Final Degree Thesis

Software tool in Mathematica environment for EBIS measurements modelling

By

CARLOS DOLADER RETAMAL
ABSTRACT

The electrical Bioimpedance (EBI) has become very useful for many different clinical applications. EBI measurements may contain artifacts that can lead to errors which have to be analyzed. It has to be added to the already known capacitive leakage artifact caused by a parasitic capacitance, other effects, between them can be remarked the electrode polarization impedance effect, especially when there is electrode imbalance. In this work is developed a software tool in Wolfram Mathematica environment, this tool will help to the evaluation of EBIS measurements realized in non-ideal conditions through EBIS measurements modeling. As an example of application, the effects produced by the electrode polarization impedance together with the parasitic capacitance artifacts over the estimation of the Cole parameters were analyzed. The tool will make able to identify errors in real measurements.
ACKNOWLEDGEMENTS

I would like to appreciate the dedication that my thesis supervisor, Ruben Buendia, has provided me during all the time this work have been realized, his detailed explanations for solving my problems were very useful.

I would like to thank all the support and encourage that my parents, Jose and Angela, have shown me during all these years to keep on with my studies.

I want to thank to everyone I’ve known in Borås, all of them allow this experience had been so special and so exciting. I want to mention to my flat partners and friends, Benoit, Marc, Juan and specially Margaux, all the moments I spent with you were a grateful experience that I will never forget.

Carlos Dolader Retamal
# LIST OF CONTENTS

ABSTRACT ......................................................................................................................... 2  
ACKNOWLEDGEMENTS ......................................................................................................... 3  
LIST OF CONTENTS ............................................................................................................... 4  
LIST OF ACRONYMS ............................................................................................................. 6  
CHAPTER 1: ......................................................................................................................... 7  
INTRODUCTION .................................................................................................................... 7  
  1.1. INTRODUCTION ......................................................................................................... 7  
  1.2. GOAL ........................................................................................................................ 7  
  1.3. WORK DONE ............................................................................................................ 7  
  1.4. STRUCTURE OF THE THESIS .................................................................................... 8  
CHAPTER 2: .......................................................................................................................... 9  
BACKGROUND .................................................................................................................... 9  
  2.1. BIOIMPEDANCE ........................................................................................................ 9  
      2.1.1. Electrical properties of biological tissues .............................................................. 9  
  2.2. EBI MEASUREMENTS ................................................................................................. 12  
  2.3. COLE MODEL ............................................................................................................ 12  
CHAPTER 3 ........................................................................................................................... 15  
MATERIALS AND METHODS ............................................................................................ 15  
  3.1. EBI MEASUREMENT MODELS ................................................................................. 15  
      3.1.1. Capacitive leakage Effect Model ...................................................................... 15  
      3.1.2. Parasitic Capacitance and Electrode Polarization Impedance Model ............... 17  
      3.1.3. Electrode mismatch effect ............................................................................... 19  
  3.2. NON-LINEAR LEAST SQUARES (NLLS) FOR COLE FUNCTION MODULUS FITTING 19  
  3.3. MATHEMATICA ........................................................................................................ 19  
  3.4. COLE FITTING method IMPLEMENTATION IN MATHEMATICA ........................... 20  
CHAPTER 4 .......................................................................................................................... 22  
RESULTS .............................................................................................................................. 22  
  4.1. SOFTWARE ENGINE ................................................................................................. 22  
      The model used ............................................................................................................. 22  
      The user interface ......................................................................................................... 22  
      Results visualization .................................................................................................... 26  
  4.2. EXAMPLE OF APPLICATION: STATISTIc RESULTS AND ANALYSIS ................. 28  
      4.2.1. Results modifying $C_{par}$ ............................................................................... 28  
      4.2.2. Results modifying electrode polarization impedance parameters ........................ 31  
CHAPTER 5: ........................................................................................................................... 37  
DISCUSSION, CONCLUSION AND FUTURE WORK ......................................................... 37  
  5.1. DISCUSSION AND GENERAL CONCLUSIONS ...................................................... 37  
  5.2. FUTURE WORK ........................................................................................................... 37
**LIST OF ACRONYMS**

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>BCA</td>
<td>Body Composition Assessment</td>
</tr>
<tr>
<td>EBI</td>
<td>Electrical Bioimpedance</td>
</tr>
<tr>
<td>EBIS</td>
<td>Electrical Bioimpedance Spectroscopy</td>
</tr>
<tr>
<td>MF-EBI</td>
<td>Multi Frequency Electrical Bioimpedance</td>
</tr>
<tr>
<td>TBC</td>
<td>Total Body Composition</td>
</tr>
<tr>
<td>V/I</td>
<td>Voltage / Current</td>
</tr>
<tr>
<td>ICG</td>
<td>Impedance Cardio Graphy</td>
</tr>
<tr>
<td>ICF</td>
<td>IntraCellular Fluid</td>
</tr>
<tr>
<td>ECF</td>
<td>ExtraCellular Fluid</td>
</tr>
</tbody>
</table>
CHAPTER 1:  

INTRODUCTION

1.1. Introduction

Electrical Bioimpedance (EBI) can be defined as the opposition that biological material offers to the flow of electrical charges. Examples of biological material can be the human body tissue or wood.

The amount of EBI applications have increased during the past decade, examples of that might be skin cancer detection (Aberg et al., 2005) or dry weight determination (Kuhlmann M K et al., 2005).

Fitting the EBIS measured data to the Cole function in order to estimate the Cole parameters is the base of most of EBIS applications since the Body Composition Assessment (BCA) parameters can be obtained from the Cole parameters. Between these applications are home monitoring applications such as lung edema early diagnosis (Beckmann et al., 2007) or hydration status (Medrano et al., 2007).

To obtain a good estimation of the Cole parameters reliable EBIS measurements are needed. However this is not often the case, the EBIS measurements are often tainted with noise and stray capacitances. An extensive study about how to remove the effect of stray capacitances can be found in (Buendía, 2012). However, in that job the electrode polarization impedance ($Z_{ep}$) was neglected, $Z_{ep}$ represents the impedance of the electrode-skin interface. Neglecting $Z_{ep}$ is usually acceptable in measurements obtained with conventional electrodes, otherwise, measurements obtained with textile electrodes might present a high $Z_{ep}$. Therefore $Z_{ep}$ should be considered.

A software engine which allows analyzing and quantifying the effect of $Z_{ep}$ in EBIS measurements tainted with capacitive leakage effect could help in a remarkable way to improve the reliability of Cole analysis from EBIS measurements specially in the ones obtained with textile electrodes because they are more likely tainted with stray capacitances which present a high $Z_{ep}$.

1.2. Goal

The main goal of this thesis is to, considering EBIS measurement models, create a software engine that makes possible to evaluate how the different elements of a model may affect on the Cole parameters estimation when using different fitting approaches.

1.3. Work done

A simulation core has been implemented in Wolfram Mathematica. This software tool allows evaluating how the different elements of a model might affect to the measurement. This way, it enable a study of how some parasitic effects produced by
non-ideal conditions in the EBIS measurements may affect to the Cole parameter estimation. In addition it will make able to identify the errors in real measurements.

