Optimisation of parametric processes in nonlinear optical waveguides

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Abstract

This Master’s Thesis presents an optimisation of parametric processes on two different materials used for optical communications. The main objective is to determine whether they are suitable for all-optical signal processing. Different methods for simulating the key parameters affecting their nonlinear behaviour are discussed, and a selection of them have been implemented in Matlab® to be used in the optimisation procedure. Such optimisation is based on maximising the efficiency of the parametric Four-Wave Mixing (FWM) process. This has been achieved by tailoring the waveguide geometry to engineer the dispersion properties, to minimise the propagation loss, and to maximise the nonlinear performance of the medium. The influence of the input pump power on the latter has also been discussed. Additionally, we present a comparison between different nonlinear media that resulted in a publication.
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Jaume Bohigas Nadal
# Table of Contents

Abstract i

Acknowledgements iii

List of Figures vii

List of Tables ix

List of Acronyms xi

1 Introduction 1

2 Theoretical background 3
   2.1 Optical nonlinearities . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 3
   2.2 Four-wave mixing . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 4
   2.3 Dispersion . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 5
   2.4 Conversion efficiency . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 6

3 Computational models 7
   3.1 Mode solver . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 7
   3.2 Conversion efficiency computation . . . . . . . . . . . . . . . . . . . . . . . 8
      3.2.1 Shibata et al. 1987 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 8
      3.2.2 Stolen and Bjorkholm 1982 . . . . . . . . . . . . . . . . . . . . . . . 9
      3.2.3 Coupled equations system . . . . . . . . . . . . . . . . . . . . . . . . 10
      3.2.4 Split-step computation . . . . . . . . . . . . . . . . . . . . . . . . . . 11
   3.3 Effective mode area computation . . . . . . . . . . . . . . . . . . . . . . . . 12
   3.4 Linear loss computation . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 14

4 Optimisation of III-V semiconductor optical waveguides 17
   4.1 Waveguide structure . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 17
   4.2 Optimisation procedure . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 19
      4.2.1 Maximum nonlinear coefficient $\gamma$ . . . . . . . . . . . . . . . . . . . 19
      4.2.2 Minimum propagation loss $\alpha$ . . . . . . . . . . . . . . . . . . . . . 20
      4.2.3 Dispersion effect . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 21
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2.4</td>
<td>Input power effect</td>
<td>27</td>
</tr>
<tr>
<td>4.3</td>
<td>Comparison of III-V with other nonlinear media</td>
<td>30</td>
</tr>
<tr>
<td>4.4</td>
<td>Optimisation summary</td>
<td>31</td>
</tr>
<tr>
<td>5.1</td>
<td>Conclusions</td>
<td>33</td>
</tr>
<tr>
<td>5.2</td>
<td>Future work</td>
<td>34</td>
</tr>
<tr>
<td>Appendix</td>
<td></td>
<td>35</td>
</tr>
<tr>
<td>A</td>
<td>Comparison of nonlinear media for all-optical signal processing</td>
<td>37</td>
</tr>
<tr>
<td>Bibliography</td>
<td></td>
<td>41</td>
</tr>
</tbody>
</table>
List of Figures

2.1 Frequency scheme of four-wave mixing .......................... 4

3.1 Fields output of the mode solver ............................... 8
3.2 Comparison between Shibata and Stolen conversion efficiency computation .... 10
3.3 Conversion efficiency with split-step frequency sweep model ...................... 12
3.4 Structure used for EMA comparison ................................ 13
3.5 Comparison of effective mode area computation ............................. 13
3.6 Propagation loss versus core dimensions ................................ 14
3.7 Electric field $E_x$ distribution for square waveguide .......................... 15

4.1 Waveguide structure simulated for optimisation .......................... 17
4.2 Nonlinear parameter versus width and height for AlGaAs ....................... 19
4.3 Nonlinear parameter versus width and height for InGaP ...................... 20
4.4 Linear loss versus width and height for AlGaAs ............................. 20
4.5 Linear loss versus width and height for InGaP ............................. 21
4.6 Dispersion curve at maximum $\gamma$ ................................ 22
4.7 CE at maximum $\gamma$ ............................................ 22
4.8 Dispersion curves with little variation on width ............................ 23
4.9 Phase mismatch curves with little variation on width ........................ 24
4.10 Phase mismatch curves for structures with parametric gain .................. 24
4.11 CE with parametric gain for minimum and maximum FWHM ................ 25
4.12 CE of two best AlGaAs waveguides .................................. 26
4.13 CE of two best InGaP waveguides .................................. 26
4.14 Gain saturation effect ............................................. 28
4.15 CE varying pump power for AlGaAs waveguide ............................... 28
4.16 CE varying pump power for InGaP waveguide ............................... 29
4.17 Minimum pump power needed to reach a certain CE for different materials 31
List of Tables

4.1 III-V semiconductors used for the waveguide core . . . . . . . . . . . . . . . . 18
4.2 Numerical recapitulation for the most remarkable waveguides . . . . . . . . . 27
4.3 Numerical recapitulation for two remarkable waveguides varying pump power . 30
List of Acronyms

BCB  Benzocyclobutene
CE   Conversion Efficiency
DFWM Degenerate Four-Wave Mixing
EMA  Effective Mode Area
EME  Eigenmode Expansion
FCA  Free Carrier Absorption
FWHM Full Width at Half-Maximum
FWM  Four-Wave Mixing
GVD  Group Velocity Dispersion
HNLF Highly NonLinear Fibre
NLSE Nonlinear Schrödinger Equation
SBS  Stimulated Brillouin Scattering
SFG  Sum-Frequency Generation
SHG  Second-Harmonic Generation
SOI  Silicon-On-Insulator
SPM  Self-Phase Modulation
SRS  Stimulated Raman Scattering
THG  Third-Harmonic Generation
TPA  Two Photon Absorption
XPM  Cross-Phase Modulation
Chapter 1

Introduction

In the last years, there has been a rapid increase in the use of telecommunication networks, which have been adapted to new requirements of much more bandwidth, speed, and capacity. An efficient way of transmitting signal at high speed that has been used for many years is the use of light. However, this good transportation performance becomes limited by the use of conventional electronic circuits in the system. Such electronic devices are reaching its essential limits and cannot be significantly improved to keep up with the increasing demand of capacity. In this context, all-optical signal processing appears as a promising alternative to improve the performance of high speed networks.

All-optical signal processing may involve different features such as wavelength conversion, signal amplification, and signal regeneration. These features could be obtained with silica fibres [1–3], which are characterised by a very low propagation loss. However, since they also present a very low nonlinear profile, long lengths of fibre are needed for nonlinear signal processing. Therefore, an alternative media is required to allow for optical signal processing in integrated devices. The main reason of performing this signal processing in an integrated circuit is that the devices are more compact and flexible for system design.

In the past few years, a different approach has been the use of silicon waveguides [4–8], due to its low-cost fabrication and the possibility to easily manufacture Silicon-On-Insulator (SOI) wafers on the same board used for electronic devices. Furthermore, SOI waveguides have more efficient nonlinear effects than fibres, which allows to use lower input powers and very short length devices. These nonlinear effects include Four-Wave Mixing (FWM), Stimulated Raman Scattering (SRS), parametric amplification, Self-Phase Modulation (SPM), and Cross-Phase Modulation (XPM), widely used for all-optical signal processing. However, it is difficult to manufacture a laser based on silicon due to its molecular properties, and the combination with other media becomes necessary. Moreover, silicon waveguides suffer from high nonlinear losses because of the Two Photon Absorption (TPA) effect, which additionally causes Free Carrier Absorption (FCA). This usually is an important limitation that makes silicon waveguides not the optimal solution in many applications.

In this context, it is important to study new materials to overcome this drawback.
Two III-V semiconductors, Aluminium Gallium Arsenide (AlGaAs) and Indium Gallium Phosphide (InGaP), have been recently proposed \[9, 10\]. They have an excellent nonlinear performance and, in addition, prevent TPA and FCA induced losses because of their large band-gap energy. This may allow higher power to be launched into the device to obtain better FWM performance. Additionally, both the refractive index and the band-gap energy of AlGaAs and InGaP can be varied in a wide range in order to optimise the material properties, just by altering the aluminium and gallium mole fraction \[11\]. Even though hybrid integrated optical circuits including III-V semiconductors are more expensive and complex to manufacture, these materials may present better nonlinear performance than silica fibres and silicon waveguides.

This Master’s Thesis presents a theoretical research with simulations about the efficiency of parametric processes on AlGaAs and InGaP waveguides. The main objective is to determine if a good performance for all-optical signal processing can be achieved by engineering the parameters that define the device. To determine if it has a good nonlinear behaviour, the study will be focused on the FWM effect, which is usually used to amplify and convert the frequency of the signal (wavelength conversion). More precisely, we will determine whether positive net conversion gain and parametric conversion gain are achievable with these media.

This report is divided into five chapters. Chapter 1 is an introduction to the research topic and to the main objectives of this thesis. Chapter 2 reviews the theoretical background of nonlinear optics. Chapter 3 presents the computational models that have been used to perform the simulations. Chapter 4 explains the optimisation procedure followed, the results obtained after optimisation, and a comparison with other nonlinear media performed during the course of the thesis. Finally, chapter 5 concludes the thesis and provides possible directions for future work.
This chapter consists of a theoretical background review of nonlinear optics and an explanation of the most important phenomenons treated in this thesis. Section 2.1 shows a review of optical nonlinearities, explaining its causes and the derived effects. Section 2.2 presents the Four-Wave Mixing (FWM) process derived from the third-order optical nonlinear susceptibility, and section 2.3 deals with the optical wave dispersion, essential in the FWM. Section 2.4 introduces the computation of the Conversion Efficiency (CE) and presents the most important parameters that affect it.

