Optimum Placement of Distributed Generation in Three-Phase Distribution Systems with Time Varying Load Using a Monte Carlo Approach

Juan A. Martinez and Gerardo Guerra

Abstract—**A procedure has been developed for estimating the optimum allocation of distributed generation (DG) units in distribution systems. The approach is based on a single-target model in which the objective is loss minimization. The main goal of this work is to explore the application of a Monte Carlo based method for loss minimization of a three-phase distribution system when considering the intermittent nature of some distributed resources and the time varying shape of the node loads. The procedure has been implemented in an open environment in which OpenDSS is driven from MATLAB. Some examples illustrate the scope of the proposed procedure.**

Index Terms— **Distribution System, Distributed Generation, Optimal Placement, Loss Minimization, Load Flow.**

I. Introduction

The connection of distributed generation (DG) to the distribution system can be used for supporting voltage, reducing losses, providing backup power, providing ancillary services, or deferring distribution system upgrade [1]. Aspects to be considered when embedding DG into a distribution system are the great variety of generating technologies, or the intermittent nature of some renewable sources.

Since DG units are relatively small in size, one of the criteria used to locate DG is to place generation close to load consumption. Several strategies have been proposed to optimally allocate DG; for instance, loss minimization [2], minimizing system update [3], risk minimization [4], or maximizing DG capability [5]. A significant activity has been dedicated to this purpose during the last decade; for a summary of the works related to optimum allocation of DG see reference [6], which was later updated in [7].

Given the actual configuration and operation of the distribution system (i.e., it is three-phase and may run under unbalanced conditions), the nature of the load (e.g., it is voltage-dependent and time-varying) and the intermittent nature of some generation, a rigorous study may require a probabilistic approach, and calculations performed over an arbitrary time period, that may range from several minutes to several years.

Although some works have been performed in this field using probabilistic methods [7]-[9], not much has been done with a full model; e.g., a model such as that mentioned above for the distribution system, load and DG. The Monte Carlo method is a natural approach when uncertainties are involved and some variables are random/intermittent, although it is evident that its application to a system of actual size when using an advanced representation and considering a multi-objective method is very costly in terms of computer time.

This paper is aimed at exploring the application of a Monte Carlo based method for optimum allocation of DG when the goal is to minimize losses. The simulation tool used for this purpose is OpenDSS, a distribution system simulator whose capabilities allow users to represent the most important distribution components and perform multiphase calculations. In addition, OpenDSS capabilities can be expanded with an external link to other tools; e.g., MATLAB [10], [11].

The document has been organized as follows. Section II summarizes the modeling approach followed in this work and the solution method implemented in MATLAB-OpenDSS. The test system is detailed in Section III, while the studies performed to both validate and apply the solution method are presented and analyzed in Section IV, which also includes a discussion about the present and the future versions of the method.

II. Modeling Approach and Solution Method

A. Modeling Approach

The following aspects should be considered for a realistic model of the distribution system [12].

1. Distribution systems are three-phase and radial by nature; however, single-phase lines, loads and distributed resources must be also considered. In addition, the distribution system model may require more than one voltage level (i.e., MV and LV levels) for certain studies.
2. A realistic load model must include voltage dependence, and time-varying behavior.
3. Generation units can be either stochastic or deterministic. Stochastic may be used as synonymous of intermittent and non-dispatchable, while deterministic is synonymous of non-intermittent and dispatchable.
4. Detailed models of the generating machines must be necessary in case of unbalance.
5. When voltage control from DG is not allowed, the DG node becomes a PQ node. In such case, the internal impedance matrix of the machine must be included in load-

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flow calculations. Constraints about power ramps may be also considered.

6. Renewable generation must operate to produce as much power as the energy resources permit. Energy exceeding the primary load may be sent to storage devices, deferrable loads, or dumped. A generic node with generation and storage must include dispatch strategies and constraints.
7. When energy storage is present, history terms are required and the calculations must be performed over a time period, since the states of energy storage devices at one time step affect the states at the next step, and depend on the states at the previous step.

In this paper the model used to represent the test system is three-phase (which can imply unbalance), the load is voltage-independent and time varying, and generation nodes are always PQ nodes. The term generation is used here in a very general sense, since a capacitor bank may be seen as a generator that injects only reactive power.

