OBJECT MANIPULATION WITH TWO ROBOTIC FINGERS USING TACTILE INFORMATION

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Abstract

The integration of tactile sensors in dexterous robotic hands contributes greatly to creating autonomous robots capable of interacting with their environment and manipulating various types of objects. The shape of these objects may not be previously known to the robot. In this work we realize the manipulation of unknown objects that are grasped with two fingers of the SCHUNK SDH 2 robotic hand. The fingers manipulate the object changing its orientation with the use of the tactile information provided by the fingers' tactile sensors. The different contact points in the fingertip's surface are modelled by introducing a virtual joint that adds an extra degree of freedom to each finger.
Table of Contents

Acknowledgements ......................................................................................................................... 3
Abstract ............................................................................................................................................ 5
Chapter 1 ........................................................................................................................................ 9
Introduction .................................................................................................................................... 9
  1.1 Objectives of this work ........................................................................................................... 10
  1.2 Motivation ............................................................................................................................. 10
  1.3 Related Work ......................................................................................................................... 10
  1.4 Contribution .......................................................................................................................... 12
  1.2 Outline of the Thesis ............................................................................................................. 12
Chapter 2 .................................................................................................................................... 13
Model of the Robotic Hand ............................................................................................................. 13
  2.1 SCHUNK Dextrus Hand SDH2 ............................................................................................. 13
  2.2 The Weiss Robotics Tactile Traducers ................................................................................. 15
  2.3 Forward Kinematic Model .................................................................................................... 17
  2.4 Denavit-Hartenberg Parameters ............................................................................................ 18
  2.5 The virtual joint ..................................................................................................................... 19
  2.6 Fingertip Model .................................................................................................................... 21
  2.7 Fingertip Tactile Sensor Analysis ........................................................................................ 24
  2.8 The Forces ............................................................................................................................. 32
  2.9 The Grasp ................................................................................................................................ 32
  2.10 Force Closure ....................................................................................................................... 33
  2.11 Angles to the Normal Vector ............................................................................................... 35
  2.12 Inverse Kinematic Model of Finger 1 .................................................................................. 36
Chapter 3 .................................................................................................................................... 41
Manipulation with Two Robotic Fingers ......................................................................................... 41
  3.1 Conceptual Analysis of the Manipulation Movement ............................................................ 41
  3.2 Solution Approach ................................................................................................................ 42
  3.3 Object Manipulation with Only One Finger .......................................................................... 43
    3.3.1 The Moving One Finger Algorithm ................................................................................ 45
    3.3.2 Simulation Results of One Finger Movement ................................................................. 46
  3.4 Object Manipulation Moving Both Fingers .......................................................................... 48
    3.4.1 The Moving Two Fingers Algorithm ............................................................................. 51
    3.4.2 Simulation Results for Two Fingers Movement ............................................................. 52
  3.5 Manipulation of Real Objects Using the Tactile Sensors ...................................................... 54
    3.5.1 The Tactile Information ................................................................................................. 55
    3.5.2 The Two Finger Manipulation with Tactile Feedback Algorithm ................................ 56
Chapter 4 .................................................................................................................................... 59
Experimental Results ..................................................................................................................... 59
  4.1 The Contact Force ................................................................................................................ 59
  4.2 Manipulation of Unknown Objects with Two Robotic Fingers .......................................... 63
    4.2.1 Object A: Rubber .......................................................................................................... 66
    4.2.2 Object B: Egg ................................................................................................................. 70
    4.2.3 Object C: Cylinder ......................................................................................................... 74
Chapter 5 .................................................................................................................................... 81
Conclusions and Future Work ....................................................................................................... 81
  5.1 Conclusions .......................................................................................................................... 81
  5.2 Future Work .......................................................................................................................... 81
References ...................................................................................................................................... 83
Chapter 1

Introduction

In recent years, many dexterous robotic hands have been built with the aim to carry out not only simple tasks like pick-and-place, but also dexterous manipulation. Dexterous manipulation means to move an object from an initial position and orientation to a desired final one by the robot [19]. Possible tasks are taking the newspaper from the table, opening and closing the screw cap of the bottle, pouring the water into the glass, etc. Grasping and manipulation of various objects are the fundamental functions of robotic hands. Dexterous robotic hands should be able deal with a large varieties of different objects and different kinds of tasks.

Usually in classical robot-interaction tasks the contact is expected and planned to occur at specific locations of the robot. However more advanced applications require more complex forms of interactions. The location and the characteristics of the contact cannot be predicted or modelled in advance [10]. Therefore, a tactile sensor system is required, which is capable of detecting the contact and measuring the contact forces. The integration of tactile sensors different parts of the robot greatly contributes to the creation of autonomous robots capable of interacting with the environment.

The tactile receptors that the human has in his skin allow him to sense the contact with objects, the contact force and realize when there is a slippage of grasped object. Many of the object's characteristics are also recognized with tacting. The shape of the object, the irregularity of the contact surface, the temperature of the object are some of them. Precise muscle control and dexterity of the human hand are due to the feedback information from tactile receptors [3]. The tactile information is important for dexterous robotic hands in order to recognize the objects' properties and achieve dexterity. and precise objects handling.

The tactile sensor is a robotic sensor that like human tactile receptors is able to detect the contact and measure the applied forces. It can be used so as to obtain information about the object's shape, its place, the location of the contact points and the contact force applied to the object by the robotic fingers. Slippage detection and estimation of the friction coefficient between the finger and the object are some of the common applications of the tactile sensors [5].

The object manipulation by robotic hands equipped with tactile sensors in order to be able to detect the contact and increase their autonomy is a challenging subject. Humans are capable of manipulating any unknown object without seeing it or having any information a priory about its properties. However robots normally need precise information about these properties in order to manipulate it successfully. The information humans need about the object's properties are obtained during the manipulation using the tactile sensors they have in their hands. While manipulating the unknown objects sensors of the skin provide them with valuable information about its shape and pose [6]. The robotic researchers are often inspired by this human ability to create applications for robotic hands equipped with tactile sensors which try to imitate the human way of doing the things.

In this work we model geometrically and cinematically the two coupled fingers of the SHUNK Dextrous hand SDH 2 [9].and the fingertip tactile sensor. In order to model the different points of contact in the fingertip we introduce a virtual joint that adds an extra DOF in each finger of the hand.

We propose a method to manipulate unknown objects grasped by the two coupled fingers of the
SDH 2. The objects orientation must change during the manipulation. The object may roll in the fingertips surface depending on its particular shape. The objects position while rolling will be perceived using the tactile information. An on-line adjustment of the distance between the fingers so us to maintain the contact forces around their initial value will be used. In that way the fingers will follow the changes in the object's shape while it rolls. In this way an autonomy of the fingers will be obtained by perceiving the object's shape during the manipulation. An experimental process using the SDH 2 hand has been realized to support the developed algorithm.

1.1 Objectives of this work

The object of this work is the manipulation of unknown objects with two fingers using the SDH2 robotic hand. The objects' form is not previously known and it will be perceived during the movement using the information obtained by the tactile sensors of the fingers. The proposed method is trying to imitate the human way of rotating an object grasped with the thumb and the index finger. For a dexterous manipulation the position of the object that may roll on the finger surface during the movement, and the pressure applied to it should be known at every moment of the process. This information will be obtained by the tactile sensors. An algorithm will be implemented for the SDH 2 hand. Finally real experimentation will be done where the SDH 2 should be able to manipulate different unknown objects.

1.2 Motivation

In dexterous manipulation is desirable that the robots manipulate unknown objects autonomously without the need of an object's model or its exploration through prehension, immobilization or artificially constrained motions. In this work the manipulation of unknown objects by rolling as an autonomous task is proposed. The manipulated objects are grasped with two robotic fingers. The manipulation will be realized using only the information obtained by the tactile sensors without any previous knowledge of the objects shape or exploration with additional methods. The human ability of perceiving the objects shape and being able to adapt to it is willing to be imitated.

1.3 Related Work

Dexterous robotic hands [18] [9] have been developed with the aim to imitate the functionality of the human hand in order to increase the autonomy of the robotic systems and their flexibility and adaptability to various circumstances and tasks. The installation of tactile sensors [17] or skin that integrates sensing elements [15] [16] on the robotic hand is becoming a key-technology for the implementation of dexterous robotic hands that will perform stability of object handling and manipulation and adaptability to external forces.

An intelligent dexterous hand may be able to deliver the operator from tasks like path planning, contact determination or slip detection. Fundamental methods of a grasping or manipulation process should be realized autonomously [12]. With this aim, collision avoidance algorithms for robotic
hands are frequent in actual robot research. The development of collision free path planning algorithms started with the development of computer controlled robot systems. The investigation group of KIT [13] supported by SCHUNK GmbH & Co. KG [7] describes in [2] a real-time collision detection algorithm developed for the SDH 2. The aim is to prevent its fingers to collide with each other. The developed collision detection is subdivided into three hierarchy levels making use of the nearly exact finger geometry. The same in [12] present a collision free path planning algorithm for SDH2 for the autonomous transfer of all fingers into a desired target position. The principal idea is the transfer of each phalanx into a predefined home position.

A method for dexterous manipulation planning problem of rotating object with surface of revolution using at least two fingers of a robotic multi-fingered hand is presented in [19]. The proposed method is based on finding contact points that can solve the problem of rotation the object. The relative pose between the hand and the object to achieve maximal rotational angle is also optimized. Though the rotation is realized with points of contact fixed in the object's surface. Although precision grasping and manipulation is included in dexterity, several dexterous robotic tasks may require a not-precise object manipulation. The uncertainty introduced in this case can be handled with the use of the tactile sensors. In [14] an object manipulation for transiting the object from an initial precision fingertip grasp to power grasping using a multi-fingered hand is proposed. Power grasping is characterized by multiple points of contact between the object and the surfaces of the fingers and the palm. The manipulation is composed of transitions between the steady states of stable grasping. In a proposed strategy the object is forced to roll alternatively in the fingers surfaces until it slides by leaving intentionally the friction cone. The uncertainty in the object's position is handled by recognizing the success of the intermediate steps by tactile sensors.

Although different methods of contact detection have been implemented, tactile sensing is critical in contact detection since it provides the most direct feedback to control contact forces both in voluntary and involuntary interactions with the environment. The sensors can be used for collision detection when working in unstructured environments or for human-machine interaction. Sensors with high resolution are integrated in dexterous hands enabling them for reactive gripping. They can detect collisions, locate them and by analysing the acquired pressure profiles classify them into collisions with rigid objects or moveable dynamic ones [3].

A tactile sensor system normally consists of a set of discreet measure cells called “texels”. They are ordered in homogeneous matrices with the aim to detect the applied pressure profile. The measured sizes are digitalized through local intelligence, the sensor's controller which digitalizes the signals for data acquisition. A host system process the obtained data and extracts the system characteristics [20]. This information can be used later as control feedback for grasping manipulation robot tasks. The most commonly used tactile sensor technologies are piezoresistive (rubbers or inks), piezocapacitive, piezoelectrical, and optical [3]. In [20] the construction and working principle of resistive tactile sensor cells are described.

The SDH 2 hand is used in [10] for the reconstruction of the object shape from contact points acquired from palpating sequences. A probabilistic spatial approach to build compact 3-D representations of unknown objects using the tactile sensors is presented. In [3] the tactile sensor is used to classify rigid and deformable objects. In [21] a method is proposed to estimate the contact region of the sensor, and the location of the object by using vision-based tactile sensor.
1.4 Contribution

In this work the two coupled fingers of the SCHUNK Dextrous robotic hand is modelled geometrically and cinematically, including the tactile sensor's modelling. The different possible points of contact in the sensor's surface were modelled by introducing a virtual joint that adds an extra DOF at each finger's DOFs.

Unknown objects are manipulated after being grasped with the two coupled fingers of the robotic hand, using the tactile information obtained by its sensors. The proposed manipulation is a rotational manipulation an object which is grasped in a similar to the human thumb-index finger grasp (prismatic grasp). Unlike commonly used methods of an object's rotation by robotic hands where the points of contact are fixed in the object's and the fingers' surface, this movement is trying to imitate the way the human executes tasks like opening a bottle that the object is rolling in the finger's surface and the points of contact are changing.

