Contributions to Guidance and Control of Underwater Gliders

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ABSTRACT

Autonomous underwater gliders are vehicles developed with the purpose to drive motion by means of controlling its attitude and buoyancy, with no propeller nor rudder. This thesis mainly focuses on the control of glider’s motion to make it reach a desired destination in the sea, and do it by following desired paths. With this aim both two-dimensional and three-dimensional scenarios are presented. The former states the case when motion is restricted to the vertical plane, while the latter includes the full motion.

First, a mathematical model of the glider is given. Then the control strategy used is detailed and discussed, and afterwards the guidance algorithms implemented to achieve the target point are presented. In the 2D case guidance can be accomplished focusing on pitch control, while in the 3D case turning is also needed hence introduced. Finally the whole system is implemented using Matlab/Simulink, and several representative simulations are shown and used to validate the proper behaviour of the glider.
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### NOMENCLATURE

- $\alpha$: Angle of attack
- $\beta$: Sideslip angle
- $\theta$: Pitch angle
- $\xi$: Glide path angle
- $\rho$: Water density
- $\phi$: Roll angle
- $\psi$: Yaw/heading angle
- $\omega$: New control inputs vector
- $\Omega$: Angular velocity in body frame
- $b$: Vehicle position vector in inertial frame
- $CB$: Center of buoyancy/Origin of body frame
- $CG$: Center of gravity
- $D$: Drag force
- $d_{\text{min}}, d_{\text{max}}$: Depth boundaries
- $e_1, e_2, e_3$: Unit vectors of the body frame
- $\mathbf{F}$: Total force in body frame
- $F_{\text{ext}}$: External force on vehicle in body frame
- $g$: Constant of gravity
- $I$: Identity matrix
- $i, j, k$: Unit vectors of the inertial frame
- $J$: Inertia matrix
- $J_i$: $i$th diagonal element of $J$
- $K_D, K_{D_0}$: Drag coefficients
- $K_L, K_{L_0}$: Lift coefficients
- $K_M, K_{M_R}, K_{M_Y}, K_{M_0}, K_{q_1}, K_{q_2}, K_{q_3}$: Viscous moment coefficients
- $L$: Lift force
- $M$: Mass matrix
- $M_f$: Added mass matrix
- $M_{DL}$: Viscous moment
- $m$: Mass of displaced fluid
- $\bar{m}$: Movable point mass
- $m_b$: Variable ballast mass
\begin{itemize}
\item $m_{fi}$: \textit{\textit{i}th diagonal element of $M_f$}
\item $m_h$: Hull mass (uniformly distributed)
\item $m_i$: \textit{\textit{i}th diagonal element of $M$}
\item $m_w$: Fixed point mass
\item $m_0$: Excess mass
\item $P$: Target point
\item $P_P$: Linear momentum of $\overline{m}$ in body frame
\item $R$: Rotation matrix
\item $r_p$: Position of movable mass $\overline{m}$ in body frame
\item $r_w$: Position of fixed point mass $m_w$ in body frame
\item $S$: Start position of the glider
\item $\overline{T}$: Total torque in body frame
\item $T_{ext}$: External torque in body frame
\item $u$: Control inputs vector
\item $u_4$: Buoyancy control input
\item $v$: Glider velocity in body frame
\item $V$: Glider speed
\item $x, y, z$: Vehicle position components
\end{itemize}
Chapter 1

INTRODUCTION

A large percentage of the surface of Earth is covered by water. Seas and oceans extend hiding a completely unknown world beneath. Since a lot of years ago, researchers became aware of the necessity to understand what was happening under the sea surface in order to explain the global evolution and behaviour of the planet.

Sampling the seas is one of the ways of the oceanographic research from which scientists can obtain endless information about several topics, such as temperature, salinity, conductivity, currents, seabed profile, etc. Underwater vehicles have been developed as a way for collecting data from the oceans, and different types of vehicles have been designed to accomplish this goal. Nevertheless, in recent years a new generation of vehicles emerge with the purpose of increase its autonomy. They are known as autonomous underwater vehicles (AUVs), and provide modern and reliable systems for achieving oceanic missions. Data is collected using sensors and sent via satellite, what means that there is no need of physical connection with the vehicle to get the information. Moreover, autonomy is also yield by low energy consumption, as they are designed with advanced mechanical and computational systems that reduce power requirements. Autonomous underwater gliders are one of them.

Autonomous gliders have high potential to execute oceanic missions due to its operating mode: performance is controlled by their buoyancy and attitude. They do not make use of any kind of propeller but the change of its buoyancy to induce motion, and furthermore attitude is controlled with combination of a rudder placed at the rear or with the position of movable internal masses. This yields in high-autonomy vehicles, able to carry out long-range missions with thousands of kilometers and several weeks or months long.

Within the group of underwater gliders various models have been developed in order to satisfy different requirements or with the purpose of finding new and better designs. One of the models, for instance, was developed at Princeton University for
Figure 1.1: Salinity data measured by Spray glider. (Southern California Coastal Ocean Observing System, courtesy of SIO Instrument Development Group)

laboratory studies. It is a laboratory-scale glider called ROGUE, widely detailed in [11] and which can be seen in figure 1.2.

Figure 1.2: ROGUE glider [11]

Other models have been commercially distributed, and are used for oceanic research and commercial purposes. As example of them there is SLOCUM (see figure 1.3), SPRAY and SEAGLIDER [15].

As mentioned before, different variants can be found of these models. Some of them, for instance, include a propeller to provide motion, some have a rudder or a movable tail to control the attitude, and some other advanced devices use the thermal energy of the water for propulsion [16].

However, the model taken into consideration in this thesis includes none of them.
This means that the control of the attitude of the glider here is performed only by redistribution of internal movable mass. This mass is located inside the hull of the glider, and using actuators its position is changed in order to achieve the desired behaviour. On the other hand buoyancy is controlled by pumping seawater in or out the vehicle, depending on if the purpose is to have a negatively or positively buoyant body respectively. The former yields in downwards glides while the latter in upwards glides.

One can notice then that this is a really interesting research field as far as control is concerned, with a wide range of issues and challenges to be developed and accomplished.

Actuators powered by electrical supplies installed on the glider are needed to change the position of the internal mass and to pump water. Controlling these actuators is, then, how the motion of the glider is ruled. For the sake of autonomy, one of the goals is to keep these actuators fixed whenever possible, with emphasis to pumping which has energetically high cost. Make the vehicle perform desired gliding equilibria -straight paths and spiral glides- is how this can be achieved. Guidance is another important issue of underwater vehicles, hence also for gliders. The development of guidance algorithms, as simple as possible, to lead the glider to a desired destination, and doing it by following a desired path, is also part of the work carried out in this thesis.

Following this introduction, Chapter 2 is dedicated to present a mathematical model for glider’s kinematics and dynamics [11]. A model with simplified internal masses is considered and equations are given first for a general three-dimensional
case and then for a case when motion is restricted to the vertical plane. Moreover, the hydrodynamic model used is also detailed [1].

After that, Chapter 3 is focused on the description of the control system. At first, a section with gliding equilibria can be found, followed by a section with a detailed description of the control laws implemented [2]: a nonlinear controller combined with a linear controller. A last section with some considerations regarding stability end the chapter.

Chapter 4 includes a wide explanation about guidance strategy. First the guidance for two-dimensional motion is presented. It is implemented using the sector-of-sight logic [9][10] in combination with a boundary check algorithm. Following this one can find several Matlab/Simulink simulations, with results, plots and explanations. Afterwards the same process is carried out extensively for guidance with three-dimensional motion.

Finally, the concluding remarks in Chapter 5 end the thesis.
Chapter 2

MATHEMATICAL MODELING

In the current chapter the mathematical model used to define the glider’s motion is presented. First, in section 2.1 some preliminary definitions are done, as the frames of reference and the vehicle model with the corresponding distribution of masses. Afterwards section 2.2 introduces the kinematics and dynamics equations of motion of the glider, as well as the hydrodynamic model. Following this, in section 2.3 motion is restricted to the vertical plane, resulting in a 2D scenario. In this case equations of motion and the arising hydrodynamic model are also given. Finally all the values of the parameters therein the model are given in the last section 2.4.

2.1 Underwater Glider Model

2.1.1 Frames of reference

In order to understand and work with the mathematical model presented in this section, some frames of reference are previously defined. A graphical representation of them can be seen in figure 2.1.

First an earth-fixed frame is defined, named inertial frame, with axes \((x, y, z)\) and with unit vectors \((i, j, k)\) respectively. The z-axis keeps the direction of gravity, pointing downwards, while the x and y directions lie in a plane perpendicular to gravity. Moreover \(z = 0\) is taken to be the sea’s surface level, hence \(z\) will be depth.

The second frame of reference defined, the body frame, is fixed to the glider and its unit vectors are \((e_1, e_2, e_3)\) regarding the 1,2,3-axes. Its origin is located at the center of buoyancy of the glider, CB, the 1-axis has the longitudinal direction of the glider’s body (pointing forward), the 2-axis is aligned with the lateral axis of the vehicle (pointing away of the right wing), and the 3-axis is perpendicular to the plane of the wings, pointing downwards in the vertical axis of the body.
2.1.2 Vehicle model

The model of vehicle used here corresponds to a rigid body with very simple shape, ellipsoidal or cylindrical, and with fixed wings and tail. The glider moves without any rudder, so its attitude is driven exclusively by means of its internal masses redistribution. $m_h$ is the hull mass, fixed and uniformly distributed around the hull body. Then the other internal masses are defined to be point masses, positioned into the glider’s body as an offset from the center of buoyancy CB: $m_w$ is a point mass with fixed position, for nonuniform hull mass distribution, $\bar{m}$ is a movable mass (i.e. with variable position) and $m_b$ is a variable mass with fixed position. See figure 2.2 showing the distribution of all the different masses of the glider.

The position of these masses $(m_w, \bar{m}, m_b)$ is given by vectors $r_w$, $r_p$ and $r_b$ respectively.
tively, which have the center of buoyancy as origin. Hence, the position of the masses is given in the body frame.

As it was commented before, the attitude and motion of the glider are given by the redistribution of its internal masses and by its buoyancy. The buoyancy of the vehicle can be changed by increasing or decreasing the variable mass $m_b$. The net buoyancy of the glider is denoted as excess mass $m_0$, and is computed as follows: $m$ is the mass of the fluid displaced by the body of the glider, hence the excess mass is

$$m_0 = m_h + m_w + m_b + \bar{m} - m$$

(2.1)

Notice that all terms of equation (2.1) are constant with exception of $m_b$, which is the variable used to modify glider’s buoyancy. For negative values of $m_0$ the glider is buoyant (it will go upwards), while for positive $m_0$ it will sink (go downwards).

To perform the modelization of the glider some simplifications are done related to the internal masses and its distribution (the new configuration can be seen in figure 2.3). The following points are considered:

- The fixed point mass for nonuniform hull mass distribution, $m_w$, is set to be 0
- The variable ballast mass, $m_b$, is placed, fixed, at the center of buoyancy CB

Therefore, $r_b = 0$ and the excess mass is now written as

$$m_0 = m_h + m_b + \bar{m} - m$$

(2.2)

Figure 2.3: Simplified internal masses of the glider [3]

Throughout the thesis only this simplified model of glider is considered. Nevertheless, full model taking into account all the internal masses can be found, for instance, in [3].
2.2 Dynamics in 3-D

2.2.1 Equations of motion in 3-D

The equations of motion of an underwater glider are described in [11] and presented here:

\[ \dot{\hat{R}} = R\hat{\Omega} \]  
\[ \dot{\hat{b}} = Rv \]  
\[ \dot{\hat{\Omega}} = J^{-1}\hat{T} \]  
\[ \dot{\hat{v}} = M^{-1}\hat{F} \]  
\[ \dot{\hat{r}}_P = \frac{1}{\hat{m}}P_P - v - \Omega \times r_P \]  
\[ \dot{\hat{P}}_P = \hat{\bar{u}} \]  
\[ \dot{\hat{m}}_b = u_4 \]

Before denoting all terms of equations (2.3) – (2.9), let’s define the operator \( \hat{\cdot} \), which is present for instance in equation (2.3) and will also appear several times further on in other terms of dynamics.