2.1 Optical nonlinearities

Optical nonlinearities are usually present when a nonlinear media is used for optical communications. A nonlinear media is that in which the polarization $P$ depends nonlinearly on the electric field $E$ of the optical signal. This nonlinear response of the media is only experienced for intense electromagnetic fields. In nonlinear optics, the induced polarization can be expressed as a power series \[12\],

$$P = \epsilon_0 \left[ \chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \ldots \right], \quad \text{(2.1)}$$

where $\chi^{(1)}$, $\chi^{(2)}$ and $\chi^{(3)}$ are the first, second and third-order nonlinear susceptibilities and $\epsilon_0$ is the vacuum permittivity. On the other hand, linear optics is a particular case of this equation where the induced polarization $P$ can then be expressed as a linear relation with the electric field as follows \[13\],

$$P = \epsilon_0 \chi^{(1)} E. \quad \text{(2.2)}$$

Hence, $\chi^{(1)}$ is commonly referred to as the linear susceptibility. In materials that show a center of symmetry at the molecular level, the even order susceptibilities are zero, while the odd order can be nonzero as in all materials. The second and third-order susceptibilities are the two that have been studied the most in nonlinear optics. The second-order susceptibility,
\( \chi^{(2)} \), is the responsible for effects such as Second-Harmonic Generation (SHG) and Sum-Frequency Generation (SFG). As this second-order nonlinearity only occurs in crystal materials with a non-symmetric crystal structure, it will not be present in the structures treated in this thesis. The third-order susceptibility, \( \chi^{(3)} \), occurs almost in all media and is the responsible for effects such as the Kerr effect, Third-Harmonic Generation (THG), FWM, Self-Phase Modulation (SPM), Cross-Phase Modulation (XPM), and Raman and Brillouin Scattering. In this thesis we will attempt to take benefit of the FWM phenomenon derived from the third order nonlinearity \( \chi^{(3)} \).

### 2.2 Four-wave mixing

In nonlinear optics, the FWM phenomenon is the process where two pump photons are converted to a signal and an idler photon \([12]\). This nonlinear effect is due to the third-order optical nonlinear susceptibility \( \chi^{(3)} \) and it involves the interaction of four optical fields. In such process two pump beams are launched into the media at frequencies \( \omega_{p1} \) and \( \omega_{p2} \) together with a signal at frequency \( \omega_{\text{signal}} \). Two pump photons are then converted to a signal and an idler photon, respectively, resulting in the appearance of an idler at frequency \( \omega_{\text{idler}} \) such that \( \omega_{p1} + \omega_{p2} = \omega_{\text{signal}} + \omega_{\text{idler}} \), as shown in Fig. 2.1.

![Figure 2.1: Pump, signal, and idler waves at the input (left) and output (right) of the nonlinear media.](image)

The case where \( \omega_{p1} = \omega_{p2} \) is of special interest because it could be implemented with only one pump beam. This particular case is called Degenerate Four-Wave Mixing (DFWM) and the energy is transferred from a single pump wave to two waves \([12]\). The work of this thesis will be focused on this degenerated case where the signal and idler frequencies will be equally spaced from the pump frequency.

\[
2\omega_p = \omega_{\text{signal}} + \omega_{\text{idler}}. \tag{2.3}
\]
It is worth pointing out that this process may lead to amplification of the input signal wave towards the output signal wave (signal gain) or the output idler wave (conversion efficiency). These two amplifications mainly depend on the phase-mismatch ($k$) between the involved waves, which is required to be minimal in order to obtain an efficient DFWM [14]. It is given by

$$k = \Delta k_{\text{nonlinear}} + \Delta k_{\text{linear}} = 2\gamma P_{\text{pump}} + \Delta k_{\text{linear}}.$$ (2.4)

The effective nonlinearity, $\gamma$, is written as

$$\gamma = \frac{2\pi n_2}{\lambda A_{\text{eff}}},$$ (2.5)

where $n_2$ is the nonlinear refractive index, $\lambda$ is the wavelength of light, and $A_{\text{eff}}$ is the effective mode area. The linear dispersion phase-mismatch, $\Delta k_{\text{linear}}$, can be expressed as

$$\Delta k_{\text{linear}} = \beta_2 \Omega_s^2 + \frac{1}{12} \beta_4 \Omega_s^4,$$ (2.6)

where $\beta_2$ and $\beta_4$ are the dispersion parameters at the pump frequency (widely explained in section 2.3), and $\Omega_s$ is the frequency difference between the pump and the signal, i.e. $\Omega_s = \omega_p - \omega_s$. In the cases where $\beta_2$ is not zero or very small, $\beta_4$ can be usually neglected.

The optimum case is when perfect phase-match is obtained ($k = 0$) but it can be challenging to obtain in some cases. In order to achieve phase matching, at least one of the two mismatch contributions in equation (2.4) should be negative and this can only be achieved when the dispersion parameters are negative.

### 2.3 Dispersion

Dispersion in optical communications can be a wanted or unwanted effect depending on the application. This dispersion of an optical wave is the phenomenon in which the group velocity depends on the wave’s frequency. This results in Group Velocity Dispersion (GVD), which causes the light pulse to spread in time because every frequency component is travelling with different velocities. To describe this effect the second-order dispersion parameter is often used, $\beta_2$, known as the GVD parameter. The dispersion parameter $D$ is also commonly used in practice, which is related to $\beta_2$ and to the refractive index $n$ as follows [12],

$$D = -\frac{\lambda}{c} \frac{d^2 n}{d\lambda^2} = -\frac{2\pi c}{\lambda^2} \beta_2.$$ (2.7)
As seen previously, the dispersion of the medium used in an application is a very important aspect when we want to obtain an efficient DFWM. In the previous section (2.3) it has been deduced from equations (2.4) and (2.6) that $\beta_2$ should be negative to allow phase-matching and, therefore, $D$ should be positive.

The wavelength at which both $D$ and $\beta_2$ are zero is denoted as zero-dispersion wavelength ($\lambda_0$). Working close to $\lambda_0$ is necessary when large amplifications are to be achieved. The interesting point is that the position of this wavelength $\lambda_0$ as well as the contribution of $D$ can be properly adjusted by varying the design parameters of the device such as the materials used, the difference between the refractive index of the core and the cladding, or the device dimensions among others.

2.4 Conversion efficiency

In a FWM process the CE is the power relation between the output idler wave and the input signal wave.

\[ CE = \frac{P_{\text{idler}}^{\text{out}}}{P_{\text{signal}}^{\text{in}}} \]  

(2.8)

There are many different computational models to compute or estimate it, the most relevant are discussed in the next chapter in section 3.2.

Every time more and more, the FWM process is being used for wavelength conversion in many different applications, so many efforts are being put into maximising the CE. If a positive CE can be achieved there is no need to use an optical amplifier after the device, and less noise will be added to the signal. The CE depends on many parameters, the most important being the material dispersion $D$, the effective nonlinearity $\gamma$, the material linear loss, and the pump power.

Since this thesis is focused on optimising certain waveguide structures to maximise the CE, many aspects have to be taken into account. First, the total dispersion plays a very important role. The parameter $D$ depends on the variation of the refractive index $n$ with $\lambda$, so a proper study of the chosen material and its mole fraction should be performed to obtain a positive and close to zero value of $D$. Second, a high value for $\gamma$ is required to increase the CE. As seen in equation (2.5), $\gamma$ is inversely proportional to the effective area, so a small $A_{\text{eff}}$ will be needed. There are also many different models to estimate the effective mode area, which are also shown and compared in section 3.3. Furthermore, there will always be linear losses ($\alpha$), which usually are an important drawback. These losses are much higher in optical waveguides than in fibres. Hence, a study to minimise $\alpha$ should also be done together with an optimisation of the interaction length. Finally, another important parameter that affects the CE is the pump power $P_{\text{pump}}$. Entering into saturation regime has to be avoided while taking into account that high powers in some materials can damage the device and become harmful.
For the development of this study, many simulations have been performed. There are many different models to compute some of the parameters or effects that have to be simulated so, before starting to optimise any structure, we need to be sure about what models will be the most appropriate for computing the behaviour of the device. In this chapter, we present a comparison of the different computational models for the most relevant parameters needed for the optimisation and an explanation of what model is used on each case.

3.1 Mode solver

For the study of the behaviour of any optical structure it is essential to know how the light propagates in the media. This means finding out how the electromagnetic radiation modes behave in the device. When dealing with ideal or simple configurations, this can be found analytically. However, when the case study is to obtain more realistic results or when non ideal structures are treated, numerical simulations are needed. A way for performing these numerical simulations is using an EigenMode Expansion (EME) method, which consists in rigorously solving Maxwell’s equations on each local cross-section and representing the electromagnetic fields everywhere.