As a practical case of example is performed a study to understand how the parasitic capacitance $C_{\text{par}}$, modeling the capacitive leakage effect and the electrode polarization impedance $Z_{\text{ep}}$, affect to TBC EBIS measurements,

The application has been tested with examples varying the different parameters. The evolution of the Cole parameters errors varying $Z_{\text{ep}}$ and $C_{\text{par}}$ is represented and analyzed. Figures showing this evolution of the errors have been plotted.

Apart from its utility in EBI, the software engine is a versatile virtual machine that allows the user to easily implement and analyze models.

1.4. Structure of the Thesis

The whole thesis report contains five chapters, an appendix and the references. Chapter one is an introduction which explains the current situation, the goal and the work done. Chapter two contains the background, it is explained the EBI measurements and Hook effect. Chapter three is an analysis of the capacitive leakage artifact and the electrode polarization impedance effect together with the theoretical basis of the fitting model. Chapter four presents the software engine and shows and explains the general results achieved with an example case using the software. Chapter five contains conclusions and future work is proposed. Finally, the appendix contains the code developed for the implementation of the software engine.
CHAPTER 2: BACKGROUND

2.1. Bioimpedance.

EBI is useful to study tissue composition and its electrical properties. Therefore, EBI contains information about structure, size and activity of the material measured.

2.1.1. Electrical properties of biological tissues

The human body is formed by biological tissues which are a set of cells and fluids. Extracellular fluid (ECF) and cells containing intracellular fluid (ICF) are the base of the tissue structure. The tissues and the cells contain water and electrolytes which determine the conductance capabilities. Intracellular and extracellular fluids contain free ions able to carry electricity, sodium, potassium and chlorine are the main ions promoting ionic current, their carrying charge is shown at Table 2.1.

Electrical model of a cell

The cell membrane has a capacitive behavior which has a value around $2\mu\text{F/cm}^2$. Biological tissue can be modeled as two resistances (the extra-cellular and the intra-cellular) and one capacitor (cell membrane) as shown in Figure 2.1.

<table>
<thead>
<tr>
<th>Ion</th>
<th>Extracellular fluid (ECF)</th>
<th>Intracellular fluid (ICF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Na$^+$</td>
<td>140 mEq/l</td>
<td>14 mEq/l</td>
</tr>
<tr>
<td>K$^+$</td>
<td>4 mEq/l</td>
<td>140 mEq/l</td>
</tr>
<tr>
<td>Cl$^-$</td>
<td>103 mEq/l</td>
<td>4 mEq/l</td>
</tr>
</tbody>
</table>

Table 2.1 Charge values of sodium, potassium and chlorine in ECF

Fig. 2.1 Fricke’s model. The Capacitor C models the cell membrane, the resistance Ri models the intracellular resistance and the resistance Re models the extra-cellular resistance, $R_m$ is the resistance associated to the capacitor. (Seoane 2007)
**Frequency Dispersions**

The tissue presents electrical passive properties, conductivity and permittivity, which depends on the tissue composition and environment conditions, as well as the frequency. In Figure 2.2, (Buendía, 2012), the frequency dependency of the relative permittivity ($\varepsilon_r$) and conductivity (S/m) dispersion with the frequency in muscular tissue are shown.

Conductivity [$\delta$] is given by the ability of free charges to move into the media, while permittivity [$\varepsilon$] determines the ability of bounded charges to be polarized into the media.

The admittance of a material depends on its conductivity and permittivity, as well as the shape and the volume. It is measured by applying external energy into the material. The conductivity of a tissue is a function of both, the specific conductivity of the medium and the cellular substance as well as the volume concentration.

The frequency spectrum presents four regions, $\alpha$-dispersion, $\beta$-dispersion, $\gamma$-dispersion and $\delta$-dispersion. They are known as dispersion windows and they are based on the electrical properties of the biomaterial, which depend on the frequency.

![Figure 2.2 Plot of dispersion in frequency dependence (Buendia 2011)](image)

**$\alpha$-dispersion**

The $\alpha$ -dispersion appears at low frequencies, between 10 Hz – 10 kHz. Although the elements that contribute to this frequency dependency are not clearly identified yet, (Schwan, 1993) established three main causes. First, the effect of the endoplasmic reticulum contributes to this frequency dependence. Second, the channel proteins present in the plasma membrane causes also the frequency-dependent conductance. Finally, the relaxation of counter-ions on the charged cellular surface is another mechanism that produces this frequency dependence.

**$\beta$-dispersion**

This dispersion is mainly due to the low conductivity and capacitive properties of the plasma membrane and other internal membrane structures and their interactions.
with the extra and intra-cellular electrolytes. It ranges from approximately 10 kHz to 100 kHz.

**γ-dispersion**

This frequency dependence is caused by the large content of water in cell and tissue. Tissue water is identical to normal water, which relaxes at 20 GHz, except for the presence of proteins and amino acids, etc. Tissue water displays a broad spectrum of dispersion from hundreds of MHz to some GHz.

**δ-dispersion**

The δ-dispersion is a minor additional relaxation between β and γ, it is caused in part by rotation of amino acids, partial rotation of charge side groups of proteins, and relaxation of protein-bound water that occurs between 300 and 2000 MHz.

The Table 2.2 shows the elements that contribute to the different kind of dispersions that have been mentioned above.

<table>
<thead>
<tr>
<th>CONTRIBUTING BIOMATERIAL ELEMENT</th>
<th>DISPERSION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water and Electrolytes</td>
<td>α β γ δ</td>
</tr>
<tr>
<td>Biological Macromolecules</td>
<td></td>
</tr>
<tr>
<td>Amino acids</td>
<td></td>
</tr>
<tr>
<td>Proteins</td>
<td></td>
</tr>
<tr>
<td>Nucleic acids</td>
<td></td>
</tr>
<tr>
<td>Vesicles</td>
<td></td>
</tr>
<tr>
<td>Surface Charged</td>
<td></td>
</tr>
<tr>
<td>Non-Surface Charged</td>
<td></td>
</tr>
<tr>
<td>Cells with Membrane</td>
<td></td>
</tr>
<tr>
<td>&gt;&gt; Fluids free of protein</td>
<td></td>
</tr>
<tr>
<td>&gt;&gt; Tubular system</td>
<td></td>
</tr>
<tr>
<td>&gt;&gt; Surface charge</td>
<td></td>
</tr>
<tr>
<td>&gt;&gt; Membrane relaxation</td>
<td></td>
</tr>
<tr>
<td>&gt;&gt; Organelles</td>
<td></td>
</tr>
<tr>
<td>&gt;&gt; Protein</td>
<td></td>
</tr>
</tbody>
</table>

Table 2.2. Electrical dispersions of biological matter
2.2. EBI measurements

An Ohmic Impedance establishes a relationship between the Voltage difference between two points in a volume conductor and the electrical current that flows between these two points.

The electrical current in the human body is due to cations and anions which are ions positive and negative charged respectively. Consequently, the biological tissue carries electrical current by the migration of ions.

The impedance measurement is done by measuring the voltage drop or the currents through the load as a response to an AC current or voltage applied. The figure 2.3 shows the models used to carry out the measurements.