Being a vector \( a = [a_1, a_2, a_3]^T \), then

\[ \hat{a} = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix} \]

In the equations of motion presented above, \( R \) is the application matrix that transforms vectors expressed in the body frame into the inertial frame, and therefore it is the orientation matrix of the glider relative to the inertial frame (derived from the references’ definition in section 2.1.1).

\( b = [x, y, z]^T \) is the position of the glider expressed in the inertial frame. This vector goes from the origin of the inertial frame to the origin of the body frame (center of buoyancy CB).

\( v \) and \( \Omega \) denote the velocity of the glider relative to inertial frame and expressed in the body frame. \( v = [v_1, v_2, v_3]^T \) is translational velocity, while \( \Omega = [\Omega_1, \Omega_2, \Omega_3]^T \) is angular velocity. From this it is derived that equations (2.3) and (2.4) give the kinematics of the glider.

In these equations also appear the mass and inertia matrices of the glider, \( M \) and \( J \) respectively, the total force \( \hat{F} \) and torque \( \hat{T} \), the position vector \( r_P = [r_{P_1}, r_{P_2}, r_{P_3}]^T \) of the movable point mass \( \hat{m} \) (with respect to the center of buoyancy CB), and the linear momentum \( \hat{P}_P \) of \( \hat{m} \). All of them are expressed in the body frame coordinates.
As control inputs to the system there are $\bar{u} = [u_1, u_2, u_3]^T$, which is the force on movable mass $m$, and $u_4$, which acts as buoyancy control and is equal to the variable mass rate $\dot{m}_b$.

The orientation matrix $R$ is expressed by means of Euler angles, notation of the typical literature in flight dynamics: yaw $\psi$, pitch $\theta$ and roll $\phi$.

$$R = \begin{pmatrix} \cos \psi \cos \theta & -\sin \psi \cos \phi + \cos \psi \sin \theta \sin \phi & \sin \psi \sin \phi + \cos \psi \cos \phi \sin \theta \\ \sin \psi \cos \theta & \cos \psi \cos \phi + \sin \phi \sin \theta \sin \psi & -\cos \psi \sin \phi + \sin \theta \sin \psi \cos \phi \\ -\sin \theta & \cos \theta \sin \phi & \cos \theta \cos \phi \end{pmatrix}$$

(2.11)

Euler angles convention can also be used to define the glider’s angular velocity in terms of Euler angle rates, or equivalently, derive the Euler angle rates from the angular velocity relative to the inertial frame $\Omega$.

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \frac{\sin \phi}{\cos \theta} & \frac{\cos \phi}{\cos \theta} \end{pmatrix} \begin{pmatrix} \dot{\psi} \end{pmatrix} \Omega$$

(2.12)

Procedures for obtaining matrix $R$ and the Euler angular rates can be found in Appendix A of [3].

$M$ is the mass matrix, computed as

$$M = m_h I + M_f$$

(2.13)

where $I$ is identity matrix (3x3) and $M_f$ is an added mass matrix.

Besides, $J$ is the inertia matrix, computed as

$$J = J_h + J_f$$

(2.14)

where $J_h$ is the inertia matrix for the hull mass and $J_f$ is an added inertia matrix.

$M_f$ and $J_f$ are defined by the body shape of the vehicle and the density of the fluid. In this case, due to the glider’s external shape (and neglecting the wings) it
is assumed that they are diagonal matrices with constant coefficients. Then, they can be written as follows:

\[ M_f = \begin{pmatrix} m_{f1} & 0 & 0 \\ 0 & m_{f2} & 0 \\ 0 & 0 & m_{f3} \end{pmatrix} \]  
(2.15)

\[ J_f = \begin{pmatrix} J_{f1} & 0 & 0 \\ 0 & J_{f2} & 0 \\ 0 & 0 & J_{f3} \end{pmatrix} \]  
(2.16)

therefore, for equations (2.13) and (2.14) it is seen that \( M \) and \( J \) are also constant diagonal matrices. Lets denote them as

\[ M = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \]  
(2.17)

\[ J = \begin{pmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{pmatrix} \]  
(2.18)

The expression for the total force \( \mathbf{F} \) is

\[ \mathbf{F} = (M \mathbf{v} + \mathbf{P}) \times \mathbf{\Omega} + m_0 g R^T k + F_{ext} - \mathbf{\mathbf{\bar{u}}} \]  
(2.19)

and the total torque \( \mathbf{T} \) is given by

\[ \mathbf{T} = (J \mathbf{\Omega} + \hat{r}_P \mathbf{P}) \times \mathbf{\Omega} + (M \mathbf{v} \times \mathbf{v}) + (\mathbf{\Omega} \times r_P) \times P_P + \mathbf{\bar{m}} \hat{r}_P g R^T k + T_{ext} - \hat{r}_P \mathbf{\bar{u}} \]  
(2.20)

where \( \mathbf{k} \) is the unit vector of the 3th-axis of the inertial frame (see figure 2.1), more specifically the direction of gravity. Then it can be written as \( k = [0, 0, 1]^T \).

In equations (2.19) and (2.20), \( F_{ext} \) and \( T_{ext} \) are respectively the external force and external torque on vehicle, and include the viscous forces and moments that the fluid environment induce to the glider’s body.

The hydrodynamic model is detailed in the following section 2.2.2.

2.2.2 Hydrodynamic model

In order to avoid complicating the study and analysis of the underwater glider model, in this thesis sea currents are not considered on it. As it is justified in [3], it can be assumed that sea currents exist but in scales much greater than the glider’s
dimensions, hence they can be neglected. In case of taking them into account, it would be necessary to introduce some motion to the fluid environment surrounding the vehicle, and add it to the model. Otherwise the fluid is considered to be at rest (with respect to the inertial frame).

Then, the external force \( F_{\text{ext}} \), containing the hydrodynamic forces, has the terms of drag force \( D \), lift force \( L \), and side force \( SF \), and is written as

\[
F_{\text{ext}} = \begin{pmatrix}
-D \\
SF \\
-L
\end{pmatrix}
\] (2.21)

and the external torque \( T_{\text{ext}} \), containing the hydrodynamic moments, is written as

\[
T_{\text{ext}} = \begin{pmatrix}
M_{DL_1} \\
M_{DL_2} \\
M_{DL_3}
\end{pmatrix}
\] (2.22)

At this point, several models for computing hydrodynamic forces and moments can be used. Complexity varies between them, and it is mainly introduced by the number of parameters therein. The aim is to choose a model coherent and accurate enough with reality, but as simple as possible. A model yielding a proper behaviour of flow dynamics, including its main aspects, and mathematically easy to deal with.

The hydrodynamic model chosen here is also used in [1], and is presented below:

\[
D = (K_{D0} + K_D \alpha^2)V^2
\] (2.23)

\[
SF = K_\beta V^2
\] (2.24)

\[
L = (K_{L0} + K_L \alpha)V^2
\] (2.25)

\[
M_{DL_1} = K_{MR} \beta V^2 + K_{q1} \Omega_1 V^2
\] (2.26)

\[
M_{DL_2} = (K_{M0} + K_M \alpha + K_{q2} \Omega_2)V^2
\] (2.27)

\[
M_{DL_3} = K_{MY} \beta V^2 + K_{q3} \Omega_3 V^2
\] (2.28)

where \( K_{D0} \) and \( K_D \) are drag coefficients, \( K_\beta \) is the coefficient of the side force, \( K_{L0} \) and \( K_L \) are lift coefficients, and \( K_{M0}, K_M, K_{MR}, K_{MY}, K_{q1}, K_{q2}, K_{q3} \) are coefficients of the hydrodynamic moments expressions.

All these coefficients are constant parameters, whose numeric values can be found in section 2.4. They were estimated both by theoretical methods and by experiments at sea and wind tunnels. Some references about these procedures and other models can be found, for instance, in [3], [4] and [6].

In equations (2.23)-(2.28), \( V \) is the total speed of the glider, calculated as

\[
V = \sqrt{v_1^2 + v_2^2 + v_3^2}
\] (2.29)
α is the angle of attack, given by
\[ \alpha = \tan^{-1} \frac{v_3}{v_1} \]  
(2.30)
and β is the sideslip angle, given by
\[ \beta = \sin^{-1} \frac{v_2}{V} \]  
(2.31)

More specifically, the angle of attack α can be described as the angle between the first axis of the body frame (e_1) and the projection of the velocity vector v on the 1-3 plane (e_1-e_3 plane). In the same way, the sideslip angle β can be described as the angle between the vector v and its projection on the 1-3 plane.

This model, as well as the diagonal form of added matrices M_f and J_f, are only valid at low angles of attack and sideslip [1][3].

2.3 Dynamics Restricted to the Vertical Plane

In section 2.2, general dynamics for an underwater glider moving through the three-dimensional space (3D) were presented. When the vehicle is moving performing straight glides, its motion takes place framed in a vertical plane. The glider is allowed to move up and down within this plane, but not in the 3th spatial dimension. Then, it is a case of two-dimensional motion (2D), and the dynamic model can be restricted to this plane and therefore simplified.

Concretely, motion is specified to be in the i-k plane of the inertial frame and the e_1-e_3 plane of the body frame. These restrictions introduce the following conditions to the equations:

There is only possible angular rotation in the 2nd-axis of the body frame (e_2), therefore the angular velocity vector Ω has only a component in the 2nd-axis and the Euler angles ψ = φ = 0 and θ ≠ 0 (i.e only the pitch angle may be nonzero). This gives a simpler form for rotation matrix R and vector Ω

\[
R = \begin{pmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{pmatrix}, \quad \Omega = \begin{pmatrix}
0 \\
\Omega_2 \\
0
\end{pmatrix}
\]

In the same way, the linear velocity vector v have only the terms of the 1st and 3th axes, and position b have components in x and z, as it is restricted to the i-k plane.
of the inertial frame. And analogously for $r_P$, $P_P$ and $\mathbf{u}$.

Adding these conditions to the general equations of motion (2.3)-(2.9) one gets the new equations of motion restricted to the vertical plane:

\begin{align*}
\dot{x} &= v_1 \cos \theta + v_3 \sin \theta \\
\dot{z} &= -v_1 \sin \theta + v_3 \cos \theta \\
\dot{\theta} &= \Omega_2 \\
\dot{\Omega}_2 &= \frac{1}{J_2} ((m_3 - m_1)v_1v_3 - m_b(r_{P1}\cos \theta + \\
&+ r_{P3}\sin \theta) + M_{DL} - r_{P3}u_1 + r_{P1}u_3) \\
\dot{v}_1 &= \frac{1}{m_1} (-m_3v_3\Omega_2 - P_{P3}\Omega_2 - m_0g \sin \theta + \\
&+ L \sin \alpha - D \cos \alpha - u_1) \\
\dot{v}_3 &= \frac{1}{m_3} (m_1v_1\Omega_2 + P_{P1}\Omega_2 + m_0g \cos \theta - \\
&- L \cos \alpha - D \sin \alpha - u_3) \\
\dot{r}_{P1} &= \frac{1}{m} P_{P1} - v_1 - r_{P3}\Omega_2 \\
\dot{r}_{P3} &= \frac{1}{m} P_{P3} - v_3 + r_{P1}\Omega_2 \\
\dot{P}_{P1} &= u_1 \\
\dot{P}_{P3} &= u_3 \\
\dot{m}_b &= u_4
\end{align*}

The hydrodynamic model is taken the same as in the 3-D case, presented in section 2.2.2, although now less terms appear in the dynamics. Thus, hydrodynamic forces
and moments are reduced to drag $D$, lift $L$ and a single moment $M_{DL}$ about the 2nd axis.

\[
D = (K_D0 + K_D\alpha^2)(v_1^2 + v_3^2) \quad (2.43) \\
L = (K_L0 + K_L\alpha)(v_1^2 + v_3^2) \quad (2.44) \\
M_{DL} = (K_M0 + K_M\alpha)(v_1^2 + v_3^2) \quad (2.45)
\]

### 2.4 Model Parameters

As it has been seen in the previous sections, a large number of parameters take part in the equations of motion. Parameters regarding solely to the vehicle, as mass and inertia, and hydrodynamic parameters, regarding to the interaction between the fluid and the glider.

