Study of a lunar satellite navigation system

-Annexes-

by

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Annex A.

Project Budget

This annex covers the estimated budget for the realization of this project: Study of a lunar satellite navigation system. The costs of this study are merely related to the author’s work-hours, hardware and software costs. Note that STK was used with a free educational license.

Some considerations taken during the estimation of this budget are:

- The time spent on this project and dissertation has been 6 months, investing a total of 540 hours.
- Cost per man-hour (CMR): 8€/h.
- Currency: Euro (€).
- Taxes not included.

Table A.1 shows general costs derived from hardware and software acquisitions. Table A.2 shows worked hours cost and, finally, Table A.3 shows the total cost of the project.

<table>
<thead>
<tr>
<th>Item</th>
<th>Unit cost</th>
<th>Amortization period</th>
<th>Period applicable</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>MATLAB Sutudent Version</td>
<td>71€</td>
<td>-</td>
<td>-</td>
<td>71€</td>
</tr>
<tr>
<td>Microsoft Office Professional 2007</td>
<td>100€</td>
<td>-</td>
<td>-</td>
<td>100€</td>
</tr>
<tr>
<td>Satellite Tool Kit</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0€</td>
</tr>
<tr>
<td>Laptop: Hp Pavillon dv7 6090es</td>
<td>1000€</td>
<td>3 years</td>
<td>6 months</td>
<td>167€</td>
</tr>
</tbody>
</table>

Total hardware and software cost 338€

Table A.1 Software and hardware costs
<table>
<thead>
<tr>
<th>Item</th>
<th>Number of hours</th>
<th>CMR</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Worked hour</td>
<td>540 hours</td>
<td>8 €/h</td>
<td>4320</td>
</tr>
</tbody>
</table>

Table A.2 Worked hours cost

<table>
<thead>
<tr>
<th>Item</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardware and software cost</td>
<td>338€</td>
</tr>
<tr>
<td>Worked hours cost</td>
<td>4320</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>4658€</td>
</tr>
</tbody>
</table>

Table A.3 Total costs
Annex B.

Transformation of Mean Geoequatorial into Mean Selenoequatorial Coordinates

In this annex the transformation between STK Moon Inertial frame and J2000 frame is going to be developed.

B.1. Selenocentric equatorial system

As it can be seen in Figure B.1, Moon’s orbit and Earth’s orbit don’t lie in the same plane; the angle between both orbits is 5.14°. Furthermore, both, the Moon and the Earth, have an axial tilt which defines their equators out of their orbital plane. The angle between Moon’s equator and the orbital plane is 6.68° and the angle between Earth’s equator and the ecliptic plane is the well known 23.44°. Thus, the angle between the ecliptic plane and Moon’s equator results in 1.5°. In this text, it has been assume that this angle is 1.32° instead of 1.5°. This new value and the definitions presented below can be found in [1].

![Figure B.1 Geometrical parameters of Moon-Earth’s system](image)

The selenocentric equatorial system, in which the Moon’s axis of rotation is the basic axis, has the same meaning on the Moon as the Earth’s equatorial system has for terrestrial observers. The selenoequatorial coordinates are:
Figure B.2. Difference between $\alpha^\Omega$ and $\alpha^\gamma$

- $\alpha^\Omega$: Lunar right ascension measured eastward along the celestial lunar equator from the Moon’s ascending node. The value of $\alpha^\Omega$ changes very quickly because of the precession of the Moon’s axis of rotation hence, it is sometimes more convenient to use angle $\alpha^\gamma$, which is the lunar right ascension measured from the vernal equinox on the ecliptic to the hour circle of the point in question (See Figure B.2).

- $\delta$: Lunar declination, which is the angular distance measured north or south of the celestial lunar equator, along the hour circle passing through the point in question. It has a positive sign on the Northern Hemisphere and a negative sign on the Southern.

The mean selenoequatorial coordinates of a star, $\alpha^\Omega_m$, $d_m$ or $\alpha^\gamma_m$, $d_m$, are the selenocentric equatorial coordinates without the influence of the Moon’s monthly aberration and physical libration and can be obtained from the geoequatorial coordinates in two ways: 1. Indirect transformation, by the use of the ecliptic coordinates and 2. Direct transformation.

