Particle in Cell Simulations of Electrostatic Waves in Saturn’s Magnetosphere

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Abstract

The characteristics of electrostatic waves are investigated using PIC simulations of a four component plasma: cool and hot electrons, cool ions and an electron beam. The velocities are defined by Maxwellian distributions. The system is one dimensional and simulates a collisionless, unmagnetized plasma. Langmuir waves, electron acoustic waves, beam-driven waves and ion acoustic waves are excited in the simulations. The results are analysed using the dispersion relation and compared with previous investigations and analytical results.
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Chapter 1

Introduction

Electrostatic waves, resulting from plasma instabilities, are important for the study of planetary magnetospheres. There are several reasons why the study of these waves is relevant. Electrostatic waves are related to processes that happen in magnetospheres. Thus, their study can be important, for example, for the study of the trapped energetic electrons or for the loss cone scattering. Besides, there are electrostatic waves that give information about the particle density, like Langmuir waves (Kurth et al., 1983). This thesis is focused on the magnetosphere of Saturn following up on the work done by Koen et al. (2012b), studied electrostatic waves in the Saturnian magnetosphere but in the electron plasma frequency range. The aim of this work is the study of electrostatic waves in the ion plasma frequency range. Nevertheless the waves around the electron plasma frequency will be also investigated.

Most of the knowledge of the magnetosphere of Saturn comes from the Pioneer 11, Voyager 1 and 2, and Cassini missions. The source of the plasma in Saturn’s magnetosphere is the rings and the icy planets (Gombosi and Ingersoll, 2010). Saturn’s magnetosphere is similar to Jupiter’s but not like the Earth’s which has the solar wind as a main source. The Voyager and Cassini plasma wave instruments have reported observations of electrostatic waves like Langmuir wave, ion cyclotron harmonic emissions (Kurth et al., 1983) and ion acoustic waves (Gurnett et al., 2012). The presence of electron acoustic waves in Saturn’s magnetosphere is not yet observed but predictions indicate that is highly likely. Sittler et al. (1983) showed the presence of cool and hot electrons in Saturn’s magnetosphere with the data from the Voyager plasma science experiment. Young et al. (2005) investigated the composition of the plasma in Saturn’s magnetosphere, differentiating between four zones, and agreed with the presence of cool and hot electrons. The regions described by Young et al. (2005) are a hot plasma region composed of $H^+$, the outer plasmasphere formed by $H^+$, $O^+$ and $W^+$, the inner magnetosphere composed of $O^+$ and $W^+$ and a region over the rings A and B composed by $O^+$ and $O_{2}^+$. They suggested that the electron distribution should be bi-Maxwellian, but Schippers et al. (2008) have tested the data of the Cassini mission characterising the electron populations with different distribution functions. They concluded that two kappa-distributed functions fit better.

The Cassini spacecraft observed upgoing electron beams in a downward current auroral region of Saturn associated with hydrogen cyclotron waves. These electrons may be either unidirectional upward, or bidirectional, but the ions are exclusively unidirectional upward (Mitchell et al., 2009). Carlson et al. (1998) showed a correspondence between the upward electron beams, ion upward conics and enhanced broadband electromagnetic noise power, all the phenomena located within a region of downward field aligned current in the auroral zone.

The Cassini Waveform Receiver (WFR) observed ion cyclotron waves (ICW) during a high latitude pass of Saturn’s magnetosphere. The observations of the Cassini Radio and Plasma Wave Science (RPWS) instrument and the Cassini Plasma Spectrometer Investigation (CAPS) instruments identified ion cyclotron harmonic waves associated with particle beams.

Around the magnetized planets with an atmosphere like Earth, Jupiter and Saturn, energetic electron field aligned beams are associated with auroral processes. Electrons have often been accelerated between the spacecraft location and the ionosphere going upward along the magnetic field and they are consistent with generation within or close to the atmospher loss cone, (Mitchell et al., 2009). The particles are likely accelerated closer to the ionosphere as the reason for the development of the field aligned potentials, responsible for the acceleration of the electron beams, is that the magnetospheric circuit is demanding current through what would otherwise be a vacuum region. Similar events have been observed at Earth, and the model proposed to explain it is the “pressure cooker” mechanism (Mitchell et al., 2009) (Menietti et al., 2011b). It consists of a
confinement of the ions by the field aligned potential while they are accelerated perpendicular to the field by
the stochastic wavefield.

Schippers et al. (2008) demonstrate the presence of two species of electrons in the magnetosphere of Saturn. They show that they have different temperatures and densities. In a plasma with different temperature electron species and ions, ion acoustic waves are likely excitable.

The measurements made by Cassini in 2006 (Mitchell et al., 2009) showed whistler mode emissions, since most of the wave energy appears below the electron cyclotron frequency, although there are emissions above the electron cyclotron frequency as well. That suggests an electrostatic mode. They could verify the existence of field aligned ions and a correspondence between electrons and ion beams.

In the case of Saturn the particle energies are higher that those typical at Earth by about two orders of magnitude. Mitchell et al. (2009) suggested this difference may be because of the larger scale of Saturn’s magnetosphere compared to Earth, since the standoff distance of the Saturn’s magnetopause relative to Earth is an order of magnitude larger.

Electron beams and ion conics are seen in Saturn’s magnetosphere (Mitchell et al., 2009). Most of the wave energy appears below the local electron cyclotron frequency, propagating in the whistler mode. Mitchell et al. (2009) suggest that the hydrogen emission is generated as protons are accelerated at low altitude above the auroral zone by wave particle interaction, generating ion conics.

Kasaba et al. (2010) is planning to investigate the plasma waves around Mercury with the BepiColombo Mercury Magnetospheric Orbiter (MMO) spacecraft. One of the possible plasma waves is the ion acoustic wave. Rosenberg and Shukla (2009) discuss the presence of heavy negatively charged ions in the ionosphere of Titan. They suggest that they are supported by electric fields and those create the electron drifts. Ion acoustic waves may be excited by the electron drifts. The mass of these ions may be from 10 to 200amu/charge.

Menietti et al. (2011b) discussed observations of intense hydrogen cyclotron waves associated with upward electron beams in the downward electron region and show that these waves may heat the ions. In their observations, wave electric fields indicate ion cyclotron harmonics. These waves appear as constant frequency bands of emission with the lowest energy near the local hydrogen cyclotron frequency (5.77Hz) and at least three harmonics. They estimate the growth of the observed ICWs, with the dispersion solver WHAMP, using a core electron distribution, an electron beam component, an extended tail beam electron component and a cool core ion Maxwellian plasma distribution for charge neutrality. The results obtained are close to the fundamental hydrogen cyclotron frequency and the harmonic waves are much more electrostatic. Menietti et al. (2011b) suggest that a downward electric field below the spacecraft might enhance heating of ions produced by waves below the parallel electric field.

Schippers et al. (2011) discuss the Cassini observations of the electron populations associated with the crossing of Saturn’s kilometric radiation (SKR) source region. The data that they used display unusual features, it may be a very dynamic event. Cassini measured, in the downward field aligned currents, two types of upward accelerated electron beams: a broadband energetic (1-100keV) electron population and a narrow-band (0.1-1keV) electron population sporadically. These low energy and energetic beams are likely the source of the VLF emissions. The electrons appear to be highly field aligned, with the fluxes higher in the antiparallel direction compared to the perpendicular. In the upward field aligned currents, Schippers et al. (2011) observed electron loss cone distributions and evidence of shell-like distributions.

Electrostatic waves are a type of electromagnetic wave. There are electromagnetic waves also in Saturn’s magnetosphere. Menietti et al. (2011a) used the data from the Cassini of 2008, when it flew near a SKR source region. The observations indicate the presence of extraordinary X mode, ordinary O mode and Z mode. The free energy source of the auroral radio emissions, observed at all the magnetized planets, is believed to be the cyclotron maser instability (CMI).

This thesis is structured as follows: the rest of this chapter is a review of the plasma waves expected to be seen in the simulations. Chapter 2 shows the methodology used in this thesis. Chapter 3 defines the particle-in-cell simulation and presents the characteristics of the code used. Chapter 4 shows the results assuming fixed ions and Chapter 5 shows the results assuming mobile ions. In both Chapters 4 and 5 the effect of different parameters is shown. Chapter 6 presents the simulations with the Cassini data of the magnetosphere of Saturn. Chapter 7 summarizes the results and finally Chapter 8 presents the conclusions.
1.1 Electrostatic Plasma Waves

The charged particles in a plasma respond to static and oscillatory electromagnetic fields, thus interactions can occur between these plasma waves and the charged particles and can lead to instabilities. Electrostatic waves need to be excited internally instead of being propagated from outside like electromagnetic waves. Electrostatic waves that are expected to be excited in this plasma model are Langmuir, electron acoustic waves, ion acoustic waves, and beam-driven mode. These waves may interact with other waves (wave-wave processes), or with the electrons and protons in the plasma (wave-particle interactions). Such interactions are responsible for many physical processes that take place within a plasma such as the flattening of electron beams and radio emissions.

Consider a four-component plasma, hot and cool electrons, electron beam, and ions. The distribution function of the electron beam is a drifted Maxwellian

\[ f_{eb}(v - v_b) = \frac{n_0}{\pi v_b} \exp \left[ -\frac{(v - v_b)^2}{v_b^2} \right]. \]  

(1.1)

In (1.1), \( v \) is the velocity, \( v_b \) the electron beam velocity, \( m_e \) the electron mass, \( T_e \) the electron temperature and \( n_0 \) is the background density. It is assumed \( v_b \gg v_{th,e} \), where \( v_{th,e} = \sqrt{2KT_e/m_e} \) is the thermal velocity and the subscript “e” refers to electrons. The ions and the cool electrons are also Maxwellian

\[ f_{i,c}(v) = \frac{n_0}{\pi v_{th,i,c}} \exp \left[ -\frac{v^2}{v_{th,i,c}^2} \right]. \]  

(1.2)

In (1.2), the subscript “i” means ions and “c” cool electron species. The hot electrons are initialized with a kappa distribution (Pierrard and Lazar, 2010)

\[ f_h^k(r, v) = \frac{n_h}{2\pi(kw_{th,h}^2)^{3/2}} \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa - 1/2)\Gamma(3/2)} \left( 1 + \frac{v^2}{kw_{th,h}^2} \right)^{-(\kappa + 1)} . \]  

(1.3)

In (1.3), the subscript “h” means hot electron species, \( w_{th,h} = \sqrt{(2\kappa - 3)kBT_h/m_e} \) is the thermal velocity, \( K_B \) is the Boltzmann constant, \( n_h \) the density, \( T \) the temperature, \( \Gamma(x) \) the gamma function and \( \kappa \) is the spectral index. The latter defines the tail of the distribution function and represents the slope of the energy spectrum. When \( \kappa \to \infty \) the kappa-distribution function tends to the Maxwellian distribution. Low \( \kappa \) values mean a strong non-Maxwellian tail with a surplus of suprathermal particles. The resonance between waves and particles can occur in the region of the combined distribution function with positive slope.

