Application of intelligent materials to enhance SPAR floating offshore platforms stability

Maria Rius Planas

Dissertação para obtenção do Grau de Mestre em

Engenharia Mecânica

Júri
Presidente: Prof. Mario Manuel Gonçalves da Costa
Orientador: Prof. Antonio José Nunes de Almeida Sarmento
Vogais: Prof. Luis Manuel de Carvalho Gato
Prof. José Alberto Caiado Falcão de Campos

Setembro 2011
Resumo

Esta tese apresenta um estudo teórico sobre a estabilidade de uma plataforma flutuante, composta por dois cilindros concêntricos, para aplicação de turbinas eólicas offshore.

No segundo capítulo é feita uma revisão dos conceitos básicos da teoria linear, bem como a dedução da equação do movimento. O capítulo seguinte resume os resultados do estudo das dimensões óptimas e os resultados da simulação da equação do movimento da estrutura com e sem prato de inércia. A adição do prato, assim como a variação da sua porosidade em função da frequência da onda incidente, ajuda a estabilizar a estrutura numa gama ampla de frequências, dado o aumento da massa adicionada e a consequente redução da frequência da ressonância da estrutura.

Graças à colaboração do Centro de Nanotecnologia e Materiais Inteligentes, CeNTI, uma solução para simular a variação da porosidade é proposta, assim como o material candidato para a estrutura e os tratamentos necessários contra corrosão do água do mar. Finalmente, o trabalho futuro e as conclusões da tese são resumidas no último capítulo.

Palavras-chave:
Energia eólica offshore, estabilidade, teoria linear, prato de inércia, controle da porosidade, corrosão.
Abstract

The following thesis presents a stability study of a floating wind turbine platform, composed by two concentric cylinders, for offshore application.

The theoretical bases of the linear theory and the derivation of the equation of motion are introduced in the second chapter. The following chapter resumes the results of the research of the optimum dimensions as well as the results of the simulation of the equation of motion of the spar with and without inertial plate. The addition of the inertial plate and the variation of its porosity (depending on the frequency of the incident wave) help to stabilize the structure in a large frequency bandwidth, due to the increment of the added mass coefficient and the consequent reduction of the resonance frequency.

With the collaboration of the Centre of Nanotechnology and Smart Materials, CeNTI, a solution to simulate the porosity of the inertia plate is proposed as well as the material for the body of the spar and the plate and all the necessary corrosion treatments due to the seawater. Finally, the future work and all the conclusions of the study are explained in the last chapter.

Key-words:

Wind offshore energy, stability, linear theory, inertial plate, porosity control, corrosion.
I would like thank my supervisor, Antonio Sarmento, for making this thesis possible and his collaboration with suggestions and corrections that helped me in my first research experience. I want to thank him too for the opportunity given me to work in the Wave Energy Centre that contributes to my formation and learning very positively.

Chapter 5 hasn’t been possible without the collaboration of Marco Alves (WaveEC) and Matthieu Guérinos (IST, PhD student) that helped me doing some of the necessary calculus with WAMIT. Miguel Vicente (WaveEC) helped me learning Matlab and solved all my doubts.

I would like to thank the collaboration of the Centre for Nanotechnology and Smart Materials, CeNTI, and specially to João Gomes (CeNTI) and Bruno Matos (CeNTI) in chapter 4 giving me the opportunity to stay in their installations working, providing me all the necessary bibliography and helping me to find a solution to my problem.

Finally, I appreciate the good reception of all my workmates in WaveEC that from the first moment helped me feeling like at home although I wasn’t in my origin country. All of them helped me with the Portuguese language.
Nomenclature

Lowercase Latin

\( g \)  Gravity acceleration
\( h \)  Sea depth
\( k \)  Wave number
\( m \)  Mass
\( n \)  Normal vector
\( p \)  Pressure
\( r \)  Area ratio relation
\( r_i \)  Radius of cylinder \( i \)
\( r_p \)  Radius of the plate
\( r_h \)  Radius of the hole
\( t \)  Time
\( u \)  Horizontal velocity
\( \vec{v} \)  Velocity
\( w \)  Vertical velocity
\( x \)  Horizontal displacement
\( x_g \)  Centre of gravity \( x \) coordinate
\( y_g \)  Centre of gravity \( y \) coordinate
\( z \)  Vertical displacement
\( z_g \)  Centre of gravity \( z \) coordinate
\( z_{i0} \)  Vertical average displacement
\( z_i \)  Depth from sea level of cylinder \( i \)

Uppercase Latin

\( A \)  Amplitude
$A_i$ Area of cylinder $i$

$A_p$ Area of the damping plate

$B$ Damping coefficients matrix

$C(t)$ Function term of Bernoulli equation

$Cl$ Chlorinity

$D_{pto}$ Damping power take off coefficients matrix

$F_{vi}$ Vertical force at face $i$

$F_e$ Excitation force vector

$F_f$ Friction force vector

$F_{hs}$ Hydrostatic force vector

$F_{hd}$ Hydrodynamic force vector

$F_{pc}$ Force vector due to the external pressure

$F_{pto}$ Power take off vector force

$F_r$ Radiation force vector

$G$ Hydrostatic coefficients matrix

$H$ Twice the wave amplitude

$I$ Inertia matrix

$I_{ij}$ Moments of inertia in matrix position $ij$

$K_{pto}$ Mechanical spring coefficient matrix

$L$ Length

$M$ Added mass coefficients matrix

$P$ Total pressure

$S$ Active area

$S(\circ/\circ/\circ)$ Salinity of seawater expressed in parts per thousands ($\circ/\circ/\circ$)

$T$ Period

$V$ Submerged volume

$V_0$ Average submerged volume

**Lowercase Greek**

$\eta$ Position of the free surface

$\lambda$ Wavelength

$\mu$ Friction coefficient
\( \xi(t) \)  Wave function
\( \rho \)  Density
\( \phi \)  Velocity potential
\( \varphi \)  Wave phase
\( \omega \)  Angular velocity
\( \omega_0 \)  Angular frequency for zero damping force
\( \omega_p \)  Peak frequency
\( \omega_r \)  Resonance frequency

**Subscripts and Superscripts**

\( \hat{f}_e \)  Module of the excitation force
\( \bar{z} \)  Average value of variable \( z \)
\( \dot{z} \)  Derivative of \( z \) with respect to time
\( \ddot{z} \)  Second derivative of \( z \) with respect to time
\( \hat{z} \)  Complex amplitude
\( \mathbf{z}, \mathbf{z} \)  Vector \( z \)

**Acronyms and Institutions**

AES  Auger Electron Spectroscopy
BIEM  Boundary integral equation method
CeNTI  Centre for Nanotechnology and Smart Materials
EDP  Energías de Portugal
DG-RTD  Directorate General-Magazine of European Reserch
FRP  Fibre-reinforced polymer
I &D  Innovation and Development
INORE  International Network on Offshore Renewable Energy
IST  Instituto Superior Técnico
R &D  Research and Development
EU  European Union
RAO  Response Amplitude Operator
UK  United Kingdom
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>United States</td>
</tr>
<tr>
<td>WaveEC</td>
<td>Wave Energy Centre</td>
</tr>
<tr>
<td>WECs</td>
<td>Wave energy converters</td>
</tr>
<tr>
<td>WERATLAS</td>
<td>European Wave Energy Atlas</td>
</tr>
<tr>
<td>XPS</td>
<td>X-Ray Photoelectron Spectroscopy</td>
</tr>
</tbody>
</table>
## Contents

1 Introduction .................................................. 1
   1.1 Overview of wave and offshore wind energy ................. 1
   1.2 Description of the problem .................................. 3
   1.3 Presentation of the thesis .................................... 6