1.2 Outline of the Thesis

The rest of this document is organized as follows. In Chapter 2 the Schunk robotic hand SDH2 and its tactile sensors are described and modelled. Chapter 3 presents the analysis of the manipulation of unknown objects with the use of the tactile information and the developed algorithms. The experimental results of the manipulation of different objects with the SDH2 hand are performed in Chapter 4. Chapter 5 concludes the work and gives a brief description of potential future work.
Chapter 2

Model of the Robotic Hand

2.1 SCHUNK Dextrus Hand SDH2

The robotic hand used in this work is the SDH2 Schunk hand [9] shown in Fig 1. It is a three-finger hand with a total of seven active DOF. Each finger has two active DOF. There is one rotational joint between the palm and the proximal link and another between the proximal and distal link. Two of the fingers are coupled (pivoting joints) They share a common joint (Fig 1: joint 1a and 1b) to rotate contrarywise around the finger axis perpendicular to the palm. The third finger is fixed with regard to that axis. Each finger has two tactile sensor arrays.

![Fig 1: The SCHUNK Dextrous hand SDH2. The DOFs of the robotic hand are enumerated.](image)

Using the enumeration of the joints in Fig 1 the joint angles will called $\varphi_1$, $\varphi_2$, $\varphi_3$, $\varphi_4$, $\varphi_5$ etc. respectively. The physical limits of these angles as given by the manufacturer are:
The zero positions of the joint angles and their positive directions are shown in Fig 2.

<table>
<thead>
<tr>
<th>Number of Fingers</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>DOF (active) Total</td>
<td>7</td>
</tr>
<tr>
<td>DOF (active) per Finger</td>
<td>2</td>
</tr>
<tr>
<td>DOF (active) 2-Finger Pivoting Joints</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1: SDH 2 Kinematics.

The final version of the robotic hand was introduced in 2008. The actuators are DC motors coupled with high-ratio gears and can achieve maximum torques of up to 1.4N m within the distal joints and 2.1N m within the proximal joints of the fingers [12]. The SDH2 C++ library provided by the manufacturer is used to control the SDH2. Currently, there are two different possibilities to move
the fingers. The first is to define angular velocities and let the SDH2 calculate and control the motion sequence on its own. The second is to set target angles for each joint. In this work the second way is used.

The object manipulation is realized by the two coupled fingers rotated by 90° so that they move in the same plane. The DOF of these two fingers only permit manipulation in one plane. The manipulated objects will be arbitrary with regular surfaces, such as sticks, polyhedra, cylinders, spheres and objects with curved surfaces. A geometrical model to represent the SDH 2 need to be created.

### 2.2 The Weiss Robotics Tactile Traducers

The SDH-2 hand is equipped with a total of six tactile traducers manufactured by Weiss Robotics [8]. Two of these sensors are attached to each finger. The DSA9205 is used at the proximal phalanges. Its active sensing matrix consists of 14 rows x columns of sensing cells ( Texels) in about 24 mm x 51 mm area, Fig 3. Its spatial resolution is 3.4 mm. An integrated sensor controller at each sensor digitizes the pressure signals and processes the the data which supplies at various interfaces Fig 6. The 6 x 14 pressure matrix is provided either numerically or at digital graphic interfaces. The technical specifications of the sensor are given in Fig 3.

![Fig 3: The rproximal tactile sensor SDH 9205 and technical specifications](image)

**Technical Specifications**

| Power supply: | 5 V, 10 mA |
| Number of sensor cells: | 6 x 14 |
| Spatial resolution: | 3.4 mm |
| Measurement range: | 250 kPa |
| Sampling rate: | 150 frames/s |
| Operational temperature range: | 0 to 40 °C |
| Shell materials: | Stainless steel, silicone rubber |
| Protection class: | IP65 (mounted) |
| Physical dimensions: | 24.4 x 51.4 x 5.4 mm |

The sensor used at the distal phalanges is the DSA9210 [8]. It is a tactile transducer with crooked measurement plate especially constructed to be used on the fingertips (Fig 4). It consists of 70 sensor cells and its spatial resolution is 3.4 mm. The sensor matrix is formed by 9 rows of 6 cells and 4 rows of 4 cells. The location of the centre of the cells is described in Fig 5 An integrated sensor controller processes the pressure data and provides a 6 x 13 pressure profile matrix. The sensor DSA 9210 mounted in the fingertips of SDH2 will be the one analysed in this work.
The sample rate of these DSA sensors is around 230 Hz. SDH 2 uses a serial bus, RS-232 for data transmission, which is shared between all sensors [9]. This effectively limits the available frame rate for a single sensor to approximate 30 Hz when all of the six sensors are used. The repetitive accuracy for each of the finger joints is 0.01°. Therefore the dimensions of one texel with area 3.4 mm × 3.4 mm are one magnitude above the other possible sources of measurement uncertainty [10] making it the principle factor of uncertainty in the experiments.
2.3 Forward Kinematic Model

The forward kinematics for a robotic finger can be defined as: Given the generalized coordinates \( q(t) = [q_1(t) \ q_2(t) \ .. \ q_n(t)] \) of the joints and geometric parameters of the finger (\( n \) is the number of DOFs) determine the position of the point of contact finger-object with respect to a reference coordinate system.

The Debavit- Hardenberd algorithm [1] describes the way of calculation of the transformation matrices which refer each point referenced to one frame to the previous reference frame and finally to the global reference frame. The different reference systems must be defined and the corresponding Denavit-Hartenberg parameters must be calculated. Then the transformation matrices can be determined.

In order to calculate the kinematics of fingers of the SDH2 the geometrical model of the fingers must be used. Since all the fingers have the same physical dimensions, only one finger model is required. Each finger consists of the lower proximal joint, the limb, the upper distal joint and the fingertip. The deformation of the tactile sensor elements is not taken into consideration in this analysis. The kinematic analysis of the two SDH 2 fingers used in this work is presented below.

Fig 6: Graphical interface of SDH2 tactile sensors.
2.4 Denavit-Hartenberg Parameters

In order to model the kinematics of the robotic hand, the modified (Craig's) Denavit-Hartenberg algorithm given in [1] is used. A coordinate system is assigned to each finger joint as well as to the points of contact A, B of the two coupled fingers, Finger 1 and Finger 2 as seen in Fig 7. We will refer to the reference frames by 0, 1, 2, 3, 4 for Finger 1 and 0', 1', 2', 3', 4' for Finger two according to the nomenclature of their axis. The analysis for Finger 1 is described below. For Finger 2 it is done in the same way. The articulation angles of the two fingers and their positive rotation directions are those indicated in Fig 1 and Fig 2.

Since the point of contact can be any point of the sensor's surface, a virtual joint is used to model the different points of contact introducing an extra DOF in each finger. It is located in the centre of the upper joint C and it is responsible for moving the virtual phalange CA. The length \( r_1 \) of CA and the angle \( \theta_1 \), Fig 8, it introduces to the Denavit-Hartenberg table will be calculated later. The length and the angle of Finger's 2 virtual joint are \( r_2 \) and \( \theta_2 \) respectively.

The Denavit - Hartenberg parameters of the two fingers are shown in Table 2 and Table 3. The reference frame \( 0' \) of Finger's 2 lies in the lower joint-3 of this finger. It is translated 66 mm at the y0 axis with respect to the global reference frame 0 that is in the lower joint-2 of Finger 1, Fig 2. A translation matrix multiplied before its transformation matrix introduces this translation.

<table>
<thead>
<tr>
<th>( a_i )</th>
<th>( a_l )</th>
<th>( d_i )</th>
<th>( \theta_l )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>90</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>86.5</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>68</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>( r_1 )</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: Denavit- Hartenberg parameters of Finger 1

<table>
<thead>
<tr>
<th>( a_i )</th>
<th>( a_l )</th>
<th>( d_i )</th>
<th>( \theta_l )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0'</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>1'</td>
<td>90</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2'</td>
<td>0</td>
<td>86.5</td>
<td>0</td>
</tr>
<tr>
<td>3'</td>
<td>0</td>
<td>68</td>
<td>0</td>
</tr>
<tr>
<td>4'</td>
<td>0</td>
<td>( r_2 )</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3: Denavit- Hartenberg parameters of Finger 2
In the following sections will be shown that the \( r_1 \) and \( \theta_1 \) of the virtual joint only depend on the height of the contact point in the sensors surface. Therefore the virtual joint adds an extra extra variable in the Denavit-Hartenberg tables and an extra DOF in the finger. Once the Denavit-Hartenberg parameters are determined, the transformation matrices that reference the contact points of the fingers at the global frame can be created as described by the algorithm given in [1]. The transformation matrices are functions of the finger's joint angles as well as of the virtual joints' parameters.

### 2.5 The virtual joint

The virtual joint \textbf{CA} is used to model the different points of contact in the cinematic analysis of Finger 1. The joint is described by the length \( r_1 \) and its angle \( \theta_1 \) with respect to the axis \( x_3 \), Fig 8. These are used as variables in the Denavit-Hartenberg parameters of Finger 1. The analysis is
limited in the plane y0z0 which is expressed by z3=0. So the z3 axis is ignored supposing that the virtual joint is only moving in this plane. The contact point A is expressed with respect to the axis system 3 as \((A_{x3}, A_{y3}, A_{z3})\), they can be calculated as a function of the coordinates of this point, \(A_{x3}\), \(A_{y3}\) by the following formulas:

\[
\begin{align*}
  r_1(A_{x3}, A_{y3}) &= \sqrt{A_{x3}^2 + A_{y3}^2} \\  
  \theta_1(A_{x3}, A_{y3}) &= \tan^{-1}\left(\frac{A_{x3}}{A_{y3}}\right)
\end{align*}
\] (2.1)

Although the varying angle in the upper articulation of Finger 1 is \(\phi_3\), we also need to introduce \(\Phi_3\), which is the angle \(CA\) is inclined with respect x2, the x-axis of the previous reference system. \(\Phi_3\) is related to \(\phi_3\) with

\[
\phi_3 = \Phi_3 - \frac{\pi}{2} + \theta_1 \quad (2.3)
\]

The \(r_2\) and \(\theta_2\) of the virtual joint in Finger 2, are calculated in the same way. Although the joint angle can be measured directly the calculation of the coordinates of A and B requires the information obtained by the tactile sensors. A further geometrical analysis has to be done in order to extract from this information A and B positions. Since Finger 1 and Finger 2 are

![Fig 8: Representation of the virtual joint and its parameters r1, \(\theta_1\).](image)
identical in their physical dimensions and the reference systems assigned to them in the rest of this work only the analysis of Finger 1 will be described. The equations of Finger 2 are calculated in the same way.

### 2.6 Fingertip Model

Since the touching surface of the finger is not planar, an equation that expresses this surface is needed to be developed. In the plane $x_3y_3$ the curve is described by a part of a circle with radius 60 mm and a part of a straight line as seen in Fig 9. The fingertip's physical dimensions are determined from the CAD data of the manufacturer.

It is necessary to express the $A_{y_3}$ as a function of $A_{x_3}$ for every contact point $A$ in the fingertip's surface. Two different cases can be distinguished: $A$ is in the planar surface, $A$ is in the cylindrical surface with centre $K(33.5, -45)$. The following formulas express $A_{y_3}$ as a function of $A_{x_3}$ for each case:

\[
A_{y_3} | A_{x_3}| = 15, \text{ for } 17.5 < A_{x_3} \leq 33.5 \quad (2.4)
\]

\[
A_{y_3} | A_{x_3}| = K_{y_3} + \sqrt{60^2 - (A_{x_3} - K_{x_3})^2}, \text{ for } 33.5 < A_{x_3} < 66.402 \quad (2.5)
\]

Using the equations (2.1) (2.2) (2.4) (2.5), $r_1$ and $\theta_1$ are expressed as a function only of $A_{x_3}$ as follows:

\[
r_1 | A_{x_3}| = \sqrt{A_{x_3}^2 + A_{y_3} | A_{x_3}|^2} \quad (2.6)
\]

\[
\theta_1 | A_{x_3}| = \tan^{-1}\left(\frac{A_{x_3}}{A_{y_3} | A_{x_3}|}\right) \quad (2.7)
\]
It is also necessary to calculate the equation of the tangent surface to the curved surface of the finger for each potential point A as well as the angle between this tangent surface and CA. In the plane the equation of the tangent line to the curve that describes the sensor at the contact point A and the angle $\varphi_1$ between the tangent line and the virtual joint are used. The same equations can be used in the 3D space supposing that the contact point has always the $z_3$ coordinate equal to zero.