In a plasma composed of cool, hot, and beam electrons and ions the linear dispersion relation is (Baumjohann and Treumann, 1997)

\[ 1 + \frac{2}{k^2\lambda D_e} [1 + \zeta_c Z(\xi_c)] + \frac{2}{k^2\lambda D_h} [1 + \zeta_h Z(\xi_h)] + \frac{2}{k^2\lambda D_b} [1 + \zeta_b Z(\xi_b)] + \frac{2}{k^2\lambda D_t} [1 + \zeta_t Z(\xi_t)] = 0 \]  

(1.4)

In (1.4), \( \xi_c = w/kv_{th,c}, \xi_h = w/kv_{th,h}, \xi_t = w/kv_{th,t}, \xi_b = (w - v_b)/kv_{th,b} \) and \( Z(\xi) \) is the plasma dispersion function. (Baumjohann and Treumann, 1997).

1.1.1 Langmuir Waves

Langmuir waves, also called electron plasma waves, are one example of electrostatic waves and are observed in many space regions, e.g. upstream from planetary bow shocks. This mode can be excited by a tenuous electron beam through beam-plasma interactions. The mode has a frequency close to the electron plasma frequency. In the case considering Maxwellian electrons without any beam components, the mode is Landau damped by electrons through resonant wave-particle interaction. The Landau damping is due to the imbalance in the energy exchange between electrons and electric field. The Langmuir dispersion relation is

\[ w_l^2 = v_p^2 + \frac{3}{2} k^2 v_{th,e}^2 . \]  

(1.5)

In (1.5) the subscript “l” refers to Langmuir, the group velocity \( v_g = \frac{3}{2} v_{th,e} \) and \( v_p = \left( \frac{m_e^2}{e^2 n_0} \right)^{1/2} \) is the plasma frequency. If the plasma is cool, where the thermal velocity can be neglected, the oscillation will only be at the
electron plasma frequency but if there is a hot component in the plasma, Langmuir waves present a thermal correction in its shape. This behaviour is observed in simulations.

If we consider a scenario where \( n_0 < n_0, \xi_h > 1 \) and fixed ions, (1.4) may be expressed as

\[
w^2_p = \omega^2_p \left( 1 + 3k^2 \lambda_D^2 \right) + \omega^2_h \left( 1 + 3k^2 \lambda_D^2 \right)
\]

(1.6)

In (1.6), \( \lambda_D = \left( \frac{m_e e^2}{n_0 e^2} \right)^{1/2} \) is the Debye length. At large \( k \), waves travel at the thermal velocity, it may be because at high frequencies, when the wavelength \( \lambda \) is smaller, particles are closer, then there are more interactions between them for the thermal motion and thus more momentum carried. On the other hand, at small \( k \) waves travel slower than \( v_{th} \), since the density gradient is small at large \( \lambda \).

If we excite the plasma waves with an electron beam, then electrons are bunched so that they pass by any fixed point at a frequency \( w_p \), they would generate an electric field at that frequency and excite plasma oscillations. If they are not bunched from the beginning, when oscillations arise, these will bunch electrons and the oscillations grow by positive feedback.

If we consider an unmagnetized, cool plasma, with \( \mathbf{E} \) and \( \mathbf{k} \) in the same direction, since the spatial variation of \( \mathbf{E} \) and \( \mathbf{v} \) are along the direction of oscillation, all wave numbers are allowed. The plasma can undergo plane parallel oscillations at \( w_p \). Although, these waves cannot transport energy because the group velocity vanishes since \( \omega \) is not dependent on \( k \).

Now if we consider thermal motions, the thermal pressure of the warm electrons converts the longitudinal plasma waves into energy transporting modes called Langmuir waves. Ions oscillate electrostatically with an amplitude less than electrons do, so that the ion thermal pressure can be neglected. In the analysis the proton motion can be ignored because the proton thermal speeds are smaller than the electron thermal speeds. In order to analyze ion acoustic waves, the proton motion cannot be ignored.

\[
w^2(k) = \omega^2_p \left( 1 + \frac{m_e}{m_i} \right) \left[ 1 + \frac{3k^2 \lambda_D^2}{1 + m_e/m_i} \left( 1 + \frac{m_i^2 T_i}{m_e^2 T_e} \right) \right]
\]

(1.7)

\[
w^2(k) = \omega^2_p F \left[ 1 + \frac{3k^2 \lambda_D^2 c}{1 + m_e/m_i} \left( 1 + \frac{m_i^2 T_i}{m_e^2 T_e} \right) \right] + \omega^2_{ph} F \left[ 1 + \frac{3k^2 \lambda^2 \lambda_D^2 h}{1 + m_e/m_i} \left( 1 + \frac{m_i^2 T_i}{m_e^2 T_h} \right) \right]
\]

(1.8)

\[F = \left( 1 + \frac{m_e}{m_i} \right)
\]

(1.9)

The motion of ions also affects Langmuir waves. The dispersion relation of Langmuir waves has another term because of the new species. Thus the dispersion relation with the effects of ions results in (1.7) with only one electron species and (1.8) with cool and hot electrons. The modifications introduced by ions are small. The term of the electron plasma frequency has a correction for the mass ratio. Debye length, \( \lambda_D c \), is corrected by the mass and the temperature ratio. The latter correction is only significant if the temperature of ions is high enough. The growth rate of Langmuir waves is also modified and depends on the temperature and mass of ions.

The (1.8) is the dispersion relation of Langmuir waves as (1.7) but for a plasma with cool and hot electrons.

1.1.2 Electron Acoustic Waves

Electron acoustic waves oscillate at a frequency between \( w_{pa} \) and \( \omega_p \). Electron acoustic waves may be excited in a two-electron plasma with different temperatures where thermal motions are present. An electron beam can create an unstable behaviour of these waves. Its phase velocity is between the cool and the hot thermal velocity

\[
\frac{K_B T_i}{m_i} \sim \frac{K_B T_e}{m_e} \ll \frac{\omega_p^2}{k^2} \ll \frac{K_B T_i}{m_e}.
\]

(1.10)

Gary and Tokar (1985) studied the electron acoustic mode in an unmagnetized plasma with cool and hot electrons and ions, all Maxwellian-distributed. Mace et al. (1999) investigated a plasma with also the three species but with hot kappa-distributed electrons. They showed that suprathermal electrons maximize the damping effect to electron acoustic waves.

If we consider a plasma with only hot and cool electrons, assuming fixed ions and \( \xi_h < 1 \), the dispersion relation is (Lu et al., 2005)
\[ w_{ea}^2 = w_{pc}^2 \left( 1 + \frac{3k^2\lambda_{Dc}^2}{1 + k^2\lambda_{Dh}^2} \right). \]  

The dispersion relation of electron acoustic waves in a plasma with hot and cool electrons and mobile ions is:

\[ w_{ea}^2 = w_{pc}^2 \left( 1 + \frac{1}{1 + \frac{3k^2\lambda_{Dc}^2}{1 + \frac{\lambda_{Dh}^2}{\lambda_{Dc}^2}}} \right) \left[ 1 + 3 \left( k^2\lambda_{Dc}^2 + \frac{n_{eh}T_c}{n_{oc}T_h} \right) \right]. \]

At small wavenumber, electron acoustic waves propagate like sound waves, in the acoustic regime. This zone with linear dispersion is strongly damped by electron Landau damping. However in the range

\[ \frac{n_{eh}T_c}{n_{oc}T_h} < k^2\lambda_{Dc} < 1 \]

is weakly damped. (1.12) shows how electron acoustic waves oscillate at a low frequency at big wavenumbers. The dependency of electron acoustic waves with the density and temperature ratios between cool and hot electrons is shown in Fig. 1.1. At high cool electron’s density, there is no damping. In the case of low cool electron’s density, the ratio \( T_h/T_c \) should be at least 10 to enter in the weakly damped regime.

![Figure 1.1: Weakly damped region of electron acoustic waves (Baumjohann and Treumann, 1997).](image)

Fig. 1.1 agrees with Gary and Tokar (1985) since they concluded that the appropriate conditions for electron acoustic wave propagation are \( 10 \leq T_h/T_c \) and \( 0 < n_c < 0.8n_0 \).

### 1.1.3 Electron Beam-Driven Mode

The effect of an electron-beam in a plasma is the excitation of another mode. The dispersion relation (Lu et al., 2005) when the beam term cannot be neglected has another root which corresponds to electron-beam-driven mode. (1.4) when beam velocity is large enough and neglecting the term of ions may be approximated as

\[ \frac{w_{pc}^2 - w_{eh}^2}{w^2} - \frac{w_{pb}^2}{(w - kv_d)^2} = 0. \]

In (1.14) the subscript "b" refers to beam component. One root of this equation, assuming \( n_b \ll n_0 \), is (Sydora et al., 1988)

\[ w = \frac{kv_d}{1 + n_b/n_0} \approx kv_d. \]
1.1.4 Ion Acoustic Waves

At low frequencies ions should be taken into account. Ion acoustic waves are called acoustic because they have the same properties as sound waves in a gaseous medium (Chen, 1984). The Navier-Stokes equation describes sound waves as

\[
\rho \frac{dv}{dt} = -\nabla p = -\gamma p \nabla \rho. \tag{1.16}
\]

(1.16) tells us that the electron motion depends on pressure, where \(\gamma\) is the ratio of specific heats and \(\rho\) the mass density. Since we work in one-dimensional system, \(\gamma\) is 3. If we linearize (1.16) with the continuity equation

\[
\frac{\delta \rho}{\delta t} + \nabla (\rho v) = 0 \tag{1.17}
\]

and we assume a stationary equilibrium with uniform \(p_0\) and \(\rho_0\), these lead to

\[
\frac{w}{k} = \sqrt{\frac{\gamma p_0}{\rho_0}} = \sqrt{\frac{\gamma KT}{M}} = c_s. \tag{1.18}
\]

(1.18) defines the velocity of sound waves in a neutral gas and mainly depends on temperature. Sound waves are pressure waves and they propagate by collisions among gas molecules.

