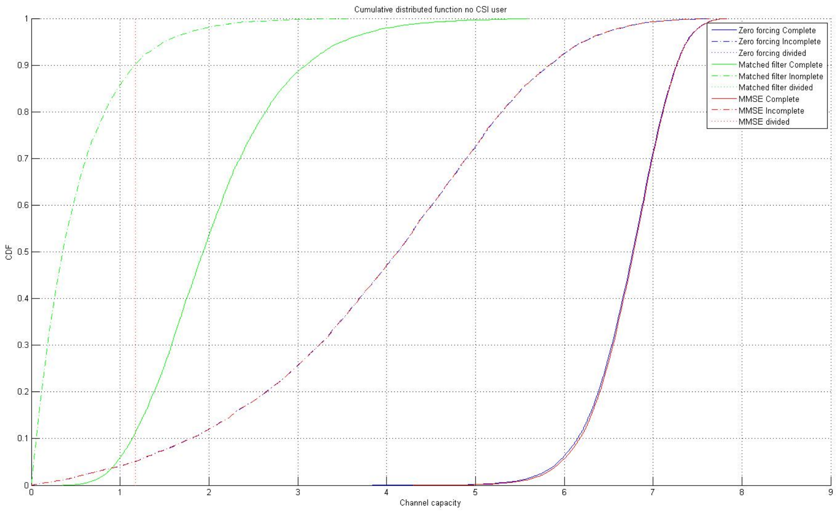

Figure 7.10: cdf of the capacity. User 2 for 3 precoders and incomplete CSI.

The main thing you can see in this plots is the slope of the different techniques. For example the complete CSI and the splitting users techniques have a steep slope. So the variance of this random variables is small. It means that the capacity is not very affected by the randomness of the channel. Even if the channel is changing very much, the capacity has a very constant value. But the assumption technique have a less steep slope. It means that the capacity is more variable.

Comparing the users with CSI and the user with incomplete CSI it can be seen that the cdf for the matched filter and the assumption technique have the same shape for all the users. It is only shifted to lower capacities. It means that the randomness properties

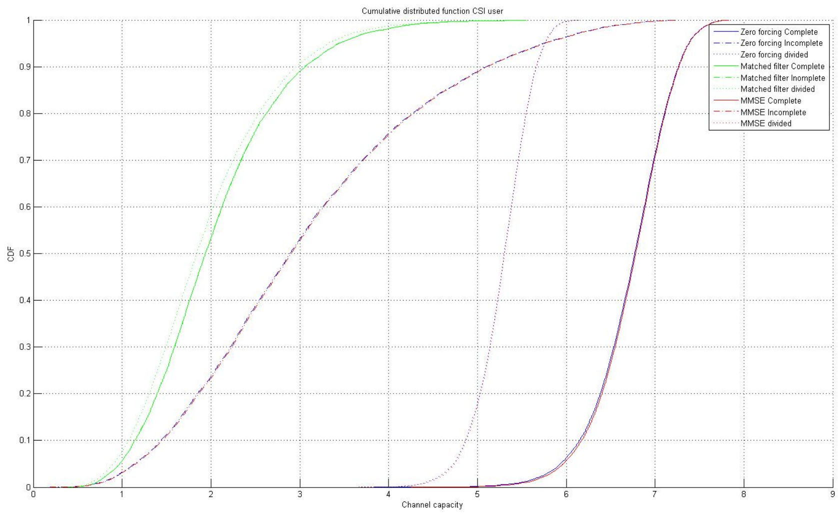

FIGURE 7.11: cdf of the capacity. User 3 for 3 precoders and complete CSI.

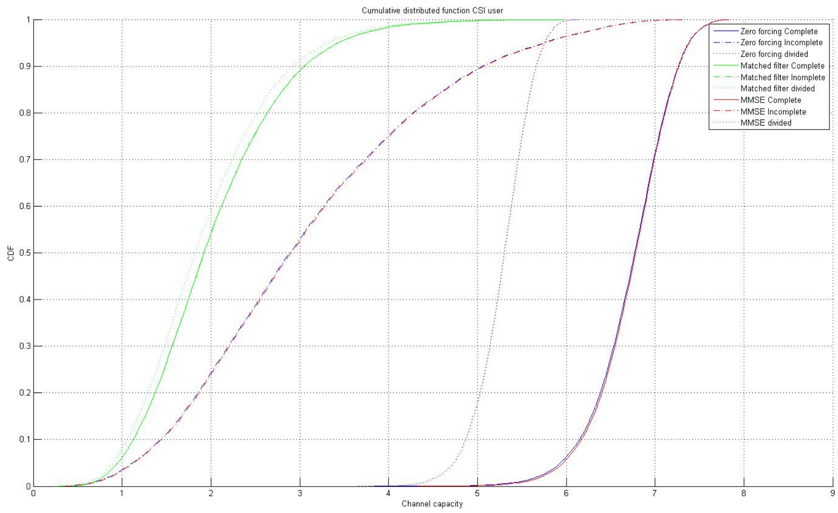

FIGURE 7.12: cdf of the capacity. User 4 for 3 precoders and complete CSI.

are the same but the mean has a lower value.

The last result that is important to notice in this plots is that the user with incomplete CSI has a constant capacity with the technique of dividing the users. This happens for all the precoders. As all the lines are at the same place it can only be seen the red one. This is because this user is not using MIMO, so it is not affected by the precoder used.

Another possibility is to get the cdf plot of the sum capacity, as it is shown in the figure 7.13. The results obtained from this plot are the same as the previous cdf plots, but with less details over the users. It can be seen that the shape of the curves is preserved and the result is the sum of the previous plots.

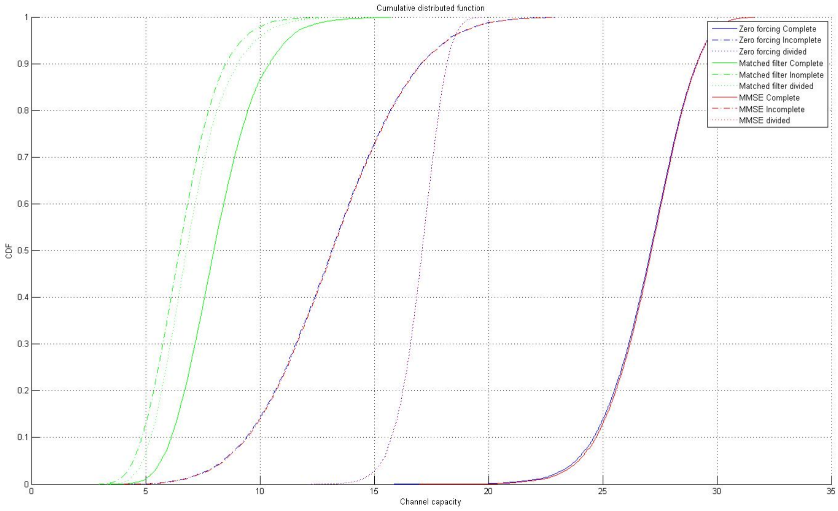

FIGURE 7.13: cdf of the sum capacity. For 3 precoders.

7.3.3 pdf plot

In the pdf plots it can be seen the mean of the capacity random variable as well as the variance. This is a better way to compare the mean of the capacity, because in the cdf plots it is difficult to do such comparison.

This simulation has been performed with the same parameters as the previous one 7.3.2.

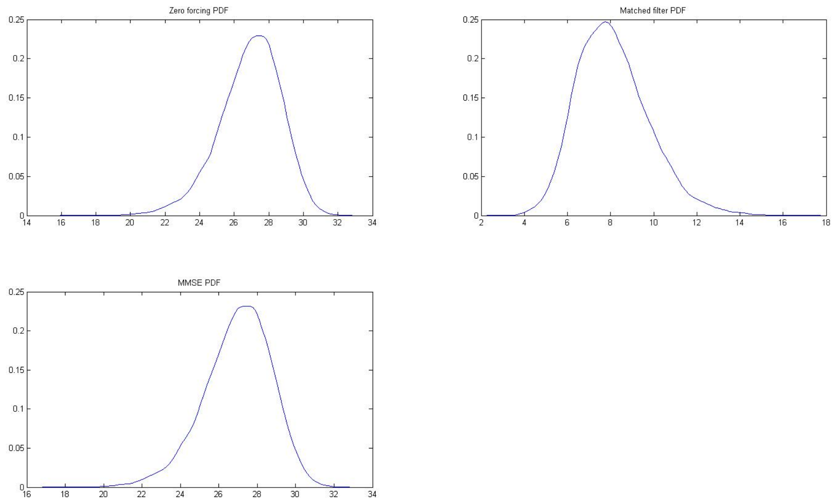

FIGURE 7.14: pdf of the sum capacity. 3 precoders and complete CSI.

In this plots it can be corroborated that the zero forcing and the MMSE have the same behavior. While the matched filter has a worst mean and a higher variance.

7.3.4 Bars plot for the sum capacity varying the number of users with incomplete CSI

This simulation has been performed with an SNR of 20 dB. It is used 128 antennas at the base station and 64 users receiving the information. The type of assumption is mean and the number of realizations is 10000. In each simulation the number of the users with incomplete CSI has been modified.

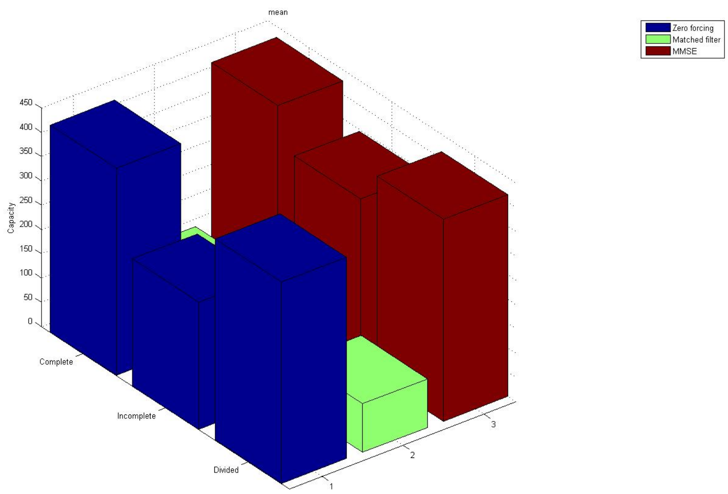

FIGURE 7.15: Bar plot for the sum capacity of all users. For three precoders and three techniques. 1 user with incomplete CSI.

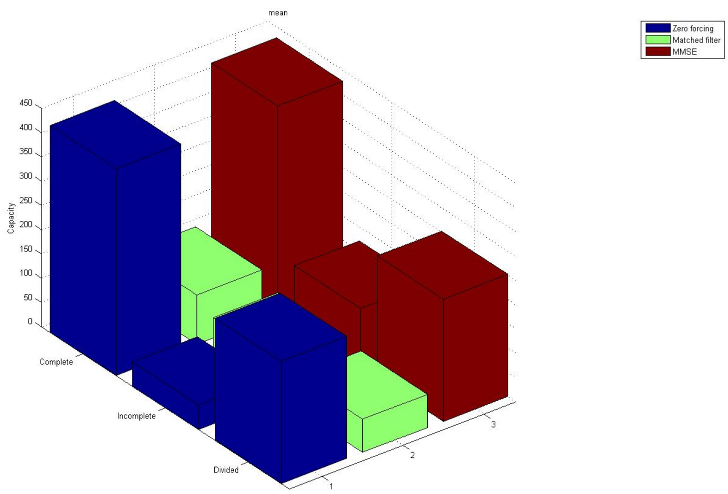

FIGURE 7.16: Bar plot for the sum capacity of all users. For three precoders and three techniques. 16 user with incomplete CSI.