In Fig 10.a the tangent line and the angle $\varphi_1$ are shown in case the point A is in the cyclical part of the curve and in Fig 10.b when it is in the linear part.

![Fig 9: Fingertip dimensions.](image)

![Fig 10: Tangent line and angle $\varphi_1$ when A is: a) on the cyclical part of the curve and b) on the linear part.](image)
The tangent line is described by its slope that is the derivative of the curve $Ay_3(A_{x3})$. It is given by:

$$\frac{dA_3}{dA_{x3}}(A_3) = 0, \text{ for } 17.5 < A_{x3} \leq 33.5 \quad (2.8)$$

$$\frac{dA_{y3}(A_{x3})}{dA_{x3}} = -\frac{A_{x3}-K_{x3}}{\sqrt{60^2-(A_{x3}-K_{x3})^2}}, \text{ for } 33.5 < A_{x3} < 66.402 \quad (2.9)$$

The angle $\psi_1$ between the tangent line at the point of contact and the virtual joint is actually associated to the normal to the surface of the finger. It will be later used to calculate the direction of the contact forces and ensure that they are inside the friction cone. It only depends on the angle $\theta_1$ that the virtual joint is inclined with respect to the axis $y_3$, thus it is also a function of the coordinates of $A$.

At the cyclical part of the curve, $\phi_1$ is calculated as the external angle of a triangle that is formed by $CA$, the tangent at $A$ and the $x_3$ axis. $\psi_1$ is the sum of the two opposite angles $g_1, g_2$. The angle $g_1$ is the complementary angle to $\theta_1$. The angle $g_2$ is calculated using the slope of the tangent $dA_{y3}/dA_{x3}$. In fact it is given by the inverse tangent of the slope at this point of touch. The angles are calculated as:

$$\psi_1 = g_1 + g_2,$$

$$g_1(A_{x3}) = \pi - \theta_1(A_{x3}) \quad (2.11)$$

$$g_2(A_{x3}) = \tan^{-1} \left( \frac{dA_{y3}(A_{x3})}{dA_{x3}} \right) \quad (2.12)$$

From (2.10) (2.11) (2.12) we conclude that $\psi_1$ is a function of $A_{x3}$ and is calculated by:

$$\psi_1(A_{x3}) = \frac{\pi}{2} - \theta_1(A_{x3}) + \tan^{-1} \left( \frac{dA_{y3}(A_{x3})}{dA_{x3}} \right) \quad (2.13)$$

At the linear part of the curve its derivative is zero so the angle $g_2$ is also zero. The expression (2.13) is converted into:

$$\psi_1(A_{x3}) = g_1(A_{x3}) + g_2(A_{x3}) = g_1(A_{x3}) = \frac{\pi}{2} - \theta_1(A_{x3}) \quad (2.14)$$

that is the expression of the internal angles of a right triangle shown in Fig 10.b formed of the linear part of the curve, $CA$ and the $y_3$-axis. Thus the expression (2.13) is also valid for this part of the
finger.

In fact, the coordinates of A are never given directly. The tactile sensor provides information about at which height of its surface the contact is detected. Since the sensor is curved, the provided height of contact is measured on the curved surface. The length $L_1$ from the bottom of the sensor I to A, Fig 11, can be calculated using the sensor's information. H is the point where the sensor's curve changes from line to circle thus the segments IH and GE are equal. Two cases are distinguished: A is in the planar surface when $L_1 < 16$ mm and A is in the cyclical surface when $L_1 > 16$ mm. Thus the vertical distance of A from the origin of the coordinate system 3, $Ax_3$ is calculated as:

$$A_{x3}(L_1) = CE + L_1 = 17.5 + L_1, \text{ for } L_1 < 16 \text{ mm} \quad (2.15)$$

$$A_{x3}(L_1) = CG + |L_1 - IH| = 33.5 + r_1 \sin \frac{L_1 - 16}{r_1}, \text{ for } L_1 \geq 16 \text{ mm} \quad (2.16)$$

![Fig 11: Definition of the variable $L_1$ in the fingertip's surface.](image)

### 2.7 Fingertip Tactile Sensor Analysis

The position in mm of all tactile elements having non-zero force response are utilized unto the creation of a pressure profile. The DSA 9210 sensor applies an internal pressure threshold thus no further filtering is required. The texels that receive pressure under this threshold return zero value. A 6 x 13 pressure matrix is returned.

The height of the contact point in the sensor's surface, $S$ is calculated from this matrix. The function given with the manufacturer's software calculates it as:
\[ S = \frac{\sum_{i=0}^{5} \sum_{j=0}^{5} j \cdot \text{pressure}_{ij} \cdot \text{texelheight}}{\sum_{i=0}^{5} \sum_{j=0}^{5} \text{pressure}_{ij}} \] (2.17)

where \( \text{pressure}_{ij} \) is the pressure measured in the texel that lies in the \( i \)-th column and in the \( j \)-th row. We will refer to the contact point on the sensor of Finger 1 and 2 with \( S_1 \) and \( S_2 \) respectively.

\( S \) takes numerical values in the range [0-40.8], that corresponds to the distance between the centre of the first and the last texel of a column. The first row of cells is considered to be the upper one at the sensor's surface giving the zero value at the top of the fingertip, as seen in Fig 12. These results can have a deviation of \( \pm 1.7 \text{ mm} \) due to the texel edge size. As discussed section in 2.2 the texel uncertainty is greater than other possible sources of measurement uncertainty and is the principle factor of uncertainty at the calculations.

In order to calculate \( L_1 \), first an assignation need to be made between the values the distal tactile sensor gives and their significance in mm on its surface. The sensor's external surface has a total height of 48.5 mm. However the height of the sensor's active area is 44.2 mm, as it is formed of 13 rows of cells (texels) with approximate height 3.4 mm. The position of the centre of each texel is given by the manufacturer [8] so considering this texel height the position of the edges of the active area is determined as shown in Fig 13.

**Fig 12:** Sensor's active area and range of the provided values.
An empirical measure of the values that the sensors gives for several points of contact in its surface indicates that the increment of these values for a constant increment of the real distance in mm is not totally linear. We start counting from the top of the whole sensors surface and we make a matching between the real point of contact in its surface and the values provided. The obtained results are shown in Table 4.

The increment of the sensor's values for every 2 mm of increment of the real distance in the whole sensors surface is presented in Fig 14. At first look we can see that the relation between the sensor's values and the real distance is not linear. Applying a constant increase of 2 mm in the sensor's surface the difference of the values tend to be smaller while we get closer to the sensor's extremes, fig. In the central part of the sensor's surface, between 5.5 mm and 42.5 mm the increment of the results for constant real distance increase is approximately linear considering possible uncertainties introduced by realizing the process manually. Therefore a non-linearity is observed at the extremes.

Fig 13: DSA 9210 location of texels in the active area as given by [8]
Table 4: Results of the empirical measure: distance in mm of the contact from the top of the sensor’s surface, the values given by the sensor, the difference between each sensor’s value and the previous.

<table>
<thead>
<tr>
<th>distance (mm)</th>
<th>sensor values</th>
<th>d(sensor values)</th>
</tr>
</thead>
<tbody>
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<td>48.50</td>
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</tr>
<tr>
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<td>0.50</td>
</tr>
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<td>1.06</td>
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<td>38.94</td>
<td>1.86</td>
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<td>2.12</td>
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<tr>
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<td>0.00</td>
</tr>
</tbody>
</table>

Fig 14: Increment of the sensor’s values for every 2 mm of increment of the real distance of the point of contact on its surface.
The sensor reacts in contact from its 1,5 mm to 45,5 mm counting from the top, as expected, which gives 44 active mm in the sensor. This is compatible with the expectable active area considering that the texel's height was approximately 3,4 mm. Then we can locate the active area 1,5 mm below the upper part of the sensor and 3 mm above its lower part as seen in Fig 12.

So we will match sensor values to mm only at the active area. Besides in the linear central part of the sensor the real distance values are given by the sensor values increased by 3 mm, which would be the distance between the top edge of the sensor's surface and the centre of a cell in the first row. Thus shifting the real distance values down by 3 mm, the sensor's values give the real distance between the point of contact and the centre of a cell in the first row, as shown in Table 5.

By real distance we will now refer to the shifted distance since the only thing that changes is the point of the sensor from which the measure starts. Then the two curves of the real (blue) and the calculated by the sensor (red) distance -the sensor values- are shown in Fig 15. In Fig 16 the error between those two is performed for every test value. The sensor calculates well the real distance for the area between 1,5 mm and 39,5 mm, with an error less than 1 mm. This means that there are about 38 mm of the active area's height where the distance is well calculated with an error less than ±1mm, while 42 mm are calculated with an error less than the expected error of ±1mm (texel's resolution). Outside of this area the distance deviations are great.

<table>
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<th>sensor values</th>
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Table 5: Results of the empirical measure: distance in mm of the contact from the top of the sensor's surface, the same distance shifted by 3 mm, the values given by the sensor, the error calculated as the difference between each sensor's value shifted distance.
However it is desirable to obtain sufficiently good results for the whole active area of the sensor and if possible outside of it. It is possible that a contact is detected outside the active area since some force is observed to be applied to some texels when we push the rubber at a close point. In an effort to map the sensor's values to real distances for a greater distance range we found out that a good mapping of the distance was obtained when the sensor values are multiplied with the active distance in mm and divided by the range of the tactile sensor's values. The distance calculated is the distance between a contact point and the upper edge of the active sensor area,

\[
\text{measured}\_\text{distance} = S \frac{44\text{mm}}{40.8} \tag{2.18}
\]

The calculated distance using the tactile sensor with respect to the real distance from the active
area's upper edge is presented in Table 6. Despite the non-linearity the results approximate well the reality giving an error smaller than \( \pm 1 \) mm. Fig 17 shows the two curves of the real (red) and the measured (blue) distance. It can be seen in Fig 18 that the error between those two curves for every test value is always bordered between \( \pm 1 \) mm.

With this method we have obtained a good matching with an error less than \( \pm 1 \) mm for an area with height 46 mm (for real distance values between -1 and 45 mm). Although the error of the calculated distance in the central part of the sensor has been augmented compared with the previous calculation technique, its still smaller than 1 mm. So in the experiments the second method will be used in order to take advantage of the greater sensing range.

<table>
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<tr>
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<td>-1,50</td>
</tr>
</tbody>
</table>

*Table 6: Real distance of the point of contact in mm, calculated distance by the sensor after the adjustment tactile values adjustment and error between them.*
Finally the curved distance $L_1$ is calculated, taking into account the 2.5 mm between the fingertip's base and the sensor's area and the 3 mm of inactive area in the sensor's surface. $L_1$ is measured beginning from the fingertip's base $I$, while the measured distance is measured from the upper point of the sensor's surface to the point of contact $A$. It is given by the formula:

$$L_1 = |44 - \text{measured\_distance}| + 5.5 = (44 - S \frac{44}{40.8}) + 5.5$$

$$L_1 = 49.5 - S \frac{44}{40.8} \text{ (2.19)}$$

---

*Fig 17: Real (red) and the calculated by the sensor (blue) distance in mm after the tactile values' adjustment for every test value.*

*Fig 18: Error between the real distance and the calculated by the sensor distance in mm, after the adjustment of the sensor values. The horizontal axis represents the real distance.*
2.8 The Forces

The total contact force at the sensor's surface when a contact is detected is calculated as the sum of the pressure measured at each one of its texels multiplied by the texel's area, as

\[ F_k = \sum \sum \text{pressure}_{ij} \cdot \text{texelheight} \cdot \text{texelwidth}, \quad k = 1.2 \quad (2.20) \]

where \( F_1 \) and \( F_2 \) are the measured contact forces in Finger 1 and Finger 2 respectively.

During the manipulation, it is desired to adjust the distance between the fingers and specifically between the contact points in order to control the contact force and maintain it around a fixed value. In this aim it is needed to have a model of the way the values of the contact forces provided by the sensor change when the object is pressured or relaxed. Specifically it is desirable to match the contact force increment or decrease to a distance in mm that the fingers should approach each other or fend off in order to bring the force back to its previous value.