B. Procedure for Optimum Allocation of DG

The target of the study is minimizing losses; therefore, the developed procedure may be defined as a Monte Carlo method aimed at estimating the location and size of one or more generation units to minimize the system losses.

Input data includes system parameters (i.e., network topology, component parameters), and yearly variation of loads and generations. Random variables to be generated during the application of the Monte Carlo method are locations and sizes of the generation units. Note that this can be rigorously made by considering that the generation pattern depends of the area/node where the generator is located. In case of photovoltaic generation, it can be assumed that the solar radiation is the same for each system node, but this can be untrue in case of wind generation; e.g., when the altitude over sea level is not same at each system node.

Although DG units can inject both active and reactive power, in this work generators will only inject active power. However, the procedure can be also applied to allocate capacitor banks by simple changing active by reactive power.

The procedure developed for this work may be summarized as follows:

1. Generate the random values for locations and sizes of generators, and assign a shape to each generator, depending on the location.
2. Perform the load flow calculation. Neglect the case if one of the following conditions is satisfied:
    - the voltage at one or more nodes is below or above the respective limits,
    - the current through one or more system sections is above the thermal limit,

    Another condition to be considered is the inversion of he load flow in the substation (see next section).
3. Stop the procedure after reaching the number of runs.

C. Implementation of the Procedure

The procedure summarized above has been implemented in OpenDSS, a simulation tool for electric utility distribution systems, which can be used as both a stand-alone executable program and a COM DLL that can be driven from some software platforms. In this study, the program is driven from MATLAB (see Fig. 1), which is used to calculate the random variables and control the execution of the procedure [10].

Off-line support tools are used to generate load and intermittent generation (wind, solar) curves. In this work, the curves have been generated using capabilities of HOMER (Hybrid Optimization Model for Electric Renewables) [13], [14]. See reference [15] for a previous work related to these aspects. Note, however, that MATLAB capabilities could be also used for this purpose, compacting in this way the procedure.

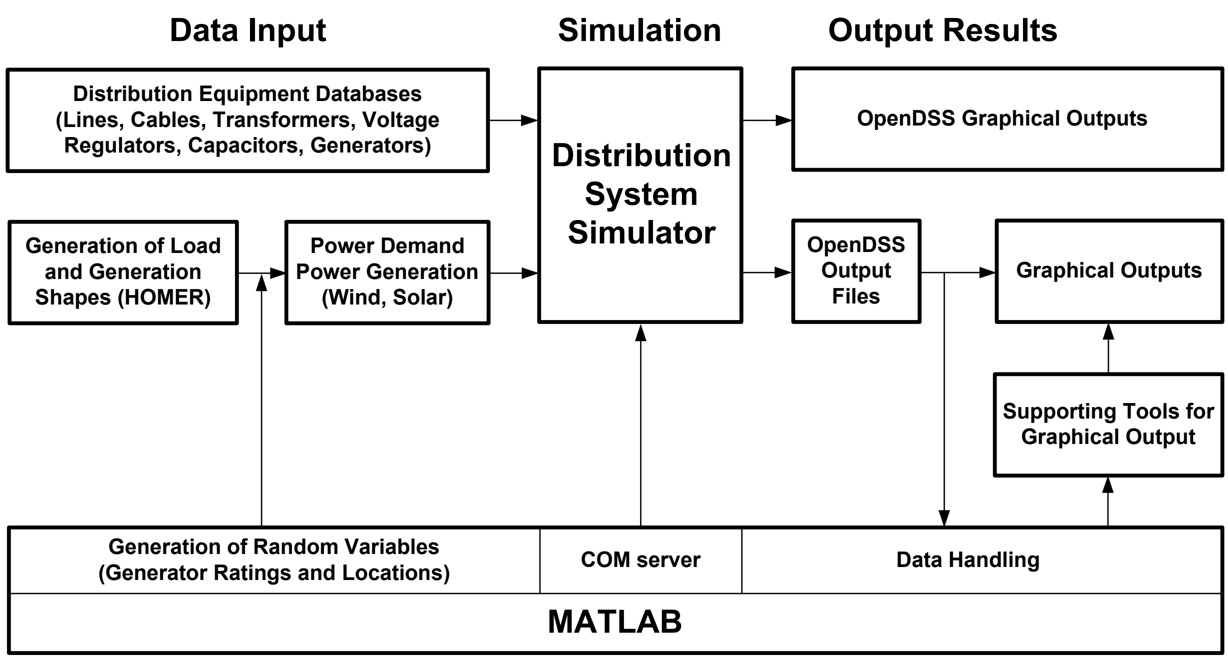


Fig. 1 Block diagram of the implemented procedure.