![Figure 2.3 Impedance measurements. (Seoane 2007)](image)

2.3. Cole Model

The Cole function accurately fit bioimpedance measurements inside the β-dispersion region (Cole, 1940). It is very useful to represent and analyze EBIS data. The Cole equation generates a complex value presenting a nonlinear relationship with the angular frequency. It produces a depressed semicircle in the impedance plane. The magnitude of this depression is determined by the parameter α, obtaining a complete semicircle when α equals 1. The equation contains three other parameters: $R_\infty$, $R_0$, and τ. and the angular frequency ($\omega$) is the independent variable. $R_0$ is the resistance at zero frequency, $R_\infty$ is the resistance when frequency tends to $\infty$ and τ is the time constant (RC) and the inverse of the characteristic frequency.

The depressed semicircle representing the Cole equation, see Equation 2.1, is the Cole plot shown in Figure 2.4, (Buendía, 2012). The horizontal axis of the plot represents the real part of the impedance, the resistance, and the vertical axis represents the imaginary part, the reactance. (Seoane et al., 2010).

\[
Z(\omega) = R_\infty + \frac{R_0 - R_\infty}{1 + (j\omega\tau)^\alpha}
\]  

Equation 2.1
It is shown in Figure 2.5 a three dimensions plot which shows how the real part decreases with higher frequencies.

The Cole model correspond, considering $\alpha=1$, with the Fricke’s model shown in Figure 2.1. The equivalent circuit shown in Figure 2.6 can be easily drawn from the Cole model as long as $\alpha=1$.

The electrical model is based on the circuit shown in Figure 2.6. The relations between the Cole equation parameters and the circuit elements let understanding of how the Cole model is obtained.

$$R_0 = R_s + R_p, \quad R_\infty = R_s \quad \text{and} \quad \tau = R_p \cdot C$$
It would be useful knowing how each parameter affects to the Cole equation for understanding the fitting results. In the Figure 2.7 is shown that:

- \( R_\infty \) acts by moving up and left the Cole diagram at high frequencies.
- \( R_0 \) acts by moving down and left the Cole diagram at low frequencies.
- \( \alpha \) acts by moving up the Cole diagram at central frequencies.

![Figure 2.6 Equivalent circuit](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_\infty )</td>
<td>296.7</td>
</tr>
<tr>
<td>( R_0 )</td>
<td>449.6</td>
</tr>
<tr>
<td>( \tau )</td>
<td>( 5.2727 \times 10^{-6} )</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.7186</td>
</tr>
</tbody>
</table>

Figure 2.7 Evolution when each parameter is increased. The original values of the parameters are: \( R=296.7, R_0=449.6, \tau=5.2727\times10^{-6} \) and \( \alpha=0.7186 \). They evolve to higher values with darker plots and lower values with lighter plots.

In the Figure 2.8 we can see the evolution of the Cole plot when the different parameters are modified.

![Figure 2.8 Evolution when increasing different parameters](image)

Figure 2.8 Evolution when increasing different parameters. The parameters are: \( R=296.7, R_0=449.6, \tau=5.2727\times10^{-6} \) and \( \alpha=0.7186 \). They evolve to higher values with darker plots and lower values with lighter plots.
CHAPTER 3

MATERIALS AND METHODS

3.1. EBI Measurement Models

3.1.1. Capacitive leakage Effect Model

The Capacitive leakage effect is observed mainly at high frequencies as a deviation that the reactance suffers. However it affects to resistance and reactance, as well as module and phase of the impedance at every frequency.

Based on the model shown in Figure 3.1 the electric current $I_0$, intended for stimulating the charge $Z_{TUS}$, partially leaks away through the capacitor in parallel $C_{par}$ instead of circulating through $Z_{TUS}$. This effect is frequency dependent and it is more noticeable at high frequencies because the parasitic leakage pathway becomes more conductive. This fact produces the deviation aforementioned (Buendia et al., 2010).

In Figures 3.2 and 3.3 can be observed how based on the model of Figure 3.1, from frequencies above 300 kHz the reactance increases when it would decrease. In the Figure 3.2 it can be observed the reactance function of frequency and in the Figure 3.3 the impedance plot, both with and without a $C_{par}$ of 50pF in parallel.
Figure 3.2 Reactance plot with and without Capacitive leakage effect. Ideal conditions (Blue). Parasitic capacitance in parallel of 50pF (Red)

Figure 3.3 Cole plot in ideal condition (Blue) and with a parasitic capacitance of 50pF in parallel (Red)
3.1.2. Parasitic Capacitance and Electrode Polarization Impedance Model

The model considered for the case of study, see Figure 3.4, is drawn from the one presented in Figure 3.1 but considering $Z_{ep}$.

$Z_{meas}$ can be obtained as a function of $Z_{tissue}$, $Z_{ep}$ and $C_{par}$ by analyzing the circuit of Figure 3.4, see Equations 3.1-3.6.

$$Z_{meas}(\omega) = \frac{V_{meas}}{I_0}$$ \hspace{1cm} Equation 3.1

$$\frac{V_0}{I_0} = \left(\frac{Z_{tus} + Z_{ep}}{Z_{tus}}\right) || (jX_{C_{par}})$$ \hspace{1cm} Equation 3.2

$$I_{tissue} = \frac{V_0}{Z_{tus} + Z_{ep}}$$ \hspace{1cm} Equation 3.3

$$V_{meas} = I_{tissue} \times Z_{tus} = V_0 \times \frac{Z_{tus}}{Z_{tus} + Z_{ep}}$$ \hspace{1cm} Equation 3.4

$$I_0 = V_0 \times \frac{Z_{tus} + Z_{ep} + jX_{C_{par}}}{(Z_{tus} + Z_{ep}) + jX_{C_{par}}}$$ \hspace{1cm} Equation 3.5

$$Z_{meas}(\omega) = \frac{V_{meas}}{I_0} = \frac{Z_{tus}}{Z_{tus} + Z_{ep} + jX_{C_{par}}} = \frac{Z_{tus} + jX_{C_{par}}}{Z_{tus} + Z_{ep} + jX_{C_{par}}} = \frac{Z_{tus}}{1 + \frac{Z_{tus} + Z_{ep}}{jX_{C_{par}}}}$$
\[
\{X_{Cpar} = \frac{-1}{w_{Cpar}} \} = \frac{Z_{tus}}{1+jw_{Cpar}(Z_{tus}+Z_{ep})} \\
\]

**Equation 3.6**

For the study \(Z_{ep}\) has been modeled with three elements, a resistance (RE11) in parallel with a series combination of a capacitor (CE11) and another resistance (RE1), as proposed in (Bogónez-Franco P et al., 2009). This way, the model of Figure 3.5 is gotten. \(Z_{ep}\) model can be observed in Figure 3.6 and its analytic expression in Equation 3.7.

\[
Z_{ep}(\omega) = (RE11)||\left(\frac{1}{w_{CE11}}\right) = \frac{RE11(RE1-\frac{j}{w_{CE11}})}{RE11+RE1-\frac{j}{w_{CE11}}} = \frac{RE11}{1+(\frac{RE11}{RE1-\frac{j}{w_{CE11}}})}
\]

**Equation 3.7**

Combining equations 3.6 and 3.7 the analytic expression of \(Z_{meas}\), considering the \(Z_{ep}\) model, can be obtained, See Equation 3.8. This is the \(Z_{meas}\) of the EBIS measurement model utilized in the study and presented in Figure 3.5.
3.1.3. Electrode mismatch effect

The mismatch of electrodes occurs when there is a large difference between the $Z_{ep}$ of the electrodes used in the EBIS measurement, (Bogónez-Franco P et al., 2009). It cannot be analyzed with the model used previously. More complex EBIS measurements models would allow it.