However lacks a physical model of the values given by the sensor when a force is applied to it. The sensor is not giving identical values when identical stress is applied to it two different times. Additionally the sensor's values are not constant when constant stress is applied to it [22]. The KIT investigation group [13] that collaborates with the manufacturer company [7] reports that the textel output value is rising in spite of unchanging charge. Consequently a model of the sensor's performance is difficult to be obtained.

During the manipulation process, the pressure values provided by the sensor for the same charge are not constant. We also observed that these values change depending of the time the hand is in use. An empirical method was realized to investigate the way of changes of the contact force. Specifically an experiment was realized in which an object was grasped with a desired contact force. Then one finger was moved in order to translate the contact point inwards and outwards the object by 1 mm in the y0 axis and the contact force was measured. This process was repeated 3 times. The obtained results are performed in Chapter 4.

2.9 The Grasp

Before the manipulation process the object is grasped using a prismatic precision grasp [19] [10]. This grasp only utilizes the two of the three fingers and specifically the coupled ones. It is comparable with a human grasp with the thumb and index finger opposed. The coupled angle \( \phi_1 \) is constantly set at 90 degrees and the two fingers have moved in a “mirror” pose. The third finger is not used and it is positioned so as not to interrupt the movement or collide with the other fingers. This grasp position will be maintained during the whole experiment.

The actual DOFs being used in this work are two for each one of the fingers. Thought an extra DOF is added by the virtual joint. The four DOFs are expressed through the varying joint angles. When the two fingers touch an object the points of contact in the two fingers and the intermediate points \( A, B, C, D \) can be calculated knowing the angles \( \phi_2, \phi_3, \phi_4, \phi_5 \) and \( Ax_3 \). The coordinate \( Ax_3 \) is calculated using the tactile information.
With $\phi_1$ set to $\pi/2$ the actual coordinates of the points $A$, $B$, $C$, $D$ with respect to the global coordinate system-$\theta$ are:

$$
A = \begin{bmatrix}
0 \\
86.5\sin[\phi_2 + r_1 \cos[\phi_2] \cos[\phi_3 - \theta_1] - \sin[\phi_2] \sin[\phi_3 - \theta_1]] \\
86.5\cos[\phi_2 + r_1 \cos[\phi_3 - \theta_1] \sin[\phi_2] - \cos[\phi_2] \sin[\phi_3 - \theta_1]] \\
1
\end{bmatrix}
$$ (2.21)

$$
B = \begin{bmatrix}
0 \\
66 - 86.5\sin[\phi_4 + r_2 \cos[\phi_4] \cos[\phi_5 - \theta_2] + \sin[\phi_4] \sin[\phi_5 - \theta_2]] \\
86.5\cos[\phi_4 + r_2 \cos[\phi_5 - \theta_2] \sin[\phi_4] - \cos[\phi_4] \sin[\phi_5 - \theta_2]] \\
1
\end{bmatrix}
$$ (2.22)

$$
C = \begin{bmatrix}
0 \\
86.5\sin[\phi_2] \\
86.5\cos[\phi_2] \\
1
\end{bmatrix}
$$ (2.23)

$$
D = \begin{bmatrix}
0 \\
66 - 86.5\sin[\phi_4] \\
86.5\cos[\phi_4] \\
1
\end{bmatrix}
$$ (2.24)

while $r_1$, $\theta_1$, $r_2$, $\theta_2$ are calculated from the tactile information.

In the 2D space the object is represented by the segment $AB$ that connects its points of contact $A$, $B$ at the two fingers. Since in 3D we may have contact areas instead of contact points, the centres of gravity of these areas are used as points of contact.

### 2.10 Force Closure

When there is a punctual contact with friction between the robotic finger and the object, in order to avoid sliding the set of forces must be located within a cone centred about the normal to the object's surface at the point of contact. The angle of this cone, called the friction cone, is given by

$$
\alpha = \tan^{-1} \mu
$$

where $\mu$ is the coefficient of static friction. Any force applied within the friction cone will not produce slippage (Coulomb friction model).
When we refer to a force-closure grasp we mean a grasp capable of preventing any motion of the objects due to external forces and torques. A planar grasp with two contact points with friction, is force-closure, when the line connecting the contact points lies inside both friction cones Fig 20. The friction cone at each different contact point on a fingertip's surface lies around the fingertip's normal at this point. The normal at a point of the fingertip's curved surface is the perpendicular vector to this surface for this point.

When an object is grasped and manipulated with two punctual contacts A and B at the plane (y0z0) the angle between the normal and AB should always be smaller than the friction angle $\alpha$. In this work we use the angle between the object and the tangent to the fingertips surface. This is the orthogonal to the normal. Then the above condition can be expressed as

$$\pi/2 - \alpha < \omega < \pi/2 + \alpha \quad (2.24)$$

where $\omega$ is the angle between the segment AB and the tangent to each fingertips surface as shown in Fig 20.
Fig 20. The equation above represents a condition that must be met during all the manipulation process. With $\omega_A$ and $\omega_B$ we will refer to the angle between the segment $\mathbf{AB}$ and the tangent at the contact points $\mathbf{A}$ and $\mathbf{B}$ respectively.

### 2.11 Angles to the Normal Vector

When a linear object $\mathbf{AB}$ is grasped and manipulated using the two fingers it is necessary to ensure that the segment $\mathbf{AB}$ is always inside the friction cone. As explained in the previous section the angle between $\mathbf{AB}$ and the tangent at each point of contact $\mathbf{A}$ and $\mathbf{B}$ are used in order to accomplish this condition.

It must be ensured that $\omega_A$ and $\omega_B$ are inside the range $[\pi/2 - \alpha, \pi/2 + \alpha]$. The following conditions must be met during the whole manipulation movement:

$$\pi/2 - \alpha < \omega_A < \pi/2 + \alpha \quad (2.25)$$

$$\pi/2 - \alpha < \omega_B < \pi/2 + \alpha \quad (2.26)$$

where $\alpha$ is the friction angle.

The angle $\omega_A$ of Finger 1 can be calculated reducing the angle between $\mathbf{CA}$ and the tangent, $\delta_1$ from the angle between $\mathbf{CA}$ and $\mathbf{AB}$, $\delta_2$, as seen in Fig 21.

$$\omega_A = \delta_2 - \delta_1 \quad (2.27)$$

![Diagram showing angles and tangents for manipulation](image)

*Fig 21: The angle between $\mathbf{AB}$ and the tangent $\omega_A$, the angle between $\mathbf{CA}$ and the tangent $\delta_1$ and the angle between $\mathbf{CA}$ and $\mathbf{AB}$ $\delta_2$.]*
The angle $\delta_2$ is the angle between the virtual phalange and the segment AB that connects the two points of contact and indicates the orientation of the object. $\delta_2$ is calculated using the law of cosines in the triangle CAB. Given the joint angles, $\phi_2, \phi_3, \phi_4, \phi_5$ and the tactile information $S_1 S_2$, the coordinates of A, B, C and D with respect to the global frame-0, are calculated with forward kinematics. Therefore the angle $\delta_2$ and consequently the angle $\omega_A$ is a function of the fingers' angles and $L_1$.

$$\delta_2(\phi_2, \phi_3, \phi_4, \phi_5, L_1) = \cos^{-1}\left(\frac{-|CB|^2 + r_i |L_i|^2 + |AB|^2}{2 \cdot r_i |L_i| \cdot |AB|}\right)$$

$$\omega_A(\phi_2, \phi_3, \phi_4, \phi_5, L_1) = \cos^{-1}\left(\frac{-|CB|^2 + r_i |L_i|^2 + |AB|^2}{2 \cdot r_i |L_i| \cdot |AB|}\right) + \frac{\pi}{2} + \theta_1 - \tan^{-1}(A_{y_3} |L_i|)$$

Following the same process we determine $\omega_B$

$$\omega_B(\phi_2, \phi_3, \phi_4, \phi_5, L_2) = \cos^{-1}\left(\frac{-|AD|^2 + r_j |L_2|^2 + |AB|^2}{2 \cdot r_j |L_2| \cdot |AB|}\right) + \frac{\pi}{2} + \theta_2 - \tan^{-1}(B_{y_3} |L_2|)$$

2.12 Inverse Kinematic Model of Finger 1

The inverse kinematics for a robotic finger could be defined as: Given a position and orientation of the contact point A determine the different possible values that each joint of the finger should have. The inverse kinematics can be determined with various methods. In this work since only two DOFs are used for each finger the fingers only move in the plane y0z0 [9] A solution can be directly calculated geometrically.

The angles $\phi_2, \phi_3$ can be expressed as a function of the position of A ($A_{x_3}, A_{y_3}, A_{z_3}$), when this is expressed in the global frame,

$$\phi_2 = f_1(A_{y_0}, A_{z_0})$$

$$\phi_3 = f_2(A_{y_0}, A_{z_0})$$

In order to calculate the angle of the upper articulation $\phi_3$ first the angle of the virtual joint $\Phi_3$ need to be calculated. Then $\phi_3$ is given by (2.3) that is repeated here for convenience:

$$\phi_3 = \Phi_3 - \frac{\pi}{2} + \theta_1$$
The angles of the triangle OCA, c and o shown in Fig 22 are calculated using the law of cosines at this triangle. Then the angles φ2 and φ3 will be expressed through c, o and γ for each configuration of Finger 1.

\begin{align*}
c &= \cos^{-1}\left(\frac{-|OA|^2 + |OC|^2 + |CA|^2}{2 \cdot |OC| \cdot |CA|}\right) \\
o &= \cos^{-1}\left(\frac{-|CA|^2 + |OC|^2 + |OA|^2}{2 \cdot |OC| \cdot |OA|}\right) \\
γ &= \tan^{-1}\left(\frac{A_{z3}}{A_{y3}}\right)
\end{align*}

Since 2 DOFs are used in the plane, sometimes the same point A can be reached with two different angle combinations as shown in Fig 23 and Fig 24. They can be distinguished by the sign of the angle Φ3. Fig 23 shows the three possible configurations that may occur if the angle Φ3 is positive. The three configurations that may occur if the angle Φ3 is negative are shown in Fig 24.

When Φ3 is positive three different cases are distinguished. In case 1 the whole finger (A and C) is at y0 positive, in case 2 it is in y0 negative and case 3 C is in y0 negative and A in positive.

Only the absolute value of the angles o and c are used, while γ, φ2, Φ3 take positive and negative values. When Φ3 is positive the angles φ2, φ3 are given by:

\begin{align*}
φ2 &= -o - γ + \text{sign}(A_{y0}) \ast \frac{π}{2} \\
φ3 &= Φ3 - \frac{π}{2} + \theta_1 = -c + \frac{π}{2} + \theta_1
\end{align*}
When $\Phi_3$ is negative $\varphi_2$ and $\varphi_3$ are expressed by:

$$
\varphi_2 = o - \gamma + \pi/2 \quad \gamma > 0 \\
\Phi_3 = \pi - c
$$

$$
\varphi_3 = \Phi_3 - \pi/2 + \theta_i = c - \frac{3\pi}{2} + \theta_i 
$$

where “sign(x)” represents the function:

Fig 23: Inverse kinematics for the 3 different possible configurations with $\Phi_3$ positive.

Fig 24: Inverse kinematics for the 3 different possible configurations with $\Phi_3$ negative.

When $\Phi_3$ is negative $\varphi_2$ and $\varphi_3$ are expressed by:

$$
\varphi_2 = o - \gamma + \text{sign}(A_{y_0}) \times \pi/2 
$$

$$
\varphi_3 = \Phi_3 - \pi/2 + \theta_i = c - \frac{3\pi}{2} + \theta_i 
$$

where “sign(x)” represents the function.
\[
\text{sign}(x) = \begin{cases} 
1 & \text{if } x \geq 0 \\
-1 & \text{if } x < 0
\end{cases}
\]

So for these points of the workspace of Finger 1 that are accessible by two different configurations, there are two solutions of two different angle combination. Normally in a process that the fingers moves in a continuous way the one that is closer to the previous configuration is chosen.
Chapter 3
Manipulation with Two Robotic Fingers

3.1 Conceptual Analysis of the Manipulation Movement

Beginning from an initial know grasping position, we want to move the two fingers in order to manipulate the grasped object and variate its orientation. The object is grasped by the two coupled fingers, Finger 1 and Finger 2.

In the conceptual analysis of the movement, it is always useful to be inspired by the human way of doing it. Humans usually realize this kind of movements using the index finger and the thumb, that have more DOFs than the fingers of the SDH 2 robotic hand. However we can focus on the essence of the movement and try to analyse it.