In a gaseous medium, collisions are necessary for the excitation of sound waves. Instead of that, ions are still influenced by the forces acting on their charge. Plasma oscillations are basically constant frequency waves, with a correction due to thermal motions, but ion acoustic waves are constant velocity waves and they only exist when there are thermal motions. The physical explanation for electron plasma oscillations is that in this scenario ions are fixed. On the other hand in the scenario of ion acoustic waves, ions are not fixed. Electrons are pulled along with ions and tend to shield out electric fields arising from the bunching of ions. Ions form regions of compressions and rarefaction, and the compressed regions tend to expand into rarefactions. Ions thermal motions spread out ions and the ion bunches are positively charged and tend to disperse because of the resulting electric field. This electric field is largely shielded out by electrons, and only a fraction proportional to \(KT_e\) acts on the ion bunches. The inertia of ions produces their overshoot, and compressions and rarefactions are regenerated to form a wave.

Quasi-neutrality is assumed \(n_i = n_e = n\). Using \(E = -\nabla \phi\) and the state equation in an unmagnetized plasma, we can describe the plasma with ion fluid equation

\[
m_i n \left[ \frac{\partial v_i}{\partial t} + (v_i \nabla) v_i \right] = e n E - \nabla p = -e n \nabla \phi - \gamma_i KT_i \nabla n. \tag{1.19}
\]

(1.19) linearized is

\[
-\iota w n_i \n_0 v_{i1} = -e n \phi_1 \nabla n_1. \tag{1.20}
\]

The electron mass can be neglected. The Boltzmann relation may be used to define the electron density assuming very low frequency and a slow phase velocity for ion acoustic waves (Brambilla, 1998), which leads to

\[
n_e = n_0 e^{\phi_1 / k_B T_e}. \tag{1.21}
\]

The Taylor expansion of (1.21) is

\[
n_e = n_0 (1 + \frac{e \phi_1}{K_B T_e} + ...). \tag{1.22}
\]

We can thus define the perturbation density as

\[
n_1 = n_0 \frac{e \phi_1}{K_B T_e}. \tag{1.23}
\]

In (1.23) \(n = n_0 + n_1\) and \(\nabla n = \nabla n_0 + \nabla n_1 = \nabla n_1\). The subscript “0” means equilibrium part and “1” perturbed part. Equilibrium quantities express the state of plasma without oscillations. With ion continuity equation linearized

\[
wn_1 = n_0 \kappa v_1 \tag{1.24}
\]

we can substitute \(\phi_1\) and \(n_1\) and ion acoustic phase velocity \(c_{ia}\) is obtained.
The ion pressure term can be neglected if $T_e \gg T_i$ and then (1.25) results in

$$c_{ia} = \left(\frac{K_B T_e}{m_i}\right)^{\frac{1}{2}}$$

and the dispersion relation

$$w = k c_{ia}.$$  \hspace{1cm} (1.27)

(1.26) indicates that inertia is provided by ions while electrons provide pressure. At high frequencies, electron damping always dominates over ion damping. But at low frequencies, $w \ll w_{pe}$, and at phase velocities between $v_{the}$ and $v_{thi},$

$$\frac{K_B T_i}{m_i} \ll \frac{w^2}{k^2} \ll \frac{K_B T_e}{m_e}$$

the ion contribution should be taken into account and it yields a dispersion relation (Baumjohann and Treumann, 1996)

$$w^2_{ia} = \frac{2}{1 + 1/k^2 \lambda^2_{De}} \left[1 + \frac{3T_i}{T_e} \left(1 + k^2 \lambda^2_{De}\right)\right].$$

Although the expression has changed, the behaviour at long-wavelength also obeys an acoustic regime. In order to check this, let us derive the equation assuming $k^2 \lambda_{De} \ll 1$ and $n_e = n_i.$

$$w^2_{ia} = \frac{w^2_{pi} k^2 \lambda_{De}}{1 + k^2 \lambda^2_{De}} \left[1 + \frac{3T_i}{T_e} \left(1 + k^2 \lambda^2_{De}\right)\right] = \frac{n_i e^2 k^2 T_e e_0 K_B}{m_i e_0} \left(1 + \frac{3T_i}{T_e}\right)$$

$$w^2_{ia} = k^2 \left(\frac{K_B T_e}{m_i}\right)^{\frac{1}{2}} \left[1 + \frac{3T_i}{T_e}\right] = k^2 c_{ia} \left(1 + \frac{3T_i}{T_e}\right).$$

The new ion acoustic speed is

$$c'_{ia} = c_{ia} \left(1 + \frac{3T_i}{T_e}\right).$$

(1.32)

For short-wavelengths, $k^2 \lambda_{De} \gg 1$, and taking into account that $T_i/T_e = \lambda_{Di}/\lambda_{De},$ (1.31) yields a dispersion relation

$$w^2_{ia} = w^2_{pi} (1 + 3k^2 \lambda^2_{Di}).$$

There is a weak ion Landau damping when $T_i \ll T_e,$ and

$$\frac{w}{k v_{hi}} = \frac{c_{ia}}{v_{hi}} \left(\frac{T_e}{T_i}\right)^{\frac{1}{2}} \gg 1.$$  \hspace{1cm} (1.34)

The second term of (1.29) may be neglected if the ratio $T_i/T_e$ is very small, which leads to the expression

$$w^2_{ia} = \frac{w^2_{pi}}{1 + 1/k^2 \lambda^2_{De}}.$$  \hspace{1cm} (1.35)

In order to see the temperature ratio effect, this expression should be normalized as

$$\frac{w^2_{ia}}{w^2_{pi}} = \frac{k^2 \lambda^2_{Di} T_i}{1 + k^2 \lambda^2_{Di} T_i}.$$  \hspace{1cm} (1.36)

The dispersion relation of ion acoustic waves at very small wavenumbers $k \lambda_D$ grows with a slope proportional to the ion acoustic phase velocity $c_{ia}.$

The plasma considered here is a four-component plasma with two electron species: hot and cool. This means that there are two temperature ratios: $T_h/T_i$ and $T_c/T_i$. Bharuthram and Shukla (1986) worked also on a model with ions and two electron species with different temperatures. They did not consider the inertia of ions. In
order to compare our results with a theoretical expression with two temperature ratios, an effective temperature is defined as

$$T_{eff} = \frac{T_h T_e}{\frac{n_s}{n_h} T_h + \frac{n_h}{n_s} T_e}$$  \hspace{1cm} (1.37)$$

and (1.36) becomes

$$\frac{w_{\text{ia}}^2}{w_{\text{pi}}^2} = \frac{k^2 \lambda_D^2 T_{eff}}{1 + k^2 \lambda_D^2 T_{eff}}$$  \hspace{1cm} (1.38)$$

Instability arises in the region where Landau damping is not dominant, $|w/k| > v_{thi}$, and where distribution function shows a positive slope, $|w/k - v_{bi}| < v_{thi}$. In this scenario ion acoustic waves propagate with $v_{ph} \sim v_{bi}$ (Baumjohann and Treumann, 1997). The growth rate, considering a weak instability $\gamma/w \ll 1$, is defined in (1.39) and it describes the relation of the temperatures, the mass ratio and the drift velocity. This equation is negative if the second term in the brackets is greater than the first one. Thus if the ratio $\mu = m_i/m_e$ is too high, ion acoustic waves are damped. The temperature of electrons $T_e$ should be greater than the temperature of ions $T_i$.

$$\gamma = \left( \frac{\pi}{8} \right)^{1/2} \frac{1}{k^2 \lambda_D^2} \left[ \left( \frac{m_e}{m_i} \right)^{1/2} \left( \frac{k v_{bi}}{w_{\text{ia}}} - 1 \right) \exp \left[ -\left( \frac{w - k v_{bi}}{k^2 v_{thi}} \right)^2 \right] - \left( \frac{T_{eff}}{T_i} \right)^{3/2} \exp \left[ -\frac{w_{\text{ia}}^2}{k^2 v_{thi}^2} \right] \right]$$  \hspace{1cm} (1.39)$$

Gary and Tokar (1985) showed that if the temperatures of cool electrons and ions are equal, the density of hot electrons should be $n_h < 0.3 n_e$ in order that ion acoustic waves not be strongly damped and the temperature of hot electrons should be greater than 5 times the temperature of cool electrons $5T_e < T_h$.

1.1.5 Summary of Dispersion Relations

**Langmuir waves**

- Plasma with one electron component (motionless ions):
  $$w^2 = w_{\text{pe}}^2 + \frac{3}{2} v_{thi}$$  \hspace{1cm} (1.5)$$

- Plasma with hot and cool electrons (motionless ions):
  $$w_i^2 = w_{\text{pe}}^2 (1 + 3 k^2 \lambda_D^2 \rho) + w_h^2 (1 + 3 k^2 \lambda_D^2)$$  \hspace{1cm} (1.6)$$

- Plasma with one electron component and mobile ions:
  $$w_i^2(k) = w_{\text{pe}}^2 \left( 1 + \frac{m_e}{m_i} \right) \left[ 1 + \frac{3 k^2 \lambda_D^2}{1 + m_e/m_i} \left( 1 + \frac{m_i^2 T_i}{m_e^2 T_e} \right) \right]$$  \hspace{1cm} (1.7)$$

- Plasma with hot and cool electron component and mobile ions:
  $$w_i^2(k) = w_{\text{pe}}^2 \left( 1 + \frac{m_e}{m_i} \right) \left[ 1 + \frac{3 k^2 \lambda_D^2}{1 + m_e/m_i} \left( 1 + \frac{m_i^2 T_i}{m_e^2 T_e} \right) \right] + w_{\text{ph}}^2 \left( 1 + \frac{m_e}{m_i} \right) \left[ 1 + \frac{3 k^2 \lambda_D^2}{1 + m_e/m_i} \left( 1 + \frac{m_i^2 T_i}{m_e^2 T_h} \right) \right]$$  \hspace{1cm} (1.8)$$
Electron acoustic waves

- Plasma with hot and cool electrons (motionless ions):

\[
w_{ea}^2 = w_{pe}^2 \frac{1 + 3k^2 \lambda_D^2}{1 + \frac{k^2 \lambda_D^2}{\gamma_{ph}}}
\]  

(1.11)

- Plasma with hot and cool electrons and mobile ions:

\[
w_{ea}^2 = w_{pe}^2 \left( \frac{1 + \frac{n_h n_e}{m_e n_c}}{1 + \frac{k^2 \lambda_D^2}{\gamma_{ph}}} \right) \left[ 1 + 3 \left( \frac{k^2 \lambda_D^2}{\gamma_{ph}} + \frac{n_{th} T_e}{n_{nc} T_h} \right) \right]
\]  

(1.12)

Beam-driven waves

\[
w = kv_b
\]  

(1.15)

Ion acoustic wave

\[
\frac{w_{ia}^2}{w_{pi}^2} = \frac{k^2 \lambda_D^2 T_e/T_i}{1 + k^2 \lambda_D^2 T_e/T_i}
\]  

(1.36)

Dispersion diagrams are used to identify them and study different features that characterize them. Fig. 1.2 shows the theoretical dispersion diagrams of these waves.