3.2. Non-Linear Least Squares (NLLS) for Cole Function modulus fitting

A fitting consists in constructing a function that has the best fit to a series of data points, normally subjected to restrictions.

NLLS fitting approach aims to obtain the best coefficients for a given model that fits the curve. The method given by eq. 3.10 aims to minimize the summed squared of the error between the measured data value $Z_i$ and the fitted value $|\tilde{Z}_i|$. $N$ is the number of frequency data points included in the fitting. This approach was validated in (Ayllon et al., 2009) as working approach to estimate the Cole parameters from the resistance spectrum. In this case, the minimization cost function has been built with the modulus of the complex EBI. Thus, the term $Z_i$ is the modulus of the measured impedance, and $\tilde{Z}_i$ is the modulus of the Cole function as shown in 3.10. Performing the curve fitting using a Cole-based function allows the estimation of the values for the four Cole parameters.

$$\min \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} ( |Z_i| - |\tilde{Z}_i| )^2 $$  \hspace{1cm} \text{Equation 3.10}

3.3. Mathematica

Mathematica is one of the most spread software tools for mathematical analysis, it has been chosen for this job due to its versatility to work with mathematical models and dynamic graphs representation.
3.4. Cole Fitting method Implementation in Mathematica

The process followed to obtain the fittings by using Mathematica software is:

Obtaining data by giving values to the different parameters: Cole parameters, RE11, RE1, CE11 and $C_{par}$. Thereafter the fitting to the Cole equation is performed on, in this case, the modulus. The modulus of the Cole equation is expressed in Equation 3.11

$$|Z_i| = \sqrt{\left[ \frac{R_\infty + (R_0 - R_\infty)(1 + (\omega \tau)^\alpha \cos(\frac{\omega \tau}{2}))}{1 + 2(\omega \tau)^\alpha \cos(\frac{\omega \tau}{2}) + (\omega \tau)^{2\alpha}} \right]^2 + \left[ \frac{(R_0 - R_\infty)(\omega \tau)^\alpha \sin(\frac{\omega \tau}{2})}{1 + 2(\omega \tau)^\alpha \cos(\frac{\omega \tau}{2}) + (\omega \tau)^{2\alpha}} \right]^2}$$

Equation 3.11

Fit and FindFit are the Mathematica functions utilized to perform the fitting, they both produce least-squares fittings which are defined to minimize the error as shown in Equation 3.12

$$x^2 = \sum_{i=0}^{n} |r_i|^2$$

Equation 3.12

In Equation 3.12, $X$ is the error and $r_i$ are residuals giving the difference between each original data point and its fitted value.

The different values of the Cole parameters: $R_\infty$, $R_0$, $\tau$ and $\alpha$ are obtained using this process. An example is found in Figure 3.11.

Values: \(R_\infty \rightarrow 294.478, R_0 \rightarrow 451.597, \tau \rightarrow 5.29988 \times 10^{-6}, \alpha \rightarrow 0.700861\)

Constraints: \(200 < R_\infty' < 400,400 < R_0' < 600,0 < \alpha' < 1, \frac{1}{200000} < \tau' < 0.0000055\)

Figure 3.11 Example of the fittings performed on the modulus using Fit and FindFit Mathematica functions
Finally, in Figure 3.12 the differences between the original plot, the plot with the parasitic elements and the fitted plot can be observed through the impedance plot.

Figure 3.12 Cole plot with parasitic elements (Red), ideal plot (Green) and fitted plot (Blue).
CHAPTER 4

RESULTS

4.1. Software engine

The creation of the software engine allows the estimation of Cole parameters considering EBI measurements models. The value of the elements of the models can be modified with the graphic interface showed in Figure 4.1. The model implemented for the study takes into account the main artifacts that may affect EBIS measurements in TBC. They are the parasitic capacitance and the electrode polarization impedance.

The engine is flexible and versatile so, the parameters included in the model can be changed. This way the user may not consider any of the artifacts implemented here as well as adding other artifacts.

4.1.1 The model used

For a deeper analysis and more options choosing parameters, the model chosen was the one in Figure 4.2:

![Figure 4.1 Model implemented](image)

In addition to the previously modeled $Z_{ep}$, it contains another block to modify values of more parameters of the electrode polarization impedance which will allow simulating scenarios with a more complex $Z_{ep}$.

4.1.2 The user interface

The user can modify all the parameters, $R_0$, $R_\infty$, $\tau$, $\alpha$ (Initial cole parameters), $C_{par}$, $RE_{11}$, $RE_1$ and $CE_{11}$ ($Z_{ep}$) and $RE_{11b}$, $RE_{1b}$ and $CE_{11b}$ ($Z_{ep'}$). Modifying the initial cole parameters is interesting due to the simulation of measurements will not be done always in the same part of the body, each part needs their own parameters to be measured with more precision.
Figure 4.2 shows the panel where the different parameters can be modified sliding the buttons. They can be modified also by writing the values manually. When the parameters are modified the different graphics: Cole plot (Figure 4.3), conductance plot (Figure 4.5), module of the impedance (Figure 4.4), real part (Figure 4.7) and imaginary part (Figure 4.6) are changing in real time.
Figure 4.4 Graphic of the module plot

Figure 4.5 Graphic of the conductance plot
Figure 4.6 Graphic of the imaginary part in logarithmic scale

Figure 4.7 Graphic of the real part
Figure 4.8 is the final result of the graphic interface.

![Figure 4.8 Graphic interface](image)

**4.1.3 Results visualization**

The final result of the tool provide the errors in percentage regarding the measurement under ideal conditions and a figure that includes three plots: the Cole impedance built out of the Cole parameters obtained under ideal conditions, the impedance plot of the model including $C_{par}$ and $Z_{ep}$, and finally the Cole plot drawn from the parameters obtained as a result of the fitting done from the measurement with artifacts.

Figure 4.9 is the final result of the tool which is formed by several parts. Firstly, the parameter analyzed and its actual value can be observed. Secondly, the values of the different Cole parameters are written with its relative errors respect the original ones. The fitting is shown with the plot of the module with Cole parameters previously obtained. Finally the ideal impedance plot (green), the impedance plot with the parasitic elements (red) and the fitted impedance plot (blue) are shown allowing the user to compare them.
Figure 4.9 Final result of the script done (Loop of two iterations)
4.2. Example of application: statistic results and analysis

Analyzing the results obtained with the tool performed may help to realize how each parameter of the $Z_{ep}$ model and the $C_{par}$ affect to real measurements. In this part it will be analyzed and it will be found the maximum value of parasitic elements, $Z_{ep}$ and $C_{par}$, that would let us to estimate the value of the Cole parameters of the tissue impedance with an acceptable error.