In this research we have used the mode solver WGMODES - Optical Eigenmode Solver for Dielectric Waveguides, based on an EME method\(^1\). It is fully written in Matlab\(^\circledR\) and allows the definition of particular non-uniform structures, which makes it very flexible and adaptive. With this mode solver both the electric and magnetic field can be computed all over the cross-section of the waveguide, as well as the effective refractive index \(n_{\text{eff}}\). The distribution of the fields will be essential when computing the effective area \(A_{\text{eff}}\), and the effective index will be needed when computing the total dispersion of the waveguides. In figure 3.1 there is an example of the output from the mode solver for a 200 x 400 nm\(^2\) SOI waveguide with silica lower cladding and with air for the upper cladding.

\(^1\)For further details about the method and the algorithm used in this mode solver, refer to the article by Thomas E. Murphy et al. (2008) [15].
3.2 Conversion efficiency computation

Computing the estimated CE is not always an easy task since many effects should be taken into account and it may be involve the solution of complex systems of equations that might require a lot of computation time. To avoid this time consuming there are two models which can give a first approximation of the CE, and they are of immediate computation. These two models are usually called Shibata and Stolen, presented in the following two subsections (3.2.1 and 3.2.2). The third subsection (3.2.3) explains a model that consists on solving a system of differential coupled equations. Finally, a numerical computation for the CE which is much more accurate but also much more time consuming is also discussed in the fourth subsection (3.2.4).

3.2.1 Shibata et al. 1987

The first analytical expression that could be used is the one introduced by Shibata et al. [16], expressed as

\[
CE = \eta \gamma^2 L_{eff}^2 e^{-\alpha L} P_{pump}^2,
\]  

(3.1)

where \( L_{eff} = (1 - e^{-\alpha L})/\alpha \) is the effective device interaction length, \( \alpha \) is the linear loss, \( L \) is the physical length, and \( \eta \) is a wave efficiency parameter that depends on the loss \( \alpha \) and
the linear phase mismatch $\Delta \beta = \Delta k_{\text{linear}}$, written as

$$
\eta = \frac{\alpha^2}{\alpha^2 + (\Delta \beta)^2} \left(1 + \frac{4e^{-\alpha L}}{(1 - e^{-\alpha L})^2} \sin^2\left(\frac{\Delta \beta L}{2}\right)\right).
$$

This theory assumes that there is no saturation for high pump powers, so it will not be valid in case of pump depletion. It neither considers nonlinear phase matching as it only takes the linear contribution of the phase mismatch factor into account. This model is compared with two more analytical approximations in Fig. 3.2, where the CE is computed for a 10 cm long 300 x 500 nm$^2$ SOI waveguide with a pump power of 100 mW, a linear loss coefficient of 1.1 dB/cm, and an effective nonlinearity of $\gamma = 202$ W$^{-1}$m$^{-1}$.

### 3.2.2 Stolen and Bjorkholm 1982

Another analytical expression can be considered to estimate the conversion efficiency in the DFWM process. From the results of the theory by Stolen and Bjorkholm [17], the CE can be expressed as follows,

$$
CE = \left[\frac{\gamma P_{\text{pump}}}{g} \sinh(gL)\right]^2,
$$

(3.3)

where

$$
g = \sqrt{(\gamma P_{\text{pump}})^2 - \left(\frac{k}{2}\right)^2},
$$

(3.4)

and $k$ is the phase-mismatch shown in equation (2.4). In this case it is also assumed that there is no pump depletion. In contrast to the model by Shibata, this assumes that there are no losses, hence it will not be a realistic model in the cases where there are high losses such as in waveguides. However, it considers both contributions, linear and nonlinear, of the phase mismatch so parametric gain can be obtained.

It can be seen in Fig. 3.2 that with the Stolen model parametric gain is obtained at a detuning of 5 nm while for Shibata it is not. Furthermore, higher values of CE are obtained with Stolen model. This difference is due to the linear losses that are not being considered. If linear losses are considered, for instance using $L_{\text{eff}}$ in equation (3.3) instead of $L$, both curves coincide for small detunings, which is also represented in the figure. Moreover, this correction of the Stolen curve by the linear loss is not enough to get a match between the two methods. In addition, these two analytical models also lack the effect of nonlinear losses that might be present in some media, which are induced by Two Photon Absorption (TPA) and, consequently, by Free Carrier Absorption (FCA).
CHAPTER 3. COMPUTATIONAL MODELS

Figure 3.2: Comparison of CE versus wavelength detuning for a 10 cm long 300 x 500 nm² SOI waveguide using Shibata et al. (1987) and Stolen and Bjorkholm (1982) models. Pump power used is 100 mW, $\alpha = 1.1$ dB/cm and $\gamma = 202 \text{ W}^{-1}\text{m}^{-1}$.

3.2.3 Coupled equations system

The DFWM can be described by a set of coupled equations in terms of the complex envelopes of the interacting waves ($A_j$), under the assumption of continuous wave as described in [18]:

\[
\frac{dA_p}{dz} = -\frac{1}{2}(\alpha + \alpha_{TPA} + \alpha_{FCA})A_p + j\gamma \left[ |A_p|^2 + 2|A_s|^2 + 2|A_i|^2 \right] A_p + 2j\gamma A_p^*A_s A_i e^{j\Delta \beta z},
\]  

(3.5a)

\[
\frac{dA_s}{dz} = -\frac{1}{2}(\alpha + \alpha_{TPA} + \alpha_{FCA})A_s + j\gamma \left[ 2|A_p|^2 + |A_s|^2 + 2|A_i|^2 \right] A_s + j\gamma A_s^*A_p^2 e^{-j\Delta \beta z},
\]  

(3.5b)

\[
\frac{dA_i}{dz} = -\frac{1}{2}(\alpha + \alpha_{TPA} + \alpha_{FCA})A_i + j\gamma \left[ 2|A_p|^2 + 2|A_s|^2 + |A_i|^2 \right] A_i + j\gamma A_i^*A_p^2 e^{-j\Delta \beta z},
\]  

(3.5c)

where $A_p$, $A_s$, and $A_i$ are the amplitudes of the pump, signal, and idler waves, and $\alpha_{TPA}$ and $\alpha_{FCA}$ are the nonlinear loss coefficients due to TPA and FCA. These nonlinear loss coefficients can be expressed as
3.2. CONVERSION EFFICIENCY COMPUTATION

\[ \alpha_{TPA_j} = \frac{\beta_{TPA}}{A_{\text{eff}}} \left( |A_j|^2 + 2 \sum_{m \neq j} |A_m|^2 \right), \]  
(j = p,s,i for the pump, signal and idler waves)

\[ \alpha_{FCA} = \frac{\sigma \beta_{TPA} \tau}{2hcA_{\text{eff}}^2} \left( \sum_m \lambda_m |A_m|^4 + 4 \sum_{m \neq n} \frac{\lambda_m \lambda_n |A_m|^2 |A_n|^2}{\lambda_m + \lambda_n} \right), \]  
(3.7)

where \( \sigma \) is the FCA cross section, \( \beta_{TPA} \) is the TPA coefficient, and \( \tau \) is the carriers’ lifetime.

Solving this system of differential equations, the amplitudes of all the interacting waves are found. Thus, the CE can be obtained by computing the relation between the output idler wave and the input signal wave.

In the cases where the media that is being used has the presence of nonlinear losses due to TPA and FCA, this method will be much more reliable than the previous two models, since it takes into account the nonlinear loss coefficients. Moreover, in this case, the complex field amplitudes are being computed instead of directly using an analytical formula to find the CE. It is reasonable to think that we will obtain more accurate results. However, solving the system of coupled equations will not be an easy nor a fast procedure, thus it is going to be a slower computation when implementing it with Matlab®.

3.2.4 Split-step computation

Another numerical model for computing the CE is using a split-step frequency sweep solving the basic scalar nonlinear Schrödinger equation (NLSE) [12] for each frequency. Dispersion and linear propagation loss are considered in this model. However, other effects such as nonlinear loss due to TPA and FCA are not included in this computation. Depending on the resolution (frequency step size) wanted, this computation is going to be the slowest one of the four presented. Nevertheless, it is the one that gives the best fit when the material does not present nonlinear losses. An example of the computation using this model can be seen in Fig. 3.3, where the CE is computed again for a 10 cm long 300 x 500 nm² SOI waveguide with a pump power of 100 mW, a linear loss coefficient of 1.1 dB/cm and an effective nonlinearity of \( \gamma = 202 \text{ W}^{-1}\text{m}^{-1} \).

One can notice that the same CE level is reached for detunings close to zero as in the Shibata model, because both models consider propagation losses. On the other hand, it has a more similar shape to the Stolen model with \( L_{\text{eff}} \) (Fig. 3.2), where a little bit of parametric conversion gain is achieved. Nevertheless, the bandwidth in this case is more similar to the one attained with Shibata. Finally, it can be concluded that when the analytical approaches are used to estimate the CE, each model gives different information about its behaviour, but this numerical solution gives the most reliable solution in absence of nonlinear loss. Hence, this will be usually used because the media simulated in this thesis is chosen not to have TPA nor FCA.
CHAPTER 3. COMPUTATIONAL MODELS

Figure 3.3: Conversion Efficiency versus wavelength detuning for a 10 cm long 300 x 500 nm$^2$ SOI waveguide using split-step frequency sweep computation. Pump power used is 100 mW, $\alpha = 1.1$ dB/cm and $\gamma = 202$ W$^{-1}$ m$^{-1}$.