**B.2. Indirect transformation**

First, the mean geoequatorial coordinates $\alpha$, $\delta$ are transformed into the mean ecliptic coordinates (Figure B.3) $\lambda$, $\beta$ by the well-known formulas
Study of a lunar satellite navigation system

\[
\begin{align*}
\sin \beta &= \cos \epsilon \sin \delta - \sin \epsilon \cos \delta \sin \alpha \\
\cos \beta \sin \lambda &= \cos \delta \cos \alpha \\
\cos \beta \sin \lambda &= \sin \epsilon \sin \delta + \cos \epsilon \cos \delta \sin \alpha
\end{align*}
\]  
(B.1)

where \( \epsilon \) is the obliquity of the ecliptic.

**Figure B.3.** Transformation from geoequatorial to ecliptic coordinates. Calaf, Jaume, “Astrodynamic’s lecture notes”, October 2011

Next, the transformation formulas of the ecliptic coordinates into selenoequatorial coordinates can easily be obtained from the astronomical triangle on the selenocentric celestial sphere (Figure B.4):

\[
\begin{align*}
\sin d_m &= \sin \beta \cos I + \cos \beta \sin I \sin(\lambda - \Omega) \\
\cos d_m \sin(\Omega - a_m^\gamma) &= \sin \beta \sin I - \cos \beta \cos I \sin(\lambda - \Omega) \\
\cos d_m \cos(\Omega - a_m^\gamma) &= \cos \beta \cos(\lambda - \Omega)
\end{align*}
\]  
(B.2)

where \( I \) is the inclination of the lunar equator to the ecliptic and \( \Omega \) is the ecliptic longitude of the ascending node of the lunar orbit on the ecliptic. (\( \sigma \) is neglected).

**Figure B.4.** Spherical triangle showing the relation of the ecliptic to the selenoequatorial coordinates of a star.
The reverse transformation of the mean selenoequatorial into geoequatorial coordinates is give by the formulas:

\begin{align*}
\sin \beta &= \cos I \sin d_m + \sin I \cos d_m \sin (\Omega - a_m^y) \\
\cos \beta \sin (\lambda - \Omega) &= \sin I \sin d_m - \cos I \cos d_m \sin (\Omega - a_m^y) \\
\cos \beta \cos (\lambda - \Omega) &= \cos d_m \cos (\Omega - a_m^y) 
\end{align*} \tag{B.3}

and

\begin{align*}
\sin \delta &= \cos \epsilon \sin \beta + \sin \epsilon \cos \beta \sin \lambda \\
\cos \delta \sin a &= - \sin \epsilon \sin \beta + \cos \epsilon \cos \beta \sin \lambda \\
\cos \delta \cos a &= \cos \beta \cos \lambda. 
\end{align*} \tag{B.4}

**B.3. Direct transformation**

The second method is the transformation of the geoequatorial into selenoequatorial coordinates by the use of quantities, given in the almanacs, characterizing the mutual positions of these two systems:

- \( i \): inclination of the mean equator of the Moon to the true equator of the Earth.
- \( \Delta \): arc of the lunar mean equator from its ascending node on the Earth’s equator to its ascending node on the ecliptic of date.
- \( \Omega' \): arc of the true equator of the Earth from the true equinox of date to the ascending node of the mean equator of the Moon.

According to Figure B.5, we can write the transformation formulas of these two systems:

\begin{align*}
\sin \delta &= \cos i \sin d + \sin i \cos d \sin (a^\Omega + \Delta) \\
\cos \delta \sin (a^\Omega - \Omega') &= - \sin d \sin i + \cos i \cos d \sin (a^\Omega + \Delta) \\
\cos \delta \cos (a^\Omega - \Omega') &= \cos d \cos (a^\Omega + \Delta) 
\end{align*} \tag{B.5}

and, conversely,

\begin{align*}
\sin d &= \cos i \sin \delta - \sin i \cos \delta \sin (a^\Omega - \Omega') \\
\cos d \sin (a^\Omega + \Delta) &= \sin i \sin \delta + \cos i \cos \delta \sin (a^\Omega - \Omega') \\
\cos d \cos (a^\Omega + \Delta) &= \cos \delta \cos (a^\Omega - \Omega').
\end{align*} \tag{B.6}
The transformation of the apparent coordinates is also possible by use of the formulas given above. However, it is necessary to take into account the influence of the lunar monthly aberration and physical libration separately, by replacing $I$ and $\Omega$ in Eqs. B.2 and B.3 by their true values $I+\rho$ and $\Omega+\sigma$, where $\rho$ and $\sigma$ are the physical librations in the inclination and in the node, respectively. Precession of the moon needs to be corrected too.