Fig 25: Examples of objects's rotation in one plane with the thumb and the index finger.

Empirically observing the human executing movements with the index finger and the thumb like opening a bottle, leads to the following conclusions. The thumb's anatomy makes its movement in one plane difficult. When these two fingers rotate an object in one plane, as shown in Fig 25, the thumb's way of move is a simple open-close movement. The index finger has more DOFs in the plane and is charged with doing the most complicated part of the movement. It moves in a “circular way” around the other and adjusts the distance between the fingers so that the pressure applied to the object don't relaxes the object doesn't fall. The thumb's role seems to be only to facilitate a fluent movement the other finger, and moves in order to help the index remain in its workspace.

In this way the movement of the two fingers is being perceived as if the thumb is following an
independent track while the index finger intends to rotate around it. The movement must stop when
the fingers can no longer maintain the object because the applied force exceeds the friction cone
limits.

Therefore we will try to do a manipulation in the same way. One of the two fingers is given the role
of the thumb following an independent route and the other is trying to follow it. While the first
moves the second one is constantly trying to change the orientation of the object and maintain a
certain distance between the fingers so that the object is not over-pressured or let to slip. The
distance between the contact points need to be adjusted using the tactile information about the
contact force. In this way the fingers with follow the object's shape changes while it rolls between
them. The two fingers don't move sequentially. The dependent one knows beforehand the next move
of the independent, so its response is simultaneous as if there were a brain that coordinates them.
The movement has to stop just before the friction cone limits are exceeded. We will discuss later
how an optimal route of the independent finger could be defined.

3.2 Solution Approach

In this work we realize the manipulation of an object grasped by two robotic fingers with two free
DOFs in a prismatic grasp (Section 2.9). When an object is grasped by the two fingers the
manipulation process consists of two tasks. Make one finger follow an independent route and make
the other finger respond to maintain the object without falling and change the orientation of the
object. These two tasks are coordinated and carried out simultaneously. The independent movement
of the first finger could be adopt to satisfy some other constraint or to follow a particular goal.

The tactile information obtained by the sensors will be used in order to adjust the distance between
the fingers and hold the object firmly. During the manipulation the distance between the two points
of contact at the object's surface may vary. Also when a curved object is manipulated it will roll in
the fingertip's surface. The tactile information is used to know at any time at which point of the
fingertip’s surface the object touches and to detect a variation of the distance when the value of the
applied forces change.

To determine how each one of the fingers should move in order to realize the manipulation of the
object we first study the movement of the dependent finger while the other remains static. In the
next section we will describe how this finger should move in order to vary the orientation of the
object. Then we will combine it with the other finger's movement.

The analysis begins considering that the manipulated object is a thin stick. In this case the distance
between the points of contact in the two fingers doesn't change and there is no rolling in the
fingertip's surface, so the points of contact in the sensors surface remain the same during all the
manipulation. At first the tactile information is used only to determine the initial point of contact.
Later this information will be used to extend the analysis towards the manipulation of more
complicated objects through rolling detection. The tactile information will indicate when the points
of contact in the sensor change and the fingers distance need to be adjusted so that the applied
forces remain around a certain value.
3.3 Object Manipulation with Only One Finger

In this section we will study the movement of the dependent finger when the other is remaining still during the process. Since the independent finger could follow different routes, remaining still would be one of them. In this way it is possible to analyse the movement of the dependent finger.

Consider the simple case in which the grasped object is a thin stick whose length is given by the distance between the two points of contact in the two fingers, \( A \) and \( B \).

\[
A_{AB} = \sqrt{(A_{x0} - B_{x0})^2 + (A_{y0} - B_{y0})^2 + (A_{z0} - B_{z0})^2} \quad (3.1)
\]

In order to achieve the desired movement we demand changing the inclination of the object while maintaining the distance between the two points of contact constant. Finger 2 remains still in space and there is no rolling, thus the position of point \( B \) is unchanging. Then \( A \) should run a circle around \( B \) with radius equal to \( A_{AB} \) as seen in Fig 26 where \( A^c \) is the position of \( A \) at the current configuration. The equation of this circle is given by:

\[
(A_{y0} - B_{y0})^2 + (A_{z0} - B_{z0})^2 = A_{AB}^2 \quad (3.2)
\]

We demand a consecutive change in object's inclination. Given the \( A, B \) the inclination of the grasped object with respect to the absolute coordinates system-\( 0 \) is given by:

\[
\sigma_{AB} = \tan^{-1}\left(\frac{B_{z0} - A_{z0}}{B_{y0} - A_{y0}}\right) \quad (3.3)
\]

**Fig 26:** The solutions of the equation system.
Actually, the two DOFs of each finger only permit A to move on the left semicircle. In this case we have \( A_{y0} < B_{y0} \), so the possible position of A is given by the solution of (3.1):

\[
y_{0} = B_{y0} - \sqrt{A_{x0}^2 - (z_{0} - B_{z0})^2} \quad (3.4)
\]

Given the coordinates of point B, \((B_{y0}, B_{z0})\), the distance \(\Lambda_{AB}\) of the object and a desired inclination of the object \(\sigma_{AB}^{d}\), the coordinates of A are calculated as the solution of the system of equations (3.4) and

\[
\tan^{-1}\left(\frac{B_{z0} - A_{z0}}{\sqrt{A_{x0}^2 - (A_{z0} - B_{z0})^2}}\right) = \sigma_{AB}^{d} \quad (3.5)
\]

Replacing (3.4) in equation (3.5) and solving the second for the variable \(z_{0}\) we get following solutions of \(z_{0}\):

\[
A_{x01} = \frac{B_{z0} + B_{z0} \cdot \tan(\sigma_{AB}^{d})^2 - \sqrt{A_{x0}^2 \cdot \tan(\sigma_{AB}^{d})^2 + A_{x0}^2}}{1 + \tan(\sigma_{AB}^{d})^2} \quad (3.6)
\]

\[
A_{x02} = \frac{B_{z0} + B_{z0} \cdot \tan(\sigma_{AB}^{d})^2 + \sqrt{A_{x0}^2 \cdot \tan(\sigma_{AB}^{d})^2 + A_{x0}^2}}{1 + \tan(\sigma_{AB}^{d})^2} \quad (3.7)
\]

The above equations are used to calculate the \(z_{0}\)-coordinate of the desired new position of A, \(A_{x0}\). If \(A^{1}\) is the desired position the equations (3.6) and (3.7) give the \(z_{0}\)-coordinate of \(A^{1}, A_{x0}^{1}\) and of its diametrically opposite point \(A^{2}, A_{x0}^{2}\) as shown in Fig 26.

Replacing the two solutions \(z_{01}\) and \(z_{02}\) in (3.4) that calculates the \(A_{y0}\) the same value is obtained for both of them. In fact since \(A_{y0}\) is limited in the left semicircle, this equation gives the \(A_{y0}^{1}\) and the \(A_{y0}^{2}\). \(A^{3}\) is the symmetrical of \(A^{2}\) at the left semicircle. So the \(y_{0}\)-coordinate of the frame-0 is calculated by

\[
A_{y0} = A_{y0}^{i} = B_{y0} - \sqrt{A_{x0}^2 - (A_{z0} - B_{z0})^2} \quad i = 1,2 \quad (3.8)
\]

Only one of the two calculated points \(A_{y0}^{1}(A_{y0}, A_{z0}), A_{y0}^{2}(A_{y0}, A_{z0})\) satisfies the equation (3.3). So this equation will be used as a condition in order to choose the right solution of \(A_{z0}\),

\[
IF \tan^{-1}\left(\frac{B_{z0} - A_{z0}}{B_{y0} - A_{y0}}\right) = \sigma_{AB}^{d} \quad THEN \quad A_{z0} = A_{z0}^{1}, i = 1,2 \quad (3.9)
\]

The desired inclination is calculated as the sum of the current inclination and a desired increment \(d\sigma\).
\[ \sigma_{AB}^d = \sigma_{AB} + d\sigma \] (3.10)

### 3.3.1 The Moving One Finger Algorithm

The manipulation is a repeating process during which the configuration of Finger 2 remains fixed while the configuration of Finger 1 is calculated every time so as to change the inclination of the object without changing the distance between the two points of contact A and B. This distance must remain constant during the process. The rotation direction changes when the friction cone limits tend to be exceeded or when the limits of the finger's workspace are reached. The steps of the algorithm are as follows.

1) The two fingers are closed until they grasp the object. The object's length ($\Lambda_{AB}$), the points of contact at the fingertips' surface ($S_1, S_2$) and the position of points B and D are unchanging and are calculated once at this initial position. The initial position of A is also calculated.

2) A desired change in inclination for every step ($d\sigma$) is defined.

**While** (Inside_Friction_Cone)

2a) The desired position of A is calculated in order to move in circle around B varying the inclination of the object by $d\sigma$.

2b) The new values of the angles $\phi_2, \phi_3$ that move A to the desired position are calculated with inverse kinematics.

2c) The new position of point C is calculated with direct kinematics.

2d) For this configuration the angles between the object and the tangent surface of the fingertips, $\omega_A$ and $\omega_B$, are calculated in order to ensure that the friction cone restrictions are satisfied.

2e) If (the new configuration is inside the fingers' workspace and the forces don't exceed the friction cone limits and the contact is not lost)

   
   \{ 
   \text{the finger is driven to this new configuration by setting its articulation angles at the calculated values.} 
   \}

Else

   
   \{ 
   \text{Inside_Friction_Cone=False} 
   \text{the rotation is changing direction.} 
   \}
3.3.2 Simulation Results of One Finger Movement

A simulation of this process has been executed. The manipulated objects is a stick with length 73.8 mm. The initial upper and proximal joint angles for each of the two fingers are given close values as it would happen in a real grasp. The points of contact on the sensors surfaces \( S_1 \) and \( S_2 \) are also given close values.

As shown in Finger 1 starts from an initial configuration (red lines). It moves changing the orientation of the object first at the direction that decreases its inclination and then at the direction that increases it. When it reaches the extremes of the friction cone the moving direction is reversed. The initial (red) and the two extreme configurations of the fingers and specifically of their proximal and virtual joint are represented. The extreme positions at the direction that the inclination increases (anticlockwise rotation) and at the direction it decreases (clockwise rotation) are represented with green and blue lines respectfully.

In Fig 27.a the initial and the extreme configurations of Finger 1 as well as the movement of A are presented. The least is also presented in Fig 27.b where part of the circle A runs around B is observed. The three configurations as well as the object position at each is demonstrated in Fig 27.c, while in Fig 27.d the variation of the object's inclination can be observed. The change of the angles, \( \varphi_2, \varphi_3 \) of the angles between the object and the tangent surface of each finger are seen in Fig 27.e and Fig 27.f.

As seen in Fig 27.b, while Finger 1 is moving the object's inclination changes towards the desired direction. The distance between the fingers remains constant since A is drawing a circle around B. However the range of the permitted movement of Finger 1 is small due to the configuration where Finger 2 remains still that does not facilitate its movement.

It can be observed in Fig 27.f that the variation of the angle \( \omega_A \) in Finger 1 is significantly greater than the variation of \( \omega_B \). \( \omega_A \) takes all the range of the permitted values limits during the movement. At both of the extreme configurations that the forces tend to exceed the friction cone limits \( \omega_A \) is the one that tends to get out of its range. Possibly if Finger 2 could move in a way that its angle with the tangent surface would take angles in a greater range, then the range of the movement would be augmented.

Concluding, it was verified that when the independent finger moves in the way described in the previous section, it actually executes the desired task. The object is rotated as wished. So the assumption about the way the independent finger should move looks to be correct.
Fig 27: Simulation results for an object with length 73.8 mm. a) the initial configuration (red) and the extreme configurations just before the forces exceed the friction cone (green), (blue), b) the route of point A, c) The configurations of “a)” where the object is also represented, d) object’s orientation for each of the previous configurations, e) the joint angles of Finger 1 and f) the angles ω for each repetition of the algorithm.
3.4 Object Manipulation Moving Both Fingers

When the independent finger is following a certain route, the dependent finger is moving around it in the way described in the previous section. The difference is that point A is no longer rotating around a fixed point, but plays the role of a satellite of point B that moves around it and follows its movement at the same time. Point A should not only react by doing this “circular” movement around B, but also by rotating counterwise when Finger 2 reverses the movement.