![Dispersion diagram of electrostatic waves.](image)

Figure 1.2: Dispersion diagram of electrostatic waves.

In order to see the theoretical effect of each component in the waves dispersion relation, Fig. 1.3 to 1.6, show each case. The parameters for these plots are \(n_h/n_c = 1\) in case of two electron components, \(n_b/n_c = 0.06\), and \(T_h/T_i = 100\).
Figure 1.3: Dispersion diagram of Langmuir wave for $T_e/T_i = 1$ in case of two electron components.

Figure 1.4: Dispersion diagram of Langmuir wave with $T_e/T_i = 10$ in case of two electron components.
Figure 1.5: Dispersion diagram of electron acoustic waves with $T_e/T_i = 1$ in case of two electron components.

Figure 1.6: Dispersion diagram of electron acoustic waves with $T_e/T_i = 10$ in case of two electron components.

From these graphs it can be seen that Langmuir waves are excited at higher $k\lambda_D$ if the temperature ratios are low. Electron acoustic waves are excited at higher frequencies if ions are mobile and the temperature ratios control the slope after the acoustic regime.
Chapter 2

Methodology

The aim of this thesis is the simulation of different scenarios of plasma in order to study electrostatic plasma waves. The method employed is particle-in-cell (PIC). Koen et al. (2012a) performed a PIC simulation to investigate electrostatic plasma waves in an unmagnetized, collisionless plasma with hot, cool and beam electrons and fixed ions. Since the work of this thesis is following Koen et al. (2012a) work, his code has been used as an initial point. The aim of modifications of Koen et al. (2012a)’s code has been the introduction of mobility of ions.

Once the code was modified, basic scenarios have been simulated but forcing ions to be fixed. The comparison of these results with Koen et al. (2012a) results verifies the code still works as it was doing before. The simple case of plasma frequency oscillation is also performed. The successful results confirm the reasonable running of the code. Basic scenarios with mobile ions are also performed in order to see if they have a reasonable behaviour too. The modified code has allowed us to investigate the effect of ions in the behaviour of electrostatic waves.

The effect of several parameters in a plasma assuming fixed ions has been studied too in this thesis. The study has not been very deep because is not the scope of this thesis but it is a way to see how the plasma waves of interest behave. These results are also useful to compare if the parameters have the same effect with fixed and mobile ions.

After that the plasma with mobile ions is simulated, testing different parameters. Different types of ions, with different mass, are tested in order to see the results with ions more and less mobile. The other important parameter is the ratio between the temperatures of ions and electrons. This comparison has been done with two steps. First only one electron component has been considered in order to have only one temperature ratio. Once that has been performed, two electron components with different temperatures have been used, since it is the scenario that we are interested in.

Finally, once Langmuir, electron acoustic and ion acoustic waves have been studied with different parameters, the model is applied to the magnetosphere of Saturn. Three cases are considered, corresponding to three different regions: the inner magnetosphere, the extended plasma sheet or intermediate region and the outer magnetosphere. An electron beam was not included to perform these simulations. Although the electron beam is not taken into account in our performance of the magnetosphere of Saturn, this component has been used in the rest of simulations to see its effect. It is interesting to observe how the presence of the beam modifies the rest of waves and it is responsible for another electrostatic mode also.
Chapter 3

Particle in Cell Simulation

Computer simulations test a theory with a numerical experiment. The simulation starts from some initial conditions and evolves. Even though it is a complex physical system, it can be simulated by high speed computers. There are several models to study a plasma and one of them is the particle model. The particle model consists on the tracking of the motion of particles which are under the effect of electric and magnetic fields.

PIC simulations model the scenario of the motion of charged particles describing it with the kinetic theory, with periodic boundary condition (Matsumoto and Omura, 1993). The code may include electric fields, magnetic fields, Coulomb forces, collisions between charged species, background neutral species, etc. depending on the plasma that we want to describe. The tracking of the motions of particles allow us to study instabilities or other aspects of the system.

The code of Koen et al. (2012a) consists in an one-dimensional, electrostatic system with two species, cool and warm electrons, and an electron beam. The cool electrons and electron beam speeds are initialized with a Maxwellian distribution, while the warm electron speeds are initialized with a kappa distribution.

The system is defined with a grid of 1024 points. Each cell size $\Delta x$, the distance between two grid points, is the plasma Debye length. This size is chosen because then individual effects can be neglected in favour of collective effects. Thus it should be always lower than Debye length. Coulomb force from any given charged particle causes all the nearby charges to move, thereby electrically polarizing the medium. These nearby particles move collectively to shield out the electric field due to any one charged particle, which in the case of no shielding would decrease as the inverse square of the distance from the particle. The long range Coulomb force in a plasma is limited to a distance around Debye length.

The grid has an electric field defined for each grid point. Each particle has its mass, charge, position, velocity, and acceleration. The particle motion equation is based on the Lorentz force

$$\frac{dv}{dt} = \frac{e}{m}(E + v \times B). \quad (3.1)$$

The different steps in the simulation are shown in Fig. 3.1.

![Figure 3.1: The conceptual layout of the process used in the particle-in-cell (PIC) algorithm.](image)
Yi et al. (2010) have done a two dimensional simulation of the beam-instability. The simulation code developed in this thesis can be used to model the scenario present in Saturn’s magnetosphere. Baluku et al. (2011) have already studied this model but analytically and without looking at the lower frequencies around ion plasma frequency $\omega_{pi}$. Baluku et al. (2011) considered a plasma with two electron species described with a combination of two kappa distributions. The same study can be done for ion acoustic waves, using also the two kappa distribution for electrons and the Cassini data. Koen et al. (2012a) also studied electrostatic waves in Saturn’s magnetosphere. They used PIC simulations to characterize the properties of these waves.

3.1 Computation Time

PIC simulations should be run with the number of particle as large as possible to reduce the noise and to be more close to the large particle model that we would like to simulate. The length of computation time increases considerably with the number of particles. The experiments in laboratories or in a natural environment present a number of particles of the order of $10^{18} m^{-3}$ or $10^{9} m^{-3}$ respectively. Thus each particle in a simulation is equivalent to a set of particles in the real case, which is named ”superparticle” (Dawson, 1983).

The simulations in this thesis are run with 4 CPUs and computation times are summarized in Table 3.1 depending on the time of simulation $w_{pe} t$. There are two cases because electron and ion waves require different values for this parameter. Fig. 3.2 shows the evolution of these times. The values presented here are just to show the sensitivity of the computation time to the number of particles and the time of simulation but they depend on other factors too, e.g., resolution.

<table>
<thead>
<tr>
<th>Number of particles</th>
<th>400</th>
<th>4 000</th>
<th>40 000</th>
<th>400 000</th>
<th>4 000 000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{pe} t = 100$</td>
<td>28&quot;</td>
<td>45&quot;</td>
<td>2'15&quot;</td>
<td>18'</td>
<td>3h 5'</td>
</tr>
<tr>
<td>$w_{pe} t = 1000$</td>
<td>5&quot;</td>
<td>7'15&quot;</td>
<td>25&quot;</td>
<td>2h 58'</td>
<td>1day 2h 11' 24&quot;</td>
</tr>
</tbody>
</table>

Table 3.1: Computation times of the PIC program depending on the number of particles.

Figure 3.2: Evolution of the computation time depending on the number of particles with $w_{pe} t = 100$ (left) and with $w_{pe} t = 1000$ (right).

3.2 Noise in the Simulations

The PIC method carries a statistical noise because of the low number of particles. It has problems to resolve relatively small changes of the system. The poor signal-to-noise-ratio achieved solving the kinetic distribution function with discrete particles causes limitations for noise. This statistical noise creates random statistical fluctuations in the number of particles per cell and then causes field fluctuations. The noise can be seen, e.g., with Fourier transformations which add harmonics to the signal or the spread. The statistical noise decreases as $N^{-1/2}$ for random loading. The ideal case would be with an infinite number of particles. Thus for a good simulation the number of particles should be really high but that has a high computation cost. In the case that
physical fluctuations of the kinetic distribution, $\delta f$, are much smaller than the total distribution function, it can be hidden by the noise.

### 3.2.1 Average of Samples

The random property of the noise allow us to reduce it by averaging samples. We have run the option averaging 10 and 20 samples and comparing with non-averaged result and that is shown in Fig. 3.3.

This method is not useful because the time taken to average enough samples to get a good result is the same or even higher than the time using enough particles to get a good result. Another method may be used to reduce the standard error $\epsilon$: the variance reduction method (Hesterberg and Nelson, 1998).

### 3.2.2 Variance Reduction

This method is based on the correlation between the observed variable $x$, and a control variable $y$. The control variable is created for us, then $E[y] = \bar{y}$ is known analytically. The aim is the estimation of the expected value $E[x] = \bar{x}$. The desired standard deviation would be less than $\text{var}[x]$. There is a variable $z$ declared as

$$z = x - \alpha(y - \bar{y}) = \bar{z} + \alpha \bar{y}. \tag{3.2}$$

In (3.2) $\alpha$ is an optimization parameter. The variable $z$ is defined to have the expected value equal to the expected value of $x$, then

$$E[z] = E[\bar{z}] + \alpha \bar{y} = E[x] - \alpha(E[y] - \bar{y}) = E[x] \tag{3.3}$$

and the variance is

$$\text{var}[z] = \text{var}[\bar{z}] = \text{var}[x - \alpha y] = \text{var}[x] - 2\alpha \text{Cov}[x, y] + \alpha^2 \text{var}[y]. \tag{3.4}$$

In (3.4) $\text{Cov}[x, y] = E[(x - \bar{x})(y - \bar{y})]$. To process $z$ we only need to compute $\bar{z}$ because the value $\bar{y}$ can be known analytically.