B.4. Influence of precession

According to Cassini’s law:

1. The Moon rotates eastward, about a fixed axis, with uniform angular velocity and a period equal to the sidereal period of the Moon’s revolution around the Earth.
2. The inclination $I$ of the Moon’s equator to the ecliptic is constant and is approximately $1^\circ32'.1$.
3. The ascending node of the lunar orbit on the ecliptic coincides with the descending node of the lunar equator on the ecliptic; therefore, the poles of the

\[\text{Figure B.5. Spherical triangle showing the relation of the geoequatorial to the selenoequatorial coordinates of a star.}\]
Moon’s equator, of the ecliptic and of the Moon’s orbit, lie, in that order, on one great circle.

The Moon’s axis of rotation and the plane of the Moon’s equator make one revolution about the axis of the ecliptic poles in approximately 18.6 years. The angle of precession cone of the Moon’s polar axis is equal to the inclination of the lunar equator to the ecliptic. Hence, the Moon’s precession is approximately 1360 times faster than that of the Earth.

The daily precession of the ascending node of the lunar orbit is

\[ P_0^d = -0^\circ .0529539222 \]  \hspace{1cm} (B.7)

The daily precessional motion in the lunar right ascension and declination \((\alpha^\Omega \text{ and } d)\) measured from the ascending node of the lunar equator on the ecliptic are

\[ M_0^d = -P_0^d \cos I, \quad N_0^d = P_0^d \sin I \]  \hspace{1cm} (B.8)

Finally, the influence of the precession in selenoequatorial coordinates is:

\[ \alpha^\Omega - a_0^\Omega = \left( M_0^d + N_0^d \tan d_0 \sin a_0^\Omega \right) t \]  \hspace{1cm} (B.9)

\[ d' - d_0 = N_0^d t \cdot \cos a_0^\Omega \]  \hspace{1cm} (B.10)

where \(t\) is a number of the ephemeris days from the epoch \(t_0\) of the mean coordinates \(a_0^\Omega\) and \(d_0\).

**B.5. Orbital elements and state vector**

The method presented in last section allows us to transform selenoequatorial to geoequatorial coordinates and the other way around. However, our starting point is always the orbital elements of our designed orbit during a period of time in one of the above coordinate frames and, what we want to obtain is the orbital elements of the same orbit in the other coordinate frame, during the same period of time.

For this reason the state vector is needed. It is the bridge between orbital elements and equatorial coordinates (declination and right ascension). There are several strategies to
transform orbital elements to the state vector and the other way around. An example of them can be found on [2].

The transformation between the state vector and the equatorial coordinates is a simple Cartesian-spherical transformation.

By knowing declination and right ascension:

\[
\begin{align*}
    x &= r \cos \delta \cos a \\
    y &= r \cos \delta \sin a \\
    z &= r \sin \delta .
\end{align*}
\] (B.11)

By knowing the state vector:

\[
\begin{align*}
    r &= \sqrt{x^2 + y^2 + z^2} \\
    a &= \arctan \left( \frac{y}{x} \right) \\
    \delta &= \arcsin \left( \frac{z}{r} \right) .
\end{align*}
\] (B.12)