In this study the analysis will be done with the results obtained with half the nominal value of the parameters, the nominal value, twice the value, five and ten times the value of each parameter observed. For example, the Table 4.1 shows the errors (%) with these different values of $C_{par}$.

4.2.1. Results modifying $C_{par}$

At first the impact of $C_{par}$ was analyzed, which nominal value was set in 50 pF. The constraints of the Cole parameters to be fitted can be modified and it will let to analyze how with different constraints the results can be similar but never the same. In the next example, Initial values of Zep elements and Cole parameters are showed in Table 4.1A and the results of the different errors of Cole parameters in percentage obtained are showed in Table 4.1B and Figure 4.10.

The fitting are realized with the next constraints:

$$100< R_\infty <500, 200< R_0<600, 5*10^{-6}< \tau<5.5*10^{-6} \text{ and } 0.5<\alpha<1$$

<table>
<thead>
<tr>
<th>$C_{par}$</th>
<th>$R_{inf}$</th>
<th>$R_0$</th>
<th>$\tau$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25pF</td>
<td>0.17</td>
<td>1.4</td>
<td>5.09</td>
<td>3.11</td>
</tr>
<tr>
<td>50pF</td>
<td>0.91</td>
<td>1.3</td>
<td>5.17</td>
<td>1.3</td>
</tr>
<tr>
<td>100pF</td>
<td>2.81</td>
<td>1.26</td>
<td>5.17</td>
<td>1.14</td>
</tr>
<tr>
<td>250pF</td>
<td>13.7</td>
<td>2.1</td>
<td>5.17</td>
<td>17</td>
</tr>
<tr>
<td>500pF</td>
<td>32.34</td>
<td>7.63</td>
<td>5.17</td>
<td>30.42</td>
</tr>
<tr>
<td>5nF</td>
<td></td>
<td></td>
<td></td>
<td>It's not possible getting a good fitting</td>
</tr>
</tbody>
</table>

Table 4.1B Results
It can be observed how for $C_{par}$ higher than 100pF the errors obtained start increasing to values that will not let to know the real parameters of Cole equation. If the constraints are modified and we narrow them to closer values of the real parameters, the results are improved as it can be seen in Table 4.2 and Figure 4.11, but there is the problem that in real cases the parameters expected will not be previously known.

The fitting obtained when the constraints are close to the Cole parameters values shows how it is possible to obtain good fittings for $C_{par}$ values until 250pF, for higher values the results are not good enough.

$$250<R_{\infty}<350, \ 400<R_0<500, \ 5*10^{-6}<\tau<5.5*10^{-6} \ \text{and} \ 0.5<\alpha<1$$

Table 4.2 Results of the different errors of Cole parameters in percentage obtained with different values of $C_{par}$. Note closer values of Cole parameters as a constraints than the simulation of table 4.1 and Fig 4.10
A comparison of Cole plot with different values of $C_{par}$ is showed in the Figure 4.12. The red plot is the evolution of Cole plot when $C_{par}$ is increasing and the rest of the parameters are fixed, less color intensity means higher values of $C_{par}$. The green plot shows the original plot, it is without capacitive leakage. The blue plots are the results of the parameters gotten.
4.2.2. Results modifying electrode polarization impedance parameters

In following tests the constraints are the first one used, it is because the real parameters will not be known in real measurements and it is needed a wide range of values to be seek in

Individual parameter changes:

The electrode polarization impedance, $Z_{ep}$, is modeled as a block of three elements, RE1, RE11 and CE11. Firstly the parameters were modified individually. Table 4.2 and the Figure 4.13 show the results modifying the value of RE11 and fixing the values of RE1, CE11 and $C_{par}$.

Initial values of $Z_{ep}$ elements (without RE11, the analyzed element) and Cole parameters are showed in Table 4.2A and the results of the different errors of Cole parameters in percentage obtained are showed in Table 4.2B and Figure 4.13.

Nominal value of RE11=10KΩ

<table>
<thead>
<tr>
<th>Cpar</th>
<th>5,00E-11</th>
</tr>
</thead>
<tbody>
<tr>
<td>RE1</td>
<td>250</td>
</tr>
<tr>
<td>CE11</td>
<td>1,50E-08</td>
</tr>
<tr>
<td>Rinf</td>
<td>296,7</td>
</tr>
<tr>
<td>R0</td>
<td>449,6</td>
</tr>
<tr>
<td>T</td>
<td>5,27E-06</td>
</tr>
<tr>
<td>α</td>
<td>0,7186</td>
</tr>
</tbody>
</table>

Table 4.2A Initial conditions

<table>
<thead>
<tr>
<th>RE11</th>
<th>Rinf</th>
<th>R0</th>
<th>T</th>
<th>α</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>0,89</td>
<td>1,26</td>
<td>5,17</td>
<td>1,32</td>
</tr>
<tr>
<td>10000</td>
<td>0,91</td>
<td>1,3</td>
<td>5,17</td>
<td>1,3</td>
</tr>
<tr>
<td>20000</td>
<td>0,93</td>
<td>1,31</td>
<td>5,17</td>
<td>1,29</td>
</tr>
<tr>
<td>50000</td>
<td>0,94</td>
<td>1,32</td>
<td>5,17</td>
<td>1,29</td>
</tr>
<tr>
<td>100000</td>
<td>0,95</td>
<td>1,33</td>
<td>5,17</td>
<td>1,28</td>
</tr>
<tr>
<td>1000000</td>
<td>0,95</td>
<td>1,33</td>
<td>5,17</td>
<td>1,28</td>
</tr>
</tbody>
</table>

Table 4.2B Results of the different errors of Cole parameters in percentage obtained with different values of RE11.
The same process is done with RE1 being modified and the rest of elements fixed, the results are showed in the Table 4.3B and the Figure 4.14.

**Nominal value of RE1 =250Ω**

Initial conditions:

<table>
<thead>
<tr>
<th>Cpar</th>
<th>5E-11</th>
</tr>
</thead>
<tbody>
<tr>
<td>RE1</td>
<td>10000</td>
</tr>
<tr>
<td>CE11</td>
<td>1.5E-8</td>
</tr>
<tr>
<td>Rinf</td>
<td>296.7</td>
</tr>
<tr>
<td>R0</td>
<td>449.6</td>
</tr>
<tr>
<td>T</td>
<td>5.27E-6</td>
</tr>
<tr>
<td>a</td>
<td>0.7186</td>
</tr>
</tbody>
</table>

Table. 4.3A Initial conditions

<table>
<thead>
<tr>
<th>RE1</th>
<th>Rinf</th>
<th>R0</th>
<th>τ</th>
<th>α</th>
</tr>
</thead>
<tbody>
<tr>
<td>125</td>
<td>0.22</td>
<td>1.4</td>
<td>5.09</td>
<td>3.25</td>
</tr>
<tr>
<td>250</td>
<td>0.21</td>
<td>1.3</td>
<td>5.09</td>
<td>2.98</td>
</tr>
<tr>
<td>500</td>
<td>0.77</td>
<td>0.8</td>
<td>5.17</td>
<td>1.01</td>
</tr>
<tr>
<td>1250</td>
<td>1.13</td>
<td>0.39</td>
<td>5.17</td>
<td>0.25</td>
</tr>
<tr>
<td>2500</td>
<td>1.61</td>
<td>0</td>
<td>5.17</td>
<td>1.51</td>
</tr>
<tr>
<td>25000</td>
<td>0.67</td>
<td>0.7</td>
<td>5.17</td>
<td>3.86</td>
</tr>
</tbody>
</table>