It is worth mentioning that there is an OpenDSS capability, namely the *autoadd* solution mode [10], which can be used to test each bus of the system and determine whether it is the optimum bus for placement of the DG unit; see also [16]. This feature has not been used in this work.

III. Test System

Fig. 2 shows the diagram of the test system. It is a three-phase single-feeder distribution system with 100 nodes and the same load at each node. It is, therefore, possible to assume that the load is uniformly distributed, which will allow validating some cases presented below. Note that the model includes the substation transformer and a simplified representation of the high-voltage system. The phase conductors of the overhead line are in a flat configuration. The normal thermal limit is 400 A, while the emergency limit is 600 A.

As mentioned above, a generator injecting only reactive power behaves as a capacitor bank. The optimum allocation of capacitor banks in a distribution feeder with uniformly distributed load has been thoroughly analyzed [17], so this will allow comparing the theoretical results with those derived from the procedure detailed above.

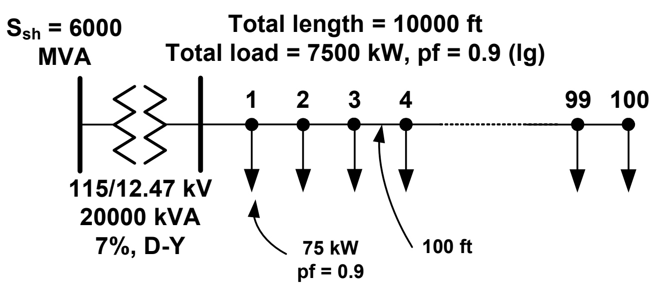


Fig. 2 Test system configuration and data.

IV. Studies and Simulation Results

The studies and results have been grouped into two parts:

1. Studies performed to obtain the optimum allocation of capacitor banks with both constant and time-varying load.
2. Studies aimed at obtaining the optimum placement of renewable distributed generation units (i.e., photovoltaic generation) injecting only active power, with some constraints for voltages, losses and power flow.

A. Optimum Allocation of Capacitor Banks

A solution to the optimum allocation of n capacitor banks of equal sizes on a distribution feeder with a uniformly distributed load has been known for many years [17]. The per unit distance for the optimum location of the i-th capacitor bank is

$$x_{i,opt} = 1 - \frac{(2i-1)c}{2} \tag{1}$$

where c is the compensation ratio of each capacitor and is obtained from the following expression:

$$c = \frac{2}{2n+1} \tag{2}$$

For n = 1, these expressions provide the well-known 2/3 rule; that is, c = 2/3 of the total reactive load and x = 2/3 of the feeder length. For n = 2, the compensation ratio of each capacitor bank is 2/5 of the reactive load, giving a total compensation of 4/5, while the optimum locations for both capacitor banks are respectively 2/5 and 4/5 of the feeder length.

Fig. 3 shows the results obtained after using the Monte Carlo method for estimating the optimum location of a single capacitor bank on a feeder with a constant load. Uniform distributions are assumed for all random variables in this study; the maximum values of the capacitor bank power and distance to the substation are respectively the total reactive power of the load and the length of the feeder.

The minimum of the surface shown in Fig. 3 corresponds to a capacitor bank of 2892 kvar located at 6177 ft from the substation. The losses are reduced from 107.7 kW without capacitor bank to 85.8 kW with the optimum allocation of the capacitor bank.

It is important to keep in mind that the theoretical results were derived for a single-phase feeder with some simplifications about voltage drop and load model. On the other hand, the random distance of the distance must be corrected to match the closer node distance; that is, the final distance can be only varied in steps of 100 ft.