3.3 Effective mode area computation

In a similar way as in the previous section, there are many formulations for estimating the value of the Effective Mode Area (EMA) $A_{\text{eff}}$. The effective area is mainly characterised by the electric field $E(x,y)$ and the magnetic field $H(x,y)$. If taking a full vectorial model, a definition for $A_{\text{eff}}$ as in [19] can be,

$$A_{\text{eff}} = \left( \int_{-\infty}^{\infty} S_z \, dx \, dy \right)^2 \int_{-\infty}^{\infty} S_z^2 \, dx \, dy,$$

(3.8)

where $S_z = (E \times H) \cdot \hat{z}$ is the $z$ component of the Poynting vector. This model takes into account the contribution of both electric and magnetic fields all over the cross section.

Another definition which is more commonly used is the one found in [12] and [20], which is expressed by

$$A_{\text{eff}} = \frac{\left( \int_{-\infty}^{\infty} |E(x,y)|^2 \, dx \, dy \right)^2}{\int_{-\infty}^{\infty} |E(x,y)|^4 \, dx \, dy},$$

(3.9)

where $E(x,y)$ is the modal electric field. These two expressions might present very different results, specially in the case of high refractive index contrast and on waveguides with small core dimensions (sub-wavelength dimensions). In contrast with the previous one, this only considers the contribution of the electric field.

A comparison between the expressions (3.8) and (3.9) is done in [21] together with a new definition of EMA that they introduce as
3.3. EFFECTIVE MODE AREA COMPUTATION

\[ A_{\text{eff}} = A_{NL} \frac{\iint_{-\infty}^{\infty} S_z dx dy}{\iint_{NL} S_z dx dy} \]

where \( A_{NL} \) is the area of the nonlinear region, and NL denotes the nonlinear region. A comparison has also been performed between the three models with the same waveguide structure as in [21] (Fig. 3.4) and we obtain the plots in figure 3.5. It can be observed that with expression (3.9) very different results are obtained for small dimensions, so it might not be right for such structures. It is also noticed that when augmenting the waveguide dimensions, the three formulations diverge from each other. This is because each one considers different field contributions and for big dimensions the field confinement inside the core changes.

Figure 3.4: Slot waveguide simulated for the effective mode area comparison. In the computation it has been used \( h = 0.5 \mu m, t = 100nm, H = 0.3 \mu m, W = 1.4 \mu m, n_{Si} = 3.48, n_{SiO2} = 1.45 \) and \( n_{NC} = 1.60 \).

Finally, it has to be decided what expression should be used for the optimization of nonlinear optical waveguides, taking into account that both big and small core waveguides with high refractive index contrasts between the core and cladding will be studied. So, regarding this, and as reported in [22], the use of the expression (3.8) will be the most suitable in this case.

Figure 3.5: Computation of the EMA using the three equations compared.
3.4 Linear loss computation

Optical waveguides usually suffer from very significant linear loss, so it is necessary to engineer and optimize its parameters and try to reduce the effect of propagation loss. Depending on the size of the waveguide and its fabrication procedure, it will have sidewall roughness which will result into strong scattering effects. In the article by Grillot et al. (2004) [23] an investigation about the size influence on propagation losses is reported. It is concluded on their study that propagation loss strongly depends on the size of the cross-section of the device. The loss coefficient can be found using the expression

\[ \alpha_{(dB/m)} = \frac{4.34}{2\pi} \frac{\sigma^2}{k_0 \sqrt{2d^4 n_c}} g(V) f(x, \gamma), \]  

(3.11)

where the waveguide width is \(2d\), \(g(V) = U^2 V^2 / (1 + W)\) depends only on the geometry with \(U = k_0 d \sqrt{n_c^2 - n_{eff}^2}\), \(V = k_0 d \sqrt{n_c^2 - n_{cl}^2}\), and \(W = k_0 d \sqrt{n_{eff}^2 - n_{cl}^2}\). The sidewall roughness is considered in the function \(f(x, \gamma)\),

\[ f(x, \gamma) = \frac{x \sqrt{1 - x^2 + \sqrt{(1 + x^2)^2 + 2x^2 \gamma^2}}}{\sqrt{(1 + x^2)^2 + 2x^2 \gamma^2}}, \]  

(3.12)

where \(x = W \frac{L_c}{\sigma}\), \(\gamma = \frac{n_{cl} V}{n_c W \sqrt{\Delta}}\), and \(\Delta = \frac{n_c^2 - n_{cl}^2}{2n_{eff}^2}\). The parameters \(L_c\) and \(\sigma\) represents the correlation length and the standard deviation respectively, as stated in [23]. These two last parameters depend on the fabrication process. In equation (3.11), the \(2\pi\) factor in the denominator was found to be missing in the original expression and it has been added here. In figure 3.6 it is shown how the propagation loss coefficient varies with the core dimensions for a square Silicon-On-Insulator (SOI) waveguide with Silica cladding.

![Figure 3.6: Propagation loss versus core dimensions for roughness parameters \(\sigma = 10\,nm\) and \(L_c = 50\,nm\).](image)
The propagation losses against the waveguide width are calculated using equation (3.11). The effective index varies from $n_{\text{eff}} = 1.437$ to $n_{\text{eff}} = 2.926$ for waveguide width from 180 nm to 500 nm. The loss coefficient decreases when the waveguide size is reduced below 260 nm because the optical confinement decreases, as shown in figure 3.7. The field is much less concentrated inside the core for small sizes (Fig. 3.7(a)), while for bigger dimensions (Fig. 3.7(b)) it is much more intensive inside and near the core and it leads to higher propagation loss.

![Figure 3.7: Plots of the $x$ component of the electric field ($E_x$) over the cross-section for square waveguide core of 180 nm and 260 nm. The figures only show half of the waveguide’s section as it has a symmetric behaviour.](image)
In this chapter the optimisation procedure followed on this research is developed. Section 4.1 explains the waveguide structures optimised and the materials that have been used. Section 4.2 details the optimisation procedure itself, specifying each step. Section 4.3 compares different commonly used nonlinear media that have also been done during the course of this project. Finally, section 4.4 summarises the optimisation performed.

4.1 Waveguide structure

The waveguide structure considered in this research is a buried channel waveguide (figure 4.1). The core, which contains the nonlinear media, is made of a III-V semiconductor with high refractive index. This core is all surrounded by a Benzocyclobutene polymer (BCB) cladding. A higher refractive index could be obtained using silica (SiO$_2$) or air claddings, but this is not feasible due to fabrication limitations. The use of III-V materials for optical waveguides is a quite new technique, so there are still some manufacturing limitations.

Figure 4.1: Waveguide structure simulated for the optimisation. The $n_2$ cladding is made of BCB polymer and the $n_1$ core is a III-V semiconductor.
We have considered two different III-V semiconductors, Aluminium Gallium Arsenide (Al\(_x\)Ga\(_{1-x}\)As) and Indium Gallium Phosphide (In\(_{1-x}\)Ga\(_x\)P), where \(x\) is the concentration of Aluminium and Gallium, respectively. These two media have a refractive index between 3.1 and 3.4 while the BCB cladding has an index of 1.575 that will provide a significant index contrast. AlGaAs and InGaP also have a relatively high nonlinear refractive index \(n_2 (\sim 10^{-17} \text{m}^2/\text{W})\), which is an advantage to obtain higher values of the effective nonlinearity \(\gamma\) (Eq. (2.5)).

The refractive index of the III-V materials depends on the working wavelength (\(\lambda\)) and on the alloy \(x\). Regarding the concentration \(x\), the one that gives the highest refractive index has to be found. Nevertheless, different alloy compositions will result on different bandgap energy [24] and we have to be careful and avoid some effects such as TPA so as to avoid nonlinear losses. TPA only occurs if the photon energy is at least half the bandgap energy [25]. Consequently, to make sure such effect is circumvented, the band gap energy (\(E_g\)) must be higher than two times the photon energy, which, at \(\lambda = 1550\) nm, is equal to:

\[
E_g > 2\hbar \omega = 1.6 \text{ eV},
\]

where \(\hbar\) is the reduced Planck constant, and \(\omega = \frac{2\pi \varepsilon}{\lambda}\) is the angular frequency. When increasing \(x\), the refractive index decreases, so it has to be chosen the minimum concentration that has a bandgap energy over 1.6 eV. For AlGaAs it is found that \(x = 0.18\) will be a good choice as it has a bandgap energy of 1.65 eV. In the case of InGaP the concentration of Gallium has been set to \(x = 0.5\). In table 4.1 we put together the main properties of both semiconductors.

<table>
<thead>
<tr>
<th>Material</th>
<th>Alloy (x)</th>
<th>(n) at 1550nm</th>
<th>(n_2)</th>
<th>(E_g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al(<em>x)Ga(</em>{1-x})As</td>
<td>0.18</td>
<td>3.34</td>
<td>(1.45 \times 10^{-17} \text{m}^2/\text{W})</td>
<td>1.65 eV</td>
</tr>
<tr>
<td>In(_{1-x})Ga(_x)P</td>
<td>0.5</td>
<td>3.16</td>
<td>(1 \times 10^{-17} \text{m}^2/\text{W})</td>
<td>1.86 eV</td>
</tr>
</tbody>
</table>

Table 4.1: III-V semiconductors used for the waveguide core
4.2 Optimisation procedure

The procedure followed for both materials is focused on optimising two main parameters, which are the nonlinear coefficient $\gamma$ and the linear propagation loss coefficient $\alpha$. These parameters are modified by changing the dimensions of the waveguide’s core, height ($h$) and width ($w$). The following subsections show the results of this optimisation and the resulting CE achieved for the two semiconductor materials.