The first thing that you can see is corroborating the first simulation 7.2. As the SNR is 20 dB the capacity (with complete CSI) for the precoders zero forcing and MMSE is

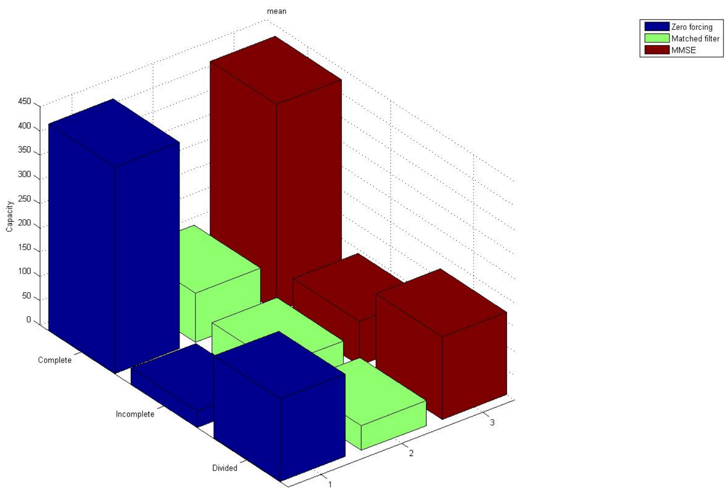

FIGURE 7.17: Bar plot for the sum capacity of all users. For three precoders and three techniques. 25 user with incomplete CSI.

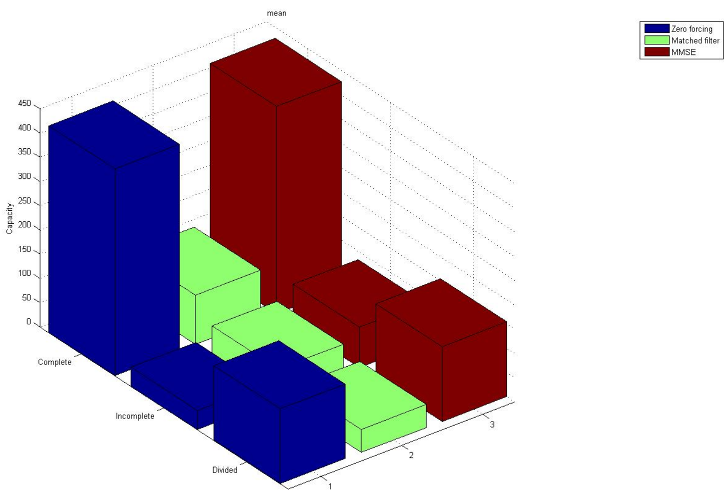

FIGURE 7.18: Bar plot for the sum capacity of all users. For three precoders and three techniques. 27 user with incomplete CSI.

nearly the same. And the capacity for the matched filter is significantly smaller. For all the number of users with incomplete CSI the same result is obtained. This result is also valid for the different techniques used in this simulation. So to obtain results from this simulation the key parameter is the difference of capacity between the options instead of the whole capacity, because it is already known that the MMSE will perform the best.

Focusing on the before mentioned, another result is that the matched filter precoder is much more robust to the incompleteness of the CSI for the users with CSI than the other filters. The difference of capacity between the complete CSI option and the others

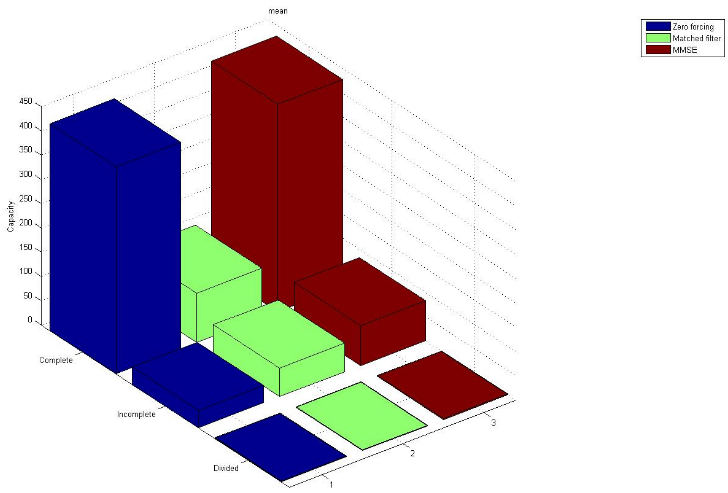

FIGURE 7.19: Bar plot for the sum capacity of all users. For three precoders and three techniques. 28 user with incomplete CSI.

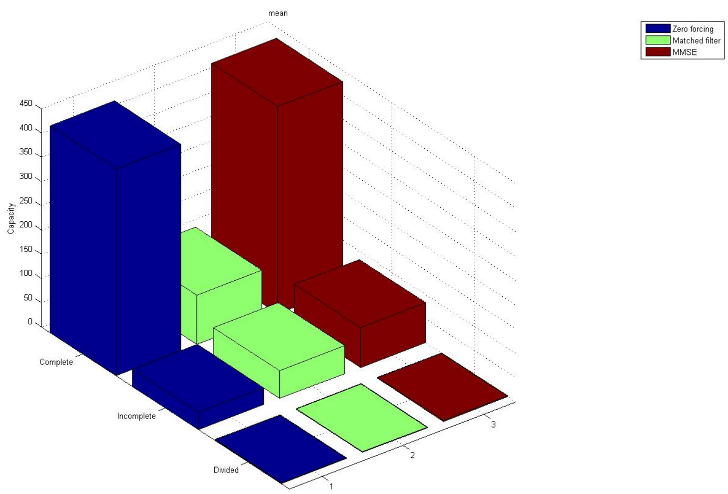

FIGURE 7.20: Bar plot for the sum capacity of all users. For three precoders and three techniques. 29 user with incomplete CSI.

options in the matched filter is less than in the zero forcing and the MMSE precoders. At first sight it might seem that it is a not very important result. But it is important because the companies wants to have a fixed capacity in their systems, and if the number of users with incomplete CSI is unknown or very random can bring the system to high variations of the capacity. Otherwise if the precoder used is the matched filter this variations on the incomplete CSI users are not affecting so much at the capacity. The price that it is paid is to have less capacity.

Finally it can be analyzed the relation between the number of user with incomplete CSI and the performance of the the techniques used. While using the mean assumption when the number of users with incomplete CSI is small, the performance of dividing users is much better than making an assumption. It can be seen in the figures 7.15 7.16 7.17 7.18. But if the number of user with incomplete CSI increases the technique of dividing the users becomes worse. For example in the figure 7.19 7.20. So it can be said that for the technique of assuming with mean the CSI the capacity is reduced very much when a few users don't transmit all the CSI, but then is not so affected if more user stop transmitting its CSI. On the other hand, for the technique of dividing the users, if a few users don't transmit all the CSI, the capacity is not harmed very much. But if more users stop transmitting the CSI, then the capacity decreases significantly. For the zero forcing and MMSE precoders the result is different. In such a case, the technique of dividing users is always better than making an assumption.

It could be useful to find in which number of users is the threshold that makes one technique better than the other for the means assumption. It can be seen in the simulation that this threshold is between 27 and 28 users. But it is not an absolute parameter and it depends on the configuration of the system. In the simulation 7.21 7.22 it has been decreased the number of users in the system to 32 users. In this case the threshold is between 16 and 17 users. To have half the users don't mean the threshold to be the half 7.21 7.22.

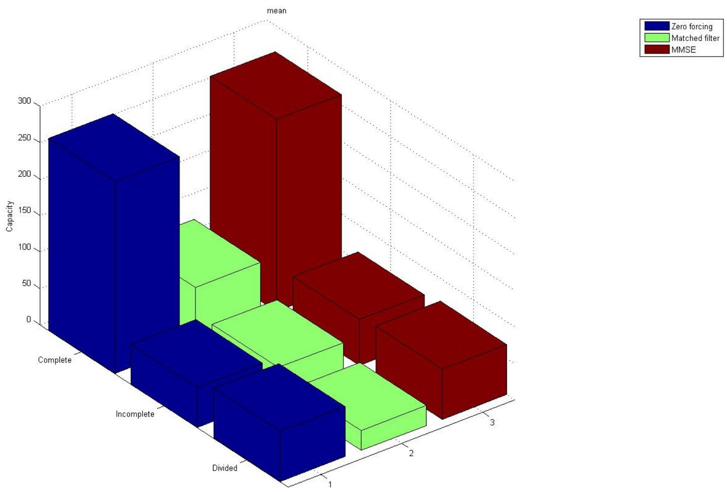

FIGURE 7.21: Bar plot for the sum capacity of all users. For three precoders and three techniques. 16 user with incomplete CSI.

It can also be seen that if the SNR is modified the threshold varies. In the simulation 7.23 7.24 7.25 it has been performed with the same parameters as the previous one, but decreasing the SNR to 10 dB 7.23 7.24 7.25.

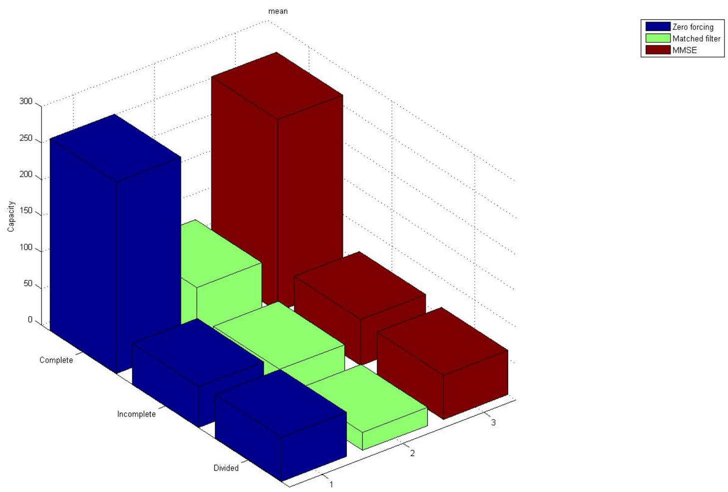

FIGURE 7.22: Bar plot for the sum capacity of all users. For three precoders and three techniques. 17 user with incomplete CSI.

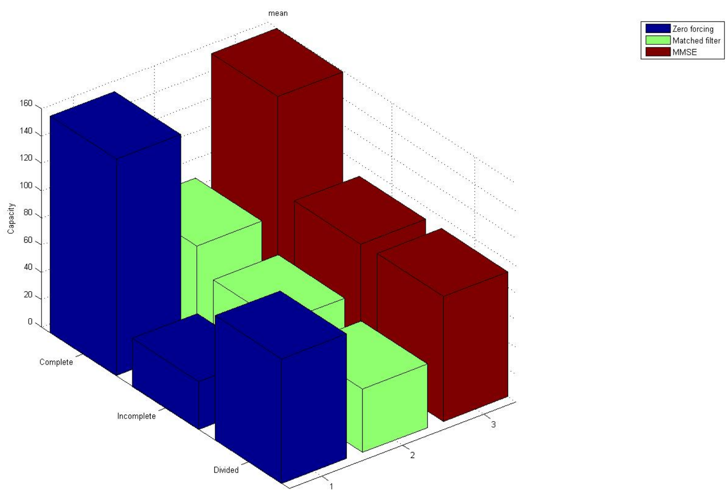

FIGURE 7.23: Bar plot for the sum capacity of all users. For three precoders and three techniques. 8 user with incomplete CSI.