The velocity vector is part of the state vector so, it can be found from the orbital elements, however, it cannot be found from the declination and the right ascension. To find it without the orbital elements, we need to find the Euler angles of the transformation between the geoequatorial to the selenoequatorial frame. To find the two rotating angles we need to solve the following equations system:

\[
\begin{pmatrix}
    x_{\text{seleno}} \\
    y_{\text{seleno}} \\
    z_{\text{seleno}}
\end{pmatrix} =
\begin{bmatrix}
    1 & 0 & 0 \\
    0 & \cos \theta_1 & \sin \theta_1 \\
    0 & -\sin \theta_1 & \cos \theta_1
\end{bmatrix}
\begin{pmatrix}
    \cos \theta_3 & \sin \theta_3 & 0 \\
    -\sin \theta_3 & \cos \theta_3 & 0 \\
    0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
    x_{\text{geo}} \\
    y_{\text{geo}} \\
    z_{\text{geo}}
\end{pmatrix} =
\begin{pmatrix}
    x_{\text{geo}} \\
    y_{\text{geo}} \\
    z_{\text{geo}}
\end{pmatrix}
\] (B.13)

where \( x_{\text{seleno}}, y_{\text{seleno}} \) and \( z_{\text{seleno}} \) are the selenoequatorial coordinates founded from the declination and right ascension (Eq. B.11); \( x_{\text{geo}}, y_{\text{geo}} \) and \( z_{\text{geo}} \) are the geoequatorial coordinates founded from the orbital elements\(^{[2]} \) and \( \theta_1 \) and \( \theta_3 \) are the desired angles.

Once the Euler angles are found, the selenoequatorial velocity vector can be found

\[
\begin{pmatrix}
    v_{x_{\text{seleno}}} \\
    v_{y_{\text{seleno}}} \\
    v_{z_{\text{seleno}}}
\end{pmatrix} =
\begin{pmatrix}
    v_{x_{\text{geo}}} \\
    v_{y_{\text{geo}}} \\
    v_{z_{\text{geo}}}
\end{pmatrix}
\] (B.14)

and then, by using the selenoequatorial velocity plus the already obtained selenoequatorial position, the final orbital elements on the selenoequatorial frame are found.
The process to change the coordinates between both equatorial frames is represented in Figure B.6:

Figure B.6 Block diagram of the coordinates transformation process
Annex C.

Complete Constellation Selection Data

In this annex are all the results from every constellation that have been simulated in order to find the best one for the purpose of the project. All the constellations are Walker constellations and follow the structure shown below:

\[ i: t/p/f \ (f=1) \]

Where \( i \) is the inclination, \( t \) the number of satellites, \( p \) the number of planes and \( f \) is an integer that determines relative spacing between satellites in adjacent planes.

Others parameters that can be found are:

- \( s=\)number of satellites per plane \((s=t/p)\)
- \( PU=\)pattern unit \((PU=360^\circ/t)\)
- Spacing between satellites= \(PU\cdot p\)
- Node/Planes spacing= \(PU\cdot s\)
- Phase difference between adjacent planes= \(PU\cdot f\)

The value chosen for \( f \) is 1 so the pattern unit and phase difference are equal to 360°. The value chosen for \( f \) is 1 so the pattern unit is equal to the phase difference between adjacent planes \((0\leq f\leq (p-1))\).

A total of ten different constellations have been simulated with different number of planes, satellites, height and inclination. To calculate the orbit radius it has been used 1738 km as the value of the Moon radius. It has been simulated four different heights; 200km, 2000km, 4000km and 6500km and two different inclinations; 90° and 60°. The maximum number of planes that has been allowed is three and the constellation with more satellites has got 18 of them.

Constellation 6.1 is the final chosen because its performance is the best with some difference. In the last section of this appendix it is presented an uncompleted constellation 6.1. The reasons why an incomplete constellation is taken into account are explain in the main report of this project.

The values of the different DOP are an average value and they are compute with the information of the satellites best located in order to have a better DOP.
C.1. Constellation 1

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<td>6</td>
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<td>PU</td>
<td>$360/t=12$ deg</td>
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<td>Spacing between satellites</td>
<td>60 deg</td>
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<td>Planes spacing</td>
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<td>Phase difference between adjacent planes</td>
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<td>i</td>
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Table C.1 Parameters for constellation 1

Figure C.1 3D view of constellation 1
Figure C.2 Ground track of constellation 1

Figure C.3 Satisfaction criteria: At least 4 satellites on sight. Constellation 1
Figure C.4 Number of satellites always in sight by latitude. Constellation 1

Figure C.5 Number of satellites always in sight by longitude. Constellation 1
C.2. Constellation 2