Two full sets of parameters are used in this thesis, one of them for the 2-D scenario and the other one for the 3-D scenario.

First, parameters when motion is restricted to the vertical plane are presented, in the following Tables 2.1 and 2.2. They belong to a model of underwater glider called ROGUE, a laboratory-scale glider developed at Princeton University [11]. See picture 1.2.

For the general case when the glider moves through the whole space, parameters are taken from [1] and can be found in Tables 2.3 and 2.4. These parameters belong to the operational model of glider SLOCUM. See picture 1.3.

<table>
<thead>
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<th>Unit</th>
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Table 2.1: 2D Vehicle parameters

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Table 2.2: 2D Hydrodynamic parameters
### Table 2.3: 3D Vehicle parameters

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<td>11</td>
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### Table 2.4: 3D Hydrodynamic parameters

<table>
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Chapter 3

GLIDER CONTROL

This chapter aims to describe a control system to be implemented in the dynamics in order to control the behaviour of the underwater glider. Two important issues need to be noticed: no rudder is used in the vehicle, as we defined in the glider model, neither propeller is installed. This yields in the fact that the behaviour of the glider is defined mainly by its attitude and its buoyancy, hence, controlling glider’s behaviour can be extrapolated in controlling its attitude and buoyancy together. With no rudder, the attitude has to be controlled by means of changing the position of the internal movable point mass $m$, while the buoyancy is controlled by changing the mass of variable $m_b$.

When we wish the glider to go downwards under the ocean, we need to let sea water get into the vehicle’s body, hence the vehicle will become heavier and will sink in a controlled way. This is, $m_b$ increases until the glider is negatively buoyant, i.e. according to (2.2) until $m_0 > 0$. Instead, if we want the glider to go upwards to the surface, the sea water (or part of it) inside the body must be pushed out. Then, as air is lighter than water, the glider will become buoyant and hence go upwards. In this case the value of the control variable $m_b$ decreases until $m_0$ is negative. Depending on how negative $m_0$ is, i.e. how buoyant the glider is, it will go upwards faster or slower. And analogously for the downward case.

In section 3.1 steady equilibria are derived from the general equations of motion presented in the previous chapter. After this, section 3.2 is exclusively dedicated to the controller, with detailed explanations about the control laws implemented. Some considerations regarding the stability of the system are discussed in section 3.3, and end the chapter.

3.1 Gliding Equilibria

Underwater gliders belong to the group of vehicles known as Autonomous Underwater Vehicles (AUV), and as its name tells, it is desired for them to be as much autonomous as possible. Low energy consumption is then one of the main goals, leading, among other things, longer missions. See, for example, [15].
A way of increasing autonomy is not to use propeller but attitude and buoyancy control instead, as we saw. Although this procedure is energetically more efficient, some actuators and hence energy supply are still needed in order to move the point mass $m$ and to pump water in and out the vehicle. Notice that this energy consumption happens only when the glider’s configuration needs to be changed. Therefore, the next goal is to describe steady glide paths, where once the glider has reached them, it can remain with its motion without changing its internal configuration and hence with no energetic cost.

In [1] it is analytically proven that the only steady equilibria for underwater gliders are straight glide paths and spiral glides.

3.1.1 Straight paths

One of the typical motions of interest performed by underwater gliders, then, is to follow desired straight paths. When the glider moves through a straight line, its motion is framed in two dimensions, so in a case where dynamics are restricted to the vertical plane this will be the only realizable equilibria. The equilibria is determined below following the methodology stated in [11].

![Figure 3.1: Simulation of a straight path (Downwards glide with $\xi = -30^\circ$)](image)

The desired straight path to be reached by the glider will be defined by two variables: the glide path angle $\xi_d$ and the glider’s speed $V_d$. Here subscript ‘d’ denotes ‘desired’. As these values are the ones that the glider will have at equilibria, further on subscript ‘d’ will be used to specify the value of all the variables at the
gliding equilibria.
The glide path angle is a representation of the slope of the path seen in the vertical plane. As the angle of attack $\alpha$ is nonzero, $\xi$ is similar to the inclination with which the glider goes downwards or upwards (pitch angle $\theta$), but not equal. Then, it can be written in terms of this notation as: $\xi = \theta - \alpha$. A sketch is also added in figure 3.2.

![Figure 3.2: Relationship between $\xi$, $\alpha$ and $\theta$](image)

At equilibria it is known that all the dynamic variables are constant, then their time derivatives must be zero. This is translated to the equations of motion (2.34)-(2.42) by replacing

$$\dot{\theta} = \dot{v}_1 = \dot{v}_3 = \dot{r}_{P1} = \dot{r}_{P3} = \dot{P}_{P1} = \dot{P}_{P3} = \dot{m}_b = 0$$

Then by equations (2.34) and (2.40)-(2.42) we immediately get

$$\Omega_{2d} = u_{1d} = u_{3d} = u_{4d} = 0$$

which can also be replaced in the remaining equations of motion (2.35)-(2.39). Doing this we finally obtain the following equations:

$$0 = \frac{1}{J_2}((m_{f3} - m_{f1})v_{1d}v_{3d} - m g(r_{P1d} \cos \theta_d + r_{P3d} \sin \theta_d) + M_{DLd} - r_{P3d}u_1 + r_{P1d}u_3)$$

(3.1)

$$0 = \frac{1}{m_{1d}}(-m_{0d}g \sin \theta_d + L_d \sin \alpha_d - D_d \cos \alpha_d)$$

(3.2)

$$0 = \frac{1}{m_{3d}}(m_{0d}g \cos \theta_d - L_d \cos \alpha_d - D_d \sin \alpha_d)$$

(3.3)

$$0 = \frac{1}{m}P_{P1d} - v_{1d}$$

(3.4)

$$0 = \frac{1}{m}P_{P3d} - v_{3d}$$

(3.5)
where $L_d, D_d$ and $M_{DL_d}$ can be substituted by expressions (2.43)-(2.45) evaluated at equilibria.

The expressions of the dynamic variables $\theta_d$, $v_{1d}$ and $v_{3d}$ can be described easily, and using (3.4) and (3.5) we also find $P_{P_{1d}}$ and $P_{P_{3d}}$.

$$\theta_d = \xi_d + \alpha_d \quad v_{1d} = V_d \cos \alpha_d \quad v_{3d} = V_d \sin \alpha_d$$

$$P_{P_{1d}} = m v_{1d} \quad P_{P_{3d}} = m v_{3d}$$

Mixing equations (3.2) and (3.3) one may obtain an expression for the equilibrium angle of attack $\alpha_d$. Being $\xi_d \neq \pm \frac{\pi}{2}$ and $V_d \neq 0$,

$$\alpha^2_d + \left( \frac{K_L}{K_D} \tan \xi_d \right) \alpha_d + \frac{1}{K_D} (K_{D0} + K_{L0} \tan \xi_d) = 0 \quad (3.6)$$

Notice that this is an equation depending only on variable $\alpha_d$ as all the other terms are known, and due to its quadratic form, it can only be solved with a real number as solution if

$$\left( \frac{K_L}{K_D} \tan \xi_d \right)^2 - 4 \frac{1}{K_D} (K_{D0} + K_{L0} \tan \xi_d) \geq 0 \quad (3.7)$$

As $K_L, K_{L0}, K_D$ and $K_{D0}$ are constant hydrodynamic parameters, the previous equation gives a threshold of feasible glide path angles $\xi_d$. More specifically, $\xi_d$ must satisfy

$$\xi_d \in \left( \tan^{-1} \left( 2 \frac{K_D}{K_L} \left( \frac{K_{L0}}{K_L} + \sqrt{\left( \frac{K_{L0}}{K_L} \right)^2 + \frac{K_{D0}}{K_D}} \right) \right), \frac{\pi}{2} \right), \ i f \ \xi_d > 0 \quad (3.8)$$

and

$$\xi_d \in \left( -\frac{\pi}{2}, \tan^{-1} \left( 2 \frac{K_D}{K_L} \left( \frac{K_{L0}}{K_L} - \sqrt{\left( \frac{K_{L0}}{K_L} \right)^2 + \frac{K_{D0}}{K_D}} \right) \right) \right), \ i f \ \xi_d < 0 \quad (3.9)$$

Then, when choosing a desired glide path angle one should be sure that the value is within the feasible range.

Continue with the resolution of the angle of attack at equilibria: in [11] is justified to take the smaller value of $\alpha_d$ given by equation (3.6) as the validity of drag model is limited to small angles of attack. This results in
\[ \alpha_d = \frac{1}{2} \frac{K_L}{K_D} \tan \xi_d \left( -1 + \sqrt{1 - 4 \frac{K_D}{K_L} \cot \xi_d (K_{D0} \cot \xi_d + K_{L0})} \right) \] (3.10)

Now that \( \alpha_d \) is determined, the ballast mass can also be found by combining equations (3.2) and (3.3), as \( m_{bd} \) appears implicitly in the mass terms therein.

\[ m_{bd} = (m - m_h - \bar{m}) + \frac{1}{g} (-\sin \xi_d (K_{D0} + K_D \alpha_d^2) + \cos \xi_d (K_{L0} + K_L \alpha_d)) V_d^2 \] (3.11)

With equation (3.1), not used until now, we can finally obtain a set of variables \((r_{P1_d}, r_{P3_d})\) that satisfy the desired equilibria. A smart option is to fix the value of \( r_{P3_d} \) and then compute \( r_{P1_d} \) according to it, through

\[ r_{P1_d} = -r_{P3_d} \tan \theta_d + \frac{1}{mg \cos \theta_d} \left((m_{f3} - m_{f1}) \nu_{1_d} \nu_{3_d} + (K_{M0} + K_M \alpha_d) V_d^2\right) \] (3.12)

We have then computed the values of the ballast mass \( m_{bd} \) and the position \((r_{P1_d}, r_{P3_d})\) of the movable mass \( \bar{m} \) at the desired equilibria, controlling the pitch angle of the glider. By choosing a negative value of the desired glide path angle \((\xi_d < 0)\), this set of values will yield in a downwards straight glide. Instead, if the angle is chosen to be positive \((\xi_d > 0)\) it will yield in an upwards glide.

Here these expressions were derived from motion restricted to the vertical plane. Anyways one may take this vertical plane to be a plane of the 3D space, hence the values computed this way can also be used for straight paths in the full motion case.

A last consideration is taken into account. Referring to [11], we know that controllability is not lost although the movable mass \( \bar{m} \) is restricted to have a single degree-of-freedom, i.e. fixed \( r_{P1} \) or \( r_{P3} \). In spite of this, if the position along the longitudinal axis \((e_1)\) is fixed, the vehicle will be configured to perform only upwards or downwards glides, but not both. Then, for sake of simplicity but without compromising motion, in the simulations shown at the end of this report the position \( r_{P3} \) of mass \( \bar{m} \) is kept constant.