This threshold is also dependent of the type of assumption to make the CSI complete. This is because the type of assumption changes the capacity of the communication. As a result, the number of users that makes both techniques to have similar performances is different. For example in the ones assumption the technique of splitting the users is always better than making an assumption. It can be seen in the figures 7.26 7.27 7.28 7.29.

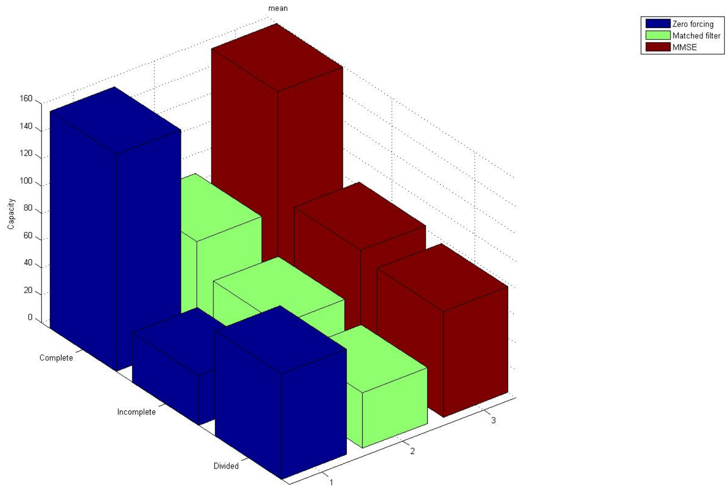

FIGURE 7.24: Bar plot for the sum capacity of all users. For three precoders and three techniques. 10 user with incomplete CSI.

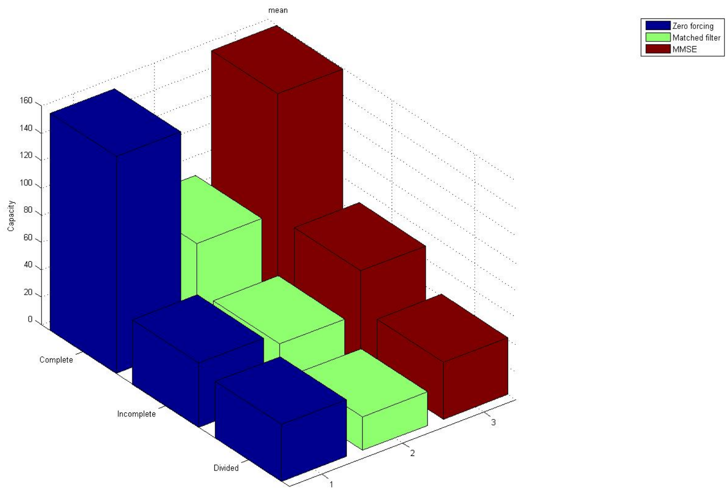

FIGURE 7.25: Bar plot for the sum capacity of all users. For three precoders and three techniques. 16 user with incomplete CSI.

7.3.5 Bars plot for each user

To know the sum capacity of a system is very important. And most of the times, the systems try to maximize this parameter because it offers better performance in a global point of view. But sometimes it is more important to focus on the performance of each user. Because in the same system it can be different types of users with different QoS parameters assigned. Or perhaps the transmission of one user is affecting the other users. For this reason it can be plotted the capacity of a scenario specifically for each user.

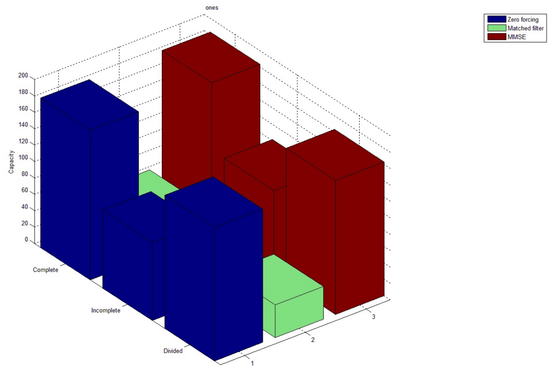

FIGURE 7.26: Bar plot for the sum capacity of all users. For three precoders and three techniques. 2 user with incomplete CSI.

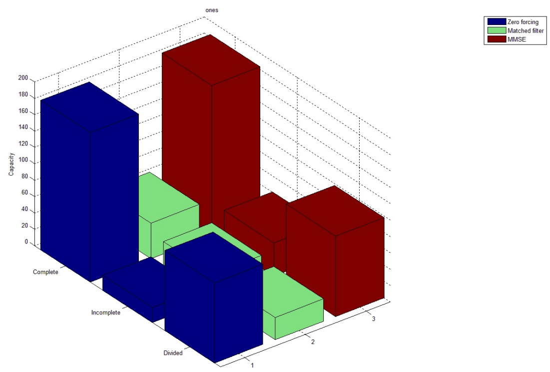

FIGURE 7.27: Bar plot for the sum capacity of all users. For three precoders and three techniques. 9 user with incomplete CSI.

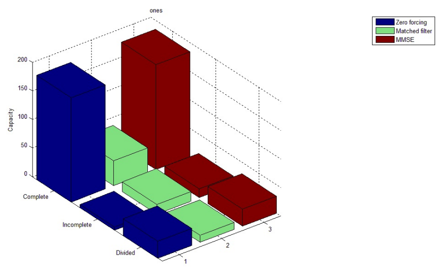

FIGURE 7.28: Bar plot for the sum capacity of all users. For three precoders and three techniques. 19 user with incomplete CSI.

It is performed a simulation with the same parameters as the cdf plots 7.3.2. With a 20 dB SNR, 8 antennas at the base station, 4 users, and 1 of this users do not transmit all the CSI. The type of assumption is ones and it has been performed 10000 realizations.

The easiest thing to see in this plot 7.32 is that the user 3 is the one with incomplete CSI. In the three precoders the behavior of the users 1, 2, 4 is the same, but the user

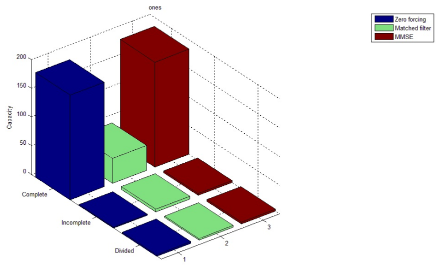

FIGURE 7.29: Bar plot for the sum capacity of all users. For three precoders and three techniques. 29 user with incomplete CSI.

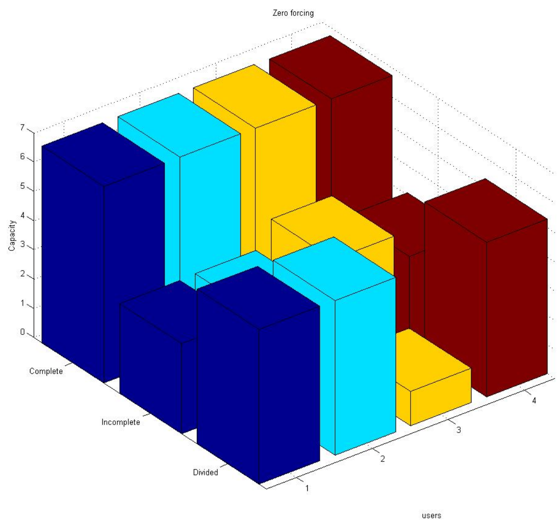

FIGURE 7.30: Bar plot for the capacity of each user. For zero forcing precoder and three techniques.

3 is always performing different. Also it can be concluded that the effect of this user over the other users is not dependent on which user has incomplete CSI. It is reasonable because the channel is uncorrelated, so there is no relationship between users. The only thing that make one user interfere the others is the precoder, and it actuates the same way for all the users.

In this plot it can also be seen the reason why the matched filter is more robust in front of the users with incomplete CSI. It is easy to see that in the matched filter the CSI of one user is not affecting the other users. So the capacity for the users with CSI is the same that in the case of all the users with complete CSI. But on the other hand the user with incomplete CSI has a worse performance than with other precoders. As a result it can be said that if there are just a few users with incomplete CSI the matched filter loses less capacity respect the all users with complete CSI case. But if there are a lot of users with incomplete CSI the degradation of the capacity becomes more important in the matched filter than in the others precoders.

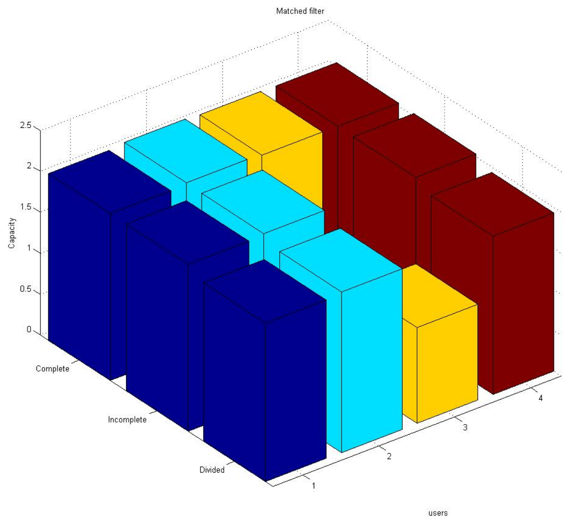

FIGURE 7.31: Bar plot for the capacity of each user. For matched filter precoder and three techniques.

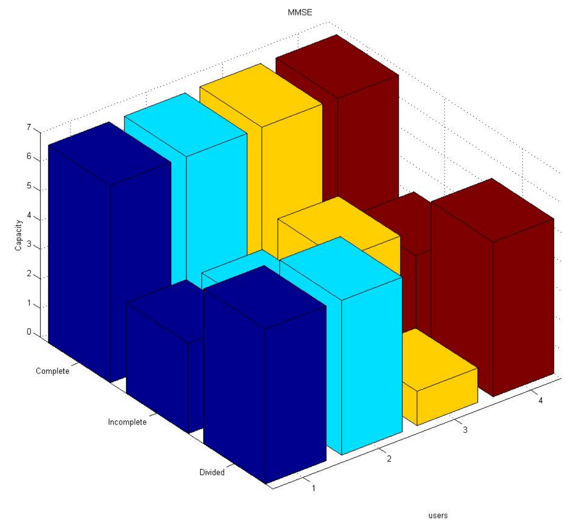

FIGURE 7.32: Bar plot for the capacity of each user. For MMSE precoder and three techniques.

In contrast with what has been said in the previous paragraph, in the zero forcing and MMSE, the user with incomplete CSI is affecting so much the other users that has better capacity than them. This is a really bad behavior, because it means that a user who is in good conditions and could have a high capacity is affected by a user that has worse conditions. This is something that always should be avoided.

It is important to notice that using the technique of dividing the users, the users with incomplete CSI has always the same capacity because it is performed a plain transition (no MIMO), so it is not affected by the precoder. And it can also be seen that for this user it is better to use the division technique than the MIMO.

In the Appendix B it can be seen the same results changing the user with incomplete CSI. It is ovious that the results are the same. So it is concluded that the results are independent of which user has incomplete CSI.

7.3.6 Using correlation

All the simulations showed in the previous sections has been performed using a channel model that has no correlations. But this model is far from the reality. In the next simulations it will be showed the performance of the system when it is exploded the correlation of the channel.

As it is explained in the chapter 2, it is used a single parameter correlation model. In the considered model the users are spread through the cell so there is no correlation between them. However, on the transmitter site, the antennas are very near, so there is a big correlation.