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<td>( t )</td>
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<td>( p )</td>
<td>2</td>
</tr>
<tr>
<td>( s )</td>
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</tr>
<tr>
<td>( \text{PU} )</td>
<td>( 360/t=60 \text{ deg} )</td>
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<td>Planes spacing</td>
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<td>( \text{i} )</td>
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Table C.2 Parameters for constellation 2

Figure C.6 3D view of constellation 2
Figure C.7 Ground track of constellation 2

Figure C.8 Satisfaction criteria: At least 4 satellites on sight. Constellation 2
Study of a lunar satellite navigation system

Figure C.9 Number of satellites always in sight by latitude. Constellation 2

Figure C.10 Number of satellites always in sight by longitude. Constellation 2
C.3. Constellation 3

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Table C.3 Parameters for constellation 3

Figure C.11 3D view of constellation 3
Study of a lunar satellite navigation system

Figure C.12 Ground track of constellation 3

Figure C.13 Satisfaction criteria: At least 4 satellites on sight. Constellation 3
Figure C.14 Number of satellites always in sight by latitude. Constellation 3

Figure C.15 Number of satellites always in sight by longitude. Constellation 3
C.4. Constellation 4

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<td>i</td>
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Table C.4 Parameters for constellation 4

Figure C.16 3D view of constellation 4
Figure C.17 Ground track of constellation 4

Figure C.18 Satisfaction criteria: At least 4 satellites on sight. Constellation 4
Study of a lunar satellite navigation system

Figure C.19 Number of satellites always in sight by latitude. Constellation 4

Figure C.20 Number of satellites always in sight by longitude. Constellation 4
C.5. Constellation 5

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Table C.5 Parameters for constellation 5

Figure C.21 3D view of constellation 5
Figure C.22 Ground track of constellation 5

Figure C.23 Satisfaction criteria: At least 4 satellites on sight. Constellation 5
Figure C.24 Number of satellites always in sight by latitude. Constellation 5

Figure C.25 Number of satellites always in sight by longitude. Constellation 5
C.6. Constellation 6

C.6.1. Constellation 6.1

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Table C.6 Parameters for constellation 6.1

Figure C.26 3D view of constellation 6.1
Figure C.27 Ground track of constellation 6.1

Satisfaction criteria: At least 4 satellites on sight. Constellation 6.1

Figure C.28 Satisfaction criteria: At least 4 satellites on sight. Constellation 6.1
Figure C.29 Number of satellites always in sight by latitude. Constellation 6.1

Figure C.30 Number of satellites always in sight by longitude. Constellation 6.1
Figure C.31 GDOP by latitude of constellation 6.1

Figure C.32 GDOP by longitude of constellation 6.2
Study of a lunar satellite navigation system

Figure C.33 PDOP by latitude of constellation 6.1

Figure C.34 PDOP by longitude of constellation 6.1
Figure C.35 HDOP by latitude of constellation 6.1

Figure C.36 HDOP by longitude of constellation 6.1
Study of a lunar satellite navigation system

Figure C.37 VDOP by latitude of constellation 6.1

Figure C.38 VDOP by longitude of constellation 6.1
Figure C.39 TDOP by latitude of constellation 6.1

Figure C.40 TDOP by longitude of constellation 6.1
C.6.2. Constellation 6.2

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Table C.7 Parameters for constellation 6.2

Figure C.41 3D view of constellation 6.2
Figure C.42 Ground track of constellation 6.2

Figure C.43 Satisfaction criteria: At least 4 satellites on sight. Constellation 6.2
Study of a lunar satellite navigation system

Figure C.44 Number of satellites always in sight by latitude. Constellation 6.2

Figure C.45 Number of satellites always in sight by longitude. Constellation 6.2
Figure C.46 GDOP by latitude of constellation 6.2

Figure C.47 GDOP by longitude of constellation 6.2
Study of a lunar satellite navigation system

**Figure C.48** PDOP by latitude of constellation 6.2

**Figure C.49** PDOP by longitude of constellation 6.2
Figure C.50 HDOP by latitude of constellation 6.2

Figure C.51 HDOP by longitude of constellation 6.2
Study of a lunar satellite navigation system

![Graph](image1)

**Figure C.52** VDOP by latitude of constellation 6.2

![Graph](image2)