2 The dynamics of floating structures ........................... 8
   2.1 Assumptions .................................................. 8
   2.2 Linear wave theory .......................................... 9
   2.3 Equation of motion ......................................... 13

3 Design of a variable geometry spar for heave stability ........ 17
   3.1 Preliminary design based on Froude-Krylov forces .......... 17
   3.2 RAO of selected geometry based on diffraction-radiation forces .... 22
   3.3 Spar with inertial plate .................................... 28
   3.4 Conclusions ................................................ 32

4 Corrosion treatments for offshore structures and case study of the inertial plate ... 34
   4.1 Introduction to seawater properties .......................... 34
   4.2 Corrosion treatments in offshore structures ................. 36
   4.3 Case study: the inertial plate ................................ 39
   4.4 Candidate materials and processes for the inner cylinder .... 44
   4.5 Conclusions ................................................ 44

5 Conclusions and future work .................................... 46

Bibliography .................................................... 49

A Appendix: Matlab code ........................................... 50
   A.1 Preliminary dimension design ............................... 50
Chapter 1

Introduction

1.1 Overview of wave and offshore wind energy

During the last years many European countries have been investigating and developing systems for obtaining energy in the sea. The European Union is the most important host of collaborative research in the field of ocean energy, mainly driven by the European Commission through the Framework Programmes. Between 1991 and 2008, 41 projects granted fund from DG RTD for marine energy research & research support programmes and from DG TREN for marine energy demonstrators, have spent in a total around 60M€, [3, 22] European research programs have chiefly allowed the development of a diversified European scientific community on ocean energy.

In the international context, there is also a concern of a need for a global approach. The International Network on Offshore Renewable Energy (INORE) made by, for and with PhD students and Post Docs working with issues related to Offshore Renewable Energy is an example of reinforcement of links between the young researchers.

On a national level, the Wave Energy Centre (WaveEC) was founded in 2003 as a non profit organization that took up residence in Lisbon. WaveEC collaborates with Portuguese and international companies, R & D institutions and public entities developing projects of all kinds of sea technologies. Moreover, Portugal has perfect conditions for the deployment of Wave Energy Converters (WECs) due to several reasons like: (a) High wave energy resource available at the west coast; (b) suitable geological conditions (sandy seabed) for subsea cable laying; (c) relatively short continental platform, which allows having suitable depths relatively near the shore; (d) existence of strong electrical grid near the coast. Therefore, there has been in Portugal a growing interest in research and, more recently, in the commercial development of converters in the sea.
In this context the creation of the Portuguese pilot zone (in São Pedro de Moel) was an important initiative to facilitate the first trials of new technologies in real sea conditions.

The most important technological development in that area is related with wave energy and wind offshore. Wave energy is a renewable and endless source with a huge potential in which the energy is more concentrated than in most of the other renewable energy sources being considered nowadays. This amount of energy makes the ocean waves to be economically attractive in the future by developing new technologies and to have a destructive potential able to compromise the efforts in harvesting the energy they carry. On a long time scale, some studies indicate that the electricity production obtained with wave energy can be 70% of hydroelectrical world nowadays production. [7]

Wave energy is the energy contained in the waves generated by the wind on the surface of the sea. This energy, associated to the motion of the water particles, is mostly located between the free surface and a depth around a quarter of the wavelength. Waves generated in one location can travel long distances, in deep waters, without significant loss of energy. European wave energy resource is available in WERATLAS or European Wave Energy Atlas, produced within an EU I&D project. Figure 1.1 represents the average flux in $kW/m$ of wave front. The largest average flux between 20 and 70 kW/m can be found in latitudes between 30 and 60° (north and south hemisphere). Moreover, the most promising offshore areas in Europe are the coastlines of UK, Ireland, Norway, Portugal, Spain and France.

Figure 1.1: Flux average in kW/m of wave front in the world (left) and in Europe (right). Source: European Wave Energy Atlas.

Offshore wind turbines are being used in a number of countries to harness the energy of the moving air over the oceans and convert it to electricity. Offshore winds tend to flow at higher
speeds than onshore winds, thus allowing turbines to produce more electricity. European wind offshore energy source is available in European Atlas of Wind Offshore as figure 1.2 shows. Furthermore, the most significant source is in the North Sea, where the offshore wind started to expand, and the UK.

Nowadays, the first offshore floating wind turbine in Portugal is becoming operational. Several national entities are participating in a project, lead by EDP (the major Portuguese electrical company) to built and test an offshore floating wind turbine prototype developed by the US company Principle Power. The project involves the construction over the next six months of a 2 MW wind turbine prototype. The estimated investment for the construction and installation is about 4.1 million euros per MW installed. [12]

1.2 Description of the problem

The offshore wind structures are different depending on the depth of the sea where they are installed. Shallow water turbines are supported by fixed foundations on the seabed using generally tubular towers or trusses. From around 60m depth a fixed structure is not economically feasible due to the complexity of the project and installation. Therefore, the alternative solution is floating structures which have high expectations for development. Figure 1.3 presents some different types of wind turbines foundations depending on the sea depth and a plot with the increase of the cost as a function of the sea depth.
The different concepts of floating structures under development use different methods to ensure stability. Therefore, the objective of this work is to define a particular platform geometry trying to reduce the loads and deformations (and thus fatigue) in order to improve the platform stability. Moreover, the geometrical simplicity of the platform was also considered to facilitate the maintenance and the complexity of the installation. In that area is where a big R&D effort is needed bared in numerical and experimental modeling. These are the most critical aspects of the development of offshore wind.

Furthermore, the incentives to develop offshore wind technology are:

- Highest wind speed in deep water.
- Reducing impacts on human activities and ecosystems.
- Potential for expansion especially in countries with steep slopes along the coast.
- Ability to perform the assembly of the system on land, avoiding high costs of offshore construction, compared with fixed structures at sea.

The support structure of floating offshore wind turbines should be as most stable as possible for a wide range of frequencies. The present project concerns the use of smart materials or other equivalent solutions to change the shape of floating structures so that they become more stable under different wave periods. Depending on the frequency of the incident wave the spar stability can be affected significantly, especially close to the resonance frequency. Thus, the spar should be adjustable to each sea state to prevent high motion amplitudes.
The structure analysed is a spar defined by two concentric cylinders, as is shown in figure 1.4. The justification of the design will be exposed in chapter 3 as well as the procedure to choose the best dimensions that minimize the vertical forces. In order to increase the stability of the spar for lower frequencies, enlarging, this way, the range of stability, a submerged horizontal plate will be added to the bottom of the wider cylinder. The plate improves the stability by reducing the motions, within the most typical frequencies bandwidth, due to the increase of the inertia, mainly added mass.

![Diagram of the spar structure with variables of study](image)

**Figure 1.4: Schematic representations of the spar structure with the variables of study.**

Besides the chosen technology, represented in figure 1.4, there are other proposed alternatives for floating platforms as, for instance, the WindFloat, schematically represented in figure 1.5, developed and patented by the US company Principle Power. It consists of a floating triangular structure, topped by three void pillars that allow it to float stably. On one of the three pillars, the tower that supports the turbine is installed. The entire structure is anchored to the seabed and it can be installed virtually in any sea with a depth ranging between 50 meters and several hundred meters.

A 2MW prototype of the WindFloat is going to be tested in Portugal in the autumn of 2011 by a consortium including EDP and Principle Power.
Nowadays, the use of smart materials for that application is an open field, which increases the difficulty in the research of the best solution. In this context, the Centre for Nanotechnology and Smart Materials (CeNTI), located in Vila Nova de Famalicão, Portugal, is a new public-private institute for materials research, development and prototyping that is going to collaborate in the present project.