4.2.1 Maximum nonlinear coefficient $\gamma$

To find where the maximum nonlinear coefficient is obtained, we have done a sweep of both dimensions, height and width, and computed the value of $\gamma$ for each configuration. The nonlinear coefficient $\gamma$ is computed using Eq. (2.5) and the EMA $A_{eff}$ using Eq. (3.8). The sweep for both dimensions is set from 50 nm to 1 $\mu$m so it covers a wide enough range.

Figure 4.2 shows the geometric dependence of the nonlinear parameter for the AlGaAs waveguide. It results to strongly depend on the dimensions of the core as it varies one order of magnitude (from less than 50 $W^{-1} m^{-1}$ to more than 480 $W^{-1} m^{-1}$) in the range of computation. The maximum $\gamma$ is found to be of 480 $W^{-1} m^{-1}$ at a core dimension around 300 x 400 nm$^2$ (height x width).

For the InGaP semiconductor the geometric dependance is shown in Fig. 4.3. In this case the maximum $\gamma$ is of about 280 $W^{-1} m^{-1}$ at the size of 300 x 450 nm$^2$. For InGaP it is obtained a lower nonlinear coefficient but it can not be assesses that the CE will be lower as there are other parameters like propagation loss that have still to be studied. Moreover, this maximum point found will not probably be the point where the best CE is obtained as it does not coincide with the point of minimal losses (seen on next subsection 4.2.2).
CHAPTER 4. OPTIMISATION OF III-V SEMICONDUCTOR OPTICAL WAVEGUIDES

4.2.2 Minimum propagation loss $\alpha$

After having computed the dependence of $\gamma$ with the waveguide’s geometry, a similar procedure is done for the linear propagation loss parameter $\alpha$. The range of study is the same (from 50 nm to 1 $\mu$m) and the computations are made using Eq. (3.11).

The results for AlGaAs are shown in Fig. 4.4. At the point where the maximum $\gamma$ was obtained (300 x 400 nm$^2$) it results to have significant losses (3.16 dB/cm). For InGaP, shown in Fig. 4.5, a lower value of losses (2.27 dB/cm) is obtained, but the maximum $\gamma$ was also lower.

For both media the losses could be reduced and still have a good enough $\gamma$. If looking at
the $\gamma$ contour level plots (Figs. 4.2(b) and 4.3(b)) it is observed that, if the dimensions are increased, the variation has a small slope. By doing that, it will be also obtained a lower value for $\alpha$ while having a decent $\gamma$ so better results of CE might be obtained outside the point of maximum nonlinear coefficient.

![3D plot](image1.png) ![Contour lines](image2.png)

**Figure 4.5:** $\alpha$ as a function of core width and height for InGaP waveguide.

Another important parameter directly connected with the CE and related to the loss coefficient is the length $L$ of the device. With the obtained computations of $\alpha$ it can be computed the optimum length by maximizing $L_{\text{eff}}^2 e^{-\alpha L}$ in terms of $\alpha$. In the next subsection 4.2.3 it is done a study of the most efficient CE that could be achieved using the obtained values for the nonlinear parameter and the linear loss coefficient.

### 4.2.3 Dispersion effect

With the values for $\gamma$ and $\alpha$ a study can now be performed about the most efficient CE achievable. For this study it is also needed to compute the dispersive behaviour of the device. Using the dispersion parameter $D$, some information can be predicted about how it will behave.

First, as an initial approach, the point of maximum $\gamma$ is treated to see what it is obtained without optimising $\alpha$. The dispersion curves for both materials are computed and represented in Fig. 4.6. The dispersion obtained for AlGaAs is not bad in the sense that it is close to zero at the working wavelengths but it would be better if it was flatter and with the point of zero dispersion closer to the pump wavelength, which is 1550 nm. For the InGaP, the curve has a similar shape but at lower level, which will result on a worse CE in comparison with AlGaAs because it is below zero.
CHAPTER 4. OPTIMISATION OF III-V SEMICONDUCTOR OPTICAL WAVEGUIDES

Figure 4.6: Dispersion curves at the dimensions of maximum $\gamma$

The conversion efficiency is then computed versus the wavelength detuning between the pump and the signal. The pump beam is set to 1550 nm and the split-step computation is used (Section 3.2.4). The lengths used are the optimum ones which are, in this case, 1.5 cm for AlGaAs and 2.1 cm for InGaP. The plots in Fig. 4.7 show the results obtained for the two materials. A positive CE is not obtained, nor parametric gain on any of the two media. However, it is probable that more efficient CE could be obtained at other dimensions with lower $\alpha$, even though it has lower $\gamma$. Despite of that, the CE bandwidth, or Full Width at Half-Maximum (FWHM), is almost 50 nm for AlGaAs, which is a reasonable value. In the case of InGaP core the CE obtained is worse than AlGaAs as predicted, with a FWHM of 20 nm.

Figure 4.7: Conversion efficiency at the dimensions of maximum $\gamma$

The following step is to try to find a configuration where more efficient CE is obtained. More efficient CE refers to obtaining higher values of gain, wider bandwidth and to achieve parametric gain. In order to do so, it has to be computed the CE at different dimensions from the point of maximum $\gamma$.

To increase the maximum value of conversion gain, a lower value of $\alpha$ needs to be obtained by choosing other core dimensions. Moreover, in order to obtain parametric conversion gain, phase matching has to be achieved. The phase mismatch directly depends
on the dispersion of the device where the ideal case would be when the curve $D$ is positive, as flat as possible and close to zero or concave with two points of zero dispersion. It is also known that the bandwidth depends on the dispersion properties as follows \cite{26}:

$$FWHM \sim \sqrt{4\gamma P/|\beta_2|}. \quad (4.2)$$

So, in order to maintain or increase the conversion bandwidth it is needed to maintain a relatively high value for $\gamma$ and try to reduce the dispersion to values near zero.

When varying the core dimensions it has been noticed that even with no so big size changes (50 nm), very different dispersion curves might be obtained, which can lead to very different CE response. An example of this is in Fig. 4.8 where, by just varying the width 50 nm in an AlGaAs waveguide, it can be appreciated a significant difference of the shape between the two curves. The blue one has a sharp slope while the green one is much flatter and very close to zero. However, it will not lead to parametric gain as it has very few points of positive dispersion.

![Figure 4.8](image)

**Figure 4.8**: Dispersion curve for two AlGaAs waveguides with variation of 50 nm on the width.

To closely look at how the CE will be, it is better to look directly on the phase mismatch $k$ (Eq. (2.4)) and see whether it cuts the line of zero phase mismatch or not. There will be parametric gain if phase match is achieved ($k = 0$) so, to predict the presence of parametric gain it is needed to look at the curve of $k$ versus wavelength. For the two previous dimensions, the phase mismatch is represented in Fig. 4.9. It can be observed again that by only varying 50 nm one of the two dimensions the phase mismatch varies significantly its shape. None of the two structures will have parametric gain as they do not have any point of perfect phase match.

To continue with the research, it is computed the dispersion and the phase mismatch for different configurations to see if parametric conversion gain could be achieved. The computations of the dispersion $D$ and the CE take some time, so the range of computation is reduced to only a limited number of selected points with lower linear losses. Among
CHAPTER 4. OPTIMISATION OF III-V SEMICONDUCTOR OPTICAL WAVEGUIDES

Figure 4.9: Phase mismatch curve for two AlGaAs waveguides with variation of 50 nm on the width.

The computed points, the dimensions where best phase match curves were obtained are represented in Fig. 4.10. All the curves have two points of zero phase mismatch so, parametric gain will be achieved. Between these devices with parametric gain, better CE will be obtained in those which the curve is flatter as the conversion bandwidth will be wider.

Figure 4.10: Phase mismatch curves for structures with parametric gain

The distance between the two points of zero dispersion determine the distance between the two peaks of parametric gain. Regarding this, the devices represented with the blue line will have a higher FWHM than the others. In the other hand, the devices depicted with yellow starred line (AlGaAs) and point-dash cyan (InGaP) will have a narrower FWHM as the zero dispersion points are closer. This difference on the conversion bandwidth can be clearly seen in Fig. 4.11 with the plot of the CE for the two extreme cases from Fig. 4.10 for both materials. It can be predicted that the blue one will have a FWHM around 90 nm for AlGaAs and 80 nm for InGaP. In the other hand, the other extreme cases will be of around 40 nm for both. In Fig. 4.11 it can be confirmed this bandwidth prediction.

Overall, it has been tried for both materials to find the dimensions that give the most
4.2. OPTIMISATION PROCEDURE

Figure 4.11: Conversion efficiency with parametric conversion gain with maximum and minimum conversion bandwidth for $P_{\text{pump}} = 200 \text{ mW}$ and $P_{\text{signal}} = 1 \text{ mW}$. 

For AlGaAs, at 300 x 700 nm$^2$ (Fig. 4.12(a)) it is obtained the widest FWHM (95 nm) with a maximum conversion gain of 0 dB which already is a good result. However, at bigger dimensions with lower losses $\alpha$ it can be obtained higher values of conversion.

efficient CE. For both materials it is difficult to achieve parametric conversion gain in small devices. The smallest dimensions where parametric conversion gain is observed is at 300 x 500 nm$^2$ for AlGaAs and at 400 x 500 nm$^2$ for InGaP. A quite good performance for AlGaAs is obtained for big structures (700 x 800 nm$^2$) where positive CE is obtained with a good FWHM (60 nm). For the InGaP waveguide, it has not been obtained a behaviour as good as for AlGaAs but still acceptable CE is achieved for smaller devices (400 x 600 nm$^2$). Two of the best CE found for each material are depicted in Fig. 4.12 and Fig. 4.13.
An example of this is at 700 x 800 nm\(^2\) (Fig. 4.12(b)) where parametric conversion gain, positive conversion and enough FWHM is achieved.