To demonstrate that the MIMO techniques use the randomness of the channel to perform the spatial division, it is simulated the same system increasing the correlation at the transmitter side. The simulation has been performed at 20 dB, 8 antennas transmitting, 4 users, 1 user with incomplete CSI. The number of realizations is 10000. It is written a table with the exact values of the results to show more precisely the degradation of the capacity.

If the correlation parameter is set to its maximum value, i.e., 1, the channel matrix has the same value for all the antennas of the same user. So the matrix is rank deficient and it can not be computed the inverse. For this reason it has been used a correlation parameter of 0.99 to see the performance at completely correlated scenarios.

	Zero forcing	Matched filter	MMSE
Complete	26.9106	8.1221	26.9616
Incomplete	13.2387	6.6012	13.2760
Divided	17.0201	7.0008	17.0349

TABLE 7.1: Sum capacity, with 0 correlation at the transmitter.

The first thing that can be seen in this simulations is that when the correlation in the transmitting side increases, the capacity of the channel decreases. This is because the MIMO techniques uses the independence of the channel to create different transmission paths. But if the channel between the transmitting antennas and a determined user is

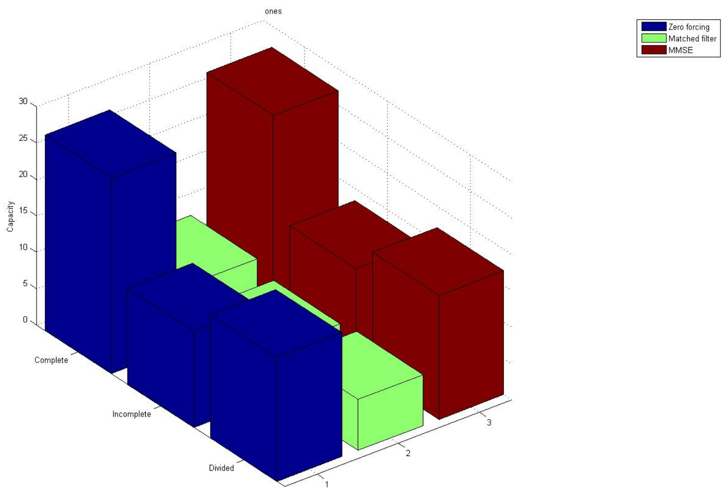

FIGURE 7.33: Sum capacity, with 0 correlation at the transmitter.

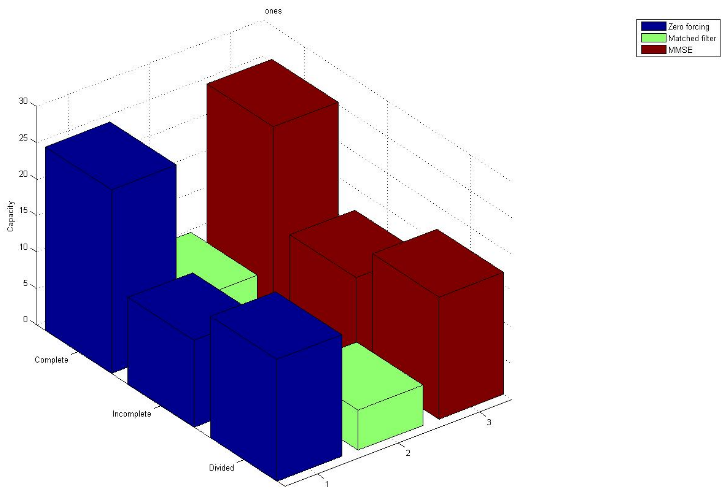

FIGURE 7.34: Sum capacity, with 0.2 correlation at the transmitter.

not independent, is not possible to have different paths. The hypothesis that sustains the MIMO techniques is that the user is placed in a highly scattering environment. So it makes that even if there is a strong correlation on the signals in the transmitting side, this correlation is broken by the scatters around the user.

To analyze this results it is important to realize that using the technique of dividing the users with complete and incomplete CSI, the users without CSI is not using MIMO. So this users are not affected by the correlation of the channel. As a result of this, when the correlation is increased the capacity of the MIMO channel is reduced, but not the capacity for the users without CSI. This is shown in the figures 7.39 7.40 7.41.

	Zero forcing	Matched filter	MMSE
Complete	25.2148	5.8062	25.3361
Incomplete	11.9214	4.8394	11.9908
Divided	16.7329	5.5296	16.7624

TABLE 7.2: Sum capacity, with 0.2 correlation at the transmitter.

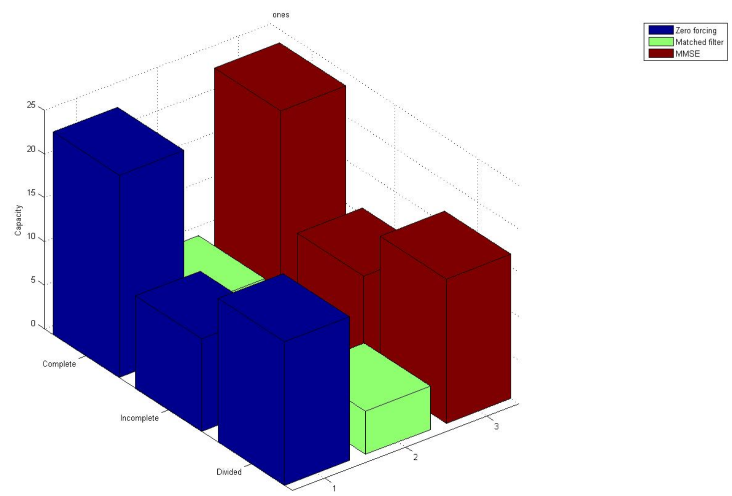

FIGURE 7.35: Sum capacity, with 0.4 correlation at the transmitter.

	Zero forcing	Matched filter	MMSE
Complete	23.1444	4.8845	23.3980
Incomplete	10.5727	4.1545	10.6696
Divided	16.4508	4.9406	16.4964

TABLE 7.3: Sum capacity, with 0.4 correlation at the transmitter.

	Zero forcing	Matched filter	MMSE
Complete	19.1779	4.0756	19.8583
Incomplete	8.7398	3.5411	8.9255
Divided	15.5735	4.3931	15.6583

TABLE 7.4: Sum capacity, with 0.6 correlation at the transmitter.

In this plots it has been represented the capacity for each user in a different bar to see the behavior of the specific user. There is one plot for the zero forcing precoder, another one for the matched filter and the last one for the MMSE. In each plot is shown the performance of the three different techniques studied in this project, complete CSI, assuming the incomplete CSI and dividing the users with complete and incomplete CSI.

In the figure 7.39 it can be seen that the zero forcing precoder is the most affected by

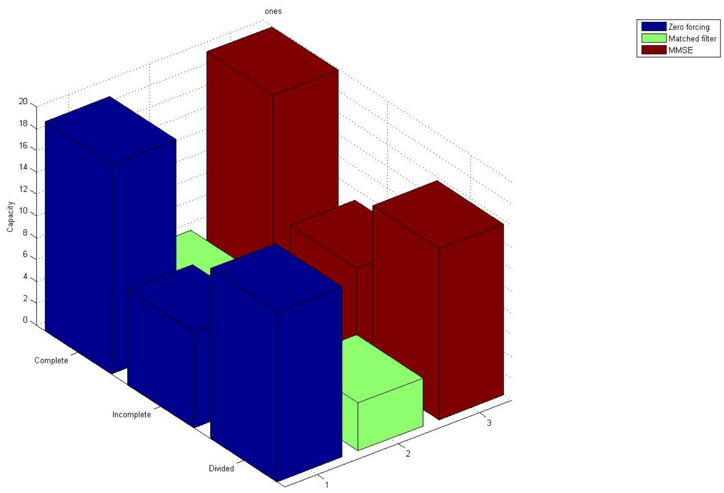

FIGURE 7.36: Sum capacity, with 0.6 correlation at the transmitter.

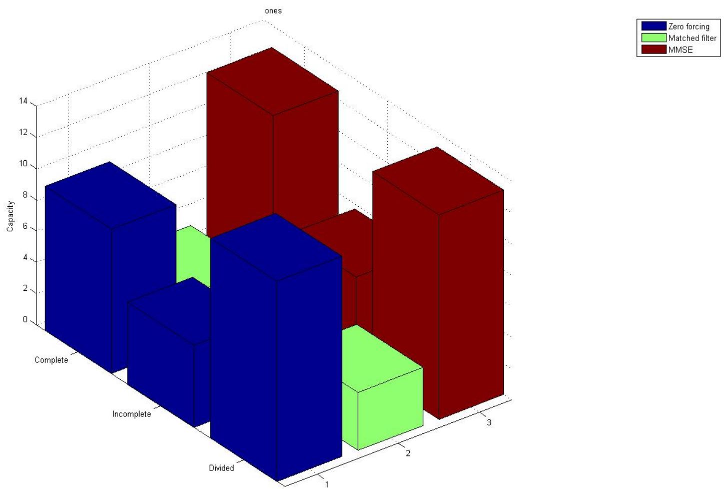

FIGURE 7.37: Sum capacity, with 0.8 correlation at the transmitter.

the correlation of the channel. All the users that are using MIMO techniques, do not have capacity to transmit information. The only user that can transmit something is the one with incomplete CSI. The reason for this behavior is because the zero forcing precoder uses the inverse of the channel matrix to obtain the precoding matrix. And if the channel matrix is rank deficient this inverse can not be computed. The values of the precoding matrix tend to a singularity. So it is not possible to transmit information for this users.

For the case of the matched filter the result is slightly better than the zero forcing. But there is still a big difference between the user with incomplete CSI and the users with

	Zero forcing	Matched filter	MMSE
Complete	9.2343	3.1385	12.5394
Incomplete	5.2580	2.8298	5.6656
Divided	12.8206	3.7242	13.1054

TABLE 7.5: Sum capacity, with 0.8 correlation at the transmitter.

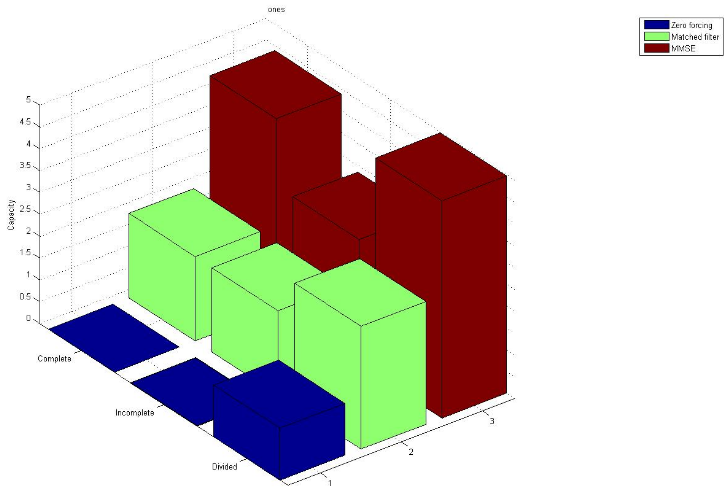

FIGURE 7.38: Sum capacity, with 0.99 correlation at the transmitter.

	Zero forcing	Matched filter	MMSE
Complete	0.0000	1.9360	4.3799
Incomplete	0.0137	1.9317	2.8444
Divided	1.1901	2.8063	4.9720

TABLE 7.6: Sum capacity, with 0.99 correlation at the transmitter.

complete CSI. It means that the correlation is affecting the MIMO transmission.