1.3 Presentation of the thesis

The work program defined the following objectives: (a) To define the best dimensions of the spar according to the assumptions of the linear theory to get more stability in the interval of common frequencies, (b) To increase the stability bandwidth changing the plate porosity, (c) To study the effect on the inner radius variation to adapt the stable frequencies to the incident wave, (d) To search smart materials able to change the porosity property or another equivalent solution to apply in the inertial plate.

The thesis is organized in five chapters, including this introduction. The first chapter has introduced the problem under study and an overview of the potential of wave and offshore wind energy.
The second chapter explains the assumptions taken in the project as well as the description of the theoretical concepts and the basic principles applied in chapter 3. The assumptions under the application of the linear theory to calculate the forces and the spar motions will be explained, too.

The spar presented in section 1.2 will be dimensioned in chapter 3 in order to minimize the displacement of the structure in the most common range of frequencies. The Froude-Krylov vertical forces acting on the spar will be calculated and the best dimensions will be obtained. Subsequently, the motions of the body will be determined to identify if the interval of stable frequencies correspond to the most common incident waves frequencies. WAMIT will be used to compute the hydrodynamic coefficients (added mass, damping and excitation forces) required to analyse the spar motions. For this propose a post-processing numerical tool was developed, which basically consists in the MATLAB implementation of the motion equation. Moreover, the design of the spar with two concentric cylinders will be justified doing a comparison of the displacement with a spar with just one cylinder.

To increase the stability, and consequently to obtain a wider stable bandwidth, is one of the purposes of the project. One can do it adding a plate to the bottom of the spar to increase the added mass which reduces the natural frequency of the spar. Thus, changing the porosity of the plate the structure can be adapted to the incident wave in order to improve the stability. Another way to obtain this effect is changing the radius of the inner cylinder. These two cases will be studied and compared obtaining some useful conclusions included in chapter 3.

In chapter 4 the seawater properties and the corrosion problems of offshore structures will be studied as well as the treatments against it used currently. The best material for the spar and the one able to satisfy the requirements about the change of porosity of the inertial plate will be found. In the case of not finding any existing material, another equivalent mechanical solution will be searched.

Finally, the last chapter will include all the conclusions of the project and an explanation of the future work to do in that area.
Chapter 2

The dynamics of floating structures

2.1 Assumptions

The spar will be floating in the sea and supporting the windmill. To create a model of the body some assumptions should be accepted henceforth.

Here it is assumed that the fluid is inviscid and the flow irrotacional allowing the following Euler equations to apply:

\[
\begin{align*}
\frac{Du}{Dt} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} \\
\frac{Dw}{Dt} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} - g
\end{align*}
\]  

(2.1)  
(2.2)

where \( u \) is the \( x \) direction velocity, \( w \) is the \( z \) direction velocity, \( x \) is the horizontal direction, \( z \) is the vertical rising direction which is 0 in the middle position of the free surface, \( p \) is the pressure, \( g \) is the gravity, \( t \) is the time, \( \rho \) is the density.

Moreover, the mass must be conserved. The mathematical equation in two dimensions to express the assumption that water is incompressible (\( \rho \) constant \( \rightarrow \nabla \cdot \vec{v} = 0 \)) is given by the following form of the continuity equation (2.3):

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0
\]  

(2.3)

Water will be assumed as an inviscid and incompressible fluid and so the Euler equations are valid. Thus, there are only normal stresses (pressures) acting on the surface of a fluid particle;
since the shear stresses are zero, there are no stresses to impart a rotation on a fluid particle. Therefore, the motion is irrotational:

\[ \nabla \times \vec{v} = 0 \quad (2.4) \]

The velocity vector \( \vec{v} \) can be represented as:

\[ \vec{v} = -\nabla \phi \quad (2.5) \]

where \( \phi \) is the velocity potential. Hence, introducing eq. 2.5 into 2.4 the Laplace equation is obtained, given by:

\[ \nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (2.6) \]

The Laplace equation has numerous solutions and it's necessary to select only those which are applicable to the particular motion of water waves.

The Bernoulli equation is simply an integrated form of Euler equations of motion and provides a relationship between the pressure field and kinematics. The resulting equation is:

\[ -\frac{\partial \phi}{\partial t} + \frac{1}{2}(u^2 + w^2) + \frac{p}{\rho} + gz = C(t) \quad (2.7) \]

where the function \( C(t) \) is referred to as the Bernoulli term and is a constant for steady flows. That equation will be simplified for small-amplitude water waves in next section 2.2. [10, 6-36]

The problem will be studied for small-amplitude waves and the consequence of this assumption will be detailed in the next section 2.2.

Finally, in accordance with the incident waves, also the spar motions will be described by a sinusoidal function (only the heave mode (axis \( Z \)) will be analysed).

## 2.2 Linear wave theory

Linear wave theory considers the problem of small-amplitude water waves. To describe this problem it is necessary to define the boundary conditions. In general, the lower boundary of our interest region is described as \( z = -h(x) \) for a two-dimensional case where the origin is located at the still water level and \( h \) represents the depth. Assuming that the bottom is impermeable, one can expect that:
\( \vec{v} \cdot \mathbf{n} = 0 \) \hspace{1cm} (2.8)

where \( \mathbf{n} \) is the normal to the bottom. Therefore, for a horizontal impermeable bottom the vertical velocity is:

\[- \frac{\partial \phi}{\partial z} = 0 \quad \text{on} \quad z = -h\] \hspace{1cm} (2.9)

The free surface can also be considered as impermeable, assuming that the velocity of the fluid is equal to the velocity of the free surface, obtaining:

\[- \frac{\partial \phi}{\partial z} = \frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial x} \quad \text{on} \quad z = \eta(x,t)\] \hspace{1cm} (2.10)

where \( \frac{\partial \eta}{\partial t} \) is the rate of rise of the free surface and \( \eta \) is the position of the free surface.

The dynamic free surface boundary condition translates the fact that the pressure on the free surface is uniform along the wave form (atmospheric pressure). Applying that condition to solve the Bernoulli equation for infinitesimally small waves, where for small amplitude waves the kinetic energy is negligible as a second order term, it results:

\[\left( - \frac{\partial \phi}{\partial z} + g \eta \right)_{z=0} = C(t)\] \hspace{1cm} (2.11)

Assuming that \( \eta \) is translated by a sinusoidal time and space dependence it may be written by:

\[\eta = \frac{H}{2} \cos kx \cos \omega t\] \hspace{1cm} (2.12)

The velocity potential is now:

\[\phi(x, z, t) = \frac{H g \cosh k(h+z)}{2 \omega \cosh kh} \cos kx \sin \omega t\] \hspace{1cm} (2.13)

Moreover, the hyperbolic functions have convenient shallow and deep water asymptotes. As the spar will be floating in a depth \( h \gg 75m \) the deep water simplification can be used. Thus, for deep waters,

\[kh \gg 1 \rightarrow e^{-kh} = 0\] \hspace{1cm} (2.14)
Furthermore, the function $\cosh kh$ is defined as:

$$
cosh kh = \frac{e^{kh} + e^{-kh}}{2}
$$

(2.15)

Hence, for deep water:

$$
\frac{\cosh k(h + z)}{\cosh kh} = e^{kz}
$$

(2.16)

The simplified potential velocity for an stationary wave is then given by:

$$
\phi(x, z, t) = \frac{H}{2} e^{kz} \cos kx \sin \omega t
$$

(2.17)

Moreover, the dispersion relation for deep waters (where $\tanh kh$ is 1) is simplified into:

$$
\omega^2 = gk \tanh(kh) \simeq gk
$$

(2.18)

where $k$ is the wave number, $g$ is the gravity, $\omega$ is the angular velocity and $h$ is depth. Thus, the wave number becomes:

$$
k = \frac{2\pi}{\lambda} \simeq \frac{\omega^2}{g}
$$

(2.19)

The pressure field associated with a progressive wave was determined from the unsteady Bernoulli equation (eq. 2.7) developed for an ideal fluid. For waves of small amplitude the second order term $\frac{1}{2}(u^2 + w^2)$ can be neglected.