![Conversion efficiency plots for AlGaAs waveguides](image)

**Figure 4.12:** Conversion efficiency for two of the best dimensions found for AlGaAs with \(P_{\text{pump}} = 200\) mW, \(\lambda_{\text{pump}} = 1550\) nm, and \(P_{\text{signal}} = 1\) mW.

For the InGaP waveguide two of the best structures found are at 400 x 500 nm\(^2\) and at 400 x 600 nm\(^2\). Both configurations show similar values of peak CE and a similar parametric conversion gain. In terms of FWHM, The second one (Fig. 4.13(b)) have a wider bandwidth because it has the two points of phase match more spaced (Fig. 4.10(b)).

![Conversion efficiency plots for InGaP waveguides](image)

**Figure 4.13:** Conversion efficiency for two of the best dimensions found for InGaP with \(P_{\text{pump}} = 200\) mW, \(\lambda_{\text{pump}} = 1550\) nm, and \(P_{\text{signal}} = 1\) mW.

As it has not been done a sweep for all the possible dimensions there might be other configurations where better conversion is achieved. No more efforts or time have been dedicated into finding more dimensions with better results as the main goal of this project is to see whether parametric conversion gain and efficient CE is achievable or not with III-V semiconductors AlGaAs and InGaP, and this has been already demonstrated with the results shown above. However, in table 4.2 there is a numeric recapitulation of the best or most remarkable dimensions found with parametric conversion gain, its linear loss...
coefficient $\alpha$, the effective nonlinearity $\gamma$, the optimum length $L_{opt}$, the resulting FWHM, and the peak CE. It has to be noticed that in some cases, even if we have a lower $\gamma$, the presence of low losses can lead us to a better performance. In the other hand, even if we have a higher nonlinearity, the presence of higher losses can result in worse performance.

<table>
<thead>
<tr>
<th>Core material</th>
<th>height</th>
<th>width</th>
<th>$\alpha$</th>
<th>$\gamma$/$W^{-1}m^{-1}$</th>
<th>$L_{opt}$</th>
<th>FWHM</th>
<th>CE$_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AlGaAs</td>
<td>300</td>
<td>700</td>
<td>1.5</td>
<td>339.7</td>
<td>95</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>700</td>
<td>800</td>
<td>0.6</td>
<td>179.7</td>
<td>60</td>
<td>4.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>300</td>
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<td>1.8</td>
<td>386.6</td>
<td>55</td>
<td>-0.9</td>
<td></td>
</tr>
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<td></td>
<td>500</td>
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<td>258.8</td>
<td>50</td>
<td>2.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>500</td>
<td>2.3</td>
<td>440.5</td>
<td>50</td>
<td>-2.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>600</td>
<td>1.5</td>
<td>339.2</td>
<td>40</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>InGaP</td>
<td>400</td>
<td>600</td>
<td>1.3</td>
<td>219.5</td>
<td>85</td>
<td>-3.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>500</td>
<td>1.7</td>
<td>248.2</td>
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<tr>
<td></td>
<td>450</td>
<td>450</td>
<td>1.9</td>
<td>247.1</td>
<td>60</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>500</td>
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<td>1.1</td>
<td>193.9</td>
<td>55</td>
<td>-3.2</td>
<td></td>
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<tr>
<td></td>
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<td>1.0</td>
<td>172.1</td>
<td>50</td>
<td>-3.2</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2: Numerical recapitulation of the most remarkable dimensions found with parametric conversion gain for $P_{pump} = 200$ mW, $\lambda_{pump} = 1550$ nm, and $P_{signal} = 1$ mW.

### 4.2.4 Input power effect

All the computations until this point have been done considering a pump power of 200 mW and an input signal power of 1 mW. This level of signal power can be pretty high when high gain is achieved. In such situation the signal becomes very strong compared to the pump power and gain saturation occurs. The visible effects of this saturation on the CE is the presence of asymmetry and distortion on the conversion peaks. An example of this effect is shown in Fig. 4.14 where it has been computed the CE for different increasing pump powers while maintaining the signal power at 1 mW. It is observed that after a pump level of 400 mW the CE is no longer symmetric and, for higher levels there is a very significant distortion.

As the input power directly affects the CE and its bandwidth, it should be done a study to see the impact of it. Such study should be done with lower input signal power in order not to enter into gain saturation, so it has been set to 1 µW. Hence, one of the best CE obtained of each material are computed again varying the input pump power. Before computing it can be predicted the effect that it will have. First of all, looking at the equation of the phase mismatch $k$ (Eq. (2.4)) it can be deduced that the $k$ curve will move up when increasing the pump power so, the points of zero dispersion will separate
CHAPTER 4. OPTIMISATION OF III-V SEMICONDUCTOR OPTICAL WAVEGUIDES

Figure 4.14: Gain saturation effect on a 700 x 800 nm$^2$ AlGaAs waveguide with $P_{\text{signal}} = 1$ mW.

Increasing the FWHM. Furthermore, an important increase of the conversion level will occur as the CE is directly proportional to the square of the pump power, as shown in the Shibata (Eq. (3.1)) and Stolen (Eq. (3.3)) approximations.

In Fig. 4.15 there is the CE for the 700 x 800 nm$^2$ AlGaAs waveguide for different pump power ($P_p$) levels. For low power such as 25 mW there is no parametric gain due to the fact that the phase mismatch curve barely reaches zero. For high powers above 800 mW there is presence of asymmetry because of gain saturation and at 900 mW it appears distortion due to the same effect.

Figure 4.15: Conversion efficiency for a 700 x 800 nm$^2$ AlGaAs waveguide for different pump powers and with $P_{\text{signal}} = 1 \mu$W.
In Fig. 4.16, one can also observe the effect of varying the pump power for the 400 x 600 nm$^2$ InGaP waveguide. In this case lower levels of CE are achieved so there is not presence of gain saturation. For powers up to 900 mW there can still be appreciated a good symmetry and no distortion appears. For low values of power there is also a lack of parametric conversion gain due to the nonlinear phase-matching term. For the InGaP waveguide, one can observe that the parametric gain attained is less pronounced.

![Figure 4.16: Conversion efficiency for a 400 x 600 nm$^2$ InGaP waveguide for different pump powers and with $P_{\text{signal}} = 1 \mu\text{W}$.](image)

Finally, in table 4.3 there is another numerical recapitulation with the results obtained by varying the input power on the 700 x 800 nm$^2$ AlGaAs and on the 400 x 600 nm$^2$ InGaP waveguides. A wider bandwidth is obtained for the InGaP example while, for the AlGaAs one, higher values of maximum CE are reached. Above 800 mW of pump power there is distortion on AlGaAS. However, at an input power of 700 mW the CE is still higher than on the InGaP one at 900 mW.
### Numerical Recapitulation of Two Remarkable Dimensions Found with Parametric Gain Varying the Pump Power with \( P_{\text{signal}} = 1 \, \mu \text{W} \)

<table>
<thead>
<tr>
<th>Waveguide</th>
<th>( P_{\text{pump}} )</th>
<th>FWHM</th>
<th>CE(_{\text{max}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>AlGaAs 700 x 800 nm(^2)</td>
<td>25 mW</td>
<td>42 nm</td>
<td>-17.7 dB</td>
</tr>
<tr>
<td></td>
<td>200 mW</td>
<td>60 nm</td>
<td>4.4 dB</td>
</tr>
<tr>
<td></td>
<td>300 mW</td>
<td>66 nm</td>
<td>12.4 dB</td>
</tr>
<tr>
<td></td>
<td>400 mW</td>
<td>75 nm</td>
<td>20 dB</td>
</tr>
<tr>
<td></td>
<td>500 mW</td>
<td>85 nm</td>
<td>27.5 dB</td>
</tr>
<tr>
<td></td>
<td>600 mW</td>
<td>90 nm</td>
<td>35 dB</td>
</tr>
<tr>
<td></td>
<td>700 mW</td>
<td>95 nm</td>
<td>42.2 dB</td>
</tr>
<tr>
<td></td>
<td>800 mW</td>
<td>100 nm</td>
<td>48.4 dB</td>
</tr>
<tr>
<td></td>
<td>900 mW</td>
<td>105 nm</td>
<td>51.7 dB</td>
</tr>
<tr>
<td>InGaP 400 x 600 nm(^2)</td>
<td>25 mW</td>
<td>70 nm</td>
<td>-22.9 dB</td>
</tr>
<tr>
<td></td>
<td>200 mW</td>
<td>85 nm</td>
<td>-3.5 dB</td>
</tr>
<tr>
<td></td>
<td>300 mW</td>
<td>95 nm</td>
<td>1.6 dB</td>
</tr>
<tr>
<td></td>
<td>400 mW</td>
<td>100 nm</td>
<td>6 dB</td>
</tr>
<tr>
<td></td>
<td>500 mW</td>
<td>110 nm</td>
<td>10.4 dB</td>
</tr>
<tr>
<td></td>
<td>600 mW</td>
<td>115 nm</td>
<td>14.6 dB</td>
</tr>
<tr>
<td></td>
<td>700 mW</td>
<td>120 nm</td>
<td>18.8 dB</td>
</tr>
<tr>
<td></td>
<td>800 mW</td>
<td>125 nm</td>
<td>22.9 dB</td>
</tr>
<tr>
<td></td>
<td>900 mW</td>
<td>130 nm</td>
<td>27.1 dB</td>
</tr>
</tbody>
</table>

Red rows have presence of asymmetry or/and distortion.