3.1.2 Spiral glides

The other feasible equilibria in 3D motion are spiral glides, where the glider follows an helical path. As there is no rudder to induce turning, it is done by displacing the movable mass \( \bar{m} \) towards \( e_2 \)-axis, i.e. introducing an offset \( r_{P2} \). This results in a roll angle \( \phi \) different from zero.

Gliding conditions derived from helical equilibria are stated in [1], and cited below:
- The total speed $V_d$ is constant
- Pitch $\theta_d$ and roll $\phi_d$ are constant. Yaw $\psi_d$ changes at constant rate
- Angle of attack $\alpha_d$ and sideslip angle $\beta_d$ are constant, then the hydrodynamic forces and moments are also kept constant
- The spiral’s axis has the direction of gravity

In [1] one can also find an in-depth analysis about dependence of helix parameters (radius, period, etc.).

Again, at equilibria the dynamic variables must be zero

$$\dot{\theta} = \dot{v} = \dot{r}_p = \dot{P}_p = \dot{m}_b = 0$$
The resulting equations for gliding equilibria in 3D, taken from [3], are

\[ 0 = J\Omega \times \Omega + Mv \times v - r_P \times [\overline{m}(v + \Omega \times r_P) \times \Omega] + \overline{m}r_P g R^T k + T_{ext} \]  
(3.13)

\[ 0 = [(M + m_b I)\dot{v} + \overline{m}(v + \Omega \times r_P)] \times \Omega + m_0 g R^T k + F_{ext} \]  
(3.14)

with \( \Omega = R^T k \omega_3 \), where \( \omega_3 \) is constant.

This system may be solved by desired glider speed \( V_d \), desired glide path angle \( \xi_d \) and desired roll \( \phi_d \), and with this get the desired glider’s configuration: \( r_{P1d}, r_{P2d}, r_{P3d} \) and \( m_{bd} \). As for the straight paths, a negative value of \( \xi_d \) will yield downwards spiral glides, while for a positive \( \xi_d \) the spiral will be performed upwards.

### 3.2 Controller Design

The control structure used is compound by one scale of two controllers (see figure 3.5). As a first and innermost level there is a nonlinear controller, and then a linear controller is applied in a second level. The former performs an input-output linearization, with a nonlinear feedback control law that linearizes the system by cancelling the no-linearities, while the later is a simple linear controller with a PID structure.

![Figure 3.5: Block diagram with control scheme](image)

Other control strategies can also be found, yielding different behaviour and stability properties. In [11] a linear-quadratic regulator (LQR) is described. A comparison between LQR and PID controllers is in [14]. In [12] control is implemented with design of servo controller and PID controller, and in reference [13] a feedforward/feedback control system is described.

#### 3.2.1 Nonlinear controller

Feedback linearization is introduced to the model in order to improve it and correct some shortcomings therein. A more detailed explanation is given in section 3.3.
The idea of this nonlinear feedback control law described below was introduced in [2].

Take equation (2.7), which defines the velocity of the movable point mass \( \overline{m} \), and derive it with respect to time to get the expression of its acceleration. That is, from equation

\[
\dot{r}_P = \frac{1}{\overline{m}} P_P - v - \Omega \times r_P
\]

get

\[
\ddot{r}_P = \frac{1}{\overline{m}} \dot{P}_P - \dot{v} - (\dot{\Omega} \times r_P + \Omega \times \dot{r}_P) \tag{3.15}
\]

The goal now is to have an expression for the acceleration \( \ddot{r}_P \) where the only derivative in it is its own velocity \( \dot{r}_P \). To achieve it lets take expressions (2.5), (2.6) and (2.8) and replace them to the previous equation (3.15). With these substitutions \( \ddot{r}_P \) can be written in terms of variables \( R \) (i.e. Euler angles: \( \psi \), \( \theta \) and \( \phi \), \( v \), \( \Omega \), \( r_P \), \( \dot{r}_P \), \( m_b \), and the control input vector \( \overline{u} \).

Let's present it in the following structure

\[
\ddot{r}_P = Z + F \overline{u} \tag{3.16}
\]

where \( Z \) and \( F \) are matrices containing all the variables cited above, and will be defined further on in this section.

Set now the control input \( \overline{u} \) to be

\[
\overline{u} = F^{-1}(-Z + \overline{w}) \tag{3.17}
\]

Equation (3.17) defines the nonlinear feedback control law set for the control vector \( \overline{u} \), in terms of the new control input \( \overline{w} \). Notice here that no modification has been done to the buoyancy control input \( u_4 \), hence just a change of variable is done to it: \( u_4 = \omega_4 \)

The whole control law defined above is given together by

\[
\overline{u} = F^{-1}(-Z + \overline{w}) \tag{3.18}
\]

\[
u_4 = \omega_4 \tag{3.19}
\]

If equation (3.18) is substituted to equation (3.16), and equation (3.19) to (2.9), results

\[
\ddot{r}_P = \overline{w} \tag{3.20}
\]
\[ m_b = \omega_1 \] \hspace{1cm} (3.21)

The relation obtained in (3.20) shows that in fact the control law introduced acts as an acceleration control to the movable point mass \( m \), while the other input is kept to be the buoyancy control as before (3.21).

Matrices \( F \) and \( Z \) that appear in previous equations are detailed below:

\[
F = \begin{pmatrix}
M^{-1} - \hat{r}_p J^{-1} \dot{\hat{r}}_p + \frac{1}{m} I & M^{-1} \\
M^{-1} & M^{-1} + \frac{1}{m_b} I
\end{pmatrix}
\] \hspace{1cm} (3.22)

\[
Z = \begin{pmatrix}
Z_P \\
Z_b
\end{pmatrix}
\] \hspace{1cm} (3.23)

where

\[
Z_P = -M^{-1} \left[ \left( (M + m_b I)v + \bar{m}(v + \Omega \times r_P + \dot{r}_P) \right) \times \Omega + m_0 g R^T k + F_{\text{ext}} \right] - \Omega \times \dot{r}_P - J^{-1} \left[ \left( J \Omega + \dot{r}_p (\bar{m}(v + \Omega \times r_P + \dot{r}_P)) \right) \times \Omega + (Mv \times v) + T_{\text{ext}} + \right.
\]
\[
+ \left( \Omega \times r_P \right) \times \left( \bar{m}(v + \Omega \times r_P + \dot{r}_P) \right) + \bar{m} \dot{r}_p g R^T k \] \times r_P
\]

\[
Z_b = -M^{-1} \left[ \left( (M + m_b I)v + \bar{m}(v + \Omega \times r_P + \dot{r}_P) \right) \times \Omega + m_0 g R^T k + F_{\text{ext}} \right] - \Omega \times \dot{r}_P
\]

Here \( F_{\text{ext}} \) and \( T_{\text{ext}} \) are the hydodynamic forces and moments presented in section 2.2.2.

Remark that, as desired, in the previous expressions the only time derivative therein is \( \dot{r}_P \). As it can be checked in matrices \( F \) and \( Z \) above, all is given in terms of \( R \), \( v \), \( \Omega \), \( r_P \), \( \dot{r}_P \) and \( m_b \). More specifically, \( m_b \) takes part of them through variable \( m_0 \), according to (2.2).

In the same way it was done with equations of motion in section 2.3, here the nonlinear controller can also be simplified when it is applied to dynamics restricted to the vertical plane.

In this case the position of the movable point mass \( \bar{m} \) has two components, \( r_{P1} \)
and $r_{P3}$, and their time derivatives (velocities) are defined by equations (2.38) and (2.39) respectively.

As in the 3D general case (3.15), differentiate now the velocities’ expressions to get accelerations

\[
\ddot{r}_{P1} = \frac{1}{m} \dot{P}_{P1} - \dot{v}_1 - \dot{r}_{P3}\Omega_2 - r_{P3}\dot{\Omega}_2
\]

\[
(3.24)
\]

\[
\ddot{r}_{P3} = \frac{1}{m} \dot{P}_{P3} - \dot{v}_3 + \dot{r}_{P1}\Omega_2 + r_{P1}\dot{\Omega}_2
\]

\[
(3.25)
\]

and analogously substitute equations (2.35)-(2.37), (2.40) and (2.41) to the ones above (3.24) and (3.25), in order to get their expressions in terms of variables $\theta$, $\Omega_2$, $v_1$, $v_3$, $r_{P1}$, $r_{P3}$, $\dot{r}_{P1}$, $\dot{r}_{P3}$, $m_b$ and inputs $u_1$, $u_3$.

Lets write it now with the same structure as in (3.16):

\[
\begin{bmatrix}
\ddot{r}_{P1} \\
\ddot{r}_{P3}
\end{bmatrix} = Z + F \begin{bmatrix} u_1 \\ u_3 \end{bmatrix}
\]

\[
(3.26)
\]

The control input is chosen to be

\[
\begin{bmatrix} u_1 \\ u_3 \end{bmatrix} = F^{-1} \left( -Z + \begin{bmatrix} \omega_1 \\ \omega_3 \end{bmatrix} \right)
\]

\[
(3.27)
\]

\[u_4 = \omega_4\]

\[
(3.28)
\]

defining the nonlinear control law.

In this case matrices $F$ and $Z$ are as follows:

\[
F = \begin{pmatrix}
\frac{1}{m} + \frac{1}{m_1} + \frac{r_{P3}^2}{J_2} & -\frac{r_{P1}r_{P3}}{J_2} \\
-\frac{r_{P1}r_{P3}}{J_2} & \frac{1}{m_3} + \frac{1}{m_3} + \frac{r_{P1}^2}{J_2}
\end{pmatrix}
\]

\[
(3.29)
\]

\[
Z = \begin{pmatrix}
-\frac{1}{m_1} X_1 - \dot{r}_{P3}\Omega_2 - \frac{r_{P3}Y}{J_2} \\
-\frac{1}{m_3} X_3 + \dot{r}_{P1}\Omega_2 + \frac{r_{P1}Y}{J_2}
\end{pmatrix}
\]

\[
(3.30)
\]

with

\[
X_1 = -m_3 v_3\Omega_2 - m(v_3 + \dot{r}_{P3} - r_{P1}\Omega_2)\Omega_2 - m_0 g \sin \theta + L \sin \alpha - D \cos \alpha
\]

\[
X_3 = m_1 v_1\Omega_2 + m(v_1 + \dot{r}_{P1} + r_{P3}\Omega_2)\Omega_2 + m_0 g \sin \theta - L \cos \alpha - D \sin \alpha
\]

\[
Y = (m_3 - m_1)v_1v_3 - \bar{m}g(r_{P1} \cos \theta + r_{P3} \sin \theta) + M_{DL}
\]
where $\alpha$ is the angle of attack given by (2.30), and $D$, $L$ and $M_{DL}$ the viscous forces and moment described in section 2.3.

Notice that in the control law stated in (3.27) there is the inverse matrix $F^{-1}$. In order to ensure that it is always well defined, $F$ must be invertible at any configuration. Let's calculate the determinant of matrix $F$ defined in (3.29):

$$
\det(F) = \left( \frac{1}{m} + \frac{1}{m_1} \right) \left( \frac{1}{m} + \frac{1}{m_3} \right) + \frac{r_{p1}^2}{J_2} \left( \frac{1}{m} + \frac{1}{m_1} \right) + \frac{r_{p3}^2}{J_2} \left( \frac{1}{m} + \frac{1}{m_3} \right)
$$

All masses are positive by definition, and inertia $J_2$ is also positive. Then, the only variables with possible negative values are $r_{pi}$, but as they appear squared, the determinant of $F$ is always different from zero and positive. Therefore matrix $F$ is always nonsingular and $F^{-1}$ exists.