**Figure C.53** VDOP by longitude of constellation 6.2
Figure C.54 TDOP by latitude of constellation 6.2

Figure C.55 TDOP by longitude of constellation 6.2
C.7. Constellation 7

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Table C.8 Parameters for constellation 7

Figure C.56 3D view of constellation 7
Gemma Saura Carretero

Figure C.57 Ground track of constellation 7

Figure C.58 Satisfaction criteria: At least 4 satellites on sight. Constellation 7
Study of a lunar satellite navigation system

Figure C.59 Number of satellites always in sight by latitude. Constellation 7

Figure C.60 Number of satellites always in sight by longitude. Constellation 7
C.8. Constellation 8

C.8.1. Constellation 8.1

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Table C.9 Parameters for constellation 8.1

Figure C.61 3D view of constellation 8.1
Figure C.62 Ground track of constellation 8.1

Figure C.63 Satisfaction criteria: At least 4 satellites on sight. Constellation 8.1
Figure C.64 Number of satellites always in sight by latitude. Constellation 8.1

Figure C.65 Number of satellites always in sight by longitude. Constellation 8.1
Figure C.66 GDOP by latitude of constellation 8.1

Figure C.67 GDOP by longitude of constellation 8.1
Figure C.68 PDOP by latitude of constellation 8.1

Figure C.69 PDOP by longitude of constellation 8.1
Figure C.70 HDOP by latitude of constellation 8.1

Figure C.71 HDOP by longitude of constellation 8.1
Figure C.72 VDOP by latitude of constellation 8.1

Figure C.73 VDOP by longitude of constellation 8.1
Figure C.74 TDOP by latitude of constellation 8.1

Figure C.75 TDOP by longitude of constellation 8.1
C.8.2. Constellation 8.2

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Table C.10 Parameters for constellation 8.2

Figure C.76 3D view of constellation 8.2
Figure C.77 Ground track of constellation 8.2

Figure C.78 Satisfaction criteria: At least 4 satellites on sight. Constellation 8.2
Figure C.79 Number of satellites always in sight by latitude. Constellation 8.2

Figure C.80 Number of satellites always in sight by longitude. Constellation 8.2
Figure C.81 GDOP by latitude of constellation 8.2

Figure C.82 GDOP by longitude of constellation 8.2
Figure C.83 PDOP by latitude of constellation 8.2

Figure C.84 PDOP by longitude of constellation 8.2
Figure C.85 HDOP by latitude of constellation 8.2

Figure C.86 HDOP by longitude of constellation 8.2
Figure C.87 VDOP by latitude of constellation 8.2

Figure C.88 VDOP by longitude of constellation 8.2
Figure C.89 TDOP by latitude of constellation 8.2

Figure C.90 TDOP by longitude of constellation 8.2
C.9. Constellation 6.1: Uncompleted

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Table C.11 Parameters for constellation 6.1 (Uncompleted)

Figure C.91 3D view of constellation 6.1 (Uncompleted)
Study of a lunar satellite navigation system

Figure C.92 Ground track of constellation 6.1 (Uncompleted)

Figure C.93 Satisfaction criteria: At least 4 satellites on sight. Constellation 6.1 (Uncompleted)
Figure C.94 Maximum number of satellites on sight by latitude

Figure C.95 Minimum number of satellites on sight by latitude
Study of a lunar satellite navigation system

Figure C.96 Maximum number of satellites on sight by longitude

Figure C.97 Minimum number of satellites on sight by longitude
Figure C.98 Coverage time by latitude (% of time with 4 satellites always on sight)

Figure C.99 Coverage time by longitude (% of time with 4 satellites always on sight)
Study of a lunar satellite navigation system

Figure C.100 Revisiting time by latitude (Time in second without coverage)

Figure C.101 Revisiting time by longitude (Time in second without coverage)
Annex D.

Complete Orbit propagation Data

In this annex are all the results of the orbit propagation done with STK of each orbit plane. The orbital elements represented are: semiaxis, eccentricity, inclination, RAAN and argument of perigee.

The parameters of the reference satellites are shown in Table D.1.