In this section it is studied the coordination of both fingers in order to manipulate the object. The way of move of the independent finger need to be determined as well as the dependent finger's response to it. It is considered again the simple case of manipulating a stick in order to obtain conclusions that could be used for different objects.

In an attempt to define the way the independent finger should move, we try to imitate the movement of the thumb when the human is realizing a similar movement. Empirically, we conclude that the thumb does a simple open-close movement, that could be translated as closing its proximal articulation and opening its upper articulation to increase the object's inclination and vice versa. In Fig 28, this open-close movement and its effect on the object's inclination is shown.

![Thumb's open-close movement.](image)

The angle of the proximal articulation $\phi_4$ will change at a lower rhythm than $\phi_5$ since its variation produces greater effect on the position of the final point B than that of $\phi_5$. Thus the new values (Fig 29) of Finger 2 angles will be given at every step of the algorithm by:

\[
\begin{align*}
\phi_4^d &= \phi_4^c - \Delta \phi \\
\phi_5^d &= \phi_5^c + k \times \Delta \phi , k > 1
\end{align*}
\]

48
where
k: a multiplication factor
Δφ: a desired change of the angle,
Δφ>0 for clockwise rotation, Δφ<0 for anticlockwise

Since Finger 2 can move in different ways, different values of the above parameters are accepted. We will discuss how this movement could be optimized.

For the good coordination of the fingers, Finger 1 must detect the tendency of changing the object inclination by Finger 2 and responds by reinforcing it. When Finger 2 tries to rotate the object at the opposite direction, this should also be detected and reinforced. For this reason the σ^d of the desired configuration is calculated as follows:

Consider a current configuration of the fingers such that the points A, B are at the positions A^c, B^c and the object's inclination is σA^cB^c (green), as seen in Fig 30. The positions of the A, B at the next desired configuration A^d, B^d with object inclination σA^dB^d (blue) are calculated as follows.
When the desired position of $B^d$, $B_d$ is defined as explained above, we calculate the difference between the inclination of the segments $A_cB_d$ (red) and the inclination of $A_cB_c$.

$$d\sigma = \sigma_{A_cB_d} + \sigma_{A_cB_c} \quad (3.13)$$

Then the inclination of the object at the desired configuration is calculated by:

$$\sigma^d = \sigma_{A_cB_d} + \lambda \cdot d\sigma \quad (3.14)$$

where $\lambda$ is a multiplication factor.

Then for this new position of $B$, $B^d$ and for this inclination, $\sigma^d$ the new position of $A$, $A^d$ is calculated as explained in section 3.3.

In this way, the tendency of Finger 2 to change the inclination or reverse the movement is detected by Finger 1, who reacts to reinforce it.

Fig 30: Calculation of the desired inclination for a current configuration of the fingers. Current configuration (green): $A^c$, $B^c$. Desired (blue): $A^d$, $B^d$. Inclination that the new position of $B$ would provoke (red)
3.4.1 The Moving Two Fingers Algorithm

In this case the algorithm of the process is a little differentiated. A repeating process is executed during which Finger 2 follows a predetermined route and Finger 1 responds by detecting the difference in the object's inclination imposed by Finger 2 and reinforcing it while maintaining the distance between A, B constant. The direction of the movement of Finger 2 changes when the forces tend to exceed the friction cone limits or when the limits of the one finger's workspace are reached. The steps of the algorithm are as follows.

1) The fingers are closed until they grasp the object. The object's length (ΛAB), the points of contact at the fingertips' surface (S1, S2) are unchanging thus calculated once at this initial position. The initial positions of A and B are also calculated.

2) A desired change in Finger 2 angles Δθ and the multiplication factors k, λ are defined.

\[ \text{While} (\text{Inside\_Friction\_Cone\&Workspace}) \]

2a) The next values of Finger 2 angles φ4, φ5 are calculated.

2b) The new positions of points B, D are calculated with direct kinematics.

2c) The desired object inclination, σd in order to reinforce Finger's 2 movement is calculated.

2d) The desired position of A is calculated as the point of the circle with centre B and radius ΛAB that gives an object inclination equal to σd.

2e) The new values of the angles φ2, φ3 that move A to the desired position are calculated with inverse kinematics.

2f) The new position of C is calculated with direct kinematics.

2e) For this configuration the angles between the object and the tangent surface of the fingertips, ωA and ωB, are calculated in order to ensure that the friction cone restrictions are satisfied.

2f) If (the new configuration is inside the fingers' workspace and the forces don't exceed the friction cone limits and the contact is not lost)

\[
\{ \\
\text{the finger is driven to this new configuration by setting its articulation angles at the calculated values.} \\
\}
\]

Else

\[
\{ \\
\text{Inside\_Friction\_Cone\&Workspace=} \text{False} \\
\text{the rotation is changing direction.} \\
\}
\]
3.4.2 Simulation Results for Two Fingers Movement

A simulation of the process is executed for the same object and initial position, for two different values of the factor $k$ that regulates the difference in increment of the two angles $\phi_4, \phi_5$.

For $k=5$ the obtained results are shown in Fig 31. For $k=2$ the results are shown in Fig 32:

**Fig 31**: Simulation results for an object with length 98.46 mm with $k=5$. a) The initial configuration (red) and the extreme configurations just before the forces exceed the friction cone (green), (blue) and Finger’s 1 workspace for this contact point (yellow), b) The configurations of a where the object is also represented, c) the angles $\omega$ for each repetition of the algorithm, d) the object inclination, e) the joint angles of Finger 1 and f) the joint angles of Finger 2.
It is observed in Fig 31.d that for a big value of k, there is a period of time around the zero inclination that the movement of Finger 2 reverses unwillingly the object rotation, provoking disorders in the inclination values. This effect is smaller for k=2 that the inclination only remains constant for some time. The same values of k are tested for two more objects with different lengths. The simulation is also executed for two more objects with different length for the two values of k. The inclination in each case is shown in Fig 33 and Fig 34.

Fig 32: Simulation results for an object with length 98.46 mm with k=2. a) The initial configuration (red) and the extreme configurations just before the forces exceed the friction cone (green), (blue) and Finger’s 1 workspace for this contact point (yellow), b) The configurations of a where the object is also represented, c) the angles \( \omega \) for each repetition of the algorithm, d) the object inclination, e) the joint angles of Finger 1 and f) the joint angles of Finger 2.

It is observed in Fig 31.d that for a big value of k, there is a period of time around the zero inclination that the movement of Finger 2 reverses unwillingly the object rotation, provoking disorders in the inclination values. This effect is smaller for k=2 that the inclination only remains constant for some time. The same values of k are tested for two more objects with different lengths. The simulation is also executed for two more objects with different length for the two values of k. The inclination in each case is shown in Fig 33 and Fig 34.
It seems that a smaller k is more appropriate to be used in the determination of Finger's 2 route. However the route of Finger 2 could be further optimized.

3.5 Manipulation of Real Objects Using the Tactile Sensors

In this section the previous theory is expanded in order to be used for the manipulation of real 3-D objects of unknown shapes. Spheroid and cylindrical objects may roll on the fingertip's surface if their orientation changes. Cuboid objects may touch at different parts of the sensor depending on which of their edges is in contact. Also the euclidean distance between two contact points at the surface of 3D object's surface varies for different positions of the points of contact.

For the above reasons the assumption of fixed points of contact at the sensors surfaces and fixed distance between A and B is no longer valid. Thought it can be assumed that for a very small variation of the object's orientation these variables don't change significantly. For such a small movement the object can be treated as if it were a stick between the its points of contact with the fingertips. The object is modelled by the line at the plane x0y0 that connects the two contact points.
So for a very small movement of the fingers the previous theory can be used. After such a small movement that the object is moved as if it were a stick the points of contact at the objects surface may change. This happens if the object rolls in the fingertips surface or if there is an alternation between edge-face and face-face contact. Then the real distance between these new contact points is not the same as before. This fact provokes that the fingers tend to compress it or relax the pressure applied to the object. So the forces applied at the fingertips by it tend to augment or decrease respectively.

We can suppose that the distance between the contact points is a little smaller or bigger than the actual, when the applied forces increase or decrease, and consider this as the real distance that the fingers should maintain. The total manipulation movement consists of a sequence of small movements, in each one of which the object can be remodelled for new points of contact and new distance. Then the previous theory can be applied for any kind of object.

3.5.1 The Tactile Information

During the manipulation the height of the points of contact in the sensors' surfaces \((S1, S2)\) and the applied forces are continuously changing. Using the tactile sensors this information can be obtained.
at every step of the algorithm. The angles of the fingers' articulations are also measured during the manipulation thus we always know the current configuration of the fingers.

The current points \( A \) and \( B \) can be calculated at every position given the angles \( \phi_2, \phi_2, \phi_4, \phi_4 \) and the heights \( S_1, S_2 \) as described in Chapter 2. Specifically the virtual joint parameters are calculated for these \( S_1, S_2 \). Then the position of \( A \) and \( B \) is calculated with direct kinematics. The length \( \Lambda_{AB} \) and inclination \( \sigma_{AB} \) of the line \( AB \) that models the object is also calculated.

Furthermore the real distance between the contact points at the two fingers can be adjusted during the manipulation movement as a function of the variation of the contact forces measured in the sensors.

Specifically, the variable \( \Delta \Lambda \) is used in order to vary the distance between the contact points \( AB \) when the contact force increases or decreases. In this way the contact force will be maintained around its initial value and the fingers will be able to follow the changes in the object's shape while it rolls. The adjusted distance \( \Lambda_{AB}^{ad} \) distance will be calculated as

\[
\Lambda_{AB}^{ad} = \Lambda_{AB} + \Delta \Lambda \quad (3.15)
\]

The values of the \( \Delta \Lambda \) in order to maintain the contact force around the same value in case the contact forces increase or decrease were determined empirically. In the experiment that was realized in order to determine a relation between the contact force increment and the \( \Delta \Lambda \) (the results are presented in section 4.1) it was observed that the absolute increment of the contact force value the sensors give is greater when \( A \) and \( B \) move away from each other by one mm than when they approach by one mm. Additionally the value of contact force provided by the sensor tends to increase with the time for constant charge [22]. For this reason a very small \( \Delta \Lambda_{in} \) will have to be added to \( \Lambda_{AB} \) in case the contact force increases. \( \Delta \Lambda_{de} \) is the chosen value of \( \Delta \Lambda \) in case he contact force decreases. Then:

If contact force increases then \( \Delta \Lambda = \Delta \Lambda_{in} \).

If contact force decreases then \( \Delta \Lambda = \Delta \Lambda_{d} \).

The contact force is calculated as the average of the force values given by the sensors,

\[
F = \frac{F_1 + F_2}{2} \quad (3.16)
\]

The increment of this force, \( \Delta F \) is calculated reducing its previous value from the current value.

### 3.5.2 The Two Finger Manipulation with Tactile Feedback Algorithm

The process followed in this part is an extension of the manipulation of a stick-object with two fingers, described in the previous section. The difference is that in every step of the algorithm the
initial conditions need to be recalculated before the routine process is executed. In every repetition the current articulation angles, the heights of the contact points at the surfaces of the sensors and the contact forces are read. The current contact points, the euclidean distance between A and B (ΛAB) and the increment of the forces are calculated for the new configuration. Then ΛAB is adjusted as a function of this increment.

The rest of the process is the same. Finger 2 moves in a predetermined way and Finger A responds by doing a “circular” movement around it with radius the adjusting length. The direction of the movement of Finger 2 changes when the friction cone limits tend to be exceeded or when the limits of the one finger’s workspace are reached.

The grasping part of the algorithm is part of the manufacturer's demo, demo_contact_grasping [9]. The moves contained in the demo, include the closing of the two coupled fingers in order to realize a prismatic grasp of an object with force control. We set a desired contact force threshold Fd in order to end the grasping process. The rest movements were developed programmed from scratch.

The steps of the algorithm are:

1) The route of Finger 2 is determined by defining a the factors Δθ, k, λ.