As seen before in equations (3.20) and (3.21), the combination of (3.26) and (3.27) becomes to relations

$$
\begin{bmatrix}
\ddot{r}_{p1} \\
\ddot{r}_{p3}
\end{bmatrix}
= \begin{bmatrix}
\omega_1 \\
\omega_3
\end{bmatrix}
\tag{3.31}
$$

$$
\dot{m}_b = \omega_4 \tag{3.32}
$$

where $\omega_1$ and $\omega_3$ are the new control inputs applied as the point mass accelerations, and $\omega_4$ is the ballast mass rate control.

### 3.2.2 Linear controller

In the preceding section an input-output linearization with nonlinear feedback control law for control inputs $\bar{u}$ is described, as well as incorporation of new control variables $\omega$, in (3.18) and (3.19). The control inputs transformation yields in acceleration control of movable mass $\bar{m}$, according to (3.20). In this section a simple linear controller with general PID structure is applied to acceleration control and buoyancy control.

To carry out the buoyancy control an input variable $\omega_4$ is matched with the ballast mass rate $\dot{m}_b$, as seen in (3.21). The linear controller in this case is chosen only with the proportinal term $P$, where the ballast mass $m_b$ tracks a desired value $m_{bd}$, resulting

$$
\omega_4 = -K_{p_m}(m_b - m_{bd}) \tag{3.33}
$$
here $K_{pm}$ is a constant gain ($K_{pm} > 0$) and $(m_{bd} - m_b)$ is error.

About acceleration control, derived in (3.20), the controller is chosen with proportional-derivative structure (PD). Therefore, being $\omega = [\omega_1, \omega_2, \omega_3]^T$

$$
\omega_1 = -K_{p1}(r_{P1} - r_{P1_d}) - K_{d1}\dot{r}_{P1} \\
\omega_2 = -K_{p2}(r_{P2} - r_{P2_d}) - K_{d2}\dot{r}_{P2} \\
\omega_3 = -K_{p3}(r_{P3} - r_{P3_d}) - K_{d3}\dot{r}_{P3}
$$ (3.34)

where $K_{pi}$ and $K_{di}$ ($i = 1, 2, 3$) are constant positive gains, and $r_{Pi_d}$ are values of the desired position for the movable mass $m$.

If dynamics are restricted to the vertical plane this linear controller has the same structure as before, although $\omega_2$ does not take part of it.

Then, it results in

$$
\omega_1 = -K_{p1}(r_{P1} - r_{P1_d}) - K_{d1}\dot{r}_{P1} \\
\omega_3 = -K_{p3}(r_{P3} - r_{P3_d}) - K_{d3}\dot{r}_{P3} \\
\omega_4 = -K_{pm}(m_{b} - m_{bd})
$$ (3.35)

### 3.3 Stability Analysis

In the introduction of this chapter we saw references to some other control strategies. A previous brief analysis of some of them will help to understand the reasons of why finally a combination of feedback linearization plus a linear controller is chosen.

We already mentioned that in [11] the authors design and use a linear quadratic regulator (LQR) as controller, for dynamics restricted to the vertical plane. By linearizing the equations of motion (2.32)-(2.42) about the gliding equilibria they study controllability and stability properties. The desired equilibrium paths studied therein are found to be unstable, as its linearization have some positive eigenvalues, although locally controllable. Accomplishment of the controllability property guarantees that by use of feedback with a linear control law it is feasible to make the glider reach a desired equilibria and to locally stabilize it, because of this, by implementing the LQR controller the authors achieve the desired objective. However, this control system has some shortcomings (already stated in [11]), which can be overcomed [2][3] by implementing the nonlinear control law complemented with the linear controller described in sections 3.2.1 and 3.2.2 respectively.
As it will be seen below, the nonlinear control law that results in feedback linearization also yields in a locally stable equilibria. Furthermore, it provides some very important advantages: it provides a much larger region of attraction for gliding equilibria, improves the behaviour when changing between different glide paths, and allows the glider to follow a sawtooth gliding, i.e. to switch between downwards and upwards glides successively (not possible with a single linear controller due to its small region of attraction). All these contributions of the control law are really critical for further work described in Chapter 4.

See [11], [2] and [3], where more information regarding the origin of the instability can be found, as well as the benefits (pointed above) of the new control system. In them it is explained that this instability is in fact an inaccuracy of the model, not existing in reality. It appears because in the model the movable point mass $\bar{m}$ is allowed to move freely inside the glider, unlike in reality where $\bar{m}$ is held by actuators. The nonlinear control law stated in (3.18) is designed to be equivalent to a suspension system applied to $\bar{m}$, and therefore eliminates this instability.

Proceed now with stability analysis of the closed-loop system. The whole system is now completely defined: On one hand the dynamic model is stated through Chapter 2, and on the other hand the control system has been detailed in the current chapter, 3. Following the reference of [2], stability will be proven by means of Lyapunov’s indirect method, using the linearization of the system. Procedure for 2-D case is detailed below.

In order to get the linearization of the complete closed-loop system, the nonlinear control law is inserted to the model. First, equations of motion (2.35)-(2.37) are rewritten in terms of new control inputs $\omega_1$ and $\omega_3$ (according to (3.27)) and in terms of $\dot{r}_{P1}$ and $\dot{r}_{P3}$ (under (2.38) and (2.39)). Finally (2.40)-(2.42) are replaced by (3.31) and (3.32). The variables are then $[\theta, \Omega_2, v_1, v_3, r_{P1}, r_{P3}, \dot{r}_{P1}, \dot{r}_{P3}, m_b]$, and equations of motion with the nonlinear feedback transformation are as follows:

$$\dot{\theta} = \Omega_2$$

$$\dot{\Omega}_2 = \frac{1}{J_2|F|}(a_1a_3Y - \frac{r_{P3}}{m_1}a_3X_1 + \frac{r_{P1}}{m_3}a_1X_3 -$$

$$- r_{P1}a_1(\Omega_2\dot{r}_{P1} - \omega_3) - r_{P3}a_3(\Omega_2\dot{r}_{P3} + \omega_1))$$

$$\dot{v}_1 = \frac{1}{m_1|F|}(- \frac{r_{P3}}{J_2}a_3Y + \frac{d_3}{m}X_1 - \frac{c}{m_3}X_3 +$$

$$+ c(\Omega_2\dot{r}_{P1} - \omega_3) - (a_3 + b_1)(\Omega_2\dot{r}_{P3} + \omega_1))$$
\[
\dot{v}_3 = \frac{1}{m_3 |F|} \left( \frac{r_{p_1}}{f_2} a_1 Y - \frac{c}{m_1} X_1 + \frac{d_1}{m} X_3 + (a_1 + b_3)(\Omega_2 \dot{r}_{p_1} - \omega_3) - c(\Omega_2 \dot{r}_{p_3} + \omega_1) \right) \\
\dot{r}_{p_1} = \omega_1 \\
\dot{r}_{p_3} = \omega_3 \\
\dot{m}_b = \omega_4
\] (3.36)

with \( a_1 = \frac{1}{m} + \frac{1}{m_1} \), \( a_3 = \frac{1}{m} + \frac{1}{m_3} \), \( b_1 = \frac{r_{p_1}}{f_2} \), \( b_3 = \frac{r_{p_3}}{f_2} \), \( c = \frac{r_{p_1} r_{p_3}}{f_2} \), \( d_1 = a_1 + b_1 a_1 + b_3 \) and \( d_3 = a_3 + b_3 a_3 + b_1 \).

Define a vector \( \zeta = (r_{p_1} - r_{p_{1d}}, \dot{r}_{p_1}, r_{p_3} - r_{p_{3d}}, \dot{r}_{p_3}, m_b - m_{bd}) \) and another one with the remaining state variables \( \eta = (\theta, \Omega_2, v_1, v_3) \). Taking as output \( m \)'s position and ballast mass \((r_{p_1}, r_{p_3}, m_b)\), and as input the control variables \( \omega = (\omega_1, \omega_3, \omega_4) \), the input-output linearized system can be presented, splitted in two parts, in the following way

\[
\dot{\eta} = q(\eta, \zeta, \omega) \quad (3.37) \\
\dot{\zeta} = A\zeta + B\omega \quad (3.38)
\]

The first four equations of (3.36) give the components of expression (3.37). Moreover, the linear system in (3.38) is defined by

\[
A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]

The next step is to linearize the system about the equilibrium point, and compute its eigenvalues in order to know, from them, the stability of the equilibria. An equilibrium point is given by the values of state variables in a configuration of gliding equilibria defined by desired \( r_{p_{1d}}, r_{p_{3d}}, \) and \( m_{bd} \), as seen in section 3.1. Thus, the equilibrium point can be easily modeled as \((\eta_d, \zeta) = (\eta_{d1}, 0)\), since at equilibria \( r_{p_1} = r_{p_{1d}}, r_{p_3} = r_{p_{3d}}, m_b = m_{bd} \) and \( \dot{r}_{p_1} = \dot{r}_{p_3} = 0 \). As before, subscript \'d' denotes \'desired\'. Furthermore, in section 3.2.2 a linear control law was defined for control input \( \omega \), so now it also has to be taken into consideration. We defined in (3.35) the linear controller to have a proportional term for buoyancy control and proportional-derivative terms for \( \dot{r}_{pl} \), then the linear control law can be shown in the actual notation as

\[
\omega = K\zeta \quad (3.39)
\]
where $K$ is a matrix with the constant gains of the linear controller

$$K = \begin{bmatrix}
-K_{p1} & -K_{d1} & 0 & 0 & 0 \\
0 & 0 & -K_{p3} & -K_{d3} & 0 \\
0 & 0 & 0 & 0 & -K_{p_m}
\end{bmatrix}$$

Now that the equilibrium point is set and the control law defined, the linearized system of (3.37)-(3.38) evaluated about the equilibrium point $(\eta_d, 0)$ can be written by use of the Jacobian matrix as

$$\begin{bmatrix}
\dot{\eta} \\
\dot{\zeta}
\end{bmatrix} = \begin{bmatrix}
(\frac{\partial q}{\partial \eta})_{(\eta_d, 0)} & (\frac{\partial q}{\partial \zeta})_{(\eta_d, 0)} \\
0 & A + BK
\end{bmatrix}$$

Notice that the matrix of the complete linearized system above is an upper triangular matrix. This yields in the fact that its eigenvalues are the eigenvalues of the blocks in its diagonal, i.e. the union of the eigenvalues of $(\frac{\partial q}{\partial \eta})_{(\eta_d, 0)}$ and $(A + BK)$.

Assume a configuration of the glider with sufficient bottom-heaviness [11]. The zero dynamics of system (3.37)-(3.38), providing output zero, are $\dot{\eta} = q(\eta, 0, 0)$. Being $\eta_d$ a locally exponentially stable equilibrium of the zero dynamics, which is true under the previous assumption, then:

By choice of a linear control law for input $\omega$, $\omega = K\zeta$ defined in (3.39), such that $A + BK$ is Hurwitz, we can conclude that $(\eta, \zeta) = (\eta_d, 0)$ is a locally exponentially stable equilibrium of the whole system.

Notice that if this is fulfilled, both $(\frac{\partial q}{\partial \eta})_{(\eta_d, 0)}$ and $(A + BK)$ have eigenvalues with negative real parts, and therefore all eigenvalues of the linearized system of the closed-loop dynamics are stable.
Chapter 4

GUIDANCE

The purpose of this chapter is to design algorithms in order to the glider can move autonomously from any start point to any final target point, and doing it with a desired behaviour. To achieve this purpose, the logic in how to choose the path followed by the glider is defined.