Finally in the MMSE precoder the correlation of the channel is affecting the capacity because it is reduced from nearly 7 bits/s/Hz/user in the case of no correlated channel (figure 7.32), to 1 bits/s/Hz/user in this correlated scenario. But in this case the capacity is comparable with the user that is not using MIMO.

After this analysis, it can be said that the MMSE precoder is the most robust to the correlation of the channel. However it has not a very good performance in this correlated scenario.

The same results are obtained for a smaller SNR. For example in the appendix C it can be seen the same plots for a 5 dB SNR. The only difference is that the zero forcing precoder collapses for a smaller correlation.

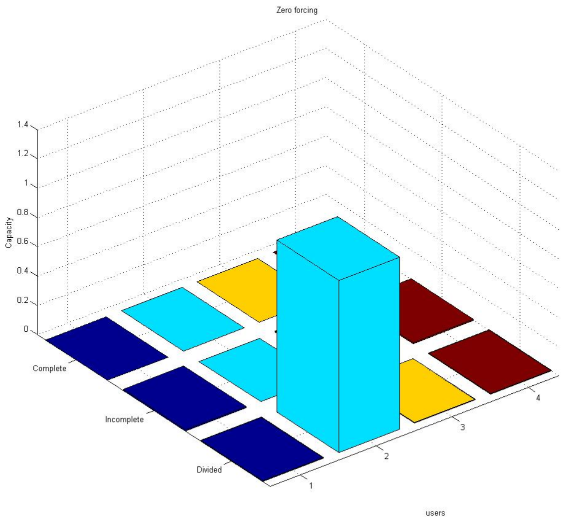

FIGURE 7.39: Sum capacity, with 0.99 correlation at the transmitter. Separated users, zero forcing.

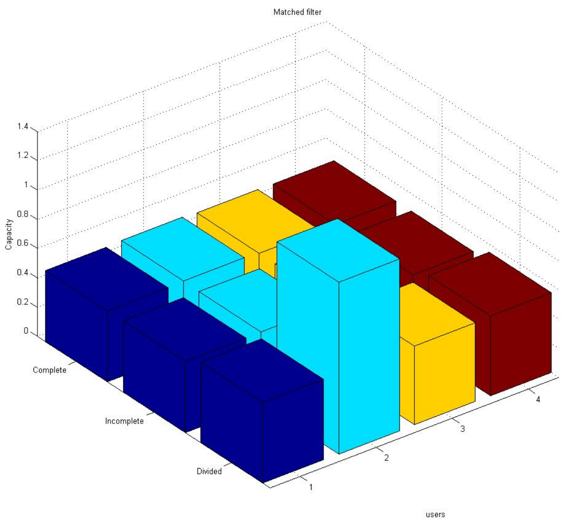

FIGURE 7.40: Sum capacity, with 0.99 correlation at the transmitter. Separated users, matched filter.

7.3.7 Assumption with correlation

In the previous section it has been explained how the correlation of the channel decreases the capacity. So it was concluded that it is better a channel without correlation. But sometimes this correlation of the channel can add some information to the CSI and it can be exploited to increase the capacity. When the channel is correlated it has more information than the simple path losses. There is an intrinsic information in the relationship between its values. It can be used this spatial correlation to obtain the missing CSI of some users. The statistical behavior of the channel can transmit

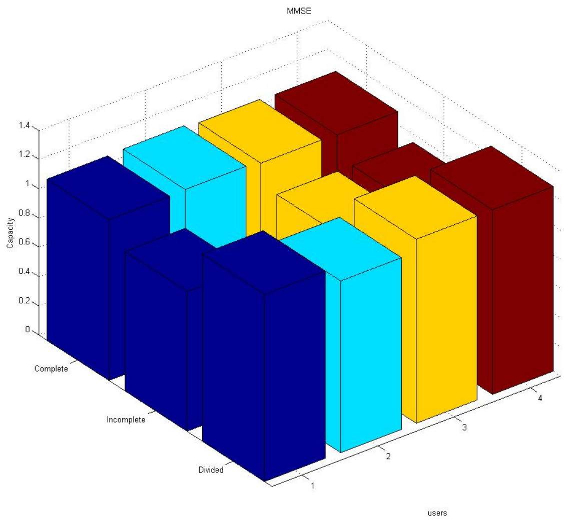

FIGURE 7.41: Sum capacity, with 0.99 correlation at the transmitter. Separated users, MMSE.

the information that the users can not. In the figures 7.42 7.43 it is analyzed the performance of an assumption designed to use the spatial correlation. This assumption is explained in the chapter 6.

The simulation has been performed with 20 dB SNR, 8 transmitting antennas, 4 users, 1 user with incomplete CSI and 10000 realizations. The correlation parameter is sweep from 0.01 to 0.99 and it is showed the result for the random assumption and the correlation assumption.

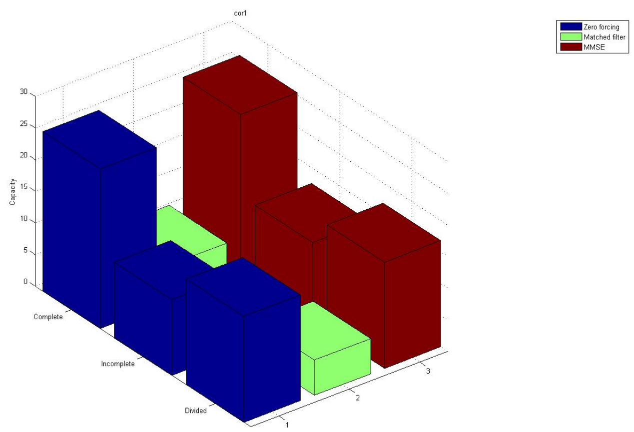

FIGURE 7.42: Sum capacity, with 0.2 correlation at the transmitter side. Using correlation assumption

The extended result can be seen in the appendix D. After analyzing the plots it is easy to see that there is no significant difference between the random assumption and

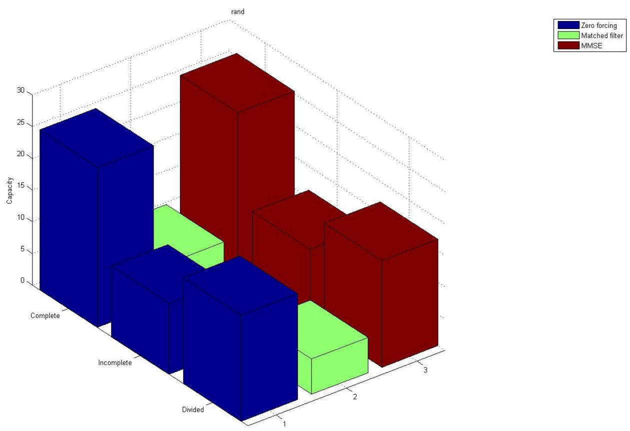

FIGURE 7.43: Sum capacity, with 0.2 correlation at the transmitter side. Using random assumption

the assumption using the correlation. So it means that this specific assumption do not perform as it was supposed. The information carried by the channel properties is not used in a good way to determine the CSI.

It is also important to realize that it has been considered the worst case. The users with incomplete CSI just transmit one path information. It could be a reason for the bad performance of the assumption.

Chapter 8

Conclusion

In this chapter it is summarized the most important results obtained after the simulations. Here are explained all the results even those that are already known. However, the explanation is focused on the new results.

The first result obtained in this project is well known and says that the capacity increases with the SNR. This can be obtained using the Shannon's equation. Also it can be said that the MMSE achieves always the higher capacity of the three studied precoders. While the zero forcing tends to be the same as the MMSE in high SNR scenarios and the matched filter tends to be the same as the MMSE for small SNR scenarios. This result can be applied when the number of users with incomplete CSI is small, but if this users increases, the three precoders becomes similar. If the number of antennas, and users is increased keeping the same proportion, the result is shocking and can not be explained by the author. In that case the capacity using the MMSE precoder has a maximum and the three precoders makes the capacity tend to a certain value for high SNR.

Another result obtained is that the users with incomplete CSI reduces the capacity of other users. It can also be said that the technique of splitting the users and having complete CSI the capacity has smaller variance, so it is not so affected by the randomness of the channel as the assumption technique.

It can also be concluded that the users with incomplete CSI in the matched filter affects less the other users than in the other precoders. But this users has worse capacity. So when there are a few users with incomplete CSI the matched filter is more robust to the incompleteness of the CSI. And if there are a lot of users with incomplete CSI the degradation of the capacity becomes more important in the matched filter than in the other precoders.

It can be added that using the means technique exists a threshold of users with incomplete CSI that makes the technique of splitting the users better than making an assumption of the CSI. This threshold is a parameter that depends on the number of antennas, users, SNR and type of assumption.

In the project is demonstrated that the MIMO systems use the independence of the channel paths. So when the correlation is increased, the capacity decreases. When there is a high correlation channel, to split the users with complete and incomplete CSI is better than making an assumption, because the users with incomplete CSI do not use MIMO and their performance is better. In fact, it would be better to not use any MIMO technique. The zero forcing precoder is the most affected by this behavior, because of its incapacity to invert a rang deficient matrix. Note that when a matrix is highly correlated tends to be rang deficient, because all the values tends to be the same. The MMSE precoder is the most robust in front of the correlation of the channel, although it has not a good performance.

Finally it can be concluded that the assumption designed for the correlated channel is not good because it does not performs better than the random assumption. So it takes no advantage of the correlation. In a future work it will be used the time correlation to increase the capacity.

Appendix A

Matlab code

Matlab Code

```
function [out] = sim2()
%sim1 Summary of this function goes here
%   Detailed explanation goes here

%Parameters definition
snr = 5; %SNR (dB)
M = 8;  %Base station antennas
N = 4;  %Users
U = 1;  %Users with incomplete CSI
rtx = 0.99; %Correlation in the transmited side
rrx = 0; %Correlation in the recieved side
S =  ones ; %Type of assumption
R = 10000; %Number of realizations

%Preallocation of variables
v = ones(N,M); %create a flag matrix of the channel, 1 means this user ...
    feedbacks CSI for this path and 0 mean it dosen t

SINRZ = zeros(R,N,3);
SINRM = zeros(R,N,3);
SINRE = zeros(R,N,3);

URZ = zeros (R,N,3);
URM = zeros (R,N,3);
URE = zeros (R,N,3);

i = 1;
while i ≤ U, %for each no feedback CSI user
```

```
    f = randi(N); %choose a random user
    e = randi(M); %choose a random path
    if mean(v(f,:)) ≠ 1, %If this user has been choosen before, the loop ... is repeated
        i = i − 1;
    end
    v(f,:) = zeros(1,M); %change the flag in the flag matrix
    v(f,e) = 1;
    i = i + 1;
end

Rtx = zeros(M);
for i=1:M,
    for j=1:M,
        Rtx(i,j) = rtxˆ((abs(i−j))ˆ2);
    end
end
Rrx = zeros(N);
for i=1:N,
    for j=1:N,
        Rrx(i,j) = rrxˆ((abs(i−j))ˆ2);
    end
end

% for snr = [−20:1:20] %Simulate the capacity for diferent snr

    ro = 10ˆ(snr/10); %change SNR from dB to linear

    CMZ = zeros (R,3);
    CMM = zeros (R,3);
    CME = zeros (R,3);


    for d=1:R, %loop for all the realizations

        %Channel generation
        D = ...
    complex(random( Normal ,0,1/sqrt(2),N,M),random( Normal ,0,1/sqrt(2),N,M)); ...
    %create random Rayleigh fading channel
        G = sqrt(Rrx)*D*sqrt(Rtx);

        for g=1:3,%If g=1 analisys for complete CSI, if g=2 analisys for ... incomplete CSI, if g=3 analisys for split users

            if g == 1,  %In the first loop analisys for complete CSI
                CSI = G;
            elseif g == 2,   %In the second loop analisys for incomplete CSI