A stationary wave can be decomposed into two progressive waves with the same frequency and amplitude that propagate in opposite directions. Therefore, the progressive wave that travel in the $x$ positive direction has the pressure:

$$
p = -\rho gz + \rho g \frac{H}{2} \frac{\cosh k(h + z)}{\cosh kh} \cos(kx - \omega t)
$$

(2.20)

The interference of the spar in the propagation of the progressive wave can be neglected in a simplified analysis. The ideal case of stability is when the vertical forces in the faces $A_1$ and $A_2$, shown in figure 1.4, of the spar are equal:

$$
F_{v1} = F_{v2}
$$

(2.21)

Through the force balance in the $z$ direction it is possible to verify that the pressure on face $A_1$ and $A_2$ are related by means of:
\[ P_1 A_1 + g \rho V = P_2 A_2 \]  \hspace{1cm} (2.22)

where \( V \) is the submerged volume, \( \rho \) is the water density, \( g \) is the gravity acceleration and \( P_i \) is the pressure in the face \( i \).

Introducing equation 2.20 into 2.22 it results:

\[-\rho g z_0 A_1 + \rho g \frac{H}{2} \cosh k(h + z_1) \cos(kx - \omega t) A_1 + \rho g V_0 = -\rho g z_0 A_2 + \rho g \frac{H}{2} \cosh k(h + z_2) \cos(kx - \omega t) A_2\]  \hspace{1cm} (2.23)

where \( H/2 \) is the amplitude, \( h \) is the sea depth, \( k \) is the wave number, \( x \) is the position in the \( x \)-axis, \( \omega \) given by \( \omega^2 = g k \tanh kh \) is the angular frequency, \( t \) is the time, \( z_{i0} \) is the average depth of face \( A_i \) and \( V_0 \) is the average submerged volume.

Moreover, the submerged volume can be represented by:

\[ z_2 A_2 - z_1 A_1 = V \]  \hspace{1cm} (2.24)

From equation 2.24, the expression 2.23 can be simplified obtaining:

\[ \rho g \frac{H}{2} A_1 \cosh k(h + z_1) - A_2 \cosh k(h + z_2) \cos(kx - \omega t) + \rho g (V_0 - V) = 0 \]  \hspace{1cm} (2.25)

The term \( \rho g (V_0 - V) \) is zero only when the spar is not oscillating. Thus, it becomes clear that if:

\[ A_1 \cosh k(h + z_1) = A_2 \cosh k(h + z_2) \]  \hspace{1cm} (2.26)

there will be no vertical forces acting in the spar and therefore, the acceleration will be zero according to the newton second law. \[11, 41-72\]

The Froude-Krylov force acts on the body when only is considered the pressure induced by the incident wave. That simplification is reasonable if the body dimension is small compared with the wavelength. By applying the simplification of equation 2.16 and according to equation 2.25 the Froude-Krylov term of the excitation force is given by:

\[ |F_t| = |F_{v1} - F_{v2}| = |\rho g \frac{H}{2} \cos(kx - \omega t) [e^{kz_1} A_1 - e^{kz_2} A_2]| \]  \hspace{1cm} (2.27)

Suitable spar geometries should minimize the Froude-Krylov force.
2.3 Equation of motion

The generic equation of motion is obtained in order to set the basis of a standard frequency domain model. For this purpose it is necessary to simplify and to linearize some of the forces involved. Hence, in the time domain, the general motion equation, according with the Newton second law, is given by:

\[ I \ddot{z}(t) = F_{pe}(t) + F_f(t) \]  

(2.28)

Here, the bold font denotes matrix or vector. In any case, \( I \) represents the inertia matrix, \( z \) the displacement vector in the \( z \) direction ( and so \( \ddot{z} \) the acceleration), \( F_{pe} \) the force vector due to the external pressure on the buoy and \( F_f \) the friction-force vector. The complete inertia matrix for the 6 rigid DoFs, is described by:

\[
I = \begin{bmatrix}
    m & 0 & 0 & 0 & m z_g & -m y_g \\
    0 & m & 0 & -m z_g & 0 & m x_g \\
    0 & 0 & m & m y_g & -m x_g & 0 \\
    0 & -m z_g & m y_g & I_{11} & I_{12} & I_{13} \\
    m z_g & 0 & -m x_g & I_{21} & I_{22} & I_{23} \\
    -m y_g & m x_g & 0 & I_{31} & I_{32} & I_{33}
\end{bmatrix}
\]  

(2.29)

where \( m \) is the mass of the body (equivalent to the mass of the displaced water in the free floating condition, i.e., \( m = \rho V \), \( x_g \), \( y_g \), \( z_g \) the coordinates of the center of gravity and \( I_{ij} \) the moments of inertia defined, in terms of the corresponding radius of gyration, by the relation

\[ I_{ij} = \rho V r_{ij} |r_{ij}| \]  

(2.30)

In accordance with the linear theory and considering harmonic the oscillatory motion of the waves and the device, it is possible and convenient to decompose each term of equation 2.28 in its spatial and temporal dependencies. Therefore, all forces acting on the device will be described by a complex amplitude and the sinusoidal time dependence, \( e^{i \omega t} \). Hence, the device displacement vector becomes,

\[ z(t) = \text{Re} \{ \hat{z}(\omega)e^{i \omega t} \} \]  

(2.31)

and, consequently, the velocity and the acceleration vectors results, respectively, in

\[ \dot{z}(t) = \text{Re} \{ i \omega \hat{z}(\omega)e^{i \omega t} \} \]  

(2.32)
and,

\[
\ddot{z}(t) = \text{Re}\left\{ -\omega^2 \dot{z}(\omega) e^{i\omega t} \right\}
\]  

(2.33)

where the hat symbol “\(\hat{}\)” denotes the complex amplitude. Therefore, in the frequency domain, equation 2.28 may be written by,

\[-\omega^2 \hat{I} \ddot{z}(\omega) = \hat{F}_{pe}(\omega) + \hat{F}_f(\omega)\]  

(2.34)

in which we vanished the exponential \(e^{i\omega t}\) in both members. The force vector due to the external pressure on the body, \(F_{pe}\), may be decomposed into two components to differentiate the sources of static and unsteady pressure. Then,

\[F_{pe} = F_{hs} + F_{hd}\]  

(2.35)

where the first term represents the vector of hydrostatic restoring forces due to gravity and buoyancy. This term is proportional to the displacement of the device if no changes in the hydrostatic coefficients matrix, \(G\), occurs. In this case, it is given by:

\[F_{hs} = -Gz\]  

(2.36)

In accordance with the prior assumptions of linearity and oscillatory harmonic motions, the vector of complex amplitudes of the hydrostatic restoring forces is:

\[\hat{F}_{hs} = -G \hat{z}\]  

(2.37)

For the heave mode (commonly identified by the index 3) element of the matrix \(G\) is simply given by:

\[G_{33} = S \rho g\]  

(2.38)
where,

\[ S = \pi r^2 \]

is the floating surface,

\[ \rho \]

the water density and

\[ g \]

the gravity acceleration.