Table. 4.3B Results of the different errors of Cole parameters in percentage obtained with different values of RE1.
The same process is done with CE11 being modified and the rest of elements fixed, the results are showed in the Table 4.4B and the Figure 4.15

**Nominal value of CE11 =15nF**

Initial conditions:

<table>
<thead>
<tr>
<th>Cpar</th>
<th>5,00E-11</th>
</tr>
</thead>
<tbody>
<tr>
<td>RE1</td>
<td>250</td>
</tr>
<tr>
<td>RE11</td>
<td>10000</td>
</tr>
<tr>
<td>Rinf</td>
<td>296,7</td>
</tr>
<tr>
<td>R0</td>
<td>449,6</td>
</tr>
<tr>
<td>T</td>
<td>5,27E-06</td>
</tr>
<tr>
<td>α</td>
<td>0,7186</td>
</tr>
</tbody>
</table>

Table 4.4A Initial values

<table>
<thead>
<tr>
<th>CE11</th>
<th>Rinf</th>
<th>R0</th>
<th>τ</th>
<th>α</th>
</tr>
</thead>
<tbody>
<tr>
<td>7,5E-09</td>
<td>0,45</td>
<td>1,51</td>
<td>5,1</td>
<td>4,2</td>
</tr>
<tr>
<td>0,000000015</td>
<td>0,21</td>
<td>1,3</td>
<td>5,09</td>
<td>2,98</td>
</tr>
<tr>
<td>0,00000003</td>
<td>0,21</td>
<td>1,18</td>
<td>5,09</td>
<td>2,79</td>
</tr>
<tr>
<td>0,000000075</td>
<td>0,31</td>
<td>0,79</td>
<td>5,17</td>
<td>1,44</td>
</tr>
<tr>
<td>0,00000015</td>
<td>0,26</td>
<td>0,75</td>
<td>5,17</td>
<td>1,43</td>
</tr>
<tr>
<td>0,0000015</td>
<td>0,07</td>
<td>0,81</td>
<td>5,15</td>
<td>1,89</td>
</tr>
</tbody>
</table>

Table 4.4B Results of the different errors of Cole parameters in percentage obtained with different values of CE11.
Evaluating $Z_{ep}$ as a block:

In the next example the fitting is obtained treating $Z_{ep}$ as a block, increasing their elements proportionally. Results are shown in Table 4.5B, Figure 4.16 and Figure 4.17. The initial values of the $Z_{ep}$ elements are: $RE1=250\,\Omega$, $RE11=10K\,\Omega$, $CE11=15nF$.

![Error plot](image)

**Figure 4.15** Plot of errors obtained in table 4.5

<table>
<thead>
<tr>
<th>Cpar</th>
<th>5.00E-11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rinf</td>
<td>296.7</td>
</tr>
<tr>
<td>R0</td>
<td>449.6</td>
</tr>
<tr>
<td>$\tau$</td>
<td>5.27E-06</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.7186</td>
</tr>
</tbody>
</table>

**Table 4.5A** Initial values of Cpar and Cole parameters

<table>
<thead>
<tr>
<th>Zep</th>
<th>Rinf</th>
<th>R0</th>
<th>$\tau$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VN</td>
<td>0.73</td>
<td>1.347</td>
<td>5.171</td>
<td>1.82</td>
</tr>
<tr>
<td>5*VN</td>
<td>1.223</td>
<td>0.238</td>
<td>5.171</td>
<td>0.645</td>
</tr>
<tr>
<td>5*VN</td>
<td>4.262</td>
<td>1.576</td>
<td>5.171</td>
<td>6.965</td>
</tr>
<tr>
<td>10*VN</td>
<td>20.52</td>
<td>11.416</td>
<td>5.171</td>
<td>30.396</td>
</tr>
<tr>
<td>20*VN</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50*VN</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is not possible getting an acceptable fitting.

**Table 4.5B** Results of the different errors of Cole parameters in percentage obtained with different values of Zep.
In the case of increasing proportionally the electrode polarization impedance parameters, the effect on the Cole parameters is similar to the capacitance leakage effect. This is because the modulus is affected in a similar way when increasing values of the parasitic capacitance and when increasing the electrode polarization impedance.

Figure 4.16 Plot of errors obtained in table 4.6

Figure 4.17 Cole plot results. Ideal conditions (green), plot with parasitic elements(red) and fitting results(blue).
Trying to get a better approach of how the $Z_{ep}$ and $C_{par}$ affect to the fitting, the results increasing $Z_{ep}$ varying $C_{par}$ were analyzed.

The Cole parameter estimation if the parasitic capacitance is reduced to its half value, 25pF, is much better. With $C_{par}$ fixed at 25 pF the errors (in percentage respect the original value) for a $Z_{ep}$ of ten times the nominal value are:

<table>
<thead>
<tr>
<th>$R_\infty$</th>
<th>$R_0$</th>
<th>$\tau$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3,516</td>
<td>1,191</td>
<td>5,171</td>
<td>5,839</td>
</tr>
</tbody>
</table>

The fitting result if the parasitic capacitance is increased to its double value, 100pF, is slightly worse than the results with $C_{par} = 50$ pF due to it is higher. The maximum errors for ten times the nominal value of $Z_{ep}$ are (in percentage respect the original value):

<table>
<thead>
<tr>
<th>$R_\infty$</th>
<th>$R_0$</th>
<th>$\tau$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>34,012</td>
<td>17.169</td>
<td>5,171</td>
<td>30.420</td>
</tr>
</tbody>
</table>
CHAPTER 5: DISCUSSION, CONCLUSION AND FUTURE WORK

5.1. Discussion and General Conclusions

The software engine may be a useful tool to analyze the effect of artifacts on real EBIS measurements, not only for TBC but also for segmental, thoracic measurements, or any application that fit the data to the Cole model.

The flexibility and versatility of this tool make it able to be adapted to more complex models, as could be a more detailed one of the electrode skin interface or more complex EBIS measurement models. Even other models, apart from the Cole model could be implemented.

If the effect of artifacts over segmental measurements are going to be analyzed the approximate values of the original Cole parameters can be easily modified in the visual interface of the tool.

The electrode mismatch effect cannot be analyzed with the model implemented in the tool. For that purpose more complex EBIS models like the one in Figure 5.1 are needed. The same can be said about the effect of Zep over voltage sensing electrodes.

About the results obtained testing with the software engine, the main conclusion that can be gotten is how the electrode polarization impedance has a similar effect as the parasitic capacitance on the modulus and consequently on the modulus fitting. If the parasitic capacitance is low the electrode polarization impedance has a negligible effect in the fitting results.