```

```
                if strcmp(S, ones )%Assumes ones in the random user CSI
                    for f=1:N,
                        for e=1:M,
                            if v(f,e)==0
                                CSI(f,e) = 1;
                            end
                        end
                    end
                elseif strcmp(S, rand ) %Assumes random Rayleigh ...
    variables in the random user CSI
                    for f=1:N,
                        for e=1:M,
                            if v(f,e)==0
                                CSI(f,e) = ...
    complex(random( Normal ,0,1/sqrt(2),1,1),random( Normal ,0,1/sqrt(2),1,1));
                            end
                        end
                    end
                elseif strcmp(S, mean )%Assumes the mean of others users CSI
                    aux=G;
                    for f=1:N,
                        for e=1:M,
                            if v(f,e)==0
                                h=0;
                                for j=1:N,
                                    if v(j,e)==1
                                        aux(f,e)=aux(f,e)+G(j,e);
                                        h=h+1;
                                    end
                                end
                                aux(f,e)=aux(f,e)/h;
                            end
                        end
                    end
                    for f=1:N,
                        for e=1:M,
                            if v(f,e)==0
                                    CSI(f,e) = aux(f,e);
                            end
                        end
                    end
                elseif strcmp(S, corl ) %Uses the correlation at the ...
    transmitter to estimate the channel
                    aux = (G.*v)*Rtx;
                    for f=1:N,
                        for e=1:M,
                            if v(f,e)==0
                                    CSI(f,e) = aux(f,e);
```

```
                                        end
                                    end
                                end
%                             elseif strcmp(S,  cor2 )
%                                 aux = Rtx∗((G.∗v) / ((G.∗v)∗(G.∗v) ));
%                                 for f=1:N,
%                                     for e=1:M,
%                                         if v(f,e)==0
%                                                 CSI(f,e) = aux(f,e);
%                                         end
%                                     end
%                                 end
                            else
                                CSI = G; %Complete channel state information
                            end
                        end

                        if g≠3,
                            ZF = (CSI ) / (CSI∗(CSI ) ); %create precoding matrix for ...
    zero forcing
                            betaz = sqrt(ro/(trace(ZF∗(ZF ) ))); %normalizing factor ...
    for zero forcing filter
                            ZF = betaz ∗ ZF;

                            MF = (CSI ) ; %create precoding matrix for match filter
                            betam = sqrt(ro/(trace((CSI ) ∗CSI))); %normalizing factor ...
    for matched filter
                            MF = betam ∗ MF;

                            F = (CSI ) ∗CSI + (N/ro)∗eye(M); %auxiliar matrix for MMSE
                            MMSE = (inv(F))∗(CSI ) ;
                            betae = sqrt(ro/(trace(MMSE∗MMSE ) ));%normalized factor ...
    for MMSE
                            MMSE = betae ∗ MMSE;%create precoding matrix for MMSE

                            for j=1:N, %SINR for each user

                                aaux = 0;
                                baux = 0;
                                caux = 0;

                                A = G∗ZF;
                                B = G∗MF;
                                C = G∗MMSE;

                                for k=1:N,

                                    if k ≠ j,
```

```
                            aaux = aaux + power((abs(A(k,j))),2);
                            baux = baux + power((abs(B(k,j))),2);
                            caux = caux + power((abs(C(k,j))),2);

                        end

                    end

                    %SINR for each realization(d), user(j), complete or ...
    incomplete CSI(g)
                    SINRZ(d,j,g) = ((power((abs(A(j,j))),2))/(1 + aaux));
                    SINRM(d,j,g) = ((power((abs(B(j,j))),2))/(1 + baux));
                    SINRE(d,j,g) = ((power((abs(C(j,j))),2))/(1 + caux));

                    %Throughput for each realization(d), user(j), ...
    complete or incomplete CSI(g)
                    URZ(d,j,g) = log2(1+SINRZ(d,j,g));
                    URM(d,j,g) = log2(1+SINRM(d,j,g));
                    URE(d,j,g) = log2(1+SINRE(d,j,g));

                end

            elseif g==3, %For the case of dividing the users with and ...
    without CSIT
                a=0;
                for f=1:N,
                    if mean(v(f,:))≠1
                            G(f-a,:) = []; %Delete the users without CSIT ...
    of the channel matrix
                            a=a+1;
                    end
                end
                CSI=G;

                ZF = (CSI )/(CSI*(CSI )); %create precoding matrix for ...
    zero forcing
                betaz = sqrt((((N-U)*ro)/N)/(trace(ZF*(ZF )))); ...
    %normalizing factor for zero forcing filter
                ZF = betaz * ZF;

                MF = (CSI ); %create precoding matrix for match filter
                betam = sqrt((((N-U)*ro)/N)/(trace((CSI )*CSI))); ...
    %normalizing factor for matched filter
                MF = betam * MF;

                F = (CSI )*CSI + (N/(((N-U)*ro)/N))*eye(M); %auxiliar ...
    matrix for MMSE
```

```
            betae = ...
sqrt((((N-U)*ro)/N)/(trace((inv(F)*inv(F))*(CSI )*CSI))); %normalized ...
factor for MMSE
            MMSE = betae * (inv(F))*(CSI ); %create precoding matrix ...
for MMSE

            for j=1:(N-U), %SINR for each user

                aaux = 0;
                baux = 0;
                caux = 0;

                A = G*ZF;
                B = G*MF;
                C = G*MMSE;

                for k=1:N-U,

                    if k ≠ j,

                        aaux = aaux + power((abs(A(k,j))),2);
                        baux = baux + power((abs(B(k,j))),2);
                        caux = caux + power((abs(C(k,j))),2);

                    end

                end

                %SINR for each realization(d), user(j), complete or ...
incomplete CSI(g)
                SINRZ(d,j,g) = ((power((abs(A(j,j))),2))/(1 + aaux));
                SINRM(d,j,g) = ((power((abs(B(j,j))),2))/(1 + baux));
                SINRE(d,j,g) = ((power((abs(C(j,j))),2))/(1 + caux));

                %Throughput for each realization(d), user(j), ...
complete or incomplete CSI(g)
                URZ(d,j,g) = ((N-U)/N)*log2(1+SINRZ(d,j,g));
                URM(d,j,g) = ((N-U)/N)*log2(1+SINRM(d,j,g));
                URE(d,j,g) = ((N-U)/N)*log2(1+SINRE(d,j,g));

            end
            aaa=(1/N)*log2(1+(ro/N)); %Capacity for users without CSIT
            i=N;
            k=N;
            while URZ(d,k,g)==0
                        k=k-1;
            end
            while i≥1,
```

```
                    if mean(v(i,:))≠1 %If user i doesn t have CSIT its ...
    capacity is aaa
                        URZ(d,i,g)= aaa;
                        URM(d,i,g)= aaa;
                        URE(d,i,g)= aaa;
                    else %If user i has CSIT its capacity is the ...
    calculated before
                        URZ(d,i,g)=URZ(d,k,g);
                        URM(d,i,g)=URM(d,k,g);
                        URE(d,i,g)=URE(d,k,g);
                        k=k-1;
                    end
                    i=i-1;
                end

            end

            %Sum throughput each realization(d), complete, incomplete or ...
    divided users CSI(g)
            for i=1:N;
                CMZ(d,g) = CMZ(d,g) + URZ(d,i,g);
                CMM(d,g) = CMM(d,g) + URM(d,i,g);
                CME(d,g) = CME(d,g) + URE(d,i,g);
            end
        end
    end

%Simulate the capacity for diferent snr
%     for g=1:2,
%         TMZ(snr + 21,g) = mean(CMZ(:,g)); %Mean of all realizations of ...
    zero forcing Complete CSI
%         TMM(snr + 21,g) = mean(CMM(:,g)); %Mean of all realizations of ...
    matched filter Complete CSI
%         TME(snr + 21,g) = mean(CME(:,g)); %Mean of all realization of ...
    MMSE Complete CSI
%     end
% end

%Plot the total (sum of all users) capacity cdf for the three precoders
% figure;
% hold all;
% a = cdfplot(CMZ(:,1));
% set(a, LineStyle , - )
% set(a, Color , b )
% b = cdfplot(CMZ(:,2));
% set(b, LineStyle , -. )
% set(b, Color , b )
% c = cdfplot(CMZ(:,3));
```

```
% set(c, LineStyle , : )
% set(c, Color , b )
% d = cdfplot(CMM(:,1));
% set(d, LineStyle , − )
% set(d, Color , g )
% e = cdfplot(CMM(:,2));
% set(e, LineStyle , −. )
% set(e, Color , g )
% f = cdfplot(CMM(:,3));
% set(f, LineStyle , : )
% set(f, Color , g )
% h = cdfplot(CME(:,1));
% set(h, LineStyle , − )
% set(h, Color , r )
% i = cdfplot(CME(:,2));
% set(i, LineStyle , −. )
% set(i, Color , r )
% j = cdfplot(CME(:,3));
% set(j, LineStyle , : )
% set(j, Color , r )
% legend( Zero forcing Complete , Zero forcing Incomplete , Zero forcing ...
    divided , Matched filter Complete , Matched filter ...
    Inomplete , Matched filter divided , MMSE Complete , MMSE ...
    Incomplete , MMSE divided );
% title( Cumulative distributed function );
% xlabel( Channel capacity );
% ylabel( CDF );

%Plot the probability density function for the three precoders
% figure;
% hold on;
% subplot(221),ksdensity(CMZ(:,1));
% title( Zero forcing PDF );
% hold off;
% subplot(222),ksdensity(CMM(:,1))
% title( Matched filter PDF );
% subplot(223),ksdensity(CME(:,1))
% title( MMSE PDF );
% hold;

%Plot the capacity for diferent snr
% figure;
% hold all;
% snr = −20:1:20;
% plot(snr,TMZ(:,1));
% plot(snr,TMM(:,1));
% plot(snr,TME(:,1));
% legend( Zero forcing , Matched filter , MMSE );
```

```
% title ( Complete CSI ) ;
% xlabel ( SNR ) ;
% ylabel ( Capacity ) ;
% hold;
% figure;
% hold all;
% snr = —20:1:20;
% plot(snr,TMZ(:,2));
% plot(snr,TMM(:,2));
% plot(snr,TME(:,2));
% legend ( Zero forcing , Matched filter , MMSE ) ;
% title ( Incomplete CSI ) ;
% xlabel ( SNR ) ;
% ylabel ( Capacity ) ;
% hold;

%Plot the cdf for each user and three precoders
% for i=1:N;
%     figure;
%     hold all;
%     a = cdfplot(URZ(:,i,1));
%     set(a, LineStyle , — )
%     set(a, Color , b )
%     b = cdfplot(URZ(:,i,2));
%     set(b, LineStyle , —. )
%     set(b, Color , b )
%     c = cdfplot(URZ(:,i,3));
%     set(c, LineStyle , : )
%     set(c, Color , b )
%     d = cdfplot(URM(:,i,1));
%     set(d, LineStyle , — )
%     set(d, Color , g )
%     e = cdfplot(URM(:,i,2));
%     set(e, LineStyle , —. )
%     set(e, Color , g )
%     f = cdfplot(URM(:,i,3));
%     set(f, LineStyle , : )
%     set(f, Color , g )
%     g = cdfplot(URE(:,i,1));
%     set(g, LineStyle , — )
%     set(g, Color , r )
%     h = cdfplot(URE(:,i,2));
%     set(h, LineStyle , —. )
%     set(h, Color , r )
%     j = cdfplot(URE(:,i,3));
%     set(j, LineStyle , : )
%     set(j, Color , r )
```

```
%     legend( Zero forcing Complete , Zero forcing Incomplete , Zero ...
    forcing divided , Matched filter Complete , Matched filter ...
    Inomplete , Matched filter divided , MMSE Complete , MMSE ...
    Incomplete , MMSE divided );
%     if(mean(v(i,:))==1)
%         title( Cumulative distributed function CSI user );
%     elseif (mean(v(i,:))≠1)
%         title( Cumulative distributed function no CSI user );
%     end
%     xlabel( Channel capacity );
%     ylabel( CDF );
% end

%Plot the sum capacity for each user and each precoder
% y=zeros(3,N);
% j=zeros(3,N);
% k=zeros(3,N);
% for g=1:3,
%     for i=1:N;
%         y(g,i) = mean(URZ(:,i,g));
%         j(g,i) = mean(URM(:,i,g));
%         k(g,i) = mean(URE(:,i,g));
%     end
% end
% figure;
% bar3(y);
% set(gca, YTickLabel ,{ Complete , Incomplete , Divided })
% title( Zero forcing );
% xlabel( users );
% zlabel( Capacity );
% figure;
% bar3(j);
% set(gca, YTickLabel ,{ Complete , Incomplete , Divided })
% title( Matched filter );
% xlabel( users );
% zlabel( Capacity );
% figure;
% bar3(k);
% set(gca, YTickLabel ,{ Complete , Incomplete , Divided })
% title( MMSE );
% xlabel( users );
% zlabel( Capacity );

% Plot the averaged capacity
figure;
y=zeros(3,3);
for g=1:3,
    y(g,1) = mean(CMZ(:,g));
```

```
    y(g,2) = mean(CMM(:,g));
    y(g,3) = mean(CME(:,g));
end
bar3(y);
set(gca, YTickLabel ,{ Complete , Incomplete , Divided })
legend( Zero forcing , Matched filter , MMSE );
title(S);
zlabel( Capacity );
out = y;

end
```