Furthermore, the second term of Eq 2.35 embodies the force vector resultant from the unsteady pressure over the body surface. This term is expressed as,

\[ F_{hd} = F_e - F_r \quad (2.39) \]

The first term of Eq 2.39 corresponds to the excitation force vector and is represented by,

\[ F_e = \hat{f}_e e^{i\omega t} \quad (2.40) \]

where \( \hat{f}_e \) is the complex amplitude of the excitation force and \( \omega \) is the angular velocity. Furthermore, the second term of Eq 2.39 corresponds to the radiation force vector. The radiation force is related with the waves produced by the body motions. It comprises two terms, one related to the velocity and another to the acceleration. Therefore, it results in:

\[ F_r = B \dot{\hat{z}} + M \ddot{\hat{z}} \quad (2.41) \]

and so the complex amplitude is given by,

\[ \hat{F}_r = i\omega B \dot{\hat{z}} - \omega^2 M \ddot{\hat{z}} \quad (2.42) \]

Here \( M \) and \( B \) represent, respectively, the symmetrical matrices of added mass and damping coefficients. The added mass coefficient corresponds to an inertial increment due to the water displaced in the body vicinity when the body moves. The energy involved in this process is reactive as it flows alternately amid the body and the fluid. Moreover, the damping coefficient is related with the energy transmitted to the fluid by the body oscillations, which gradually moves away from the body. In this case, the energy involved in the process is resistive as it is associated with a damping effect.

According with Eq 2.35 and taking into account the results displayed by Eqs. 2.37 and 2.41, the vector of complex amplitudes of the force due to the external pressure on the body, \( \hat{F}_{pe} \), becomes:

\[ \hat{F}_{pe} = \hat{f}_e - i\omega B \dot{\hat{z}} - \omega^2 M \ddot{\hat{z}} - G \dot{\hat{z}} \quad (2.43) \]
which represents the total force acting on a free floating body.

Finally, the friction force vector, $F_f$, is often described by a quadratic dependence on the velocity, but in a linearized form it is simply expressed by the proportionality:

$$ F_f = -\mu \dot{z} $$

(2.44)

where $\mu$ is the array of friction constants. Therefore, the complex amplitude of the friction force is given by,

$$ \hat{F}_f = i\omega \mu \hat{z} $$

(2.45)

In general, the friction term is neglected in accordance with the linear theory assumptions, based on small velocity amplitudes. [5]

Finally, in accordance with Eq. 2.37, 2.40 and 2.42, the motion equation, translated by Eq. 2.28, may be rewritten by:

$$ -\omega^2 I \hat{z} e^{i\omega t} = \hat{f}_e e^{i\omega t} - i\omega B \hat{z} e^{i\omega t} + \omega^2 M \hat{z} e^{i\omega t} - G \hat{z} e^{i\omega t} $$

(2.46)

Removing the common factor $e^{i\omega t}$ and isolating $\hat{z}$, it results:

$$ \hat{z} = \frac{\hat{f}_e}{(-\omega^2(I + M) + G) + i\omega B} $$

(2.47)

Equation 2.47 will be used in chapter 3.
Chapter 3

Design of a variable geometry spar for heave stability

3.1 Preliminary design based on Froude-Krylov forces

In this section the spar will be totally dimensioned assuming just the influence of Froude-Krylov force in the vertical $z$-axis. The excitation force acting on a floating body is generally decomposed in the Froude-Krylov and diffraction terms. The Froude-Krylov component is the force induced by the unsteady pressure field generated by undisturbed waves and the diffraction term is related to the disturbance of the waves due to the floating body. Therefore, for small bodies the Froude-Krylov term is an acceptable approximation of the total excitation force. For deep water the spar Froude-Krylov force is given by:

$$F_v = \rho g \frac{H}{2} \cos(kx - \omega t) \left[ e^{kz_1}A_1 - e^{kz_2}A_2 \right]$$  \hspace{1cm} (3.1)

Hence, considering $z_2 = z_1 - L$,

$$F_v = \rho g \frac{H}{2} \left[ e^{kz_1} \left( A_1 - e^{-kL}A_2 \right) \right] \cos(kx - \omega t)$$  \hspace{1cm} (3.2)

The maximum force or the force amplitude ($\cos(kx - \omega t) = 1$) is given by the expression:

$$F_v = \rho g \frac{H}{2} \left[ e^{kz_1} \left( A_1 - e^{-kL}A_2 \right) \right]$$  \hspace{1cm} (3.3)

where $z_1$ and $z_2$ are negative values, representing the depths of surface $A_1$ and $A_2$, respectively.
Thus, the vertical force will be zero, \( F_v = 0 \), when:

\[
F_v = A_1 - e^{-kL}A_2 = 0
\]

It is interesting to note that Eq. 3.4 is independent of the value of \( z_1 \).

Eq. 3.4 allows to verify that the vertical force \( F_v = 0 \) for a ratio, \( r \), between the surfaces \( A_1 \) and \( A_2 \) given by,

\[
r = \frac{A_2}{A_1} = e^{kL}
\]

The previous relation, which implementation code is written in appendix A.1, is graphically represented by Figure 3.1,

![Figure 3.1: Area ratio \( r = A_2/A_1 \) as a function of \( k \cdot L \) for \( F_v = 0 \)](image)

Figure 3.1 shows that for relations, \( r \), higher than 2.5 the slope of the curve is soaring and for a same \( k, L \) will be bigger. To calculate the dimensions of the spar, some variables have to be fixed. One of these variables is the pick spectral frequency \( \omega_p \) (from which one can calculate the \( k \)) of a predefined sea state. In this particular study the spectral density distribution selected, displayed at figure 3.2, represents a typical distribution of the sea climate of the north Portuguese coast.
Figure 3.2: Spectral density distribution of the north Portuguese coast in winter. Peak frequency $\omega_p = 0.62$ rad/s.

The peak frequency taken from figure 3.2 is $\omega_p = 0.62$ rad/s. Hence, the characteristic wave number is,

$$k = \frac{\omega^2}{g} = \frac{0.62^2}{9.81} = 0.0392 \text{ m}^{-1}$$  \hspace{1cm} (3.6)

The following table 3.1 summarizes the results for different possible dimensions of $L$ fixing an interval of area ratios $r$ between 1.3 and 3.2, which corresponds to an interval of $L$ from 6.7 to 29.6m. The area ratios are fixed in that interval because for values above 3, the cylinders have similar radius which is not interesting for the current study.

To calculate the radius from the relation $r$ it is necessary to fix one of them. Considering that a radius of 5m is a reasonable value, it will be used for the inner cylinder. Then, the expression to calculate $r_2$ may be deduced by equating two expressions of $A_1$:

$$A_1 = \pi(r_2^2 - r_1^2)$$
$$A_1 = \frac{A_2}{r} = \frac{\pi r_2^2}{r}$$

Thus,

$$r_2 = \sqrt{\frac{r r_1^2}{r - 1}}$$  \hspace{1cm} (3.7)

Results displayed in table 3.1 show that for higher depths the area ratio increases. When the depth increases the unsteady pressure in the face $A_2$ decreases and therefore the required area
$A_1$ to compensate the unsteady pressure at $A_2$ is smaller.

\[
r = \frac{A_2}{A_1}
\]

<table>
<thead>
<tr>
<th>$r$</th>
<th>$k \cdot L$</th>
<th>$L$ [m]</th>
<th>$r_2$ [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>0.262</td>
<td>6.689</td>
<td>10.408</td>
</tr>
<tr>
<td>1.503</td>
<td>0.408</td>
<td>10.401</td>
<td>8.643</td>
</tr>
<tr>
<td>1.777</td>
<td>0.575</td>
<td>14.668</td>
<td>7.561</td>
</tr>
<tr>
<td>2.019</td>
<td>0.702</td>
<td>17.918</td>
<td>7.038</td>
</tr>
<tr>
<td>2.322</td>
<td>0.843</td>
<td>21.495</td>
<td>6.627</td>
</tr>
<tr>
<td>2.503</td>
<td>0.917</td>
<td>23.403</td>
<td>6.452</td>
</tr>
<tr>
<td>2.706</td>
<td>0.996</td>
<td>25.395</td>
<td>6.297</td>
</tr>
<tr>
<td>2.935</td>
<td>1.077</td>
<td>27.474</td>
<td>6.158</td>
</tr>
<tr>
<td>3.194</td>
<td>1.161</td>
<td>29.617</td>
<td>6.033</td>
</tr>
</tbody>
</table>

Table 3.1: Results of $L$ and $r_2$ for different area ratios $r = \frac{A_2}{A_1}$ for the peak frequency $\omega_p = 0.62$ of the spectral distribution presented in figure 3.2.