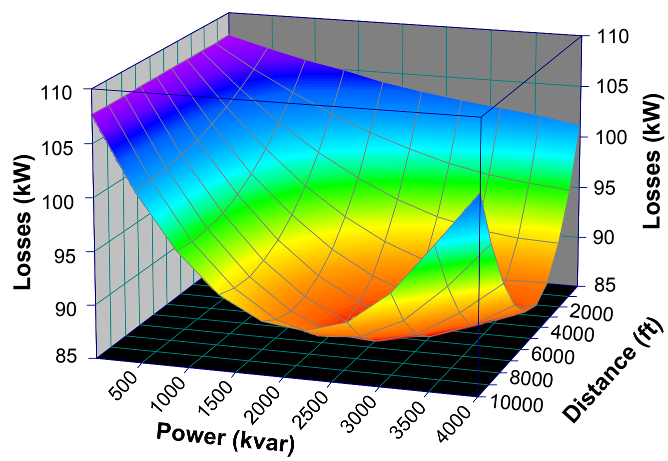


Fig. 3 Optimum location of a capacitor bank (1000 runs).

According to the 2/3 rule, the optimum capacitor bank should have 2420 kvar and should be located at 6667 ft from the substation. Although the accuracy of a Monte Carlo method can be improved by increasing the number of runs, the accuracy that can be obtained in this study after 1000 runs is in general acceptable. In fact, differences were not significant with more runs. There are some reasons to justify these differences (e.g., there can be some steady-state unbalance and some voltage drop, and line capacitances are included in calculations), although some of these aspects are not significant in this study (e.g., voltage with the optimum capacitor bank decreases from 1.004 pu at the substation secondary to 0.989 pu at the last node). Another important reason is the feeder model used to obtain the above expressions, see [17].

Fig. 4 shows the results obtained after using the method to estimate the location of two capacitor banks of the same size. In this case the random variables are three, the two distances and the reactive power of each capacitor bank. To visualize the results, the procedure has been initially applied with a fixed compensation ratio of the reactive power. The figure shows the results that correspond to a total compensation of 80%, which is the theoretical optimum compensation ratio when installing two capacitors banks.

The minimum of the surface shown in Fig. 4 corresponds to distances of 4303 and 8093 ft. The theoretical values for this compensation ratio are respectively 4000 and 8000 ft.

To estimate the optimum compensation ratio, the procedure was applied with several ratios. The results obtained are summarized in Table I. According to these results, the optimum allocation is closer to a compensation ratio of 100%, although the improvement with respect to the theoretical optimum (i.e., 80% compensation) is very small.

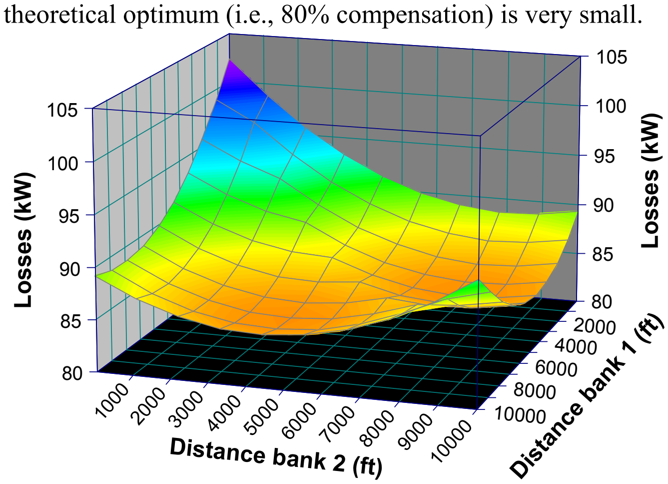


Fig. 4 Optimum location of two capacitor banks with a total compensation ratio of 80% (1000 runs).

TABLE I
OPTIMUM ALLOCATION OF TWO CAPACITOR BANKS

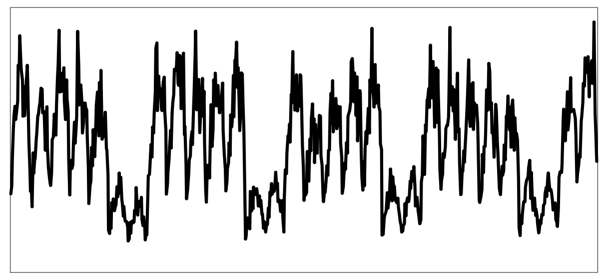
Compensation ratio	Distance Bank 1 (ft)	Distance Bank 2 (ft)	Losses (kW)
20%	8631	10000	97.39
40%	7193	9091	90.45
60%	5747	8598	86.32
80%	4303	8093	84.33
100%	2897	7590	83.83

When running the procedure with three random variables (i.e., the compensation ratio is also varied), the optimum after 1000 runs corresponds to a compensation ratio per capacitor bank of 47% and respective locations of 0.30 and 0.76 in per unit of the feeder length.