The current chapter is divided in two main sections. The former, 4.1, is dedicated to the two-dimensional scenario, when motion is restricted to the vertical plane. First the guidance strategy is described in detail, including the sector-of-sight logic and boundary check algorithms. Afterwards, several simulations of the full model implemented in Matlab/Simulink are shown and discussed. In this model are included the dynamics of the glider, the control system and guidance algorithms. For the sake of clarity, previous to the results of guidance some other simulations are given and remarked. In the latter section, 4.2, the same structure is followed for the three-dimensional scenario: first the guidance strategy is described and then simulation results are shown and discussed.

As we saw in section 3.1 of the previous chapter, we want that when the glider is on movement its motion keeps steady equilibria. Because of that, travel between the initial point and the target point will be done following steady paths. In the case when dynamics are restricted to the vertical plane, this involves performing only straight glides, while in the general case of full 3D dynamics the target point can be reached by combination of straight paths and spiral glides.

Guidance algorithm, then, is designed to rule glider’s behaviour and therefore its motion. It provides to the controller the desired path to be followed at each time and then the corresponding internal configuration of the glider \((r_{Pi_d}, m_{b_d})\) is computed. Following this method the vehicle will change its path, when necessary, until the destination point is reached. According to what was said before, these paths will always satisfy steady equilibria.

An schematic view including the whole system is shown in figure 4.1 with glider’s
dynamics, control and guidance blocks for a general case.

![General block diagram with guidance, control and dynamics](image)

**Figure 4.1: General block diagram with guidance, control and dynamics**

### 4.1 2D Scenario

If motion is restricted to the vertical plane (2D scenario), the only way the glider can go forward is by combining downwards and upwards straight glides successively, and this is what is going to be implemented here. This kind of motion is usually known as ‘sawtooth’ or ‘zig-zag’ profile, both in a vertical or horizontal plane. Specifically, in glider’s notation it is more commonly referred to as ‘sawtooth gliding’ (see figure 4.3).

As shown in section 3.1.1, the shape of a straight path can be defined only by means of the desired glide path angle $\xi_d$, and depending on if it is negative or positive it results in downwards or upwards glides respectively. From this the issue of performing sawtooth gliding is framed in controlling the glide path angle $\xi$, related to pitch angle $\theta$.

A schematic view assuming $r_{P3}$ fixed is shown in figure 4.2.

![Block diagram for 2D scenario](image)

**Figure 4.2: Block diagram for 2D scenario**

Recall here again the importance of the nonlinear control law used in the model and described in Chapter 3. As we already mentioned in section 3.3, the feedback linearization introduced by the nonlinear control law is the one that allows changing
between different glide paths and performing sawtooth gliding. When the glider has to change its path from one glide to another, it is a necessary condition for the current glide to be within the region of attraction of the incoming glide. Otherwise, switching between them will not be possible. From here arises the importance of having large regions of attraction. This region depends on the whole closed-loop system, hence also on the controller, which can induce smaller or larger regions. The nonlinear control law implemented here is the one that provides significantly larger regions of attraction. This remark is supported by several simulations and studies, as explained in [1][3] among others.

4.1.1 Guidance strategy

A smart way of implementing a logic of guidance suitable with what we want to do is by using a strategy based on sector-of-sight algorithm [9][10] and boundary check. The combination of these two methodologies to yield a guidance law was widely developed in [8], in order to maneuver an autonomous landyacht. Based on the current position of the vehicle, and relative to the destination point, the guidance algorithm provides a logic that determines the path to be followed. A powerful feature of this algorithm is its simplicity and reliability, what yields in good performance and easy computation. Furthermore, due to its versatility, it can also be applied to other vehicles. For instance, it was then integrated to an autonomous sailing platform (sailboat) [7] and now it will be used here adapted to an underwater glider.

Notice that while in the case of a landyacht and a sailboat this is applied to motion in an horizontal plane, here it will be performed in a vertical plane. This fact has no extra implications to the strategy beyond the fact that for a glider is used for pitch control instead of heading control. Nevertheless, due to obvious differences between glider’s motion and sailboat’s and landyacht’s motion, some adaptations need to be done.

**Boundary check:** Using a boundary check algorithm the aim is to restrict the movement of the glider within two depth boundaries, defined by an upper depth plane -or line, if we think just in 2D- and a lower depth plane. With this, glider’s activity is limited to the region defined within these boundaries. The glider keeps a straight path until one of the boundaries is reached. When this happens, the vehicle automatically changes its direction in order to not to leave the gliding region: If the bottom depth boundary is reached it switches to an upwards glide, and instead, if the upper boundary is overtaken it switches to a downwards glide. Notice that this procedure yields in a bounded sawtooth gliding, as shown in figure 4.3. In this simulation the depth limits are set to 1m and 40m.
Figure 4.3: Sawtooth gliding within depth boundaries (1m and 40m)

Now define $O = (x_g, z_g)$ to be the position of the glider with respect to the inertial frame ($O$ is equivalent to initially defined $b$). Here $z \geq 0$, and remember that with motion restricted to the vertical plane we set $y = 0$. Define now $d_{\text{min}}$ to be the depth of the upper boundary and $d_{\text{max}}$ to be the depth of the bottom boundary, hence $d_{\text{max}} > d_{\text{min}}$. Remember also that the inertial frame was defined in a way that $z$-axis stands for depth (section 2.1.1), then $d_{\text{min}}$ and $d_{\text{max}}$ can be matched directly (in same units) with the $z$-coordinate of glider’s position $z_g$.

A basic algorithm derived from this can be written as

\begin{align*}
&d_{\text{min}}, \; d_{\text{max}} \\
&\text{if} \; (z_g > d_{\text{max}}) \\
&\quad \; u = 1 \\
&\text{elseif} \; (z_g < d_{\text{min}}) \\
&\quad \; u = 2 \\
&\text{else} \\
&\quad \; (\text{gliding within boundaries}) \\
&\quad \; u = u0 \\
&\quad \; \text{‘sector-of-sight algorithm’} \\
&\text{end}
\end{align*}
where \( u \) is an internal variable of the guidance algorithm used as course index. \( u = 1 \) is the index for upwards straight gliding and \( u = 2 \) the one for downwards straight gliding. Furthermore \( u\theta \) keeps the value of the index at the previous computation time. Then the controller takes the index \( u \) and converts it to the corresponding desired value of the glide path angle \( \xi_d \) for upwards and downwards gliding. Afterwards, and from \( \xi_d \), the controller also computes the corresponding internal configuration of the glider (control variables \( r_{P1}, r_{P3}, m_b \)) according to the equilibria equations seen in section 3.1.1.

**Sector-of-sight:** Boundary check algorithm narrows gliding within two depth planes, but it has nothing to do about guiding the vehicle towards its destination point (target point). Sector-of-sight logic, then, is needed.

As it can be seen in figure 4.4, the sector is defined between vectors \( \vec{v}_1 \) and \( \vec{v}_2 \), in symmetry with respect the x-axis. Notation for these vectors can be simplified by taking only the angle of their directions as the magnitude of interest, \( \theta_{v1} \) and \( \theta_{v2} \) respectively. Notice that \( \theta_{v1} = -\theta_{v2} \). The desired glide path angle \( \xi_d \) will only take \( \theta_{v1} \) or \( \theta_{v2} \) as possible values.

Like it was done for the glider, define \( P = (x_p, z_p) \) to be the position of the target point with respect to the inertial frame. Notice that \( y_p \) must also be zero. With this, the line-of-sight of reference can be defined as

\[
\theta_{los} = \arg(\overrightarrow{OP}) \quad (4.2)
\]
and the angles between the line-of-sight and the limit vectors of the sector-of-sight are

\[ \theta_1 = \theta_{v1} - \theta_{los} \]  
\[ \theta_2 = \theta_{v2} - \theta_{los} \]

(4.3) \hspace{1cm} (4.4)

The line-of-sight remain within the sector-of-sight if and only if both following conditions are fulfilled

\[ \theta_1 > 0 \]  
\[ \theta_2 < 0 \]  

(4.5)

The logic procedure, then, is as follows: while the line-of-sight of the destination point is within the sector-of-sight, the glider just has to keep with its current direction (path). If not -when (4.5) are not satisfied- two possible directions may be adopted, depending on which limit vector of the sector-of-sight is closer to the line-of-sight of reference. The glider, then, takes the direction of the closer limit vector.

An algorithm to compute this logic is

\[
\text{if } (\theta_1 > 0) \& (\theta_2 < 0) \\
\text{ } u = u_0 \\
\text{elseif } (\text{abs}(\theta_1) < \text{abs}(\theta_2)) \\
\text{ } u = 1 \\
\text{else} \\
\text{ } u = 2 \\
\text{end}
\]

Under the assumption that the starting position of the glider will always be at surface, initially it should start going downwards. According to this, the glide path angle \( \xi \) at the beginning has to be set for a downwards glide, what results in the fact that the initial condition for the course index variable is \( u_{00} = 2 \).

The boundary check (4.1) and sector-of-sight algorithms presented above are mixed together to deal with the guidance of the glider. Finally, a last condition for guidance is taken into account: As we are not supposed to pretend that the glider stops when its destination is achieved, the algorithm is designed in order to after reaching the target point, the glider finally goes to the ocean surface.

4.1.2 Simulations

Finally some simulations regarding guidance restricted to the vertical plane are carried out using the Matlab/Simulink software. In this section the scenario is set
and the results are shown.

For this scenario in 2D the parameters of the vehicle are taken from Table 2.1 and the hydrodynamic parameters from Table 2.2, both can be found in section 2.4.

The movable mass $m_1$ is restricted to have a single degree-of-freedom, $r_{P1}$. Then the position along the $e_3$-axis of the glider is set fixed to $r_{P3} = 0.04m$ for all paths.

According to equations (3.8) and (3.9), evaluated at the corresponding parameters, the interval of feasible glide path angles are:

$$\xi \in (16.15^\circ, 90^\circ) \quad \text{or} \quad \xi \in (-90^\circ, -16.15^\circ)$$

In all the simulations done the glide path angle is chosen to be within these open intervals. Although here the values are given in degrees, when computing the mathematical equations all the angles are in radians.

The gains of the linear controller are tuned to yield a well behaved transition between different glides, minimizing oscillations of the pitch angle $\theta$. The gain values are set to:

$$K_{pi} = 0.2 \quad K_{pm} = 0.2$$
$$K_{di} = 1$$

We already saw on section 3.1.1 that a typical way of defining a straight glide path of equilibria is by choosing desired glide path angle $\xi_d$ and glider’s speed $V_d$. Notice that this involves a different ballast mass $m_b$ for each path, according to (3.11). Despite this, when switching between different paths it is better to do it in a smarter way: As we aim to spend as low energy as possible, the actuators that move the internal moving mass and change the ballast mass should be working only when it is strictly necessary (that is why, for instance, $r_{P3}$ is fixed). Similarly, we can restrict the change of $m_b$. Instead of using $V_d$ to define a path we will use $m_{bd}$, and it will be kept fixed for all downwards glides and fixed for all upwards glides. Therefore $m_b$ only changes when switching between downwards and upwards glides. If desired, one can solve the same equation as before, (3.11), for $V_d$ from $m_{bd}$, and get the speed of the glider for each path.

The reference value of $m_{bd}$ for all downwards glides is taken to be the one given by $\xi_d = -30^\circ$ and $V_d = 0.3m/s$. From this we get $m_{bd}=1.354kg$. For upwards glides is chosen to be such that between downwards and upwards glides $|m_0| = constant$
is satisfied. This results in $m_{bu_d} = 0.646\, kg$.

Initial conditions are also found to be critical in simulations. Starting with a configuration out of equilibria might induce to wrong behaviours. To avoid this the initial conditions are set as a downwards glide path at equilibria with $\xi_d = -45^\circ$ (remember that it was also defined $r_{P3} = 0.04\, m$ and $m_{bd} = 1.354\, kg$).