Results presented in table 3.1 were computed for null vertical forces at the peak frequency $\omega_p = 0.62$ rad/s, independent of the value of $z_1$. The analysis of the vertical forces evolution for different configurations and for a large frequencies range, from 0.25 to 2.5 rad/s, will be the criterion to choose the best geometry. The following curves in figure 3.3 are obtained based on a dimensionless force, given by $F_v/\rho g (A_2 - A_1)$, where $\rho g (A_2 - A_1)$ is the hydrostatic force. The internal radius is still fixed at $r_1 = 5m$ and the chosen initial value of $z_1$ is $-5m$. The code used in Matlab is described in appendix A.1.

![Figure 3.3: Dimensionless force as a function of T for different candidate dimensions and for $r_1 = 5m$ and $z_1 = -5m$.](image)

All the curves converge to the same period in the absence of vertical forces, corresponding to specified peak frequency (see figure 3.2).

The blue curve that corresponds to the area ratio $r = 3,1p4$ is the one that have lower values
of dimensionless force for the usual range of periods between 8 and 15s and for that reason is the chosen for the study ongoing.

In figure 3.3 the value of $z_1$ was fixed at $z_1 = -5m$. However, it is also relevant to analyze, for the chosen dimensions ($r = 3.144, L = 29.617m, r_1 = 5m, r_2 = 6.033m$), the evolution of that parameter for other frequencies besides the peak frequency. This evolution, displayed in figure 3.4, allows to see, as expected, that when $z_1$ increases the unsteady pressure in face $A_1$ decreases. The effect of $z_1$ is not very significant and for that reason, the chosen $z_1$ remains $z_1 = -5m$.

![Figure 3.4: Dimensionless force as a function of T for different values of $z_1$ for the dimensions $r = 3.144, L = 29.617m, r_1 = 5m$ and $r_2 = 6.033m$](image)

Finally, the chosen geometry is shown in figure 3.5 and the parameters of design are:

$$A_1 = \pi(r_2^2 - r_1^2) = 35.7 \text{ m}^2$$  \hspace{1cm} (3.9)

$$A_2 = \pi r_2^2 = 114.3 \text{ m}^2$$ \hspace{1cm} (3.10)

$$S = \pi r_1^2 = 78.5 \text{ m}^2$$ \hspace{1cm} (3.11)
3.2 RAO of selected geometry based on diffraction-radiation forces

To study the motions of the body the software WAMIT is used. WAMIT is a numerical code based on the linear theory for analyzing floating or submerged bodies, in the presence of ocean waves. The boundary integral equation method (BIEM), also known as the panel method, is used to solve for the velocity potential and fluid pressure on the submerged surfaces of the bodies. Separate solutions are carried out simultaneously for the diffraction problem, giving the effects of incident waves on the body, and the radiation problems for each of the prescribed modes of motion of the bodies. These solutions are then used to obtain the relevant hydrodynamic parameters including added-mass and damping coefficients, exciting forces, response-amplitude operators (RAO’s), the pressure and fluid velocity, and the mean drift forces and moments.

WAMIT works with the six rigid degrees of freedom. However, in the current study, only the heave mode (vertical direction) was considered.

The deduction of the equation of motion of the body has been explained in section 2.3. The vertical displacement is calculated as:

\[ y(t) = y_0 + v_0 t + \frac{1}{2} a t^2 \]
\[ \hat{z} = \frac{\hat{f}_e}{(-\omega^2(m + M) + G) + i\omega B} \]

where \( \hat{z} \) is the vertical complex amplitude of the displacement for unitary wave amplitude, \( \hat{f}_e \) is the complex amplitude of the excitation force, \( \omega \) the frequency, \( m \) is the mass of the body, \( M \) is the added mass coefficient, \( B \) is the damping coefficient and \( G \) is the hydrostatic coefficient.

The mass of the body is equivalent to the mass of the displaced water in the free flotation condition:

\[ m = V \rho \quad (3.12) \]

where \( V \) is the volume of the body calculated as:

\[ V = \pi (r_1^2 |z_1| + r_2^2 L) = \pi (5^2 \cdot 5 + 6.03 \cdot 29.6) = 3777.3 \, m^3 \quad (3.13) \]

The dimensions used for obtaining (with WAMIT) the hydrodynamic coefficients are the ones selected in section 3.2: \( r_1 = 5 \, m, r_2 = 6.03 \, m, L = 29.6 \, m, z_1 = 5 \, m \) and \( z_2 = 34.6 \, m \). Figure 3.6 shows the mesh structure used to run WAMIT.

![Mesh of the spar structure used to run WAMIT.](image)

WAMIT generates files with the values of the coefficients \( M \) and \( B \) and the value of the
excitation force (complex amplitudes) that can be read with MATLAB for a fix incident wave amplitude. The amplitude of the incident wave is 1m and the interval of frequencies from 0 to 1 rad/s with a step of 0.005 rad/s. The post-processing MATLAB code developed to compute the spar vertical motion is explained in appendix A.2.

Finally, the vertical displacement of the body is displayed in figure 3.7.

![Figure 3.7: Spar vertical displacement as a function of $\omega$ for the selected dimensions.](image)

From figure 3.7 one can analyse three important points which can prove that in principle the post processing code is working properly. The first one is when $\omega \rightarrow 0$, the value of the displacement is 1m, as figure 3.8 shows.

Mathematically, the value of the Froude-Krylov term of the excitation force when $\omega \rightarrow 0$ is:

$$
\hat{f}_e = | \rho g H \frac{k}{2} e^{k \zeta} (A_1 - e^{kL} A_2) |
$$

$$
k = \frac{\omega^2}{g} = 0
$$

$$
\hat{f}_e = | \rho g H \frac{1}{2} (A_1 - A_2) |
$$

(3.14)
Figure 3.8: Limiting spar vertical displacement when $\omega \rightarrow 0$.

Substituting equation 3.14 in equation 2.47 and applying $\omega = 0$, it results

\[
\hat{z} = \frac{\hat{f}}{G}
\]

\[
\hat{z} = \left| \frac{\rho g H}{2} (A_1 - A_2) \right|
\]

\[
\hat{z} = \frac{\left| \rho g S \right|}{\rho g \pi r_1^2}
\]

where $H_2 = 1$ is the wave amplitude taken as reference to compute the hydrodynamic coefficients with \textit{WAMIT}. Therefore, as $|A_1 - A_2| = \pi r_1^2$ the amplitude of the vertical displacement acquires the value

\[
\hat{z} = \frac{|\rho g (A_1 - A_2)|}{\rho g \pi r_1^2} = 1
\]

(3.16)

This result means that when $\omega \rightarrow 0$, or $T \rightarrow \infty$, the inertial forces are negligible and so the body follows the motion of the free surface (with 1m of vertical amplitude).