The last test case in this part assumes a time varying load and is aimed at estimating the optimum allocation of a single capacitor bank but using a yearly load curve. To check the accuracy of the procedure, the yearly shape of each load, as well as that of the capacitor bank, are the same, so the 2/3 rule can be also considered. Fig. 5 shows the results obtained after using the Monte Carlo procedure.

The minimum of this case corresponds to a capacitor bank of 2723 kvar located at 6323 ft from the substation. Just by chance, these results are closer to those provided by the 2/3 rule than those obtained with a constant load, which suggests that more runs may be required.

Table II shows the results obtained with a different number of runs when simulating the test system during a year. Obviously, the more runs, the better the accuracy of the results; however, the results shown in the table do not exhibit any improvement when passing from 300 to 1000 runs. Therefore, the rest of this study will be made by running the procedure no more than 1000 times.



a) Load profile

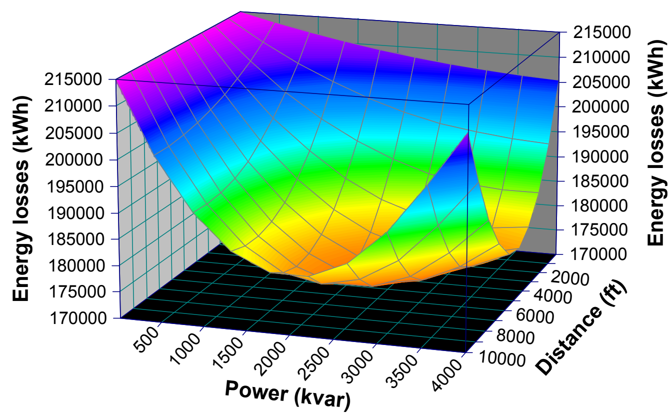


Fig. 5 Optimum location of a single capacitor bank with a time varying load (1000 runs).

TABLE II
PERFORMANCE OF THE MONTE CARLO PROCEDURE

Runs	Distance (ft)	Size (kvar)
300	6327	2720
600	6344	2712
1000	6322	2723

B. Optimum Allocation of Renewable Generation

The objective of this second part is to apply the implemented procedure for obtaining the optimum allocation of renewable generation units when considering that the shape of each node load is different, the generators only inject active power, and its generation pattern is previously known.

However, the first study is made by assuming a constant and equal load at each node; that is, the power flow is calculated in *snapshot* mode. Fig. 6 shows the results after 1000 runs. The minimum of this case corresponds to a generation unit of 5280 kW located at 6504 ft from the substation. This case proves that the 2/3 rule also applies when minimizing losses by injecting only active power, as suggested in [18]. Note that the theoretical results after applying the 2/3 rule are respectively 5000 kW and 6667 ft.

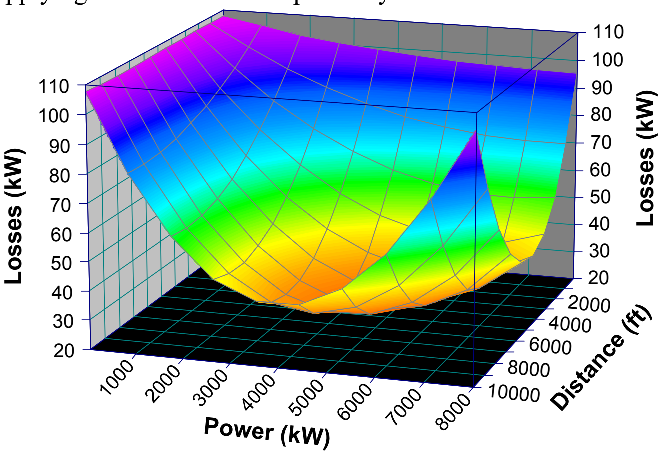


Fig. 6 Optimum location of a single generation (Power factor = 1, *snapshot* mode, 1000 runs).