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<tr>
<th>Satellite</th>
<th>Radius [km]</th>
<th>Eccentricity</th>
<th>Inclination [°]</th>
<th>RAAN [°]</th>
<th>ω [°]</th>
<th>Argument of latitude [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sat0</td>
<td>8238</td>
<td>0</td>
<td>90</td>
<td>0</td>
<td>0</td>
<td>288</td>
</tr>
<tr>
<td>Sat120</td>
<td>8238</td>
<td>0</td>
<td>90</td>
<td>120</td>
<td>0</td>
<td>24</td>
</tr>
<tr>
<td>Sat240</td>
<td>8238</td>
<td>0</td>
<td>90</td>
<td>240</td>
<td>0</td>
<td>120</td>
</tr>
</tbody>
</table>

Table D.1 Design parameters of the reference satellites in each orbit plane

The epoch and the start and stop simulation time are presented in Table D.2.

<table>
<thead>
<tr>
<th>Epoch</th>
<th>18 Mar 2012 11:00:00.000 UTCG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start time</td>
<td>18 Mar 2012 11:00:00.000 UTCG</td>
</tr>
<tr>
<td>Stop time</td>
<td>18 Mar 2013 11:00:00.000 UTCG</td>
</tr>
</tbody>
</table>

Table D.2 Epoch and start and stop simulation time

As it has been said in the main report the satellite's mass is assume to be 1000 kg and its dry mass is 750 kg.

Finally, it is important to remember that the following figures are represented in J2000 frame meanwhile the data shown in Table D.1 is in the Moon Inertial frame. For further information about this frame transformation see Annex B.
D.1. RAAN 0

D.1.1. All the perturbations

**Figure D.1** Effect of all the perturbation together on the semiaxis evolution in time, orbit plane: RAAN=0°

**Figure D.2** Effect of all the perturbation together on the eccentricity evolution in time, orbit plane: RAAN=0°
Figure D.3 Effect of all the perturbation together on the inclination evolution in time, orbit plane: RAAN=0°

Figure D.4 Effect of all the perturbation together on the RAAN evolution in time, orbit plane: RAAN=0°
Figure D.5 Effect of all the perturbation together on the argument of perigee evolution in time, orbit plane: RAAN=0°

D.1.2. Central body gravity

Figure D.6 Effect of central body gravitation on the semiaxis evolution in time, orbit plane: RAAN=0°
Figure D.7 Effect of central body gravitation on the eccentricity evolution in time, orbit plane: RAAN=0°

Figure D.8 Effect of central body gravitation on the inclination evolution in time, orbit plane: RAAN=0°
Figure D.9 Effect of central body gravitation on the RAAN evolution in time, orbit plane: RAAN=0°

Figure D.10 Effect of central body gravitation on the argument of perigee evolution in time, orbit plane: RAAN=0°
**D.1.3. SRP**

Figure D.11. Effect of SRP on the semiaxis evolution in time, orbit plane: RAAN=0°

Figure D.12. Effect of SRP on the eccentricity evolution in time, orbit plane: RAAN=0°
Figure D.13 Effect of SRP on the inclination evolution in time, orbit plane: RAAN=0°

Figure D.14 Effect of SRP on the RAAN evolution in time, orbit plane: RAAN=0°
D.1.4. Third body perturbations

**Figure D.15** Effect of SRP on the argument of perigee evolution in time, orbit plane: RAAN=0°

**Figure D.16** Effect of third body perturbation on the semiaxis evolution in time, orbit plane: RAAN=0°
Study of a lunar satellite navigation system

Figure D.17 Effect of third body perturbation on the eccentricity evolution in time, orbit plane: RAAN=0°

Figure D.18 Effect of third body perturbation on the inclination evolution in time, orbit plane: RAAN=0°
**Figure D.19** Effect of third body perturbation on the RAAN evolution in time, orbit plane: RAAN=0°

**Figure D.20** Effect of third body perturbation on the argument of perigee evolution in time, orbit plane: RAAN=0°
D.2. RAAN 120

D.2.1. All the perturbations

Figure D.21 Effect of all the perturbation together on the semiaxis evolution in time, orbit plane: RAAN=120°

Figure D.22 Effect of all the perturbation together on the eccentricity evolution in time, orbit plane: RAAN=120°
Figure D.23 Effect of all the perturbation together on the inclination evolution in time, orbit plane: RAAN=120°