The second point is the analysis of the limiting case when the $\omega \rightarrow \infty$. Mathematically, one can easily see that when $\omega \rightarrow \infty$ then $\hat{z} \rightarrow 0$:

\[
\hat{z} = \lim_{\omega \rightarrow +\infty} \frac{\hat{L}}{\rho g (m + M) + G + i\omega B} = 0
\]

(3.17)
Physically, for small periods, $T \to 0$, the pressure acting at both faces is negligible due to the exponential pressure decay. For small periods, but sufficiently high to avoid null pressure at both faces, there will be pressure cancelation because in that case the spar diameter is much higher than the wave length. The pressure on the spar, in case of $\omega \to 0$ and $\omega \to \infty$ is schematically represented in figure 3.9.

![Figure 3.9: Effect on the vertical forces of the body when $\omega \to 0$ (on the left) and $\omega \to \infty$ (on the right).](image)

The third point under analysis is when the displacement becomes unrealistic (reaching values about 26m). Between 0.4 and 0.45 rad/s the displacement is extremely high because the spar resonance frequency is within this frequencies range. However, the computed displacements are not realistic because linear theory does not take into account viscous effects. Therefore, in the reality, the displacements at that particular frequencies range will be high due to the resonance, but much lower than the values computed.

The point of maximum amplitude corresponds to the resonance frequency ($\omega = 0.42$ rad/s). The resonance effect is produced when the force acting on the body is in phase with the vertical body velocity. This frequency should be as lower as possible (to be outside of the most common local frequencies range). The resonance frequency results from

$$ (-\omega^2(m + M) + G) = 0$$  \hspace{1cm} (3.18)

$$\omega_r = \sqrt{\frac{G}{m + M}}$$  \hspace{1cm} (3.19)

Hence, it is clearly that adding mass decreases the resonance frequency. This effect will be studied at the next section 3.3.
Finally, figure 3.10 shows a zoom of the spar motions spotlighting the limits of the linear theory application.

Moreover, the design of the spar with two cylinders can be justified if it is compared with a single cylinder with same volume and draft. The volume of the spar under analysis is 3777,3 $m^3$ and so the radius of the new spar with a single cylinder and similar draft must be 5,9m. The motion comparison of both cases is presented in figure 3.11:

When $\omega \to 0$ and $\omega \to \infty$ both curves have displacements of 1m and 0m, respectively, as
explained before. Nonetheless, the resonance frequency is higher in the case of one cylinder as
the added mass coefficient has decreased and the hydrostatic coefficient has increased. Furthermore,
it is easy to see that for a displacement given, for example 5m, the interval of frequencies
that produced high displacements is wider in case of a single cylinder.

3.3 Spar with inertial plate

In this section a plate will be add to the spar in order to amplify the interval of stable frequencies
due to the increase of the added mass coefficient. The plate, located concentrically at the bottom
of the longest cylinder, has a radius of 11m. The porosity is an important factor as it allows chang-
ing the spar dynamic response. Although as WAMIT cannot simulate directly that property of
the material, another solution has been found. The methodology is based on changing the radius
of the plate, \( r_p \), with intervals of 1m from 7m to 11m; where the porosity of 0 % is represented
by the maximum radius and the porosity of 60 % by the smallest. Thus, the added mass and
damping coefficients are computed with WAMIT for each different case. Figure 3.12 shows the
discretization of the spar with the plate with maximum radius and table 3.2 the several porosities
analysed.

Figure 3.12: Mesh of the spar structure with an inertial plate \((r_p = 11\, m)\) used to run WAMIT.

Figure 3.13 displays the motion amplitudes for the different radius chosen. As the radius in-
crease, the curves are moving to the left due to the augment of mass of the spar. Varying the porosity from 60% ($r_p = 7\, m$) to 0% ($r_p = 11\, m$) the interval of high vertical motion amplitudes goes from 0,3 to 0,45 rad/s. In that interval, the response of the body is not simulated, as the results are unrealistic due to the limitations of the linear theory, as explained in the previous section.

Hence, changing the porosity of the plate the dynamic response of the spar may be significantly affected, which may be useful to reduce the body vertical motions within the most common frequencies range. Chapter 4 will try to find a material or mechanical construction able to change itself the porosity.

Furthermore, as the porosity of the plate increases the bandwidth of high motions becomes narrow and is displaced to low frequencies (both facts are consequence of the inertia augment due to the added mass increase). For example, the bandwidth of motions higher than 2m for the plate with the minimum porosity is 0,04 and, in the other side, for plate with maximum porosity is 0,07. The relative bandwidth is defined by:

<table>
<thead>
<tr>
<th>% Porosity</th>
<th>$r_p$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60%</td>
<td>7</td>
</tr>
<tr>
<td>47%</td>
<td>8</td>
</tr>
<tr>
<td>33%</td>
<td>9</td>
</tr>
<tr>
<td>18%</td>
<td>10</td>
</tr>
<tr>
<td>0%</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 3.2: Porosity variation (%) for each plate radius, $r_p$.
\[
\frac{\Delta \omega_r}{\omega_0} = \frac{B}{\sqrt{m}}
\]  

(3.20)

where \( \Delta \omega_r \) is the bandwidth, \( \omega_0 \) is the frequency for null damping force, \( B \) is the damping coefficient and \( m \) is the mass. Equation 3.20 shows that when the mass increase the interval of the resonance frequency decrease.

The increase of the added mass coefficient \( M \) provokes that the resonance frequency decreases. The resonance frequency may be also reduced by decreasing the radius of the inner cylinder \( r_1 \) (small hydrostatic coefficient).

Choosing a variation of the ratio, \( r \), until 50 %, the different radiuses obtained are summarized in table 3.3.

<table>
<thead>
<tr>
<th>% r variation</th>
<th>( r = \frac{A_2}{A_1} )</th>
<th>( A_1 ) [m²]</th>
<th>( r_1 = \sqrt{r_2^2 - \frac{A_1}{\pi}} ) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-50 %</td>
<td>1.597</td>
<td>71.57</td>
<td>3.7</td>
</tr>
<tr>
<td>-44 %</td>
<td>1.783</td>
<td>64.08</td>
<td>4</td>
</tr>
<tr>
<td>-25 %</td>
<td>2.395</td>
<td>47.72</td>
<td>4.6</td>
</tr>
<tr>
<td>0%</td>
<td>3.194</td>
<td>35.7</td>
<td>5</td>
</tr>
<tr>
<td>25 %</td>
<td>3.992</td>
<td>28.63</td>
<td>5.2</td>
</tr>
<tr>
<td>50 %</td>
<td>4.791</td>
<td>23.86</td>
<td>5.4</td>
</tr>
</tbody>
</table>

Table 3.3: % Area ratio variation and its corresponding \( r_1 \)

When the area ratio, \( r \), is increased the radius \( r_1 \) decreases and, for that reason, the mass of the structure decreases, too. However, the curves in figure 3.14 show, as expected, that the resonance frequency decreases when the radius \( r_1 \) is reduced. Thus, the variation of the hydrostatic coefficient \( G \) is more important, than the effect of the mass reduction, to change the resonance frequency. Playing with a contraction or expansion of the inner cylinder the dynamic response of the spar may be significantly affected as figure 3.14 shows.