An important aspect to be considered when running the procedure in *time* mode (i.e., during a year) is that the shapes of the generation curves correspond to those of photovoltaic units and there will be generation during daily hours only, see Fig. 7.

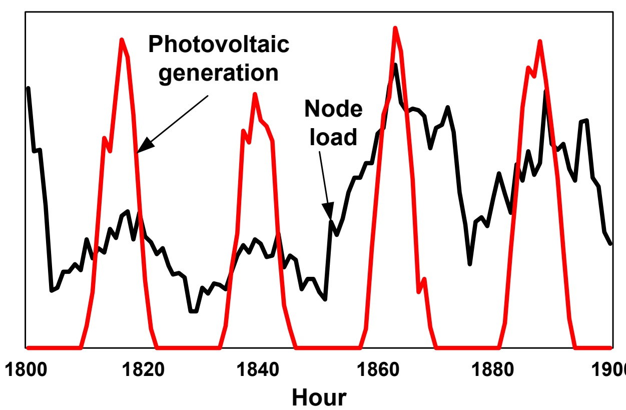


Fig. 6 Profiles of load and generation (not with the same scale).

Another important aspect of this study is that there will be constraints for some operating conditions (i.e., all voltages must be between 1.05 and 0.95 pu, the thermal limit of 400 A for line conductors cannot be exceeded), so those combinations of active power and distance for which at least one limit is exceeded are neglected.

A reverse power flow through the substation transformer is another condition that can be considered since many utilities do not accept power supply from the lower voltage level; however, this condition will significantly limit the power of the generation unit to be selected. The reason behind this conclusion is that during week ends all the loads are rather small (see Figs. 5a and 6) and only small generators would be acceptable.

Table III shows the results obtained after applying the procedure with different number of runs. Fig. 7 depicts the results corresponding to 1000 runs. It is important to remember that the 2/3 rule should not be considered now since the peak of node loads are not coincident. However, one can observe that the new surface exhibits a pattern very similar to those of Figs. 3, 5 and 6.

TABLE III
OPTIMUM ALLOCATION OF ONE GENERATION UNIT

Runs	Power (kW)	Distance (ft)
300	4629	6557
600	4656	6546
1000	4640	6553

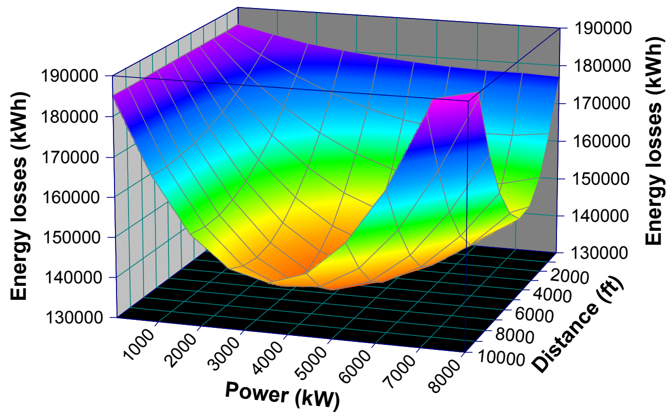


Fig. 7 Optimum location of a single generation unit (Power factor = 1, *time* mode - 1 year, 1000 runs).

The maximum rated power of the generation unit that could be obtained in the previous study was the active power that resulted from adding the peak active power of each node load (i.e., 7500 kW). The study aimed at optimally locating two generation units is made by considering that the maximum active power of each generation unit can be again this value.

Table IV shows the results obtained after applying the procedure with different number of runs. Note that this time the differences between the results derived from each study are important, and these results suggest that the accuracy can improve by increasing the number of runs.

TABLE IV
OPTIMUM ALLOCATION OF TWO GENERATION UNITS

Runs	Power Unit 1 (kW)	Distance Unit 1 (ft)	Power Unit 2 (kW)	Distance Unit 2 (ft)
300	2350	3200	3164	8200
600	3343	4100	2175	9200
1000	2817	3000	3032	7600

As mentioned above, those runs for which one operating limit or condition was exceeded were neglected. With the shapes assumed for load and generation, no cases were neglected when optimizing a single generation unit, but a significant percentage of cases had to be neglected when considering two generation units, namely 66, 134, and 211 cases when running respectively 300, 600, and 1000 cases.