Then, IC:

$$v_{10} = 0.365\, m/s, \quad v_{30} = 0.220\, m/s, \quad \theta_0 = -41.56^\circ,$$
$$m_{b0} = 1.354, \quad r_{P10} = 0.022\, m, \quad r_{P30} = 0.04\, m$$

In figure 4.5 a **first simulation** is shown. The glider starts with the initial conditions of a downwards straight path with $\xi_d = -45^\circ$, and the controls bring the glider to the desired equilibria defined by a straight path with $\xi_d = -30^\circ$.

![Graph showing variables' evolution](image-url)

Figure 4.5: Variables’ evolution switching between downwards glides
It takes about 17s to the glider to switch from a path at $\xi = -45^\circ$ to a path at $\xi = -30^\circ$. From the graphics we can also check that the pitch angle $\theta$ is not exactly coincident with the glide path angle $\xi$, due to the angle of attack $\alpha$. Furthermore, and as expected, the control variables $m_b$ and $r_{P3}$ remain constant, while the only modification requested to the glider’s internal configuration is to move the movable mass $m$ along the $e_1$-axis, changing $r_{P1}$. Just by moving $r_{P1}$ for 1.8cm the new steady motion is achieved.

Now a second simulation is presented in order to show with clarity the switch between a downwards and an upwards glide. The path followed by the glider can be seen in figure 4.6, and the evaluation of the descriptive variables in figure 4.7. In this case the controller keeps the initial path of the glider $\xi = -45^\circ$ during the first 10s, then changes the downwards angle to be $\xi = -30^\circ$ for next 30s, and finally an upwards glide is performed with $\xi = 30^\circ$. Be aware that with such short time this simulation corresponds to a shallow glide, but although it might not be useful in real operative missions it is shown to see in detail the behaviour of transition between glides.

![Figure 4.6: Path switching from downwards to upwards glides](image)

Figure 4.6: Path switching from downwards to upwards glides

Figure 4.7 shows us the clues of the motion carried out in this simulation. In this and upcoming simulations $r_{P3}$ will not be plotted anymore, as it is fixed. As desired, during the first 10 seconds the configuration of the glider is the same as initial conditions (a downward glide with $\xi = -45^\circ$). Then the angle is changed to go down with $\xi = -30^\circ$. This is done by moving $r_{P1}$ but without changing $m_b$, like in the previous simulation. 30 seconds after this, the controller has to achieve an upwards straight path with angle $\xi = 30^\circ$: the internal configuration is then changed by pumping water out of the body of the glider, in order to be buoyant.
and by moving \( r_{P1} \). Notice the following: first that the ballast mass was only changed when switching from downwards to upwards glide, and second that \( r_{P1} \) is now negative. The former was expected due to the strategy implemented to save energy, and the latter means that while previously the movable mass was located at the front of the vehicle, now it is at the rear according to body-frame definition (figure 2.1).

Figure 4.7: Variables’ evolution switching from downwards to upwards glides

The last simulations about motion restricted to the vertical plane are done including the guidance algorithms detailed at the beginning of this chapter. Remember that in these algorithms we defined depth boundaries \( d_{\text{min}} \) and \( d_{\text{max}} \), and the angles of the sector-of-sight. For the simulations these parameters are chosen aiming to represent a feasible scenario of an oceanic mission, and the same for the destination points. Then:

Depth boundaries are

\[
d_{\text{min}} = 1 \text{ m} \\
d_{\text{max}} = 200 \text{ m}
\]

and the sector-of-sight is defined with a width of 60° by two vectors with angles

\[
\theta_{v1} = 30^\circ \\
\theta_{v2} = -30^\circ
\]

In figure 4.8 it is plotted the path followed by the glider when the destination point is \( P = (700, 100) \text{m} \). Due to the strategy implemented in sector-of-sight the
target point is reached approximately, this means that the glider passes through its surroundings. Here, for instance, the closest position is \((700.6, 99.35)m\), which is about one meter away from the target.

![Figure 4.8: Path with guidance to target point (700,100)m](image)

When the glider is located more or less at \((610, 50)m\) the target point becomes out of the sector-of-sight, therefore the glider goes straight to it. After achieving the final destination the vehicle changes to go upwards and ends its mission on the sea surface, as specified in the guidance strategy (section 4.1.1).

On the other hand figure 4.9 shows the evolution of some variables describing the behaviour of the glider. The glide path angle \(\xi\) of the initial conditions is \(-45^\circ\) as before, and then the sawtooth gliding is performed with angles \(-30^\circ\) and \(30^\circ\).

Finally another simulation is slightly presented. Figure 4.10 shows the path followed to reach the target point \(P = (1200, 130)m\). In this case the glider passes for \((1201, 129.4)m\), which means \(1.2m\) far away from the desired position. Then, with these simulations the effectivity of the control and guidance strategy is proven.

The transition downwards-upwards glides may seem to be very sharp in simulations 4.8 and 4.10. Nevertheless, note that this is a consequence of the scale of the plots, as the horizontal axis represents larger distances than vertical axis. Because of this the scaled plot of a switch was previously shown in figure 4.6, where transition can be seen with the shape as performed.
4.2 3D Scenario

If the scenario is not framed only to the vertical plane, i.e. motion is extended to the whole 3D space, the transition to the destination point can be achieved by combination of both steady straight paths (sawtooth gliding) and spiral glides. Straight paths can be computed as before: fixing $r_{P2}$ to be 0 and assuming rotation only to the pitch angle $\theta$. Then, for a desired path one gets the corresponding internal configuration $r_{P1d}$, $r_{P3d}$, and $m_{bd}$.

However, to perform spiral glides some offset is needed in $r_{P2}$, resulting in nonzero roll angle $\phi$ according to what was explained in section 3.1.2.
4.2.1 Guidance strategy

The way how the destination point is reached by performing exclusively steady paths can be simplified as follows: the glider starts going forward towards the target point in a sawtooth gliding, until its proximities. Once the glider is close to it, it changes to a spiral glide around the target point up to the sea surface.

Similarly to the 2D case, here the sawtooth gliding is also narrowed within two depth boundaries. To implement this a 'Boundary check' algorithm with the same structure as in (4.1) is used. Notice that now, as it is in the 3D space, the depth limits are defined by planes.

In addition to the boundary check algorithm, a 'proximity check' logic is also needed. It was already mentioned that the glider keeps on performing a sawtooth gliding until it is close to the destination point area. A proximity check algorithm, then, is used to specify and implement these concepts.

**Proximity check:** Define \( O = (x_g, y_g, z_g) \) to be the position of the glider with respect to the inertial frame and \( P = (x_p, y_p, z_p) \) to be the coordinates of the target point also with respect to the inertial frame. Now we assume that the destination point will always be on surface, what means that \( z_p = 0 \) and \( P = (x_p, y_p, 0) \).

The way chosen to define the surroundings of the target point is by a virtual cylinder, whose axis passes through it and is aligned with the z-axis of the inertial frame (i.e. gravity). Furthermore, as the sawtooth gliding is subjected to no other restriction than depth limits, the surrounding of the destination point may be reached at any depth within the boundaries. Because of this the virtual cylinder is also defined to be as long as the distance within depth boundaries, so the glider will always reach it at some point.

A conceptual algorithm where this is implemented is:

\[
\text{if } ((x_g - x_p)^2 + (y_g - y_p)^2 > r^2) \\
\quad \text{'keep on with sawtooth gliding'} \\
\quad \text{'boundary check algorithm'} \\
\text{else} \\
\quad \text{'upwards spiral glide to the surface'} \\
\text{end}
\]

Here \( r \) is the radius of the virtual cylinder defined around the destination point.
Below both boundary check and proximity check algorithms are mixed together, linking (4.1) and (4.6), and yielding the guidance law implemented:

\[
\begin{align*}
    r, d_{\text{min}}, d_{\text{max}} & \\
    \text{if } ((x_g - x_p)^2 + (y_g - y_p)^2 > r^2) & \\
    \text{if } (z_g > d_{\text{max}}) & \\
    \quad u = 1 & \\
    \text{elseif } (z_g < d_{\text{min}}) & \\
    \quad u = 2 & \\
    \text{else} & \\
    \quad u = u_0 & \\
    \text{end} & \\
    \text{else} & \\
    \quad u = 3 & \\
    \text{end} & \\
\end{align*}
\]

Taking the same notation as in the 2D case, \( u \) is a course index variable, with values \( u = 1 \) for upwards straight path, \( u = 2 \) for downwards straight path and \( u = 3 \) for upwards spiral glide. \( u_0 \) keeps the index value of the previous sample time. First the algorithm checks whether the glider has reached the destination’s surroundings or not, i.e. if it is in or out the virtual cylinder. If it is out (first line of the algorithm above) it means that it has to continue performing the sawtooth gliding: if the bottom depth boundary is reached, go upwards \( u = 1 \), if the upper boundary is reached go downwards \( u = 2 \), and if the glider is within the limits just keep with the current path \( u = u_0 \). If instead of this the glider already reached the proximities of its destination, change to an upwards spiral glide to the surface, \( u = 3 \).

Afterwards the course index is used to obtain the corresponding internal configuration of the glider \( (r_{P_{1d}}, r_{P_{2d}}, r_{P_{3d}}, m_{bd}) \) at each computation time, according to the desired path of equilibria -straight or spiral-.

4.2.2 Simulations

The glider’s dynamics with full motion, the controller and guidance algorithms are implemented in a Matlab/Simulink model, and some simulations are shown in this section with the results of guidance in the 3D space.

For this scenario the parameters of the vehicle are taken from Table 2.3, and the hydrodynamic parameters from Table 2.4, both from section 2.4.
As it was done in simulations of motion restricted to the vertical plane, for simplicity the movable mass $m$ is fixed along the $e_3$-axis to $r_{P3} = 0.04m$. Then it has two degrees-of-freedom, $r_{P1}$ and $r_{P2}$.

For the parameters taken in this model, the feasible glide path angles are, according to (3.8) and (3.9):

$$\xi \in (8^\circ, 90^\circ) \text{ or } \xi \in (-90^\circ, -8^\circ)$$

The linear controller is tuned to yield a well behaved transition between different glides. The values of the gains are set to:

$$K_{p1} = 0.002 \quad K_{p2} = 0.002 \quad K_{pm} = 0.07$$
$$K_{d1} = 0.07 \quad K_{d2} = 0.07$$

Furthermore, the parameters of the guidance algorithm are set as follows, aiming to represent a feasible and real case:

Depth boundaries are

$$d_{min} = 2 \text{ m} \quad d_{max} = 100 \text{ m}$$

and the sawtooth gliding is performed with a glide path angle of $\xi_d = -30^\circ$ and $\xi_d = 30^\circ$ for downwards and upwards paths respectively.

The proximities of the target point are defined by a cylinder of radius

$$r = 10m$$

The simulation is realized under the assumption that the glider is initially aligned with its destination, more specifically its longitudinal axis $e_1$ is aligned with the line-of-sight vector defined by $\vec{SP}$, where $S = (x_s, y_s, 0)$ is the initial position of the glider and $P = (x_p, y_p, 0)$ is the target point (notice that both are on the surface).

For this scenario the glider is initially configured for a downwards straight path of equilibria with $\xi = -45^\circ$ and $m_b = 1.0373kg$. As in the 2D case here $m_b$ is also computed to be the same for all downwards straight glides, and the net mass $|m_0| = constant$ between upwards and downwards straight paths. The resulting value for upwards straight glides is $m_b = 0.9627kg$.