**Table 4.3:** Numerical recapitulation of two remarkable dimensions found with parametric gain varying the pump power with \( P_{\text{signal}} = 1 \, \mu \text{W} \).

## 4.3 Comparison of III-V with other nonlinear media

During the course of this thesis, a comparison between different nonlinear media for DFWM\(^1\) has been performed. The comparison includes materials of different typologies from highly nonlinear fibres (HNLF) [28, 29] to waveguides made of different materials such as Silicon based media [4, 5, 30–32], Chalcogenide [33], III-V semiconductor (AlGaAs) [9], or Hydex\(^2\) [34]. The results of this study are collected in the paper *A Comparison of Nonlinear Media for Parametric All-Optical Signal Processing*, which is attached in Appendix A.

The main aim of such work is to compare the behaviour of different nonlinear media in terms of the minimum pump power needed to obtain a given target CE. Since the comparison also includes the effect of nonlinear loss due to TPA and FCA, the model used to compute is the one that consists in solving the system of coupled equations, explained in section 3.2.3. For each material we tried to choose the best of each type that has been reported to be already manufactured, tested and used. The III-V semiconductor brought to comparison is an AlGaAs waveguide from [9], which is embedded in AlGaAs layers with different alloy concentrations. This waveguide has a very low nonlinear coefficient \( \gamma \) (∼10 W\(^{-1}\)m\(^{-1}\)) compared with the values that might be achieved with AlGaAs embedded materials.

\(^1\)For a full comparison of nonlinear media refer to [27]

\(^2\)Hydex\(^\circ\) stands for high-index doped silica gass
on BCB that are reported in this thesis. The calculations assume perfect phase matching which can be difficult or not feasible in some devices.

\[ \text{Figure 4.17: Comparison of minimum power needed to reach a certain Conversion Efficiency for different nonlinear media.} \]

In Fig. 4.17 we put together the performance of all the compared materials showing the minimum power required to achieve a certain CE. In the case of crystalline and amorphous silicon it is observed that if no FCA and TPA countermeasure is used the CE is limited to maximum values below -10 dB. For the two fibres we have also neglected the limitation that one could have due to the back reflected power because of Stimulated Brillouin Scattering (SBS) [35].

Under the consideration of zero phase mismatch and neglecting polarisation effects that might occur on HNLFs, in the paper it is reported that the one that gives the highest CE with the minimum power is the silica HNLF, but a length of about 6 km would be needed. If the effect of FCA could be suppressed, then the amorphous silicon would be in the second position of best performance. After those, other media that can give a good performance would be bismuth oxide fibre and crystalline silicon. Because of its low $\gamma$, the AlGaAs waveguide shows one of the worst performances among the compared media. If one of the optimum AlGaAs or InGaP waveguides found in this thesis could be used and put into comparison, it will probably have a similar performance to the amorphous silicon without FCA.

### 4.4 Optimisation summary

There are many parameters that influence the performance, being the most remarkable the nonlinear parameter $\gamma$, the linear loss $\alpha$, the dispersion $D$, and the input power. Apart from the input power, which is set depending on the application and the results desired, the other parameters basically depend on the physical structure of the waveguide.
First, the geometry directly affects most of the essential parameters. By varying the dimensions of the core such parameters can be widely adjusted. Both the linear loss coefficient $\alpha$ and the EMA $A_{eff}$, and with it the nonlinear coefficient $\gamma$, directly depend on the geometry. By choosing good dimensions of the core, quite high values of $\gamma$ and relatively low values of $\alpha$ can be obtained, taking into account that it is a waveguide and usually suffers from high losses. Furthermore, good dispersive behaviour can be obtained choosing suitable dimensions. The optimisation of the waveguide dimensions can also yield to a reduction of the pump power required.

Second, it has been shown that the input pump power is a significant parameter, as it varies the FWHM and moves up the CE curve. In table 4.3 it is shown that by increasing the pump power the FWHM can be more than doubled, and the maximum CE is increased by 60 dB before entering into gain saturation.

Finally, after the optimisation procedure, some deductions can be assessed for the design of an optical waveguide for optical signal processing. By engineering the dispersion properties, the FWHM and the parametric gain can be modified. Moreover, by modifying the input power, the FWHM and CE level are changed.
Conclusions and future work

5.1 Conclusions

In this Master’s Thesis a systematic optimisation of parametric processes in III-V semiconductors waveguides has been performed. This optimisation has been carried out by engineering the dispersion and nonlinear properties of two waveguide structures based on Aluminium Gallium Arsenide (AlGaAs) and Indium Gallium Phosphide (InGaP) with Benzocyclobutene polymer (BCB) cladding. The efficiency of wavelength conversion derived from the parametric process of Four-Wave Mixing (FWM) has been deeply studied to see whether parametric conversion gain and net positive conversion gain are attainable, since they are essential to be used for all-optical communications.

By tailoring the dimensions of the optical waveguides we found that dispersion properties of the media can be widely modified. At many different dimensions, positive values of dispersion were obtained, so parametric conversion gain has been achieved for both materials under study. Furthermore, relatively high power for the pump beam could be used because of the absence of nonlinear losses in these materials. The use of higher power leads to higher values of peak gain and to a wider bandwidth. The fact of being able to use higher pump power on III-V semiconductors overcomes one of the limitations present in silicon devices where high nonlinear loss is induced by free carriers when launching high pump power into the waveguide. Nevertheless, the use of high values of pump power can also result into gain saturation and distortion if high gain is obtained.

For small devices, it is difficult to obtain both high nonlinearity and a good confinement inside the waveguide, so the best performance is often given by structures bigger than 600 x 600 nm$^2$. Since the nonlinear coefficient $\gamma$ sharply decreases when shrinking the core dimensions below the point of maximum nonlinearity (300 x 400 nm$^2$), it is not possible to obtain a good nonlinear performance on small waveguides. We have also found that for small devices, the dispersion properties usually do not allow for parametric gain. Moreover, another important drawback is the presence of strong linear loss. However, such losses sharply decrease when increasing the dimensions of the core. In addition, the tolerance on the fabrication of the waveguide should be very low, as little variations on the dimensions...
can lead to very different dispersive behaviour.

To sum up, it has been theoretically demonstrated with numerical simulations that waveguides made of AlGaAs or InGaP core with BCB cladding allow to achieve parametric conversion gain and positive values of wavelength conversion efficiency, so the signal is frequency shifted and amplified. In addition, it became clear that the dimensions of the waveguide core have a very important impact on the performance of optical parametric processes.

Finally, we can say that the results obtained in this research demonstrate that III-V semiconductor based waveguides can have a nonlinear performance comparable or even better to that of reported for silicon waveguides for all-optical networks [30, 5, 4]. It is concluded that the two waveguides treated in this research could be used for all-optical signal processing with good performance. However, these results do not take into account further limitations that might be present as, for instance, manufacturing limitations or specific device size requirements.

5.2 Future work

After the work carried out in this Master’s Thesis, some possible directions of future research have been identified.

The main purpose of this thesis has been to find out if III-V semiconductor waveguides could be suitable for optical signal processing, but not to find the optimal core structure. Thus, a natural continuation of this work would be to perform a deeper optimisation of the core dimensions. An exhaustive computation may include a study of the maximum tolerance allowed in the fabrication process in order not to modify its properties. This would be interesting if there is need of very small devices, for instance, but also in other applications.

Even though we have demonstrated that AlGaAs and InGaP are very promising materials, it would be interesting to determine if the proposed waveguides can be actually manufactured and included in a chip.

Additionally, it would be useful to perform experimental research in a laboratory in order to bear out the results obtained, and to see whether there are some other effects not taken into account that might alter the performance of the studied devices.

Finally, a similar study but using different waveguide structures could be carried out to see whether they have better dispersive performance. This could be done, for instance, by employing materials with lower refractive index for the cladding or by modifying the shape of the core.
Appendix
Comparison of nonlinear media for all-optical signal processing

The paper shown in the next pages has been written in the course of the research for this Master’s Thesis. It has been already accepted to the IEEE Photonics Conference 2013, held in Seattle, USA.
Abstract

We systematically compare nonlinear media for parametric signal processing by determining the minimum pump power that is required for a given conversion efficiency in a degenerate four-wave mixing process, including the effect of nonlinear loss.

I. INTRODUCTION

All-optical signal processing relies on nonlinear media through which optical waves are made to interact. Among the available physical mechanisms, third order parametric processes are particularly attractive since they allow the realization of a wide range of functionalities, including wavelength conversion, phase conjugation, switching, sampling, logic and amplification. Traditionally, specially designed highly nonlinear fibers (HNLFs) have been largely used for the demonstration of parametric signal processing [1]. In spite of their low loss and inherent compatibility with fiber optic systems, they are however limited by a relatively modest Kerr nonlinearity.