Other fitting models can be used and easily implemented apart from the modulus fitting. The conductance fitting could be an interesting case of study due to its robustness against parasitic capacitances. It might be helpful to implement different models for the automatic realization of the fitting avoiding the manual change of the fitting model each time it is needed to use a new one.

5.2. Future Work

A more extensive and accurate study of the results could be done as a future work; it would be interesting to analyze the results when the values of the parasitic capacitance and the electrode polarization impedance are changing at the same time.

In addition more complex EBIS measurements models could be implemented and the effect of parasitic capacitances, Zep and mismatch electrodes studied. The next model, Figure 5.1, is proposed. It will allow analyzing the electrode mismatch effect.

Studying the reactance function could be interesting because it may allow getting some valuable information:
- Maximum value of the reactance.
- The characteristic frequency.
- The relation between these values, the parasitic capacitance and the electrode polarization impedance.

Figure 5.1 Model proposed for future improvements.
**APPENDIX A**

**Code**

The next Mathematica code is the realization of a graphic interface where all the variables can be modified. The first part is the code needed to reset the variables that will be used for calculating fitting values\(^1\). Next the variables are initialized and it will be used in the plots done\(^2\). The graphic interface, where all the parameters can be modified, and the graphs that will change in real time when the variables are modified, are the last part of this code\(^3\).

Clear[Rinf2, R02, T2, a2] \hspace{1cm} (1)

\[
Cpar = 1*10^\text{-}12; \quad \text{RE11} = 10000; \quad \text{RE1} = 250; \quad \text{CE11} = 15*10^\text{-}9; \quad \text{Rinf} = 296.7; \quad \text{R0} = 449.6; \quad T = 5.2727*10^\text{-}6; \quad a = 0.7186; \quad \text{RE11b} = \text{RE11; RE1b} = \text{RE1; CE11b} = \text{CE11}; \hspace{1cm} (2)
\]

Grid[{{{Column[{{Slider[Dynamic[Cpar], {1*10^\text{-}30, 10*10^\text{-}11}], Dynamic[Cpar], "Cpar"}], {Slider[Dynamic[RE11], {2000, 200000}], Dynamic[RE11], "RE11"}], {Slider[Dynamic[CE11], {10, 10000}], Dynamic[CE11], "CE11"}], {Slider[Dynamic[RE11b], {1.1*10^\text{-}12, 1.1*10^\text{-}7}], Dynamic[CE11b], "CE11b"}], {Slider[Dynamic[Rinf], {200, 500}], Dynamic[Rinf], "Rinf"}], {Slider[Dynamic[R0], {400, 800}], Dynamic[R0], "R0"}], {Slider[Dynamic[\(T\)], {2*10^\text{-}6, 10*10^\text{-}6}], Dynamic[T], "T"}], {Slider[Dynamic[a], {0.5, 0.85}], Dynamic[a], "alpha"}]}], \hspace{1cm} (2.1)

Manipulate[ListPlot[Table[Re[((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a))/1 + (((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a))) + (RE11 + (I*(w*CE11)))/(RE11 + RE1 + (-I/(w*CE11)))) + RE11b (RE1b + -l/(w*CE11b)))/(RE11b + RE1b + (-I/(w*CE11b)))))/(1 + (I*w*T)^a))/1 + (((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a))) + (RE11 + (I*(w*CE11)))/(RE11 + RE1 + (-I/(w*CE11)))) + RE11b (RE1b + -l/(w*CE11b)))/(RE11b + RE1b + (-I/(w*CE11b)))))/(I*(1/(w*Cpar))))]], \{w, 50000, 50000000, 100000\}, PlotStyle -> {Red}, Joined -> True, PlotRange -> {{200, 600}, {0, 100}}, PlotLabel -> "COLE FUNCTION f(Cpar,CE11,RE11,RE1)", AxesLabel -> {"Re", "Im"}], {None}], \hspace{1cm} (3.1)

Manipulate[ListPlot[Table[Re[1/((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a))/1 + (((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a))) + (RE11 (RE1 + (-I/(w*CE11))))/(RE11 + RE1 + (-I/(w*CE11)))) + RE11b (RE1b + -l/(w*CE11b)))/(RE11b + RE1b + (-I/(w*CE11b)))))/(I*(-1/(w*Cpar))))]]], \{w, 50000, 50000000, 100000\}, PlotStyle -> {Red}, Joined -> True, PlotLabel -> "Conductancia", AxesLabel -> {"w", "Cy"}], {None}], \hspace{1cm} (3.2)
graficaModulo = Manipulate[ListPlot[Table[{w, Sqrt[Re[((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a)))/(1 + (((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a))))) + (RE11 (RE1 + (-I/(w*CE11)))(RE11 + RE1 + (-I/(w*CE11))))) + RE11b(RE1b + (-I/(w*CE11)))(RE11b + RE1b + (-I/(w*CE11)))))]/(1 + (((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a))))) + (RE11 (RE1 + (-I/(w*CE11)))(RE11 + RE1 + (-I/(w*CE11))))) + RE11b(RE1b + (-I/(w*CE11)))(RE11b + RE1b + (-I/(w*CE11)))))]/(1 + (((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a))))) + (RE11 (RE1 + (-I/(w*CE11)))(RE11 + RE1 + (-I/(w*CE11))))) + RE11b(RE1b + (-I/(w*CE11)))(RE11b + RE1b + (-I/(w*CE11)))))^2 + (Im[((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a))))) + (RE11 (RE1 + (-I/(w*CE11)))(RE11 + RE1 + (-I/(w*CE11))))) + RE11b(RE1b + (-I/(w*CE11)))(RE11b + RE1b + (-I/(w*CE11)))))]/(1 + (((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a))))) + (RE11 (RE1 + (-I/(w*CE11)))(RE11 + RE1 + (-I/(w*CE11))))) + RE11b(RE1b + (-I/(w*CE11)))(RE11b + RE1b + (-I/(w*CE11)))))^2}], {w, 50000, 50000000, 10000}], PlotStyle -> {Red}, Joined -> True, PlotRange -> {{0, 1000000}, {300, 500}}, PlotLabel -> "Módulo", AxesLabel -> {"w", "|W|"}, {None}], (3.3)