Figure D.24 Effect of all the perturbation together on the RAAN evolution in time, orbit plane: RAAN=120°
Figure D.25 Effect of all the perturbation together on the argument of perigee evolution in time, orbit plane: RAAN=120°

D.2.2. Central body gravity:

Figure D.26 Effect of central body gravity on the semiaxis evolution in time, orbit plane: RAAN=120°
Figure D.27 Effect of central body gravity on the eccentricity evolution in time, orbit plane: RAAN=120°

Figure D.28 Effect of central body gravity on the inclination evolution in time, orbit plane: RAAN=120°
Figure D.29 Effect of central body gravity on the RAAN evolution in time, orbit plane: RAAN=120°

Figure D.30 Effect of central body gravity on the argument of perigee evolution in time, orbit plane: RAAN=120°
D.2.3. SRP

Figure D.31 Effect of SRP on the semiaxis evolution in time, orbit plane: RAAN=120°

Figure D.32 Effect of SRP on the eccentricity evolution in time, orbit plane: RAAN=120°
Figure D.33 Effect of SRP on the inclination evolution in time, orbit plane: RAAN=120°

Figure D.34 Effect of SRP on the RAAN evolution in time, orbit plane: RAAN=120°
Figure D.35 Effect of SRP on the argument of perigee evolution in time, orbit plane: RAAN=120°

Figure D.36 Effect of third body perturbation on the semiaxis evolution in time, orbit plane: RAAN=120°
Figure D.37 Effect of third body perturbation on the eccentricity evolution in time, orbit plane: RAAN=120°

Figure D.38 Effect of third body perturbation on the inclination evolution in time, orbit plane: RAAN=120°
Figure D.39 Effect of third body perturbation on the RAAN evolution in time, orbit plane: RAAN=120°

Figure D.40 Effect of third body perturbation on the argument of perigee evolution in time, orbit plane: RAAN=120°
D.3. RAAN 240

D.3.1. All the perturbations

Figure D.41 Effect of all the perturbation together on the semiaxis evolution in time, orbit plane: RAAN=240°

Figure D.42 Effect of all the perturbation together on the eccentricity evolution in time, orbit plane: RAAN=240°
Figure D.43 Effect of all the perturbation together on the inclination evolution in time, orbit plane: RAAN=240°

Figure D.44 Effect of all the perturbation together on the RAAN evolution in time, orbit plane: RAAN=240°
D.3.2. Central body gravity:

Figure D.45 Effect of all the perturbation together on the argument of perigee evolution in time, orbit plane: RAAN=240°

Figure D.46 Effect of central body gravity on the semiaxis evolution in time, orbit plane: RAAN=240°
Figure D.47 Effect of central body gravity on the eccentricity evolution in time, orbit plane: RAAN=240°

Figure D.48 Effect of central body gravity on the inclination evolution in time, orbit plane: RAAN=240°
Figure D.49 Effect of central body gravity on the RAAN evolution in time, orbit plane: RAAN=240°

Figure D.50 Effect of central body gravity on the argument of perigee evolution in time, orbit plane: RAAN=240°
D.3.3. SRP

Figure D.51 Effect of SRP on the semiaxis evolution in time, orbit plane: RAAN=240°

Figure D.52 Effect of SRP on the eccentricity evolution in time, orbit plane: RAAN=240°
Figure D.53 Effect of SRP on the inclination evolution in time, orbit plane: RAAN=240°

Figure D.54 Effect of SRP on the RAAN evolution in time, orbit plane: RAAN=240°
Figure D.55 Effect of SRP on the argument of perigee evolution in time, orbit plane: RAAN=240°

D.3.4. Third body perturbation

Figure D.56 Effect of third body perturbation on the semiaxis evolution in time, orbit plane: RAAN=240°
Figure D.57 Effect of third body perturbation on the eccentricity evolution in time, orbit plane: RAAN=240°

Figure D.58 Effect of third body perturbation on the inclination evolution in time, orbit plane: RAAN=240°
Figure D.59 Effect of third body perturbation on the RAAN evolution in time, orbit plane: RAAN=240°

Figure D.60 Effect of third body perturbation on the argument of perigee evolution in time, orbit plane: RAAN=240°
Bibliography