Recently, a wealth of alternative media presenting enhanced Kerr nonlinearity, sometimes several orders of magnitude larger than that of HNLFs, have emerged. Those include, among others, bismuth oxide fibers [2], as well as waveguides (WGs) made of crystalline [3] or amorphous [4,5] silicon, silicon nitride [6], chalcogenide glass [7], AlGaAs [8] or high-index doped silica glass (Hydex®) [9]. The motivation behind the introduction of these materials is not only to obtain higher nonlinear coefficients thanks to their enhanced nonlinear index and strong confinement, but also the possibility of dispersion engineering enabling phase matching over a wide bandwidth. However, in spite of their attractive Kerr nonlinearity, these materials may also suffer from drawbacks including linear and nonlinear loss or stimulated Brillouin scattering (SBS), which limit the achievable conversion efficiency. Although extensive literature demonstrates applications of these materials individually, their relative merits are so far unclear.

In this paper, we report a thorough and systematic comparison of nonlinear media for parametric signal processing. By estimating the pump power required in a degenerate pump wavelength conversion process, we highlight the relative strengths and weaknesses of eight popular nonlinear media.

II. DESCRIPTION OF THE MODEL

The study focuses on the estimation of the maximum attainable conversion efficiency (CE, defined as the ratio of the idler power at the waveguide output to the signal power at its input) in a degenerate four-wave mixing (FWM) wavelength conversion process. For this purpose the system of equations linking the complex envelopes of the signal \( A_s \), the pump \( A_p \), and the idler \( A_i \) is solved under continuous wave (CW) operation,

\[
\frac{dA_s}{dz} = -\frac{1}{2} (\alpha + \alpha_{\text{TPA}} + \alpha_{\text{FCA}}) A_s + j\beta A_s^2 A_p^2 + 2|A_i|^2 A_p
\]

\[
\frac{dA_p}{dz} = -\frac{1}{2} (\alpha + \alpha_{\text{TPA}} + \alpha_{\text{FCA}}) A_p + j\beta A_s A_p^2 + 2|A_i|^2 A_s
\]

\[
\frac{dA_i}{dz} = -\frac{1}{2} (\alpha + \alpha_{\text{TPA}} + \alpha_{\text{FCA}}) A_i + j\beta A_s^2 A_p^2 + 2|A_i|^2 A_p
\]

\[
\frac{dA_i}{dz} = -\frac{1}{2} (\alpha + \alpha_{\text{TPA}} + \alpha_{\text{FCA}}) A_i + j\beta A_s A_p^2 + 2|A_i|^2 A_s
\]

\[
\alpha is the linear loss, \gamma is the nonlinear (Kerr) coefficient and \Delta \beta is the linear phase mismatch between the waves. The nonlinear loss due to two-photon absorption (TPA) and TPA-induced free-carrier absorption (FCA) are

\[
\alpha_{\text{TPA}} = \frac{\beta_{\text{TPA}}}{\lambda_{\text{eff}}} \left| A_s \right|^2 + 2|A_i|^2 A_p
\]

\[
\alpha_{\text{FCA}} = \frac{\sigma_{\text{FCA}}}{2\hbar c \lambda_{\text{eff}}} \left| A_s \right|^2 + \sum_{n} \frac{\lambda_n}{\lambda_n + \lambda_s} \left| A_i \right|^2 A_p
\]

respectively. \( \beta_{\text{TPA}} \) is the TPA coefficient, \( \sigma_{\text{FCA}} \) is the FCA cross-section, \( \lambda_{\text{eff}} \) is the effective area and \( \tau_c \) is the carrier lifetime. This model is valid in the absence of pump depletion by the FWM process. In case of SBS, the threshold is defined as the value of the input pump power for which 1% of the power is back-scattered and is calculated using a shooting algorithm. The maximum CE is obtained under phase matching condition \( \Delta \beta = 0 \) at a pump wavelength of 1550 nm. The comparison is performed between the nonlinear materials in Table I.

III. RESULTS AND DISCUSSION

![Fig. 1. CE as a function of waveguide length and pump power for (a) crystalline silicon (c-Si), (b) chalcogenide (As2S3), (c) Al0.18Ga0.82As waveguides and (d) bismuth oxide (Bi2O3) fiber.](image-url)
TABLE I
NONLINEAR MATERIALS PARAMETERS

<table>
<thead>
<tr>
<th>Material</th>
<th>$\gamma$ (W$^{-1}$m$^{-1}$)</th>
<th>$\alpha$ (dB/cm)</th>
<th>$A_{\text{eff}}$ (\mu m$^2$)</th>
<th>$\beta_{\text{TPA}}$ (cm GW$^{-1}$)</th>
<th>$\sigma_{\text{FCA}}$ (m$^2$)</th>
<th>$\tau_c$ (ns)</th>
<th>$g_B$ (m W$^{-1}$)</th>
<th>Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>11.5x10$^{-10}$</td>
<td>8x10$^{-9}$</td>
<td>11</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>[1]</td>
</tr>
<tr>
<td>Bi$_2$O$_3$ HNFL</td>
<td>1.1</td>
<td>8x10$^{-1}$</td>
<td>3</td>
<td>-</td>
<td>-</td>
<td>7.9x10$^{-11}$</td>
<td>[1]</td>
<td></td>
</tr>
<tr>
<td>c-Si WG</td>
<td>2.4x10$^{-10}$</td>
<td>1.1</td>
<td>0.1</td>
<td>0.5</td>
<td>1.45x10$^{-11}$</td>
<td>1</td>
<td>-</td>
<td>[3,10]</td>
</tr>
<tr>
<td>a-Si WG</td>
<td>1.2x10$^{-10}$</td>
<td>4.5</td>
<td>0.07</td>
<td>0.25</td>
<td>1.9x10$^{-20}$</td>
<td>0.4</td>
<td>-</td>
<td>[4,5]</td>
</tr>
<tr>
<td>Si$_3$N$_4$ WG</td>
<td>0.2</td>
<td>3x10$^{-1}$</td>
<td>1.8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>[6]</td>
</tr>
<tr>
<td>As$_2$S$_3$ WG</td>
<td>1.7</td>
<td>5x10$^{-2}$</td>
<td>6.9</td>
<td>6x10$^{-4}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>[7]</td>
</tr>
<tr>
<td>AlGaAs WG</td>
<td>10</td>
<td>2</td>
<td>5.7</td>
<td>0.05</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>[8]</td>
</tr>
<tr>
<td>High index silica WG</td>
<td>0.22</td>
<td>6x10$^{-4}$</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>[9]</td>
</tr>
</tbody>
</table>

First, the CE is calculated as a function of waveguide or fiber length and pump power, as illustrated for sample materials in Fig. 1. This enables to determine the optimum length that minimizes the pump power for a given value of CE. For materials presenting nonlinear absorption, such as silicon, the optimum length depends on the pump power. In case of materials presenting only linear loss, it corresponds to the value obtained from maximizing $L_{\text{eff}} e^{-\alpha l}$, where $L_{\text{eff}}=(1-e^{-\alpha L})/\alpha$ is the effective length. The minimum pump powers resulting in a given maximum CE (i.e. at phase matching) are represented in Fig. 2. This minimum required pump power ($P_{\text{p, min}}$) is considered to be a good figure of merit to compare the intrinsic benefits of the materials since it assess the pump power efficiency of the process for the same functional performance.

The results of Fig. 2 are limited to the regime where no strong parametric gain is achieved, and where the undepleted pump approximation is valid. Parametric gain could be obtained at higher pump powers for some of the media, for instance silica HNFL. However, this would require SBS mitigation techniques to be implemented, which is outside the scope of the study. Based on our SBS threshold definition, the pump power should be limited in practice to 1 mW for silica HNFL and 390 mW for Bi$_2$O$_3$ HNFL (at the optimum fiber lengths of 5.8 km and 6 m, respectively) if no SBS countermeasure is used. For Si, in which carriers are generated via TPA, $P_{\text{p, min}}$ has also been calculated neglecting FCA, which would constitute an optimum case when the generated carriers are removed using a reverse-biased p-i-n junction.

Among the investigated media, silica HNFLs allow to achieve a given CE for the smallest pump power thanks to their low loss allowing long interaction length. Note that the calculations assume perfect phase matching, which would be challenging in practice for long HNFLs.

The linear losses of high index silica (Hydex®), silicon nitride (Si$_3$N$_4$) and chalcogenide (As$_2$S$_3$) waveguides are of the same order of magnitude, but the former two media are limited by their smaller $\gamma$. Their performance is similar to that of Al$_{0.18}$Ga$_{0.82}$As, which presents higher linear loss but a larger $\gamma$. Crystalline (c-Si) and amorphous (a-Si) silicon benefit from their high $\gamma$ but are limited to CEs to about -10 dB due to TPA and FCA. If FCA can be suppressed, a-Si offers the second best performance after silica HNFLs. The performance of bismuth oxide (Bi$_2$O$_3$) nearly matches that of c-Si (without FCA) until it reaches its SBS threshold.

The waveguide lengths corresponding to the maximum reachable CEs are also shown in Fig. 3 for the planar waveguide materials. It is clear that if TPA and FCA are present, the waveguide length needs to be optimized for each pump power.

IV. CONCLUSIONS

We have performed a systematic comparison of nonlinear media for all-optical signal processing in case of degenerate FWM wavelength conversion. The relative impact of loss (linear and nonlinear) and nonlinear coefficient has been assessed for a fixed CE target.

REFERENCES