2) While (F< Fd)
   {
     Close the fingers to grasp the object ( demo_contact_grasping [9] )
   }

3) While (Inside_Friction_Cone&Workspace)
   {
     3a) Read the inputs: Current articulation angles: φ2, φ3, φ4, φ5.
     Heights of the contact points at the surfaces of the sensors: S1, S2.
     Contact forces F1, F2,
     3b) The parameters of the virtual joints, r1, θ1, r2, θ2 are calculated.
     3c) The current contact points A, B are calculated with direct kinematics.
     3d) The distance ΛAB between A, B is calculated.
     3e) The increment of the contact forces ΔF is calculated.
     3fa) If (ΔF>0) then ΔΛ= ΔΛin
     3fb) If (ΔF<0) then ΔΛ= ΔΛde.
     3g) ΛAB is adjusted by ΔΛ.
     3h) The next values of Finger 2 angles φ4, φ5 are calculated.
     3i) The next positions of B, D are calculated with direct kinematics.
     3j) The desired object inclination and inclination σd is calculated.
     3k) The desired position of A is calculated as the point of the circle with centre B and radius the new ΛAB for the desired inclination.
     3l) The new values of the angles φ2, φ3 that move A to the desired position are calculated with inverse kinematics.
The new position of C is calculated with direct kinematics.

For this configuration the angles between the object and the tangent surface of the fingertips, $\omega_A$ and $\omega_B$, are calculated in order to ensure that the friction cone restrictions are satisfied.

If (the new configuration is inside the fingers' workspace and the forces don't exceed the friction cone limits and the contact is not lost)

{  
  the finger is driven to this new configuration by setting its articulation angles at the calculated values.
}

Else

{  
  Inside_Friction_Cone&Workspace=False
  the rotation changes direction.
}

The described algorithm has been developed in C++ programming language and a demo has been created for the SDH2 Schunk robotic hand. The obtained results are presented Chapter 4.
Chapter 4
Experimental Results

4.1 The Contact Force

An experiment was realized to examine how the contact force changes when the fingers try to move inwards or outwards the object by certain mm. In the experiment an object is grasped with the two coupled fingers of the SDH 2 with a desired contact force. The manufacturers demo, `demo_contact_grasping [9]` is used to grasp the object, as explained in section 3.5. When the object is grasped, the Finger 1 is moved in order to translate the contact point inwards and outwards the object by 1 mm in the y0 axis. The finger moves first at the direction that compresses the object (increment of $A_{y0}$) and then at the opposite direction (decrease of $A_{y0}$). The new position that $A$ is calculated by augmenting the current $A_{y0}$ by 1 mm or by decreasing it equally. This alternation is repeated three times.

In every step the joint angles and the tactile information are read and the position of the contact point $A$ is calculated. The joint angles for each position are calculated with inverse kinematics. In every iteration of the algorithm, the contact forces are measured by the two tactile sensors.

The experiment was realized for three different objects: a cylinder of hard carton, a rubber and, a plastic box. The $A_{y0}$ (Fig 36) and the average value of the forces measured in the tactile sensors of the two fingers (Fig 37) are presented for the initial and the six different configurations of the fingers for each one of the six iterations of the algorithm. The increment of this force is calculated as the difference between each value and the previous one and is shown in Fig 38.

![Fig 36: The y0 coordinate of A, at the initial configuration (1) and when the object is compressed and relaxed alternatively (2-7). The results are shown for a cylinder of hard carton (blue), for a rubber (red) and for a plastic box(yellow).](image-url)
FIG 37: Average value of the contact forces measured by the sensors of the two fingertips: \((F1+F2)/2\). The initial grasping force (1) and the force measured at each one of the configurations where the object is compressed and relaxed alternatively (2-7). The results are shown for a cylinder of hard carton (blue), for a rubber (red) and for a plastic box (yellow),

FIG 38: Increment of the average force measured by the sensors at each one of the algorithm repetitions, where the object is compressed and relaxed alternatively. The results are shown for a cylinder of hard carton (blue), for a rubber (red) and for a plastic box (yellow).
It is observed in Fig 36 that the first effort to move towards the object is not achieved since $A_{y0}$ does not change in the desired way. In the experiment with the deformable rubber, point A moves slightly toward the desired position. In the experiment with the most rigid objects it moves to the negative position. Since the experiment begins with a large grasping force it is justified because the applied forces are already great and the fingers resist to further compress. The Finger 1 tries to move point A at a position it can't access because the finger is blocked by the object. The changing of the joint angles in order to move to this new position causes that point A slightly moves to a position that is neither the previous nor the desired. Because of the independent change of the two joint angles of Finger 1, A is found at a different position than the desired when the finger's movement is blocked. The increment in the average forces in this first step is also very small (Fig 37). It seems that when the fingers try to move inwards the object in a position that is not accessible, they move to a slightly different position and the force does not increase importantly. However the new position of the A may be different than the desired. We conclude that the force when the object tends to be compressed need to be controlled in order to avoid an error in the desired position of the contact points.

In the rest of the iterations of the algorithm, $A_{y0}$ changes toward the desired direction. Its value is reduced by 1 mm when the finger is opened to relax the pressure applied to the object. When the finger is closed, its value increases less than 1 mm probably because of the object's resistance. An error in the joint angles that are set in order to move A to each new position may also cause this error. The desired displacement of point A by 1 mm is inside the range of the uncertainty introduced by the sensor's resolution.

During the movement, there is a gradual decrease of the values of $A_{y0}$. Since the position of B never changes, the distance between them gradually increases and the applied force relaxes as it is also shown in Fig 37. We conclude that this happens because the uncertainty in the position of point A during this open close movement provokes a relaxation in the force applied to the object by the fingers. The decrease of the average forces when point A is moved outwards the object by 1 mm is around the same value in all the repetitions, however this value is very different for each one of the three objects. So neither of these values can be used as a parameter in the adjustment of the fingers that is proposed in the previous chapter. The increase of the average forces when A is moved inwards the object by 1 mm does not seem to be constant.

We attempted to correlate the force increment with the translation of A in mm $\Delta y$. $\Delta y$ is calculated as the difference between each value of $A_{y0}$ and the previous one. The force increment per $\Delta y$ is shown in Fig 39 for the ultimate 5 steps of the process since the first is ignored. The force increment per $\Delta y$ and the current average force value is shown in Fig 40.
Fig 39: Force increment per $\Delta y$ for the ultimate five iterations of the algorithm where the object is relaxed and compressed alternatively. The results are shown for a cylinder of hard carton (blue), for a rubber (red) and for a plastic box (yellow).

Fig 40: Force increment per $\Delta y$ and the current average force value for the ultimate five iterations of the algorithm where the object is relaxed and compressed alternatively. The results are shown for a cylinder of hard carton (blue), for a rubber (red) and for a plastic box (yellow).
It seems that there is no correlation between the change of the \( A_{\lambda_0} \) and the provided sensor values. The variation of the force per mm is different for each object. Thus a common threshold that could be common for every object can't be used. The variation per mm referenced at the average values of the force also presents great variations not only for the different objects but also for the same object during the experiment.

For these reasons, since the provided values by the sensors tend to change significantly during the manipulation a model of the forces can't be created. The only observance that can be used is that the absolute increment in the force value is greater when the force decreases than when it increases. Thus the analogy between the distance in mm the fingers are are commanded to open or close for certain variance in the forces will not be the same.

Concluding since there is no obvious relation between the force increment and distance in mm the fingers should move inwards or outwards the object, no threshold or analogy to this increment can be used in order to decide the desired change in the distance between the contact points in order to maintain the force around the same values. This relation will have to be decided empirically.

### 4.2 Manipulation of Unknown Objects with Two Robotic Fingers

The developed algorithm for unknown object manipulation that is described in section 3.5 was used in order to realize the manipulation of two different objects using the two coupled fingers of the SDH 2 hand. The manipulated objects are mentioned in Table 7.

<table>
<thead>
<tr>
<th>object</th>
<th>shape</th>
<th>dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Rubber (approximately rectangular cuboid)</td>
<td>60,5 mm x 22 mm x 11 mm</td>
</tr>
<tr>
<td>B</td>
<td>Egg (approximately ellipsoid)</td>
<td>53 mm, 39 mm</td>
</tr>
<tr>
<td>C</td>
<td>Cylinder</td>
<td>68mm, 116mm</td>
</tr>
</tbody>
</table>

*Table 7: The manipulated objects.*

The experiment was realized as follows: The object was held between the two fingers and the fingers closed in a prismatic grasp until the detected contact force reached a desired value. Then the object is manipulated by the two fingers that move changing the object's inclination as described in Chapter 4. It is first rotated clockwise and then anticlockwise. The object is inclined until the contact forces tend to exceed the friction cone limits. Then the fingers start to rotate the object on the contrary direction. This alternation is repeated three times.

The values of \( \Delta \lambda \) that are used to adjust the distance between the contact points when the contact force changes as described in section 3.5.1, were determined empirically. For an \( \Delta F \) increment of the contact force \( \Delta \lambda \) was defined as:

If \( \Delta F > 0 \) then \( \Delta \lambda = 1 \) mm.

If \( \Delta F < 0 \) then \( \Delta \lambda = 0.2 \) mm.
In Table 8 the chosen parameters for the inclination variance and for Finger 2 predetermined movement are presented. A typical small value was assumed for the friction angle. The manipulated objects are shown in Fig 41 and Fig 42 and Fig 43. In the following sections, the obtained results for each object are presented.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Desired change of Finger 2 angles</td>
<td>Δφ</td>
<td>0,25°</td>
</tr>
<tr>
<td>Multiplication factor of Δφ</td>
<td>k</td>
<td>2</td>
</tr>
<tr>
<td>Multiplication factor of Δσ</td>
<td>λ</td>
<td>5</td>
</tr>
<tr>
<td>Friction angle</td>
<td>α</td>
<td>22°</td>
</tr>
<tr>
<td>Desired contact force</td>
<td>Fd</td>
<td>2</td>
</tr>
<tr>
<td>ΔΛ for contact force increase</td>
<td>ΔΛin</td>
<td>0,2 mm</td>
</tr>
<tr>
<td>ΔΛ for contact force decrease</td>
<td>ΔΛde</td>
<td>1 mm</td>
</tr>
</tbody>
</table>

*Table 8: Parameters of the algorithm.*

*Fig 41: Two different inclinations of Object A*
Fig 42: Two different inclinations of object B.

Fig 43: Two different views of object C.
4.2.1 Object A: Rubber

The uniform variation in the articulation angles shown in Fig 44 indicates that the movement is continuous and no unwilling reverse of the movement has occurred. In this picture it can be observed when the rotation changes direction. This happens at the iterations with number: 15, 71, 134, 203, 268.

Fig 45 shows the contact force $F$ and its increment $\Delta F$ during all the experiment. The contact force at the beginning of the manipulation is the grasping force that is measured to be 2.5. This value is expected since the grasping loop stops when the force becomes greater that the desired force. This contact force directly decreases when the manipulation starts. During the manipulations it never reaches this value. This is explained because during the grasping a continuous charge is applied on the same texels for some time. The value the sensor returns for constant charge increases with the time [22]. For this reason this contact force does not indicate a difference in the applied charge. During the experiment the force's values vary within the range of [0.5, 1.8]. The object is never over pressured or left to slip.

In Fig 46 at the moments when that the movement's direction change we can observe that there is a local maximum or minimum. What draws our attention is that between these local maxima and local minima another local minimum or maximum occurs. This is the moment in which the contact between the object and the fingertips surface changes from face-edge to face-ace and then to face-edge. This can also be observed in Fig 47 where the increment of the contact point at each of the sensors surface is shown. At these moments the contact points change while their increase during the rest of the process is close to zero. It seems that such a change of the contact type can be detected by the object's inclination and the increment of $S1$ and $S2$. This is indicative of the type of the objects surface and could be used for the recognition of the object.

It is also observed in Fig 46 that the inclination takes greater absolute values when the object is rotated anticlockwise than when it is rotated clockwise. This during the clockwise rotation the angle $\omega_A$ rapidly reaches its limit which means that the forces tend to exceed the friction cone limits as seen in Fig 48. We suppose that this is caused by the curvature of the fingertip of Finger 1 that facilitates a downward movement. Since we assume a friction angle of 22°, each $\omega$ angle can change in the range of [68,112]. During the angle $\omega_A$ is always the one that tends to exceed its limits. The contact forces applied to the object tend to exceed the limits of the friction at the fingertip of Finger 1. Additionally, the way of change of the inclination looks similar to the way $\omega_A$ changes.