The effectiveness of the control variable $y$ depends on the strength of correlations which should satisfy

$$\frac{\text{Cov}[x, y]}{\text{var}[y]} > \frac{1}{2} \Rightarrow \text{var}[\bar{z}] < \text{var}[x]. \tag{3.5}$$

If the evolution of the system does not go too far from its initial state which is known, $f(t_0)$, this one may be used to construct an effective control variate. The $\alpha$ parameter is chosen to obtain maximal reduction of $\text{var}[z]$, thus the optimal $\alpha$ value is defined as

$$\alpha^* = \frac{\text{Cov}[x, y]}{\text{var}[y]} \tag{3.6}$$

If we add the definition of $\alpha^*$, to the variance $\text{var}[z]$, it results in

$$\text{var}[z_{\alpha^*}] = \text{var}[x] - \frac{\text{Cov}^2[x, y]}{\text{var}[y]}. \tag{3.7}$$

(3.7) which tells us that in order to achieve a variance reduction, the covariance $\text{Cov}[x, y]$ should not be zero, for that is called $y$ the control variate for $x$ and the quality of $y$ depends on the correlation between $x$ and $y$. This is because if they are correlated, the covariance cannot be zero and the variance $\text{var}[z]$ is always reduced.

### Application to the code

In the code we will process $N$ observed variables $x$ and control variables $y$. The variable $z_{\alpha^*}$ is computed as

$$z_{\alpha^*} = \frac{\sum_{i=1}^{N}(x_i + \alpha^*(y_i - \bar{y}))}{N} \tag{3.8}$$

This method is applied to the charge density $\rho$ of our system. The cycle is:

1. Load particle positions and velocities
   - **Positions**: random
   - **Velocities**: Maxwellian distribution function for the speeds centered at 0 and with $T_e/T_i = v_{th}$
Figure 3.3: Dispersion diagrams with (top) 1 sample, (middle) average of 10 samples, (bottom) average of 20 samples.
2. Distribute the particles on the grid
   Compute the charge density to each grid point.
   **Variance reduction method applied to the charge density**
   Apply FFT and IFFT to the charge density and compute the electric potential \( \phi \)

3. Solve for \( \nabla^2 \phi \) and evaluate \( E \) on the grid
4. Interpolate \( E \) to particle position and compute the acceleration
5. Solve particle equation of motion
6. Step 2

Fig. 3.4 is a result using the code without the method applied and Fig. 3.5 shows the result with the same configuration but applying the variance reduction method.

The variance reduction method, as it is seen in the previous section, works well but it can be improved. Hesterberg and Nelson (1998) shows a method which may give better results. The \( \delta f \) method is based on the fact that \( \delta f \) is defined by particles but not the noise. This allows us to reduce the noise.

This method was not included in our code because it was out of our scope but it would be a good improvement.

### 3.3 Code Verification

In order to see how our code represents different scenarios and verify if results are reasonable, the following sections present the results of several combinations. In the cases with only electrons there is a background density of fixed ions to ensure quasi-neutrality.

#### 3.3.1 Plasma Frequency

A fully thermalized plasma in equilibrium state has its particles, ions and electrons, oscillating around their equilibrium position. When the equilibrium of this plasma is perturbed, charged particles are displaced in such a way as to set up electric fields. These fields react in order to restore the equilibrium. We perform a simple scenario to excite oscillations at the electron plasma frequency. It is a way to verify if our code satisfies this behaviour. We start considering an uniform, neutral charged, and unmagnetized plasma with electrons and
Figure 3.5: Dispersion diagram with $N = 40000$, $T_e = T_i$, $T_h = 100 T_e$, $n_b/n_e = 0.06$ and $v_b = 15$ and with variance reduction method.

Ions. Particles are in equilibrium positions. If we displace the electrons from their equilibrium position, charge separation produces an electric field trying to restore the neutrality. The resulting oscillation is the electron plasma frequency. Since ions are more massive, they do not have time to respond to the oscillating field. The plasma frequency does not depend on the propagation constant $k$, so the group velocity $\partial w/\partial k$ is zero. The disturbance does not propagate.

In order to check if the code is working correctly, we have simulated this slab model. The idea is a model with only cool electrons and ions, having an uniform and homogeneous plasma, and displace electrons from their equilibrium position.

In the code we introduce the following parameters: $T_i = T_e = 0$, only ions and cool electrons, no thermal velocities, no magnetic field, and electrons are displaced 2 times the Debye length.

Figure 3.6: Dispersion diagram of the slab model showing the electron plasma frequency.
Fig. 3.6 shows the dispersion diagram with the oscillation at $w_{pe}$ independent of the wave number. Although there are also harmonics of $w_{pe}$ excited in the figure, their intensity is decreasing. There is an electrostatic low-frequency noise below $w_{pe}$. In an infinite system, electric field do not disturb plasma since the plane charge sheet is zero. But if the system is finite, plasma oscillations will propagate.

3.3.2 Cool Electrons and Fixed Ions

If we simulate a plasma with only electrons the result is the electron plasma frequency excited, we can see it in Fig. 3.7. The result is as the expected because if there is only electrons and no free energy source nor anything that causes any instability, the electrons oscillate at plasma frequency only, and harmonics but with less intensity. In a low temperature plasma electrons move from the equilibrium position pushed by Coulomb force. Particles move until the kinetic energy is completely converted into potential energy. Coulomb force then restore equilibrium positions causing oscillations at $w_{pe}$.

3.3.3 Cool and Hot Electrons and Fixed Ions

The scenario with two electron species, cool and hot, is simulated and the result is shown in Fig. 3.8. The result is similar to the case with only cool electrons. This is reasonable because we only have electrons with anything more that causes any instability, but in this case the difference of temperature among them increase the intensities and add a slope in the curve of Langmuir waves. In the simulation, the hot electrons temperature is $T_h/T_c = 100$ when they are initialized with kappa distributed speeds in the range $v_{hot} \in [-4\sqrt{T_h}, 4\sqrt{T_h}] = [-40, 40]$ while the cool electrons are distributed in the range $v_{cool} \in [-1, 1]$.

If we compare Fig. 3.7 and Fig. 3.8, we can see that in a plasma with only one species the electron plasma frequency is excited while in a plasma with more than one species more modes are excited. In this case Langmuir and electron acoustic waves are excited.
3.3.4 Cool, Hot and Beam Electrons and Fixed Ions

Now a free energy source is included, an electron beam. We consider a cool electron beam with density $n_b$ and velocity $v_b$ which streams across cool and hot electrons. Electrostatic interactions between the electron beam and the other plasma species will lead to an instability. The energy stored in the streaming beam will be transferred to the other components. As we see in Fig. 3.9, the dispersion diagram changes from Fig. 3.7 and Fig. 3.8. Langmuir and electron acoustic waves and beam-driven mode are excited. These will be studied later.

Electron acoustic waves in Fig. 3.9 are slightly more damped than in Fig. 3.8. The temperature ratio $T_h/T_c$ is the same in both simulations thus the electron beam is the reason for this damping.
3.3.5 Including the Mobility of the Ions

The modification made in the code is the inclusion of the mobility of ions. The ion speed is defined with a Maxwellian distribution function, a random initialization of the position, same charge as electrons but negative and a mass ratio \(m_i/m_e = 1836\). Although mass ratio will be modified to study the sensitivity of the simulation to this parameter.

The inclusion of the mobility of ions implies a new plasma frequency: the combined plasma frequency \(w_p = \sqrt{w_{pi}^2 + w_{pe}^2}\). If mass ratio \(m_i/m_e\) is kept as the realistic value 1836 then \(w_{pi} \ll w_{pe}\). This feature allows us to express \(w_p \sim w_{pe}\) which is satisfied with all simulations. Thus the normalization may be done with \(w_{pe}\).

Besides there is another time of simulation to take into account. In order to observe waves around the electron plasma frequency the time of simulation should be proportional to \(1/w_{pe}\) and to observe waves around the ion plasma frequency the time should be proportional to \(1/w_{pi}\). Since the ion plasma frequency is considerably smaller than the electron plasma frequency, the necessary time for the observation of waves around \(w_{pi}\) is larger.

In the following sections is checked if Langmuir wave, electron acoustic waves and beam-driven mode are excited too with ions included. The simulations are done using an appropriate time in order to observe waves around \(w_{pe}\).

3.3.6 Cool and Hot Electrons and Mobile Ions

Langmuir waves and electron acoustic waves are excited in a cool and hot electrons and ions plasma. The beam-driven mode is not present since there is no beam to excite it. Fig. 3.10 shows the dispersion diagram. There is a range where Langmuir and electron acoustic waves converge, this may be because of the noise, thus the resolution is not enough. It should be taken into account that in this case more massive particles are composing the plasma thus this may be also a reason for the coupling between Langmuir waves and electron acoustic waves.

![Figure 3.10: Dispersion diagram of plasma with cool and hot electrons and mobile ions.](image)

3.3.7 Cool, Hot and Beam Electrons and Mobile Ions

The final scenario is a plasma with hot and cool electrons, mobile ions and an electron beam as a free energy source. The result obtained is a clear Langmuir wave and beam-driven mode. The electron acoustic waves are strongly damped. In this case there is no coupling between different modes.
Figure 3.11: Dispersion diagram of plasma with cool and hot electrons, ions and an electron beam with $v_b/v_c = 15$ and $T_e = T_i$ and $n_b/n_c = 0.05$. 
Chapter 4

Cool, Hot and Beam Electrons and Fixed Ions in an Unmagnetized and Collisionless Plasma

First we consider the initial state of the code, with only three electron components with an electron beam included. Table 4.1 summarizes the basic configuration. Fig. 4.1 shows the kinetic energy and the potential energy of the plasma. Fig. 4.2 shows the total energy evolution of the plasma. The phase space distribution of each species after \( w_{pe}t = 100 \) is shown in Fig. 4.3. Fig. 4.4 shows the result obtained with the basic configuration. In the following sections these parameters are varied in order to see their influence on the behaviour of waves. This part is not very deep because it is not the scope of this thesis and it is already done by Koen et al. (2012b), but it validates that our code with the changes included is still able to simulate a motionless ion plasma.

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Table 4.1: Parameters for simulations of the plasma with fixed ions, hot and cool electrons and electron beam.

The total energy of plasma should be constant during all the simulation, thus if it is not satisfied our simulation is not working well. Figures 4.1 and 4.2 show the different energies and it can be seen that the system works correctly. Since the simulation time is short, \( w_{pe}t = 100 \), it is not shown in the plots, but after \( w_{pe}t = 1000 \) the kinetic energy and the potential energy remain constant. At lower times, the kinetic energy decreases while the potential energy is increasing. At \( w_{pe}t = 60 \) they change the behaviour and even though for some oscillations, they remain constant.
Figure 4.1: Kinetic energy of the system (left) and the potential energy (right).

Figure 4.2: Total energy of the system.