Manipulate[ListPlot[Table[{w, Re[((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a)))/(1 + (((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a))))) + (RE11 (RE1 + (-I/(w*CE11)))(RE11 + RE1 + (-I/(w*CE11))))) + RE11b(RE1b + (-I/(w*CE11)))(RE11b + RE1b + (-I/(w*CE11)))))^2}], {w, 50000, 50000000, 10000}], PlotStyle -> {Red}, Joined -> True, PlotRange -> {{0, 1000000}, {300, 400}}, PlotLabel -> "Parte Real!", AxesLabel -> {"w", "Re"}, {None}], (3.4)

datosModulo = Table[{w, Sqrt[Re[((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a)))/(1 + (((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a))))) + (RE11 (RE1 + (-I/(w*CE11)))(RE11 + RE1 + (-I/(w*CE11))))) + RE11b(RE1b + (-I/(w*CE11)))(RE11b + RE1b + (-I/(w*CE11)))))]/(1 + (((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a))))) + (RE11 (RE1 + (-I/(w*CE11)))(RE11 + RE1 + (-I/(w*CE11))))) + RE11b(RE1b + (-I/(w*CE11)))(RE11b + RE1b + (-I/(w*CE11)))))^2 + (Im[((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a))))) + (RE11 (RE1 + (-I/(w*CE11)))(RE11 + RE1 + (-I/(w*CE11))))) + RE11b(RE1b + (-I/(w*CE11)))(RE11b + RE1b + (-I/(w*CE11)))))]/(1 + (((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a))))) + (RE11 (RE1 + (-I/(w*CE11)))(RE11 + RE1 + (-I/(w*CE11))))) + RE11b(RE1b + (-I/(w*CE11)))(RE11b + RE1b + (-I/(w*CE11)))))^2}], {w, 50000, 50000000, 10000}], PlotStyle -> {Red}, Joined -> True, PlotRange -> {{0, 1000000}, {-60, -20}}, PlotLabel -> "Parte Imaginaria!", AxesLabel -> {"w", "Im"}, {None}]], (3.5)

model = {Sqrt[Re[Rinf2 + ((R02 - Rinf2)/(1 + (I*w*T)^2)^a2))])^2 + (Im[Rinf2 + ((R02 - Rinf2)/(1 + (I*w*T)^2)^a2))])^2}], {200 < Rinf2 < 400, 400 < R02 < 600, 0 < a2 < 1, 5*10^-6 < T2 < 5.5*10^-6}; (5)

fit = FindFit[datosModulo, model, {Rinf2, R02, T2, a2}, w]; (6)

ZtusEstimado = Rinf2 + ((R02 - Rinf2)/(1 + (I*w*T)^2)^a2)); (7)

Expr = Sqrt[Re[Rinf2 + ((R02 - Rinf2)/(1 + (I*w*T)^2)^a2))])^2 + (Im[Rinf2 + ((R02 - Rinf2)/(1 + (I*w*T)^2)^a2))])^2]; (8)
For[$Cpar = 50*10^{-12}$, $RE11 = 10000$, $RE1 = 250$, $CE11 = 15*10^{-9}$, $RE11b = RE11$, $RE1b = RE1$, $CE11b = CE11$, $Rinf = 296.7$, $R0 = 449.6$, $T = 5.2727*10^6$, $a = 0.7186$], $Cpar <= 100*10^{-12}$, $Cpar = Cpar*2$,
{Print["Cpar=", Print[Cpar], Clear[Rinf2, R02, T2, a2], (9)}

$datosModulo = Table[[w, Sqrt[(((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a)))/(1 + (((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a)))) + (RE11 (RE1 + -(I/(w*CE11)))/(RE11 + RE1 + -(I/(w*CE11))) + RE11b (RE1b + -(I/(w*CE11b)))/(RE11b + RE1b + -(I/(w*CE11b)))))))]^2 + (Im[(Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a)))/(1 + (((Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a)))) + (RE11 (RE1 + -(I/(w*CE11)))/(RE11 + RE1 + -(I/(w*CE11))) + RE11b (RE1b + -(I/(w*CE11b))))) + (I*(-(1/(w*Cpar))))])^2], [w, 50000, 1000000, 50000]], ListLinePlot[datosModulo, Epilog -> {PointSize[Medium], Point[datosModulo]}, InterpolationOrder -> 2], (10.1)

$model = \{Sqrt[Re[Rinf2 + ((R02 - Rinf2)/(1 + (I*w*T2)^a2))]^2 + (Im[Re[Rinf2 + ((R02 - Rinf2)/(1 + (I*w*T2)^a2))]^2], 100 < Rinf2 < 500, 200 < R02 < 600, 0.5 < a2 < 1, 5*10^{-6} < T2 < 5.5*10^{-6}], (10.2)

$fit = FindFit[datosModulo, model, \{Rinf2, R02, T2, a2\}, w], (11.3)

$ZtusValoresIniciales = Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a)), (10.4)

Print[Plot[Evaluate[Expr / . fit], \{w, 100, 1000000\}, Epilog -> \{PointSize[Medium], Print[datosModulo]\}], (10)]]

Print[Grid[Table["Rinf=", Evaluate[Rinf2 / . fit], "ErrorRel(%)=", Abs[100*(Rinf - Evaluate[Rinf2 / . fit])]/Rinf], \{i, 1\}], (12.2)

Print[Grid[Table["R0=", Evaluate[R02 / . fit], "ErrorRel(%)=", Abs[100*(R0 - Evaluate[R02 / . fit])]/R0], \{i, 1\}], (12.3)

Print[Grid[Table["T=", Evaluate[T2 / . fit], "ErrorRel(%)=", Abs[100*(T - Evaluate[T2 / . fit])/T]], \{i, 1\}], (12.4)

Print[Grid[Table["a=", Evaluate[a2 / . fit], "ErrorRel(%)=", Abs[100*(a - Evaluate[a2 / . fit])]/a]], \{i, 1\}], (12.5)

$graf0 = ListPlot[Table[Re[Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a))], Im[-(Rinf + ((R0 - Rinf)/(1 + (I*w*T)^a)))]], \{w, 50000, 50000000, 10000\}], PlotRange -> \{300, 450\}, \{10, 50\}, PlotStyle -> Directive[Green, Thickness[0.012]], PlotLabel -> "IDEAL", AxesLabel -> "Re", "Im"], Joined -> True], (13)
Expression used to get the data with all the parasitic elements acting (4)

$$\begin{align*}
\text{Re} &= \left[ \text{Rinf} + \frac{\text{R0} - \text{Rinf}}{1 + (i\omega T)^a} \right]^2 \\
\text{Im} &= \left[ \text{Rinf} + \frac{\text{R0} - \text{Rinf}}{1 + (i\omega T)^a} \right]^2
\end{align*}$$

Equation 3.1

Therefore, the model used, the modulus of the Cole equation, is declared and the constraints of each parameter are fixed. (5) The function \text{Findfit} find the values of the parameters: \(R_\infty, R_0, \tau, \alpha.\) (6) \text{Z tusEstimado} gets the equation of modulus and it will let to get the Cole expression that will be plotted. (7)
Expr is the expression of the modulus which is used to get the plot obtained with the parameters found in the fitting. It will serve to test if the fitting realized is good or not. \(^{(8)}\)

![Figure 3.8](image)

The next code is a loop which can be modified to analyze with different parameters and get the results desired. \(^{(9)}\) It is important to realize that the parameters Rinf2, R02, T2 and a2 are reset in each iteration.

The code \(^{(4 \text{ to } 8)}\) is repeated inside the loop to repeat the process in each iteration. \(^{(10)}\)

Thereafter, the plot in Figure 3.8 is obtained using the expression Expr mentioned previously. \(^{(11)}\) The prints with the relatives errors of the different parameters of Cole are printed. \(^{(12)}\)

Finally, the graphs of the fitting results \(^{(15)}\), of the Cole expression with ideal conditions (without parasitic Cpar and Zep) \(^{(13)}\) and of the Cole expression with real conditions (different parameters of Cpar and Zep) \(^{(14)}\) are plotted. \(^{(16)}\)
REFERENCES