However the size $\sigma_{AB}$ is not totally representative of the object's relative orientation within the fingers. It expresses the absolute inclination of the object with respect to the global frame. It also changes when the centre of the movement is translated left or right and not necessarily implies the object's rotation. The increment of the inclination between the previous local minima or maxima and the local maxima or minima that occurs when the direction of the movement changes give a better view of the objects rotation. The increase of the inclination values through the experiment has to do with the translation of the centre of the movement. The observed local maxima and minima indicate when the object passes from the “horizontal” position that its faces are in contact with the fingertips' surfaces. It is this moment that the type of contact changes. We can see that it in the first repeat of the process this happens when the inclination is zero. Later the change of the type of contact as shown by the observed local maxima and minima, occurs when the inclination is greater than zero. So an observation of the changing of the inclination between the two local minima that surround a local maximum and vice versa reflect the changing of the orientation of the object and not the inclination's value that depends on the contact point on the object's surface and the type of contact.
Fig 49 Shows the distance $\Lambda_{AB}$ that is related to the object's length. At the initial position where the object is grasped horizontally, the distance between the contact points is 59,8 mm that is 0,7 mm smaller than the real length of the object. This error is inside the uncertainty range introduced by the sensor resolution and is justified. This distance changes during the experiment irregularly in the range of $\pm$ 1 mm. This is smaller than the uncertainty range so no conclusion can be extracted.

One can also observe in Fig 46 that the fingers move during the experiment is not a repeated process. Since the hand moves autonomously the way of movement and the maximum inclination change every time dynamically since they depend on the current contact point and the configuration of the fingers at each position.

Finally Fig 50 is shows the gradual displacement of the centre of the movement toward the positive direction of the y0 axis after the first repetition of the process. If we suppose an imaginary rotation centre of the object this seems to be translated toward the positive y0 direction. Additionally it is observed that at the second and third repeat of the manipulation movement, the reverse of the movement does not happen due to the forces that tend to exceed the friction cone limits. Neither of the angles $\omega_A$ or $\omega_B$ reaches its limit. Also variation of the inclination has a smaller range after the first circle. The other two causes that terminate the movement at one direction are: a) the object approaches the sensors limits b) the movement tends to exceed Finger's 1 workspace. Since the increment of the tactile values is small at this time, only the second case is possible. The movement of the Finger 2 displaces the centre of the movement and draw A away of its initial position. The range of A movement is reduced after the first repetition. An optimization of Finger's 2 route that considers the workspace of Finger 1 seem to be necessary for a successful repetition of this movement. However the fingers performance during the first repetition is satisfactory and the rotation of the object is realized during all the three repetitions.

*Fig 44: Object A: Joint angles (degrees): $\phi_2$ (blue), $\phi_3$ (red), $\phi_4$ (yellow), $\phi_5$ (green). The horizontal axis shows the algorithm iterations.*
Fig 45: Object A: The contact force $F$ (blue) and its increment $\Delta F$ (red) for all the iterations of the algorithm during the experiment.

Fig 46: Object A: Object inclination for all the iterations of the algorithm during the experiment.
Fig 47: Object A: Increment of the values $S1$ (blue) and $S2$ (red) given by the sensors for all the iterations of the algorithm during the experiment.

Fig 48: Object A: Angles to the normal at point A (blue) and at point B (red) for all the iterations of the algorithm during the experiment.
4.2.2 Object B: Egg

Similar conclusions can be extracted for the ellipsoidal object B. The uniform variation in the articulation angles shown in Fig 51 indicates that the movement is continuous and no unwilling reverse of the movement has occurred. The contact force shown in Fig 52 is kept around the value 1,5. A great difference between the grasping force and the values of the contact force during the

**Fig 49:** Object A: Distance between points A and B, Λ_{AB} for all the iterations of the algorithm during the experiment.

**Fig 50:** Object A: Movement of point A (blue) and point B (red) on the y0z0 plane.

4.2.2 Object B: Egg

Similar conclusions can be extracted for the ellipsoidal object B. The uniform variation in the articulation angles shown in Fig 51 indicates that the movement is continuous and no unwilling reverse of the movement has occurred. The contact force shown in Fig 52 is kept around the value 1,5. A great difference between the grasping force and the values of the contact force during the
experiment is also observed for this object. The object never slips or is over-pressured.

The points of contact at the fingertip's surface change during the whole movement that indicates that the fingers manipulate an object with curved surface, Fig 53. In Fig 54 it can be observed that the additional local minima around a local maximum or minimum at the moment of the changing of the movement's direction are very small and occur for a smaller period of time. As shown in Fig 55 the angle $\alpha_A$ moves in a great range of its permitted values but only at the clockwise movement the direction is reversed when the friction cone limits tend to be exceeded. In the anticlockwise rotation the movement changes because the object tends to roll out of the sensors surface.

The initially calculated object's length is 51.16 mm that is 2.84 mm smaller than the object's real length. The error is justified since the uncertainty of the sensor texel is $\pm 1.7$ mm in each sensor. The values of the distance $\lambda_{AB}$ shown in Fig 56 indicate that the object is an ellipsoid since the change in a regular way and decrease when the object rolls on the finger's surfaces at both movement directions. These observations could be used for the recognition of this type of objects.

It is also observed in Fig 54 that the inclination takes a importantly greater absolute values when the object is rotated anticlockwise. The angle $\alpha_A$ that is the one that takes all the range of the permitted values changes rapidly when the object rolls in the linear part of the sensor of Finger 1.

Concluding in Fig 57 is shown the gradual displacement of the centre of the movement toward the positive direction of the $y_0$ axis after the first repeat of the process. This means that Finger 2 does not return to its initial configuration after a circle but is found a little displacement since it's route depends on the number of iterations of the algorithm for every circle of increase or decrease. An optimization of Finger 2 route could solve this problem.

Fig 51: Object B: Joint angles (degrees): $\varphi_2$ (blue), $\varphi_3$ (red), $\varphi_4$ (yellow), $\varphi_5$ (green). The horizontal axis shows the algorithm iterations.
Fig 52: Object B: The contact force $F$ (blue) and its increment $\Delta F$ (red) for all the iterations of the algorithm.

Fig 53: Object B: Increment of the values $S1$ (blue) and $S2$ (red) given by the sensors.
Fig 54: Object B: Object inclination for all the iterations of the algorithm

Fig 55: Object B: Angles to the tangent at point A (blue) and at point B (red) for all the iterations of the algorithm.
4.2.3 Object C: Cylinder

The experiment was repeated for a cylinder with radius \( r \) mm. The experiment was realized with a smaller multiplicative factor: \( \lambda = 3 \). Due to the big dimensions of the cylinders, the contact point on the sensor's surfaces was changing significantly in each iteration of the algorithm. In order to conserve the condition of very small movements of the fingers the changing of the inclination in every step had to be decreased.

---

**Fig 56**: Object A: Distance \( \Delta AB \) between points \( A \) and \( B \), for all the iterations of the algorithm

**Fig 57**: Object B: Movement of point \( A \) (blue) and point \( B \) (red) on the \( y0z0 \) plane
In Fig 58 we observe that the angles change in a non perfect circle during the experiment which indicates an important translation of the centre of the contact. The contact force (Fig 59) has greater variations than the objects presented previously. The cylindrical object is the one which is less close to the linear object model. Its performance is a lot different than that of a cuboid. Although the fingers make small movements, the big volume of the object that rotates in the fingertips causes that the next point of contact not to be driven at the position that is calculated by the model. However the forces are controlled and the object never slips or is over-pressured.

Even when the diameter of the cylinder is 68 mm, initially the distance $\Delta AB$ has the value 66,3 (Fig 60). The cylinder is made of foam so the initial great grasping force compresses it and $\Delta AB$ takes smaller values than the cylinder's diameter. At the rest of the experiment that the applied forces decrease and the object is not compressed, its diameter is calculated by the $\Delta AB$ values between 67,5 and 68,5 that is very close to the real. This distance remains constant within the error ranges during all the experiment.

The performance of the cylindrical object differs a lot from that of the other objects. In this case the inclination $\sigma_{AB}$ of the object is not representative of its rotation. The changing of the object's orientation is indicated by the values of the $S1, S2$. As shown in Fig 61 the object is constantly rolling on the finger surfaces covering a great area of them during it's movement. The contact point A moves the lower part of the fingertip (values: 16-40), while B moves in a smaller range at the lower part of the fingertip (values: 4-24). The distance $\Delta AB$ and the way $S1$ and $S2$ indicate that the objects shape is cyclical (it could be a cylinder or a sphere ) and could be used for a recognition algorithm.

The inclination shown in Fig 62, does not have great variations. The fingers touch the circular surface of the object, which is totally symmetrical, thus there is no physical significance of changing its inclination. In this case the inclination reflects the relative position between the points A and B. For a long time during the manipulation the inclination does not change. When the fingers try move the object to change its inclination, the object rolls on the fingertips" surfaces and the points of contact change. The new ones give similar values of the inclination that shows that the segment AB remains close the the normal on the fingertips at the points A and B. This can be also observed at Fig 63 by the little change of the angles $\omega$. The maximum of the inclination and the angles $\omega$ is caused by translation of the centre at smaller values de $y0$. This translation is also observed at Fig 64. The gradual increase of the inclination is also caused by this gradual translation.
**Fig 58:** Object C: Joint angles (degrees): $\phi_2$ (blue), $\phi_3$ (red), $\phi_4$ (yellow), $\phi_5$ (green). The horizontal axis shows the algorithm iterations.

**Fig 59:** Object C: The contact force $F$ for all the iterations of the algorithm
Fig 60: Object C: Distance $\Lambda A\Lambda B$ between points $A$ and $B$, for all the iterations of the algorithm

Fig 61: Object C: The sensor values $S_1$ (blue), $S_2$ (red) for all the iterations of the algorithm
Fig 62: Object C: Object inclination for all the iterations of the algorithm

Fig 63: Object C: Angles to the tangent at point A (blue) and at point B (red) for all the iterations of the algorithm.
Fig 64: Object C: Movement of point A (blue) and point B (red) on the y0z0 plane.
Chapter 5

Conclusions and Future Work

5.1 Conclusions

In this work we proposed a manipulation method of an object grasped in a prismatic grasp with the two coupled fingers of the SCHUNK SDH 2 robotic hand. The hand is equipped with tactile sensors. The geometric model of the two fingers and the fingertip's tactile sensor was extracted and the kinematic model was created. The different point of contact in the tactile sensor were modelled with a virtual joint that introduces an extra DOF to each finger.

The two fingers manipulated unknown objects with the use of the tactile information. The uncertainty introduced by the lack of previous knowledge about the object's shape and the exact position of the object when it rolls in the finger's surface was handled with the use of the tactile information. The tactile sensors were used to detect at any time the contact points and consequently the position of the object. With its position known the fingers were able to rotate it until just before the contact forces exceed the friction cone limits. Furthermore the detection of the contact force was used in order to adjust the distance between the fingers during the manipulation in order to be able to adapt to the objects shape.

The realized experimentation showed that the object was manipulated as decided since the contact forces are kept around a fixed value and the friction angles are not exceeded. The experiment was realized for a cuboid and an ellipsoid object. The differences in the way the inclination, the contact point and the the length of the object change, are indicative of the particular shape of each object and could be used for its recognition.

The main difficulty we met in this work was the uncertainty in the tactile information. The uncertainty introduced by the texel size is the greater uncertainty we had to handle with and a critical factor of the error in the obtained results. Additionally measured pressure values are constantly changing and may have differences depending on the time of usage of the hand. For this reason a force model about the sensor could not be created.

5.2 Future Work

Future work has been divided in two different categories: a) Future work related with the robotic hand and b) further development of our method.

a) In terms of the robotic hand a matching between the sensor values and the real applied force should be done for better use of the tactile information. Additionally a tactile sensor with smaller texel size is desired. It would decrease significantly the uncertainty in the measure since the texel size is the greater uncertainty factor in our analysis.

b) In terms of our method it is possible to optimize the independent finger's route to use the whole workspace of the dependent finger and move accordingly. In this way a greater range of the
movements within the workspace could be obtained which is expected to improve the finger's dexterity.

An object recognition algorithm that would recognize the shape of the part of the object that is in contact with the tactile sensors while the object rolls on their surface, is also a natural extension of this work.
References


[8] WEISS ROBOTICS http://www.weiss-robotics.de. 27.08.2011

[9] SCHUNK SDH2 Documentation Overview. 19.2.2010