Figure 4.3: Phase space distribution of the cool electrons (black), hot electrons (red) and the electron beam (blue) at $w_{pet} = 100$. 
Fig. 4.4 shows Langmuir and beam-driven waves as the dominant wave. Electron acoustic waves are damped with this configuration. In the following sections the effects of density, velocity, and temperature are studied to see which is the best configuration to excite them. The simulations made to study the effect of these parameters use the basic configuration summarized at Table 4.1 but with the parameter of interest varied.

4.1 Effect of the Cool and Hot Electron Temperature Ratio

The reduction of the hot electron temperature implies a reduction of the hot electrons thermal velocity. Fig. 4.5 shows how electron acoustic waves are completely damped if we reduce the hot and cool electron temperature ratio. The extreme case with \( T_h/T_c = 1 \) represents a plasma without hot electrons, just cool electrons and electron beam.

In this case the region where electron acoustic waves should be excited is completely damped and the beam-driven and Langmuir modes are coupled. If this ratio is increased to 10, the coupling decreases but it is still present. Langmuir waves conform to the theory: the slope is lower if the temperature decreases.
4.2 Effect of Beam Properties

4.2.1 Effect of the Electron Beam Velocity

In this section the electron beam velocity $v_b/v_c$ is tested. Fig. 4.6 shows the dispersion diagram for a high beam velocity $v_b/v_c = 20$ whereas Fig. 4.7 shows the dispersion diagram for a low value $v_b/v_c = 8$. These diagrams can be compared with Fig. 4.4. Fig. 4.4 shows a dispersion diagram with $v_b/v_c = 15$.

Figure 4.6: Dispersion diagram with $v_b/v_c = 20$.

Langmuir and electron acoustic waves are strongly damped if beam velocity is high, this is shown in Fig. 4.6. Fig. 4.4 shows electron acoustic waves also completely damped and Langmuir waves strongly damped. Beam-driven mode dominates the diagram when beam velocity is high. This result agrees with the results obtained by Koen et al. (2012b). Fig. 4.7 shows the dispersion diagram in the case where the beam velocity is lower than the thermal velocity. The normalized thermal velocity of the hot electrons $v_{th}$ for $T_h/T_c = 100$ is $v_{th} = 10$. If the beam velocity is $v_b/v_c = 8 < v_{th}$, beam-driven mode is not excited and Langmuir waves and electron acoustic waves dominate the dispersion diagram.

Figure 4.7: Dispersion diagram with $v_b/v_c = 8$.

In these cases Langmuir waves are only excited for very low wavenumbers. Electron acoustic waves in the case of $v_b/v_c = 8$ are not excited for very low wavenumbers but only for a range from $k\lambda_D > 0.05$. Beam-driven
mode shows the same behaviour since it is not excited for very low wavenumbers. These results do not mean that electron acoustic and Langmuir waves cannot be excited if the rate $v_b/v_c$ is not lower than the thermal velocity. The other sections show that they can also be excited when $v_b/v_c = 15$ but depending on the rest of the parameters. This section explains how these modes are more damped or not depending on $v_b/v_c$ and the conclusion is that Langmuir and electron acoustic waves are more damped if $v_b/v_c$ increases and the behaviour of beam-driven mode is the opposite, this can be clearly seen in Fig. 4.6 and 4.7.

4.2.2 Effect of the Electron Beam Density

Fig. 4.8 and 4.9 show different dispersion diagrams depending on the electron beam density. Fig. 4.8 shows a low value $n_b/n_c = 0.04$ and a high value $n_b/n_c = 0.1$. Beam-driven waves are excited in both cases and it is the dominant mode. When $n_b/n_c = 0.1$ Langmuir and electron acoustic waves are completely damped whereas when $n_b/n_c = 0.04$ Langmuir waves are excited for very low wavenumbers and electron acoustic waves are weakly excited for a range of wavenumbers from $k\lambda_D = 0.1$ to $k\lambda_D = 0.2$. The system is saturated by the electron beam. From these results it can be seen that as the electron beam density is decreased, Langmuir and electron acoustic waves are more weakly damped.

![Figure 4.8: Dispersion diagram with $n_b/n_c = 0.04$ (left) and $n_b/n_c = 0.1$ (right).](image)

Fig. 4.9 shows a case with the ratio between the two extremes, $n_b/n_c = 0.08$, where all the waves are excited: Langmuir waves, electron acoustic waves and beam-driven waves, however Langmuir waves and beam-driven mode are still more excited than electron acoustic waves. The reference case, Fig. 4.4, with $n_b/n_c = 0.06$ present Langmuir waves and beam-driven mode excited, but electron acoustic waves are strongly damped. It may be guessed its presence for the transition from the completely damped blue region at frequencies under $w_{pe}$ to the green region where electron acoustic waves are supposed to be, even though its excitation cannot be assumed. The results with the different $n_b/n_c$ tell us that, keeping $v_b/v_c = 15 > v_{th}$, there is a range where the level of excitation of electron acoustic waves grows until it reaches a peak and decrease but being still excited. But out of this range it is damped.
4.3 Other Results

Koen et al. (2012b) is a good reference since the code for our simulations comes from his code. However it is also interesting to compare with other references. Kasaba et al. (2001) performed computer simulations of the evolution of electrostatic waves excited by electron beams with electromagnetic code. It is shown the basic beam-plasma interactions. The calculation is done in one and two dimensions. We compare our plots with the 1D run. In Fig. 4.11 is shown the result with electric field propagating to the left direction at x axis. The parameters that they used are summarized in Fig. 4.10, being the underlined parameters.

![Figure 4.9: Dispersion diagram with $n_b/n_c = 0.08$.](image)

<table>
<thead>
<tr>
<th>Table 1. Parameters for One-Dimensional and Two-Dimensional Simulations*</th>
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*Parameters in parentheses are variables in our simulations. Underlined values are the conditions in the standard case (section 3). Values in parentheses are the conditions only used in one-dimensional systems (section 5).

![Figure 4.10: Parameters for Langmuir waves simulation (Kasaba et al., 2001).](image)
As the initial condition the background electrons and ions have Maxwellian distribution functions. Our result with the same configuration, \( (T_e = 3T_i, \, v_e = 0.02, \, v_b = 0.02, \, v_b = 10v_e, \, m_i = 1600m_e) \) but with a higher number of particles, \( N = 40,000 \), is shown in Fig. 4.11 (right). The white line in Fig. 4.11 (left) is the theoretical dispersion diagram of Langmuir waves \( \omega^2 = \omega_{pe}^2 + 2k^2/v_e^2 \). In this figure are plotted both sides where is performed the forward propagating Langmuir waves at \( k > 0 \) and the backward Langmuir waves. These waves are generated by backscattering of the beam-excited Langmuir waves, but we are not looking at that. Kasaba et al. (2001) pointed out that the waves at frequencies \( \omega = 2\omega_{pe} \) and \( \omega = 3\omega_{pe} \) are weak electrostatic harmonic waves of forward propagating Langmuir wave. This feature is common with other previous simulations, as in Nishikawa and Cairns (1991) who studied the self-consistent nonlinear evolution of electron plasma waves excited by electron beams streaming along the ambient magnetic field in a long simulation system with \( L = 2048\lambda_e \), treating only electrostatic waves.

4.4 Conclusions

The waves excited with two temperature electron components and an electron beam plasma are Langmuir waves, beam-driven waves and electron acoustic waves. When the electron beam density is \( n_b/n_c = 0.08 \), the waves are more excited. The beam velocity ratio \( v_b/v_e = 15 \) excites more Langmuir waves and electron acoustic waves while the beam-driven waves are strongly excited when \( v_b/v_e \) is higher. If this ratio would be increased, it is reasonable to think that the rest of the waves would be completely damped. The results compare well with the results presented by Koen et al. (2012b). The temperature of the hot electrons should be higher than ten in order to excite electron acoustic waves and avoid the coupling between Langmuir waves and electron acoustic waves. In all the cases, independent of the configuration of the parameters, the electron driven mode is completely damped for \( k\lambda_D > 0.1 \). Electron acoustic waves used to have the higher amplitude in \( k\lambda_D = 0.1 \). In the region of \( k\lambda_D \ll 1 \) beam-driven waves and electron acoustic waves are always damped. This does not agree with Koen et al. (2012b) results. The difference between both results is the hot electrons distribution function. Koen et al. (2012b) defined the hot electrons by a Maxwellian distribution function while in our simulation they are defined by a kappa distribution function. Thus this may be the reason for the difference.
Chapter 5

Cool, Hot and Beam Electrons and Mobile Ions in an Unmagnetized and Collisionless Plasma

Now the motion of ions is taken into account. The aim of this section is to show the effect of this species with motion and validate the theory presented as the effect of mass ratios $m_i/m_e$, and temperature ratios $T_e/T_i$. The dispersion diagram helps us to study the influence of these parameters on ion acoustic waves and see if it agrees with kinetic theory. The parameters for these simulations are summarized in Table 5.1 and in the following subsections some of these parameters will be changed in order to observe the impact of them on the result. Fig. 5.1 is shows the result with the parameters of Table 5.1, where ion acoustic waves are observed on the right. Fig. 5.2 shows the ions velocity distribution. If we compare ions and electron velocity distribution, Fig. 4.3, it is clear that electrons move much faster than ions.

The next simulations are presented normalized to electron plasma frequency on the right panel and to the ion plasma frequency on the left panel.

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Table 5.1: Parameters for the simulations of plasma with ions, hot and cool electrons, and an electron beam.
5.1 Effect of the Mass Ratio

The mobility of ions depends directly on the ratio of electron, and ion mass. The higher value, the less mobile are ions. If they are heavier then the time that they need to respond to instabilities is longer. This means that ion acoustic waves in the case of very heavy ions may be not excited. The mass ratio is varied in these simulations and the results are shown in the following figures. Fig. 5.3 is a comparison of the theoretical dispersion relations of ion acoustic waves depending on mass ratio. The expression of the dispersion relation used to compare the mass ratio is \( \omega_{pi} \), thus for each different \( m_i/m_e \) the \( \omega_{pi} \) is also different.
5.1.1 Fixed Ions

If we applied a very high number like $m_i = 100000m_e$, the mass ratio may be assumed an infinite hence ions can be considered fixed. In this case it is not likely to find ion acoustic waves excited. Electron acoustic waves and beam-driven mode would be damped also, but they may vanish because of the large time of simulation.

![Dispersion diagram with $m_i/m_e \to \infty$ with $\omega_{pe} t = 1000$.](image)

Figure 5.4: Dispersion diagram with $m_i/m_e \to \infty$ with $w_{pe} t = 1000$.