Appendix B

Bar plots for each user

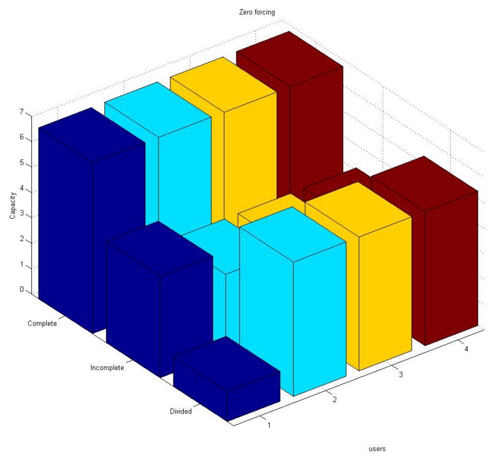

FIGURE B.1: Bar plot for the capacity of each user. For zero forcing precoder and three techniques.

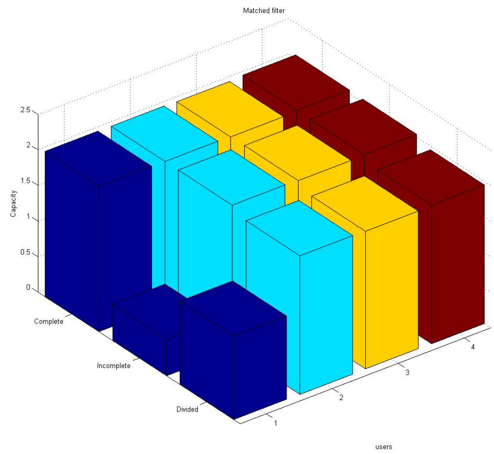

FIGURE B.2: Bar plot for the capacity of each user. For matched filter precoder and three techniques.

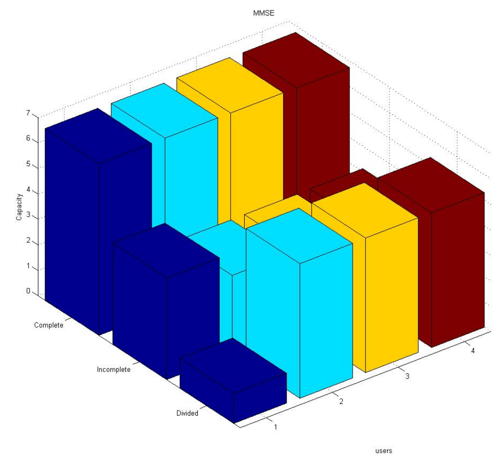

FIGURE B.3: Bar plot for the capacity of each user. For MMSE precoder and three techniques.

Appendix C

Bar plots for a correlated channel

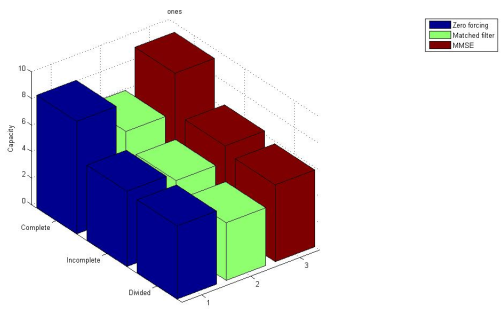

FIGURE C.1: Bar plot for the sum capacity. With a 0 correlation at the transmitter.

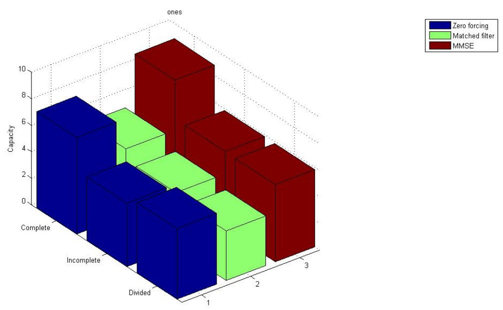

FIGURE C.2: Bar plot for the sum capacity. With a 0.2 correlation at the transmitter.

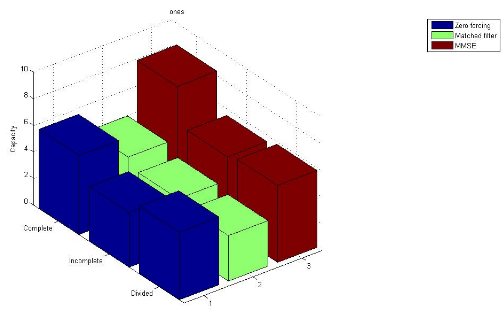

FIGURE C.3: Bar plot for the sum capacity. With a 0.4 correlation at the transmitter.

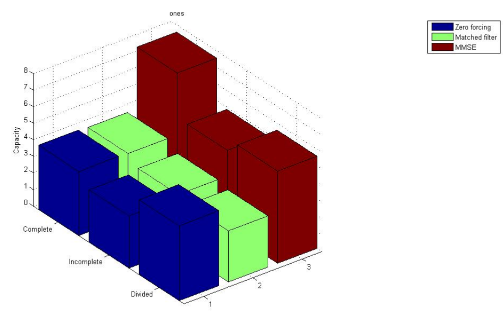

FIGURE C.4: Bar plot for the sum capacity. With a 0.6 correlation at the transmitter.

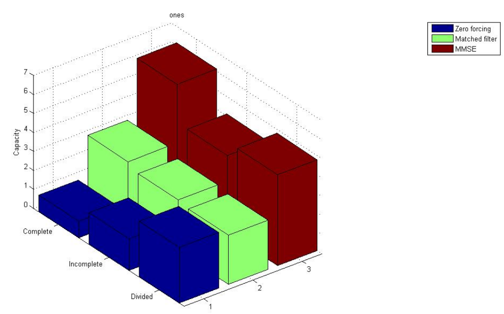

FIGURE C.5: Bar plot for the sum capacity. With a 0.8 correlation at the transmitter.

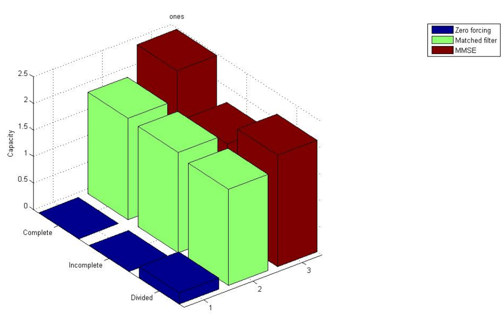

FIGURE C.6: Bar plot for the sum capacity. With a 0.99 correlation at the transmitter.

Appendix D

Comparision of the correlation assumption and the random assumption.

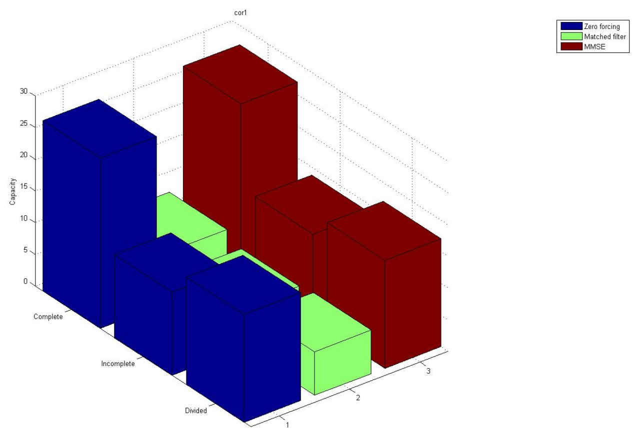

FIGURE D.1: Sum capacity, with 0.01 correlation at the transmitter side. Using correlation assumption

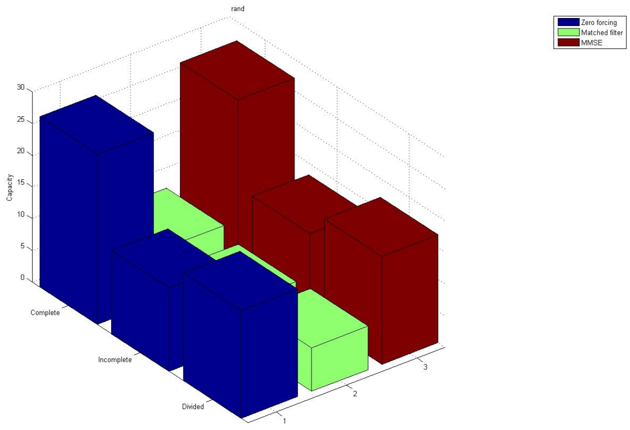

FIGURE D.2: Sum capacity, with 0.01 correlation at the transmitter side. Using random assumption.