However, when a reverse power flow through the substation transformer was avoided, then more than 80% of the runs had to be neglected when installing one generator, and more than 90% when installing two generators.

C. Discussion

The procedure presented above has some obvious advantages (e.g., it is rather simple, the simulation time does not significantly depend on the number of DG units, it can be applied for optimally locating both generation units and capacitor banks) and an obvious disadvantage (i.e., it is time consuming).

Although the test system is not a realistic one, its configuration and size can be useful for testing the procedure presented here. When the system is simulated 1000 times, the CPU time required by a normal laptop passes from about 2.5 minutes in *snapshot* mode to more than 2.5 hours in *time* mode. Note, however, that the application to systems with different configuration would be straightforward for studies similar to those presented here.

Since the goal is to analyze distribution systems of realistic size during a longer period and using a more advanced model, some refinements are needed to significantly reduce the simulation time. For instance, some simple rules that could avoid allocating DG units close to the substation if it is clear that the optimum will not be too close to the substation.

An interesting result from the above study is that (without considering the reverse power flow condition) for all the neglected cases the reason was that the thermal limit of phase conductors was exceeded, and no one case produced voltages out of limits (with or without DG units). To obtain voltages below the minimum voltage, a higher load is required; however, voltages below the minimum could be produced, depending on the loadshapes, during nightly hours, and adding DG with the generation pattern shown in Fig. 6 could not solve this problem.

Some conclusions can be derived from these results with the system used in this work: the generation size has to be significantly reduced if the reverse power flow condition is not accepted, energy storage may be used for solving the surplus of generation during weekends, and in case of large load during nightly hours another means for supporting voltage could be required.

An important aspect when using *time* mode, and even more if the study is expanded to cover several years, is the accuracy of some data, namely load and generation shapes, since it does not make too much sense running a Monte Carlo based method several thousand times to obtain a more accurate results if those shapes are not accurate enough.

To clarify the importance of some concerns consider the following study. The peak of coincident active load in the test system was increased to 10 MW, keeping the same power factor. The system was run again to find the optimum allocation of two generators, but considering two different groups of constraints. When the constraints about voltages and thermal limits were as in the previous studies, all cases have to be rejected because the thermal limit is exceeded in all of them; (2) if the voltage was allowed to be between 0.93 and 0.95 pu during 1% of the time, and the thermal limit could be exceeded during periods of up to 5 hours the solution was: Unit 1 (3306 kW, 3800 ft); Unit 2 (3837 kW, 9100 ft). This proves that unless very accurate information is available, some data must be used with caution, and it is advisable to relax some operating limits.

The above discussion about the margins to be used for any limit or constraint in the operating conditions is one of the topics for which more research can be required. For instance, the fact that regardless of its size, distributed generation (with the pattern used in this study) could be useless to avoid voltages below the minimum can force the installation of other means for voltage support (i.e., voltage regulator, capacitor bank) but also to be more accurate about the load shapes used in calculations.

V. Conclusions

This paper has presented a procedure based on a Monte Carlo method for optimum allocation of distributed generation. The procedure can be also applied for the optimum allocation of capacitor banks by simply sweeping from active to reactive power in the calculation of random values.

Advantages and disadvantages of Monte Carlo-based procedures are well known: they are rather simple and can be usually based on complete/advanced models; in turn, they usually require a high number of runs, and consequently, a long CPU time. An interesting advantage of the application analyzed here is that the simulation time does not significantly depend on the number of generation units to be allocated.

The procedure presented in this paper is using OpenDSS, a tool that offers a flexible and powerful platform for load flow analysis with capabilities for creating open systems by taking advantage of its COM DLL. In this work, this capability has been used to drive the simulator from MATLAB. This is a very important feature since further work could consider the MATLAB implementation of some tasks (e.g., 3-D graphical outputs, the generation of load and generation shapes) now released to external supporting tools, see Fig. 1.

The present procedure is single-objective, but it can be easily expanded to include other objectives (e.g., cost of energy and interruptions, system upgrading) when optimizing the allocation of generation units.

The studies performed in this work have not taken full advantage of OpenDSS capabilities, since more sophisticated load and generation models can be represented; see for instance the models for photovoltaic generation presented in [19] and [20]. In addition, OpenDSS can be also used for an accurate evaluation of the non-supplied energy, a variable that can be part of the target to be minimized.

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