Fig. 5.4 shows the result of simulation with the very high mass ratio, on the left panel the dispersion diagram is normalized with the electron plasma frequency whereas on the right panel the normalization it is done with the ion plasma frequency. Ion acoustic waves would be observed on the right panel but in this case they are not excited. The red spikes shown are noise of the simulation, this can be solved using more particles.

The results are reasonable since ion acoustic waves were not excited in the simulations shown on Chapter 4 where ions were fixed.

At the left panel of Fig. 5.4 Langmuir waves are excited for very low wavenumbers. The time of simulation $w_{pe} t$ used for the simulations is ten times the time used in Chapter 4. Some modes may be excited for $w_{pe} t = 100$ and not for $w_{pe} t = 1000$ because they may be damped after some time. In this case electron acoustic waves are not excited.
5.1.2 Oxygen Ions

A real case that we can find is a plasma with oxygen ions. The mass of the oxygen ions is 16 times the proton mass, resulting a high ratio, \( m_i = 29736m_e \). Fig. 5.5 on the right panel shows how ion acoustic waves are not excited. It shows the same result as \( m_i/m_e \to \infty \), only noise. Though there is a difference with the waves around the electron plasma frequency. Electron acoustic waves are excited in this case, Fig. 5.5 on the left panel.

![Figure 5.5: Dispersion diagram with \( m_i/m_e = 29736 \).](image)

Despite the ion acoustic waves not being excited in our results, the reason could be an insufficient time of simulation or not enough particles. Then it can not be assumed that ion acoustic waves are not excited in a oxygen ion plasma. Further simulations should be done with more particles.

5.1.3 Hydrogen Ions

The ratio \( m_i = 1836m_e \) is the realistic case that has been used in the rest of sections. The existence of atomic hydrogen in the Saturn’s magnetosphere has been shown, (H. Weiser and Moos, 1977; Shemansky et al., 1993). Ion acoustic waves, shown in Fig. 5.6 on the right panel, are strongly excited. The dispersion curve of ion acoustic waves in this case is too broad. This is for the low resolution of the frequency relative to wavenumbers. The left panels shows Langmuir waves excited but not electron acoustic waves. The difference between the dispersion diagram normalized with the electron plasma frequency (left panel) and the dispersion diagrams of Chapter 4 is for the same reason as discussed in the previous section, because of \( \omega_{pe}t \).
5.1.4 $m_i/m_e = 500$

In order to examine ion acoustic waves with lighter ions, a ratio of the order of 500 is tested and Fig. 5.7 shows the dispersion diagrams of the result. The results show that ion acoustic waves in this case are more clear than for more massive ions. If we compare the ion acoustic waves of Fig. 5.7 and Fig. ??, the difference with the broadness is noticed. This may be because in this case we work with higher ion plasma frequency. In Fig. 5.7 the acoustic regime can be seen clearly.

5.1.5 Positrons

The case with $m_i = m_e$ corresponds to the electron-positron (e-p) plasma. The first experiment in a laboratory with e-p plasma is described by Greaves and Surko (1995). They performed an e-p beam plasma with techniques that make possible the accumulation of positrons in Penning traps. There are more PIC simulations of e-p plasma, as Bessho and Bhattacharjee (2005) perform an electromagnetic simulation to study reconnection. The early universe contains this configuration, as well as the active galactic nuclei and magnetosphere of pulsars (Reza Pakzad, 2009). Besides, e-p plasma is still an interesting topic since it is applied also in the laser field (Kuznetsova, 2011). Lu et al. (2010) showed the presence of electrostatic waves in an adiabatic e-p plasma.
In this case there is no need to observe two different regions of the dispersion diagram because now both frequencies, the electron and ion plasma frequencies, are of the same order. Langmuir waves and electron acoustic waves are coupled in the region $1 < w/w_{pe} < 1.3$. These are the waves more excited. Beam driven waves are observable at $0.3 < k \lambda_D < 0.6$. Ion acoustic waves are Landau damped at small $k \lambda_D$, in the acoustic regime.

![Image](image_url)  
Figure 5.8: Dispersion diagram with $m_i/m_e = 1$.

### 5.2 Effect of the Temperature Ratio

Fig. 5.9 shows the expected dispersion diagrams for the different temperature ratios without taking in account the effects of damping.

First the effect of only one electron species is studied in order to see clearly the temperature ratio between electrons and ions. Those results are shown in Fig. 5.10, 5.11 and 5.12. It is expected that Landau damping precludes ion acoustic waves existing if electrons are not much hotter than ions. But it is also interesting to see the results with two electron species. For the latter case an effective temperature is used, mentioned before and defined as (1.37). Those results are shown in Fig. 5.14 and 5.15.

The ratio $T_e/T_i$ in the case of only one electron species is tested from $T_e/T_i = 1$ to $T_e/T_i = 100$, electron temperature is always higher than ion temperature in our simulations. This range is chosen since when electron temperature and ion temperature are equal is expected that no ion acoustic waves is excited. Although as this ratio is increased this behaviour change and at $T_e/T_i = 50$ the shape starts to be clearer like at $T_e/T_i = 100$.
5.2.1 \( T_e/T_i = 1 \)

If electron and ion temperatures are equal, Landau damping is dominant and ion acoustic waves are completely damped. If we look Fig. 5.10 (left panel), the waves excited are Langmuir and electron acoustic waves. If we make zoom around \( w_{pi} \), we can see that there are electron acoustic waves at very low wavenumber and an irregularity at around \( k\lambda_D = 0.1 \).

The results agree with theory since there are no ion acoustic waves excited like the growth rate predicted. Eq. 1.39 shows that if electron temperature is not enough higher than ion temperature, ions acoustic waves are very damped. This is because the exponentials are divided by the thermal velocity. Electron temperature should be greater to decrease the effect of the first exponential relative to the second one.
5.2.2 $T_e/T_i = 50$

When the temperature ratio is higher ion acoustic waves can be seen easily. Fig. 5.11 shows the case for $T_e/T_i = 50$. The fact that for a plasma with species with the same temperature present a coupling in Langmuir and the beam-driven mode is not shown here. Fig. 5.10 shows this coupling while Fig. 5.11 present the two waves perfectly separated. The acoustic regime is not shown, only a range from $k \lambda_D = 0.04$ to $k \lambda_D = 0.16$.

Figure 5.11: Dispersion diagram with $T_e/T_i = 50$.

5.2.3 $T_e/T_i = 100$

For a ratio of two order of magnitude, ion acoustic waves are a little bit clearer and it can be seen even from the figure normalized at electron plasma frequency, Fig. 5.12 (right). This figure also shows how Langmuir waves present a higher slope. There is a confusion if the other wave corresponds to electron acoustic waves or beam-driven waves, because there is something between both theoretical curves.

Figure 5.12: Dispersion diagram with $T_e/T_i = 100$.

Castro et al. (2010) applied a technique which creates controlled density perturbations to excite ion acoustic waves in an ultracool neutral plasma (UNP). UNP are order of magnitudes cooler than any other neutral plasma. Killian et al. (2012) presented advanced results. The dispersion relation obtained by Castro et al. (2010) experiment, Fig. 5.13, is plotted varying the initial electron temperature from $T_e = 25$K until $T_e = 105$K and the initial ion temperature is $T_i = 1$K.
In our plots we can see that when $T_e/T_i = 100$ the slope is higher in the acoustic regime than when $T_e/T_i = 50$. These results agree with the results presented in Castro et al. (2010).

Let us consider the two electron components, the cool and the hot to the following simulations. Thus we have now two ratios, $T_c/T_i$ and $T_h/T_i$. The theoretical dispersion diagrams are plotted using the highest ratio, $T_h/T_i$. This assumption is made because if (1.36) had two terms with both ratios, keeping the ratio $T_h/T_c = 100$, the highest would always dominate.

### 5.2.4 $T_c/T_i = 2$ and $T_h/T_c = 100$

This case presents a very low ratio between cool electrons and ions. In this case, shown in Fig. 5.14, ion acoustic waves are very noisy. There are a lot of spikes. This may be because of the low ratio $T_c/T_i$. Nevertheless, ion acoustic waves theoretical curve fits perfectly with the theoretical results in the acoustic regime.

### 5.2.5 $T_c/T_i = 10$ and $T_h/T_c = 100$

Once the cool electron and ion temperature ratio is high enough ion acoustic waves present a less noisy shape and fit perfectly with the theoretical curve. This is shown in Fig. 5.15 with a ratio of 1 order of magnitude.
between cool electrons and ions and three orders of magnitude between hot electrons and ions. The slope is greater also, showing that the acoustic speed is higher.

Figure 5.15: Dispersion diagram with $T_c/T_i = 10$ and $T_h/T_i = 1000$.

If we increase $T_c/T_i$ the result will be similar thus in a scenario where $T_c/T_i > 10$ ion acoustic waves can be observed. However the wavenumber range would be shorter because of a smaller Debye length.

From the cases with different $T_c/T_i$ and $T_h/T_c$ ratios it can be seen that the dominant rate, which influences more in the shape of ion acoustic waves, is $T_c/T_i$, the difference between ions temperature and the coolest electrons. If this difference is high enough, it allows ion acoustic waves to be excited better.

### 5.3 Effect of Ions on Langmuir, Electron Acoustic and Beam-Driven Waves

This section compares two different scenarios: hot and cool electron and ions plasma (shown in Fig. 5.16), and a hot and cool electrons, ions and an electron beam (shown in Fig. 5.17). Each case is simulated considering fixed and mobile ions. This allows us to easily compare the effect of the mobility of ions on the waves around electron plasma frequency.

Langmuir waves and electron acoustic waves are influenced by the mobility of ions. Fig. 5.16 shows that electron acoustic waves are more excited if ions are mobile but for smaller $k\lambda_D$. Langmuir waves have the same behaviour, this is clear in Fig. 5.17. The mobility of ions, in the case without a beam, produces the coupling of the Langmuir and electron acoustic waves.

Beam driven mode does not present any difference and this agrees with the no dependence of (1.15) on ions. Electron beam affects only the behaviour of the rest of waves. This is shown in the previous sections too. Langmuir and electron acoustic waves in Fig. 5.16 are more excited than in Fig. 5.17. Langmuir waves and electron acoustic waves, in the case where the ions are mobile, are more strongly amplified. This is more evident with Langmuir wave.
Figure 5.16: Dispersion diagram with hot and cool electrons and fixed ions (left) and with hot and cool electrons and mobile ions (right).