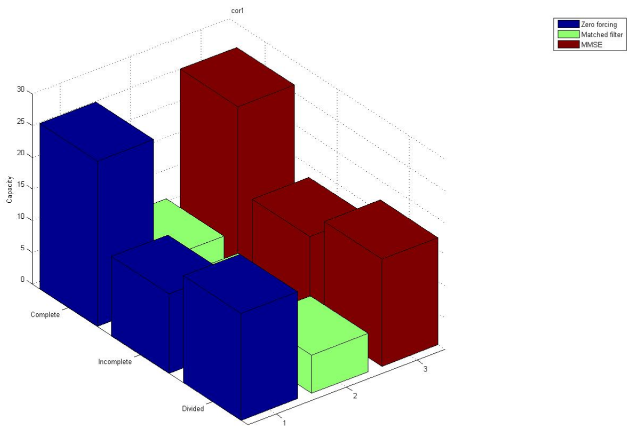

FIGURE D.3: Sum capacity, with 0.1 correlation at the transmitter side. Using correlation assumption

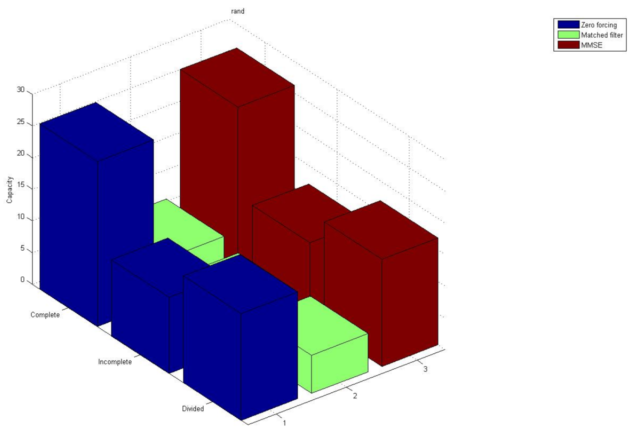

FIGURE D.4: Sum capacity, with 0.1 correlation at the transmitter side. Using rand assumption

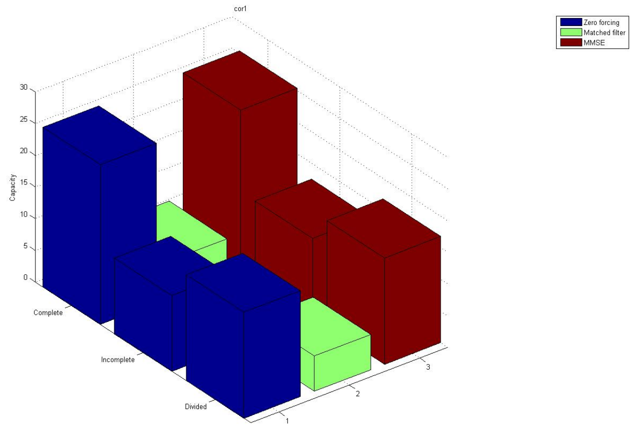

FIGURE D.5: Sum capacity, with 0.2 correlation at the transmitter side. Using correlation assumption

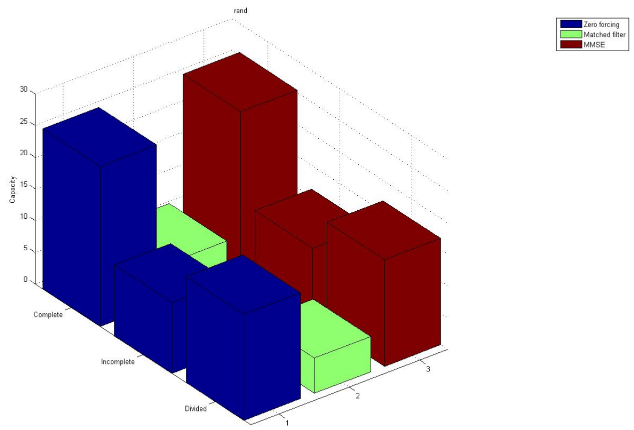

FIGURE D.6: Sum capacity, with 0.2 correlation at the transmitter side. Using random assumption

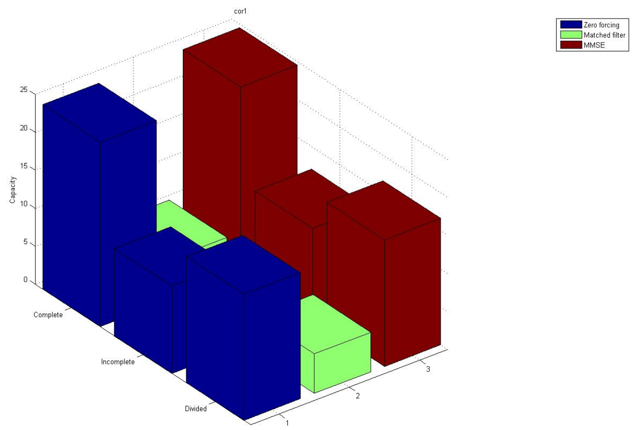

FIGURE D.7: Sum capacity, with 0.3 correlation at the transmitter side. Using correlation assumption

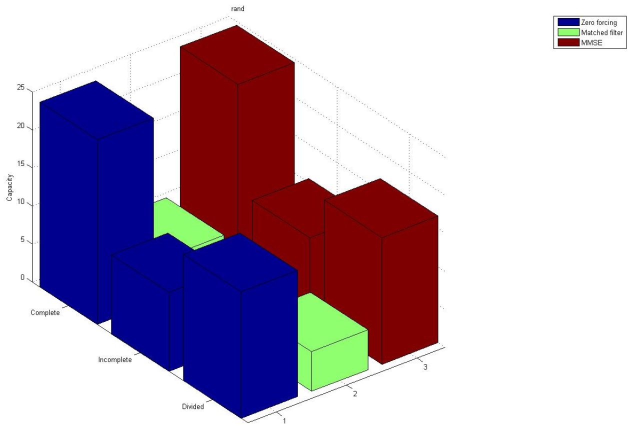

FIGURE D.8: Sum capacity, with 0.3 correlation at the transmitter side. Using random assumption

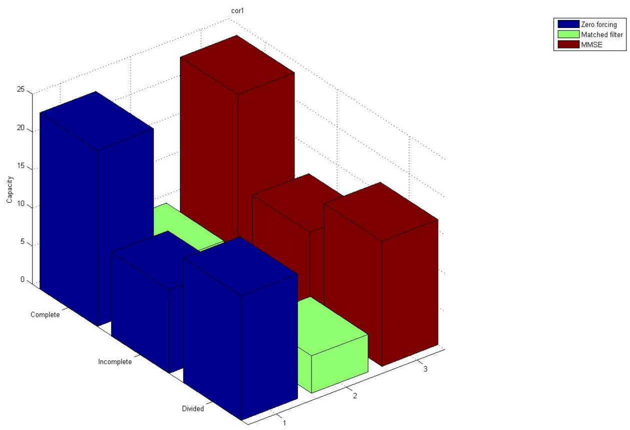

FIGURE D.9: Sum capacity, with 0.4 correlation at the transmitter side. Using correlation assumption

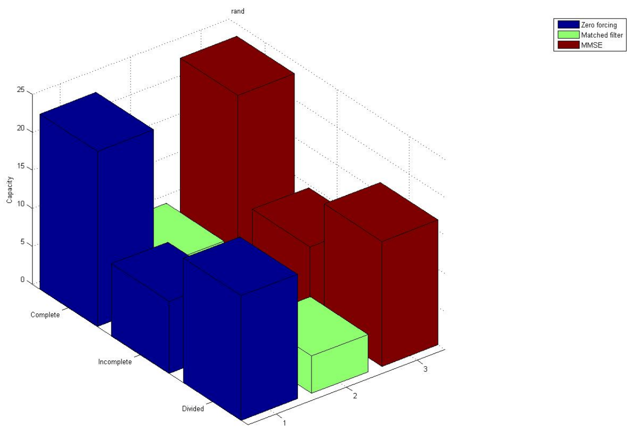

FIGURE D.10: Sum capacity, with 0.4 correlation at the transmitter side. Using random assumption

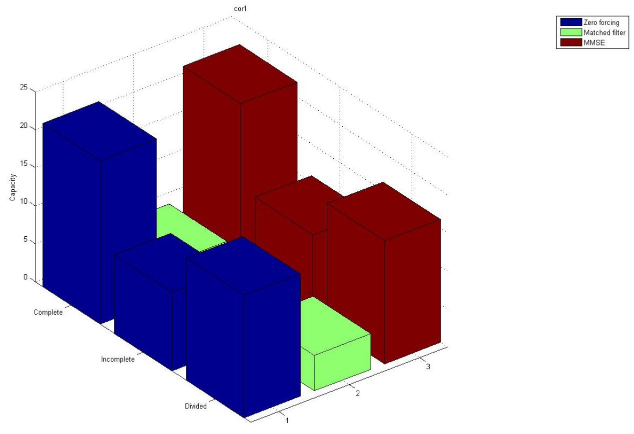

FIGURE D.11: Sum capacity, with 0.5 correlation at the transmitter side. Using correlation assumption

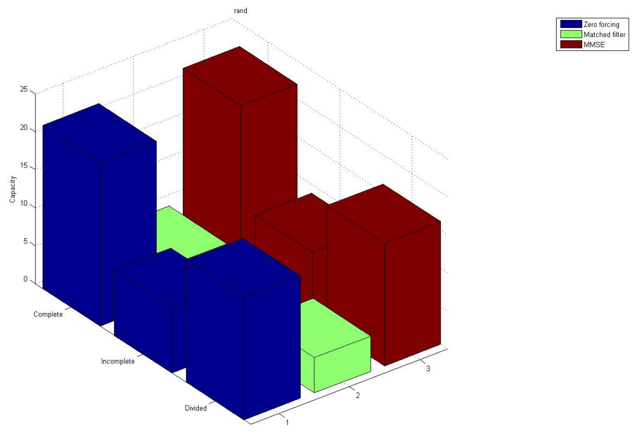

FIGURE D.12: Sum capacity, with 0.5 correlation at the transmitter side. Using random assumption

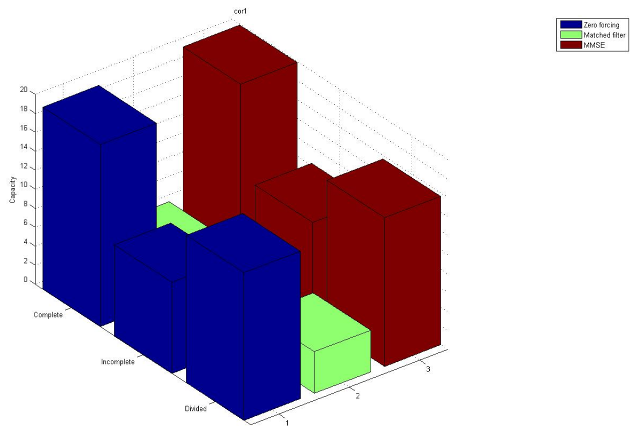

FIGURE D.13: Sum capacity, with 0.6 correlation at the transmitter side. Using correlation assumption

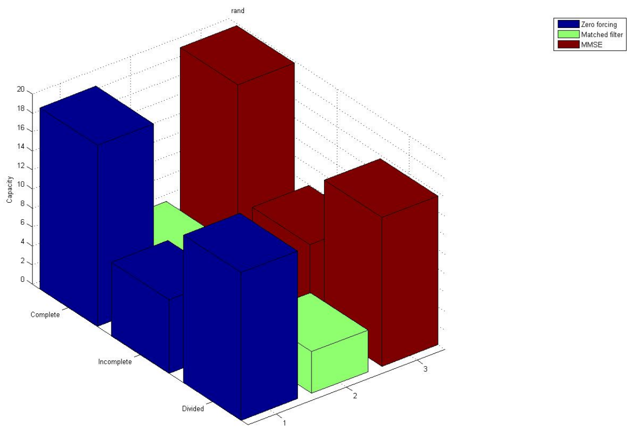

FIGURE D.14: Sum capacity, with 0.6 correlation at the transmitter side. Using random assumption

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