Resulting from this, the initial conditions IC for this simulation are then:
\[ v_{10} = 0.3587 \text{m/s}, \quad v_{20} = 0 \text{m/s}, \quad v_{30} = 0.0053 \text{m/s} \]
\[ \theta_0 = -44.15^\circ, \quad \phi_0 = 0^\circ, \quad m_{b0} = 1.0373 \text{kg} \]
\[ r_{P10} = 0.0393 \text{m}, \quad r_{P20} = 0 \text{m}, \quad r_{P30} = 0.04 \text{m} \]

As we assumed that the glider is aligned with its destination, it means that the initial yaw angle \( \psi \) depends on the initial position of the glider and the position of the target point. See figure 4.11 and the mathematical expression of \( \psi_0 \).

\[ \psi_0 = \arctan \left( \frac{y_p - y_s}{x_p - x_s} \right) \]

Figure 4.11: Initial yaw angle \( \psi \) according to start point and target point

In figures 4.12 and 4.13 it is plotted the path followed by the glider when its initial position is \( S = (20, 20, 0) \text{m} \) and the destination point is \( P = (500, -60, 0) \text{m} \). In the plots the path is showed in blue, the initial position and the final (real) position are marked with blue asterisks, while the destination point (target) is marked with a red one. Additionally the origin of the inertial frame (0,0,0) is also marked with a red asterisk.

Figure 4.12: Path with guidance from \( S = (20, 20, 0) \text{m} \) to \( P = (500, -60, 0) \text{m} \) - general view
In these figures one can check that the desired behaviour is achieved. The saw-tooth gliding is performed within depth boundaries of 2m and 100m, and when the glider gets in the virtual cylinder around the destination point it goes up to the surface following a spiral glide. Implementing the guidance algorithms, then, we can ensure that the glider will finish in a position close to the target point.

In figure 4.14 the internal configuration of the glider is shown. From the upper plot it is interesting to remark that indeed $r_{P3}$ is fixed, and $r_{P2}$ is nonzero to perform the helical motion. From the bottom plot notice that $m_b$ for the spiral glide is taken the same as for upwards straight paths.

Notice here that every time there is a change of attitude now it takes more time to achieve the steady state compared with the case when motion was restricted to the vertical plane. While before it was about 15 seconds now it is roughly 100 seconds. This is induced by the fact that the model parameters taken for the 3D case belong to a vehicle much more heavier and larger than for the 2D case (see parameters in section 2.4). Because of that, now the vehicle has more inertia and transition between glides is performed more slowly.

Meanwhile in figure 4.15 one can see the evolution of the yaw $\psi$, pitch $\theta$ and roll $\phi$ angles. The green line represents pitch ($\theta$), hence as desired initially is $-45^\circ$ and is followed by the sawtooth gliding between $-30^\circ$ and $30^\circ$. Finally the upwards spiral glide is also performed with $\theta = 30^\circ$. From roll $\phi$ remark that it is zero until the spiral glide starts, in the same way as $r_{P2}$ due to both are related.
The yaw angle $\phi$ is constant while the sawtooth gliding is performed, since motion is restricted to a vertical plane. Instead, while performing the helical motion it changes with a constant rate, yielding turning around the spiral’s axis (yaw’s plot is cut out, in order to show graphics with more clarity).

A last important and summarizing thing to point out is that, as expected, introducing an offset to $r_{P2}$ induces some roll angle $\phi$ which induces turning -yaw $\psi$- for the spiral glide.
4.2.3 Initial non-alignment

In the previous section the simulation was performed under the assumption that the glider is initially facing to the target point. Because of this it started with a downwards straight motion towards the destination and no turning was needed until the virtual cylinder was reached. In this section it is considered that at the beginning the glider is not facing the target point, hence one can see the previous scenario as a simplified case of this one. See figure 4.16 where non-alignment is represented for two typical cases: one is when initially the glider is in a perpendicular position with respect to the destination (90°), and the other when it is facing opposite to the destination (180°).

![Figure 4.16: Yaw dealignment of 90° and 180°. S = start point and P = target point](image)

Because of this new scenario, some extension is needed in the guidance algorithm, involving turning at the beginning until the vertical plane of the glider is aligned with the destination area. This initial turning is performed using a line-of-sight of reference for controlling the yaw angle of the glider.

**Line-of-sight:** We know from the model that turning is induced by a nonzero roll angle $\phi$, which at the same time is induced by introducing an offset to $r_{P2}$. This was already checked in previous simulations. Using this concept, a line-of-sight mechanism is provided to the guidance control in order to change the position of the movable mass $\overline{m}$ along the $e_2$-axis according to the desired yaw $\psi$ angle. In other words, to choose a desired $r_{P2_d}$ depending on the difference between the current yaw angle of the glider and the desired yaw angle with respect to the target point.

Being $O = (x_g, y_g)$ the current position of the glider and $P = (x_p, y_p)$ the coordinates of the target point, both on the x-y plane, define the desired yaw angle $\psi_d$
as

$$\psi_d = \arctan \left( \frac{y_p - y_g}{x_p - x_g} \right)$$

Then, as the aim here is to drive the yaw angle of the glider to be the desired one, we use the line-of-sight to finally control the desired value of $r_{P2d}$:

$$r_{P2d} = K(\psi_d - \psi) \quad (4.8)$$

A graphical description is given in figure 4.17.

![Figure 4.17: Line-of-sight for yaw control. O = glider's position and P = target point](image)

From (4.8) it can be verified that once the glider is aligned with its destination, $r_{P2}$ becomes 0 and therefore motion is performed in a vertical plane. Then, the scenario is the same as in the previous section, and the same guidance algorithms -boundary check and proximity check- can be implemented.

An important remark is that the initial turning is not performed as a steady spiral of equilibria, but driven by the controller. Nevertheless, once the right vertical plane is reached the configuration of the glider and therefore the sawtooth gliding belong to steady straight paths of equilibria, and as well the final spiral glide upwards. Summarizing, all motion is performed under equilibria conditions with exception of the initial turning.

One simulation with the glider initially non-aligned with its destination is presented below:

The guidance parameters are defined as before: Depth boundaries are set to $d_{min} = 2m$ and $d_{max} = 100m$, the sawtooth gliding is performed with a pitch angle of $\xi = -30^\circ$ and ballast mass of $m_b = 1.0373kg$ when going downwards and $\xi = 30^\circ$ and $m_b = 0.9627kg$ when going upwards, and the surroundings of the target point are defined by a cylinder of radius $r = 10m$. Finally the proportional gain for the line-of-sight control stated in equation (4.8) is set to $K = 0.02$.
The initial conditions IC for this simulation are set to:

\[
v_{10} = 0.802 \text{m/s}, \quad v_{20} = 0.014 \text{m/s}, \quad v_{30} = 0.025 \text{m/s}
\]
\[
\theta_0 = -25^\circ, \quad \phi_0 = 0^\circ, \quad m_{b0} = 1.0373 \text{kg}
\]
\[
r_{P10} = 0.03 \text{m}, \quad r_{P20} = 0.01 \text{m}, \quad r_{P30} = 0.04 \text{m}
\]

Now the initial yaw angle \( \psi \) depends on the initial position of the glider, the position of the target point, and the dealignment between them. Then, \( \psi_0 \) can be expressed as

\[
\psi_0 = \arctan \left( \frac{y_p - y_s}{x_p - x_s} \right) + O
\]

where subscript 'p' denotes the target point, 's' the start position of the glider, and 'O' is the dealignment according to figure 4.16.

The start point S is \( S = (10, -30, 0) \text{m} \), the destination point \( P = (460, 40, 0) \text{m} \), and the dealignment is taken to be as in the worst case, i.e. \( O = -180^\circ \).

In figures 4.18 and 4.19 the path followed by the glider is plotted. Keeping the same color references as before, the initial yaw angle’s direction of the glider is marked with a red line on them.

Figure 4.18: Path with guidance from \( S = (10, -30, 0) \text{m} \) to \( P = (460, 40, 0) \text{m} \) and \( O = -180^\circ \) - general view

With this simulation it is verified that the guidance control effectively drives the glider from its initial position and orientation towards the desired destination, and
it is done performing the desired motion: At the beginning, the controlled turning makes the glider achieve the right vertical plane, afterwards a sawtooth gliding is performed in this plane and within the depth boundaries, and finally, when the proximities of the target point are reached, a spiral glide is carried out to the surface.

Figure 4.19: Path with guidance from $S = (10, -30, 0)m$ to $P = (460, 40, 0)m$ and $O = -180^\circ$ - side and top views

The evolution of the position of the internal movable mass is shown in figure 4.20. The main distinctive feature compared with the simulation where the glider was already aligned with the destination point (section 4.2.2 figure 4.14) is the initial offset of $r_P$ to rule the turning, as expected. Afterwards its brought to zero, according to (4.8), until the final spiral glide starts. Then $r_P$ is set to be according the equilibria of an upwards spiral glide.

Notice the relation between this and the yaw angle in figure 4.21. $\psi$ varies in conjunction with $r_P$: At the beginning its brought from the initial value $O = -180^\circ$ to the desired value of the vertical plane. Remember that this was the goal here.

See also how the roll angle $\phi$ changes during the initial turning. It is known that indeed it is the one that relates $r_P$ and yaw $\psi$. Again, roll is zero until the start of the spiral glide towards the surface.
Figure 4.20: Position of the movable mass $\overline{m}$ for guidance from $s = (10, -300, 0) m$ to $p = (460, 40, 0) m$ and $O = -180^\circ$

Figure 4.21: Euler angles for guidance from $s = (10, -30, 0) m$ to $p = (460, 40, 0) m$ and $O = -180^\circ$
Chapter 5

CONCLUSIONS

This thesis presents a methodology to drive an autonomous underwater glider from its starting position to any desired destination point. This process is carried out by defining three main systems: glider’s dynamics, controller, and guidance strategy.

The model of glider is defined to have no propeller nor rudder, hence its motion is controlled by means of controlling its attitude -position of an internal movable mass- and buoyancy. The hydrodynamic model is chosen with the purpose of maximum simplicity but faithfully representative of flow dynamics, including its main features. For the sake of simplicity currents are not considered, then it is assumed that sea water is at rest with respect to the inertial frame.

Two different cases are derived from the model: one is when motion is restricted to the vertical plane, resulting in a 2D scenario, and the other is the 3D scenario with full motion.

The implementation of two control laws together is found to yield the best performance and benefits, for our purposes, to the system. It is based first on a nonlinear control law that introduces feedback linearization and changes the control inputs to be acceleration control of the internal movable mass. The second control law is based on a linear controller applied to these new control inputs -movable mass position- and variable ballast mass.

This control strategy yields larger regions of attraction when switching between different glide paths, and therefore are definitely necessary to perform sawtooth gliding and achieve our guidance objectives.

For the 2D scenario guidance is implemented using a sector-of-sight law, adapted to the model, together with a boundary check algorithm. Both result in the performance of sawtooth gliding by mainly controlling the pitch angle, and along with the controller successfully drive the glider to its desired destination point. Several simulations are used to check glider’s behaviour and to support the validity of the whole system.

For the 3D scenario two different cases emerge depending on if the glider is initially aligned with its destination or not. In the former case the glider starts with a saw-
tooth gliding -similar to the 2D scenario- until it arrives close to the target point, and then performs an upwards spiral glide towards the surface. The latter case implies a previous turning to offset the yaw angle and achieve the right vertical plane. Afterwards motion is performed as in the former case. Here simulations are also used to check the validity of the system. Remark that guidance algorithms have been designed to fulfill its purpose with maximum simplicity.

Future work could take into account, for instance, a model for the sea currents. However, the complexity of the actual glider’s model is quite high, what adding currents to it presumably would make it more difficult to work with. Furthermore future research could also be focused on applying the work done in this thesis to a real operative glider.

Figure 5.1: Glider model in a 3D virtual reality environment

Figure 5.2: Path followed by the glider to reach its destination point. Simulation in a 3D virtual reality environment
Bibliography