Figure 5.17: Dispersion diagram with the hot, cool and beam electrons and fixed ions (left) and with hot, cool and beam electrons and mobile ions (right).
5.4 Conclusions

The parameters chosen to be tested in the plasma with mobile ions have been the ion and electron mass ratio, the electron and ion temperature, the beam velocity and the beam temperature. The mass ratio defines the mobility of the ions. This parameter has been checked from very low mobility, close to fixed ions, to as mobile as electrons case. If the mass ratio is defined too high, like \( \mu = 100000 \), ions behave as fixed and no ion acoustic waves are excited as theory predicts. As the mass ratio is decreased the ion acoustic waves are clearer excited and the ion plasma frequency is closer to the electron plasma frequency. Electron acoustic waves when the mass ratio is too high, like in the case of oxygen ions, are excited as in the case of positrons. But they are not excited in the intermediate hydrogen case. Thus a critical mass ratio may exist, which define whether these waves are excited or not. The electron and ion temperature ratio should also be greater than one. The presence of two electron species with different temperatures makes ion acoustic waves more clearly excited. The results showed that electron acoustic waves are amplified in the cases with mobile ions, as in Fig. 5.17.
Chapter 6

Application to Saturn’s Magnetosphere

The model may be applied to several space scenarios and one of them is Saturn’s magnetosphere. Baluku et al. (2011) applied also a simulation with Saturn’s magnetosphere data in order to investigate electron acoustic waves, using a kinetic theory approach. The results of our simulations will be compared with the results by Baluku et al. (2011) and Baluku and Hellberg (2012). Baluku et al. (2011) concluded that the region out of Saturn’s magnetosphere is the region where the properties of the plasma allow a better propagation for electron acoustic waves.

The cases chosen to this simulation are Saturn’s inner magnetosphere ($R \leq 9 - 10R_S$), the extended plasma sheet ($7 - 9R_S < R < 12 - 14R_S$) and the outer region of the magnetosphere ($R \geq 12 - 14R_S$), where $R_S$ is the radius of Saturn.

Hellberg et al. (2001) described electron acoustic dispersion relation for a kappa-distributed electrons as

$$w_{ea} = w_{pc} \left( \frac{1 + 3k^2\lambda_D^2}{1 + k^2\lambda_{kh}^2} \right)$$

(6.1)

where $\lambda_{kh} \equiv \left( \frac{\kappa_{h} - 3/2}{\kappa_{h} - 1/2} \right)^{1/2}$, $\lambda_D$ is the Debye length of the hot electrons when they are kappa-distributed. This dispersion relation is the most general for the kappa distribution (Baluku et al., 2011), and assumes $3v_{tc}^2/v_{ke}^2 \ll 1$.

$v_{tc}$ is the cool electrons thermal velocity and

$$v_{sn}^2 = w_{pc}^2 \lambda_{kh}^2 = \left( \frac{n_c}{n_h} \right) \left( \frac{K_BT_h}{m_e} \right) \left( \frac{\kappa_{h} - 3/2}{\kappa_{h} - 1/2} \right)$$

(6.2)

is electron acoustic speed in a kappa plasma.

Mace et al. (1999) showed that it may also be written as

$$w_{ea}^2 = k^2 \left( \frac{v_{sk}^2}{1 + k^2\lambda_{kh}^2} + 3v_{tc}^2 \right)$$

(6.3)

where $v_{tc} = w_{pc}\lambda_D = \left( \frac{K_BT_c}{n_e} \right)$ is the cool electron’s thermal velocity.

Baluku et al. (2011) define the dispersion relation for Langmuir waves in a kappa-distributed plasma as

$$w_{l}^2 = w_{pc}^2 + 3 \left( \frac{n_{co}}{n_{eo}} \right) k^2v_{lc}^2 + 3 \left( \frac{n_{ho}}{n_{eo}} \right) k^2v_{th}^2.$$  

(6.4)

Schippers et al. (2008) illustrated the parameters of Saturn’s magnetosphere, the data which we are interested is summarized in Table 6.

<table>
<thead>
<tr>
<th>$R(R_S)$</th>
<th>$\kappa_c$</th>
<th>$\kappa_h$</th>
<th>$T_c(eV)$</th>
<th>$T_h(eV)$</th>
<th>$n_c(cm^{-3})$</th>
<th>$n_h(cm^{-3})$</th>
<th>$T_h/T_c$</th>
<th>$n_h/n_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.3</td>
<td>2.3</td>
<td>3.0</td>
<td>2.0</td>
<td>400</td>
<td>10.5</td>
<td>0.01</td>
<td>200.0</td>
<td>0.001</td>
</tr>
<tr>
<td>9.8</td>
<td>2.0</td>
<td>4.0</td>
<td>8.0</td>
<td>1100</td>
<td>2.5</td>
<td>0.07</td>
<td>137.5</td>
<td>0.027</td>
</tr>
<tr>
<td>14.0</td>
<td>2.1</td>
<td>6.0</td>
<td>30</td>
<td>900</td>
<td>0.15</td>
<td>0.10</td>
<td>30.0</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Table 6.1: Saturn’s magnetosphere parameters (Schippers et al., 2008).
Fig. 6.2, 6.3 and 6.4 show the simulation results with Saturn data. At the right panel of the figures is shown the results normalized by the ion plasma frequency in order to observe ion acoustic waves. The curve on the plots to compare ion acoustic waves follows the equation derived by Baluku and Hellberg (2012)

$$w_{ia}^2 = k^2 (C_{ik}^2 + C_{it}^2)$$ (6.5)

where $C_{ik} = w_{pi} \lambda_{Dk}$ is the ion acoustic speed applied for kappa-distributed electrons and $C_{it} = (3K_B T_i/m_i)$ is the ion thermal velocity. (6.5) is valid for $k\lambda_D \ll 1$.

Figure 6.1: Dispersion diagram with the data from the magnetosphere of Saturn (Baluku et al., 2011). From left to right, the figures are at $R = 6.3R_S$, $R = 9.8R_S$ and $R = 14R_S$. The dotted line means damped region.

### 6.1 Inner Magnetosphere

Fig. 6.2 on the left panel shows that Langmuir waves are strongly excited for wavelengths $k\lambda_D < 0.08$ and electron acoustic waves are not excited. These results agree with Baluku et al. (2011). The right panel of Fig. 6.2 shows an unclear result for ion acoustic waves. It may be because of the noise.

Figure 6.2: Dispersion diagram with the inner magnetosphere of Saturn data, $R = 6.3R_S$, $\kappa_c = 2.3$, $\kappa_c = 3.0$, $T_h/T_e = 200$, $n_h/n_e = 0.001$, $n_c/n_e = 0.999$. 

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6.2 Extended Plasma Sheet

At intermediate region of Saturn’s magnetosphere Langmuir waves are also excited but in a smaller range of wavelength and electron acoustic waves may be excited but it is not clear in the simulation either. Balluku et al. (2011) show also strongly damped electron acoustic waves. At the right panel of Fig. 6.3 are shown ion acoustic waves. (6.5) fits well in the acoustic regime but after that disagrees completely with the result. This is right because (6.5) is defined for the acoustic regime.

Figure 6.3: Dispersion diagram with the extended plasma sheet of magnetosphere of Saturn data, $R = 9.8 R_S$, $\kappa_c = 2.0$, $\kappa_e = 4.0$, $T_h/T_c = 137.5$, $n_h/n_e = 0.027$, $n_c/n_e = 0.973$.

6.3 Outer Magnetosphere

At the outer region of the magnetosphere of Saturn electron acoustic waves are excited as Balluku et al. (2011) demonstrated.

Figure 6.4: Dispersion diagram with the outer magnetosphere of Saturn data, $R = 14.0 R_S$, $\kappa_c = 2.1$, $\kappa_e = 6.0$, $T_h/T_c = 30.0$, $n_h/n_e = 0.40$, $n_c/n_e = 0.60$.

If we compare the results for electron acoustic waves with Fig. 1.1, it makes sense. Even if the temperature ratio $T_h/T_c$ is high which is appropriate for the excitation of these waves, the ratio $n_h/n_e$ is too low thus $n_c/n_e$ is
too high. Fig. 1.1 tells us that for a density ratio higher than 0.8 like in the cases for \( R = 6.3R_S \) and \( R = 9.8R_S \) these waves are strongly damped.
Chapter 7

Summary of Results

This section summarizes all the results obtained from the particle-in-cell simulations with the different scenarios. Langmuir waves are excited in most of the simulations performed. Although the beam amplifies Langmuir waves. But if it is very dense or has a high velocity, Langmuir waves will be damped. The same occurs with electron acoustic waves. However, if the beam velocity is lower than the thermal velocities, the beam-driven mode is completely damped. Langmuir waves have a different slope depending on the species in the plasma and the temperature. The high temperatures increase the slope.

The mobility of ions allows ion acoustic waves to be excited. The ion mass, which defines the ion plasma frequency, determines the mobility of ions. The cases with the ion mass for the realistic case or lower values present clearly the ion acoustic instability. Even if the mobility of ions is enough, another requirement should be satisfied for the weak damping of ion acoustic waves. First, the electron temperature should be greater than the ion temperature, $T_e \gg T_i$, and if there are two electron species this condition applies to the cool electron density.

The mobility of ions amplifies electron acoustic waves.

The application to the magnetosphere of Saturn with kappa distribution functions for the electrons has been the final aim of this thesis. Langmuir waves are present in the three cases tested. Electron acoustic waves are only excited in the outer magnetosphere. Ion acoustic waves in the outer magnetosphere are clearly excited and the theoretical curves fit perfectly. Thus we can assume the likely presence of ion acoustic waves in the outer magnetosphere of Saturn. In the other regions, the results are noisy. The results follow the slope of the theoretical curve which suggests that ion acoustic waves may be excited in these regions. Nevertheless more simulations with more particles or more time of simulation should be run.
Chapter 8

Conclusions

This thesis is an investigation of electrostatic waves in a plasma with cool, hot and beam electrons and ions using particle-in-cell method for simulation. Results have been applied to Saturn’s magnetosphere as Baluku et al. (2011), Koen et al. (2012b), Koen et al. (2012a) did. Because of the time for this project it was not enough, the collisions were not included but it is an improvement that could be done in the future. Hellinger (2004) simulated ion acoustic instability in an unmagnetized plasma with protons and electrons taking into account the particles collision. Sydora et al. (2006) studied electron-ion collisions in order to investigate the laser-plasma interaction.

The simulations should be done with more particles in order to get better results. Another improvement could be the development of a signal processing method in order to eliminate the noise. In this thesis two options have been applied. First the simulation of the same scenario have been performed several times. The average of these samples have been done. An improvement have been observed but it has not been worthy for the computational time that it lasted. The second option, more successful, has been the variance reduction method. This method does not introduce a significant delay to the computation time and the results are improved. However it is not enough, this is an important point to be improved.
Bibliography


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