As in the case of the plate study, it is visible that the bandwidth of high motions becomes narrow when it is displaced to low frequencies due to the reduction of the restoring hydrostatic coefficient.
Finally, one can combine the two parameters obtaining a spar able to change the radius of the inner cylinder and the porosity of the plate. In that case the number of combinations is high and it is hard to simulate all of them. In the following section 3.4, the critic cases will be studied to get some conclusions.
3.4 Conclusions

In this chapter the dimensions of the spar have been identified imposing null vertical forces acting on it are null for a period of 10s. Nonetheless, some of the dimensions were previously fixed: \( r_1 = 5 \text{m} \) and \( z_1 = -5 \text{m} \). Accordingly, from the relation showed in figure 3.1 the area ratios \( r \) and the lengths \( L \) have been calculated. To choose the best option the dimensionless force as a function of \( T \) was plotted showing that the best dimensions used henceforth are:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_1 )</td>
<td>( 5 ) m</td>
</tr>
<tr>
<td>( r_2 )</td>
<td>( 6.033 ) m</td>
</tr>
<tr>
<td>( r )</td>
<td>( 3.144 )</td>
</tr>
<tr>
<td>( L )</td>
<td>( 29.617 )</td>
</tr>
<tr>
<td>( z_1 )</td>
<td>( -5 ) m</td>
</tr>
<tr>
<td>( z_2 )</td>
<td>( -34.617 ) m</td>
</tr>
<tr>
<td>( A_1 )</td>
<td>( 35.7 ) m(^2)</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>( 114.3 ) m(^2)</td>
</tr>
<tr>
<td>( V )</td>
<td>( 3777.3 ) m(^3)</td>
</tr>
</tbody>
</table>

Table 3.4: Final dimensions and variables of study of the spar without the inertia plate.

Following, the spar vertical motions were analysed (Equation 3.2) and it was identified a high motions bandwidth between 0.38 and 0.45 rad/s. The use of two cylinders instead of one has been justified comparing the motions of the original design with a spar with one cylinder and the same volume. Figure 3.11 shows that the high motions bandwidth in the case of one cylinder is centered at higher frequency and is wider in the original design.

Subsequently, a plate of 22m of diameter has been added to the spar increasing the added mass coefficient and hence, decreasing the resonance frequency. Besides, it was also verified that the dynamic response of the spar can be significantly affected by changing the plate porosity. This effect may be explored to control the platform, by deviate the resonance from the frequency of the incident waves (or peack frequency in a real sea state).

Another possibility to control the platform is varying the radius of the inner cylinder. The resonance frequency decreases for smaller radius of the inner cylinder, because the effect of the reduction of the hydrostatic coefficient, \( G \), is stronger that the inertia reduction (due to the radius diminution).

These two effects and the combination of them are showed in figure 3.15 where each case is specified below:

- Case 1: \( r_1 = 3.7 \) m with plate of \( r_p = 11 \) m (porosity 0 %).
• Case 2: \( r_1 = 3.7 \) m without plate (porosity 60 %).
• Case 3: \( r_1 = 5 \) m with plate of \( r_p = 11 \) m (porosity 0 %).
• Case 4: \( r_1 = 5 \) m without plate (porosity 60 %).
• Case 5: \( r_1 = r_2 = 6.033 \) m with plate of \( r_p = 11 \) m (only one cylinder and porosity of the plate 0 %).
• Case 6: \( r_1 = r_2 = 6.033 \) m without plate (only one cylinder and porosity of the plate 60 %).

Figure 3.15: Spar motion amplitudes for the configurations: (a) Case 1: \( r_1 = 3.7 \) m with plate of \( r_p = 11 \) m; (b) Case 2: \( r_1 = 3.7 \) m without plate; (c) Case 3: \( r_1 = 5 \) m with plate of \( r_p = 11 \) m; (d) Case 4: \( r_1 = 5 \) m without plate; (e) Case 5: \( r_1 = r_2 = 6.033 \) m with plate of \( r_p = 11 \) m; (f) Case 6: \( r_1 = r_2 = 6.033 \) m without plate.

Case 1 is the one that have a narrow high motions bandwidth for the reason discussed before. Comparing case 3 and 4 one can see that adding a plate and varying the porosity of it from 0 to 60 % the resonance frequency varies 0.12 rad/s, from about 0.32 to 0.44 rad/s. Varying the radius of the inner cylinder from 3.7 (case 2) to 6.033 (case 6) a variation of 0.24 rad/s, from 0.3 to 0.54 rad/s, is achieved. Nevertheless, the maximum variation of the resonance frequency, playing with these two factors, is visible between case 1 and case 6 where a value of 0.3 rad/s, from 0.24 to 0.54 rad/s, is attained.
Chapter 4

Corrosion treatments for offshore structures and case study of the inertial plate

4.1 Introduction to seawater properties

Materials used in offshore structures are totally conditioned by the properties and problems that seawater involves. The main problem is the corrosion of the metals and usually just the part of the structure that is in contact with the water receives some treatment against it. In this section the composition of the seawater and negative effects of the corrosion will be explained. Then, in the following section 4.2, treatments used nowadays to solve that problem will be characterized and discussed for the application in the spar.

Seawater is composed by: (a) Mineral constituents which the most important in seawater is typically about 3.5% of sodium chloride, NaCl; (b) Solubility of gases like oxygen and carbon dioxide, which are particularly important in the context of corrosion; (c) Organic matter including both animal and vegetable origin; (d) Microbiological forms including types of algae and slime forming bacteria, (e) High electrical conductivity, (f) Relatively high and constant pH.

The salinity may be weakened in some areas due to the dilution with fresh water, rain or melting ice; or increased in another areas by solar evaporation. In deep water one will consider a constant salinity. Salinity is usually determined either by conductivity measurements or from the chlorinity. Chorinity, in turn, is the mass in grams of silver required to precipitate the halogens in
0.3285234 kg of seawater. Chlorinity is related to salinity as follows:

\[ S \left( \frac{\text{o}}{\text{o} \text{o}} \right) = 1.80655 \ \text{Cl} \left( \frac{\text{o}}{\text{o} \text{o}} \right) \]  

(4.1)

where \( S \) is the salinity and \( \text{Cl} \) the chlorinity expressed in parts per thousands \( \left( \frac{\text{o}}{\text{o} \text{o}} \right) \). \[13\]

Corrosion specifically refers to any process involving the deterioration or degradation of metal components and is the primary reason by which metals deteriorate properties of resistance decreasing the fatigue strength. When metal atoms are exposed to an environment containing water molecules they can give up electrons, becoming themselves positively charged ions and producing a electrical circuit. This effect can be concentrated locally to form a pit or, sometimes a crack, or it can extend across a wide area to produce general wastage. Localized corrosion that leads to pitting may provide sites for fatigue initiation and, additionally, corrosive agents like seawater may lead to greatly enhanced growth of the fatigue crack.

For corrosion to occur, three conditions must be present: (a) Two dissimilar metals, (b) An electrolyte (water with any type of salt or salts dissolved in it) like seawater and (c) A metal (conducting) path between the dissimilar metals.

If the above conditions exist, at the more active metal surface (in this case one will consider freely corroding steel which is non-uniform), the following reaction takes place at the more active sites:

\[ 2 \text{Fe} \rightarrow 2 \text{Fe}^{2+} + 4 e^- \]  

(4.2)

The free electrons travel through the metal path to the less active sites where the following reaction takes place: (oxygen gas converted to oxygen ion - by combining with the four free electrons - which combines with water to form hydroxyl ions)

\[ O_2 + 4 e^- + 2 H_2O \rightarrow 4 OH^- \]  

(4.3)

Recombinations of these ions at the active surface produce the following reaction, which yields the iron corrosion product ferrous hydroxide:

\[ 2 \text{Fe} + O_2 + 2 H_2O \rightarrow 2 \text{Fe}(OH)_2 \]  

(4.4)
This reaction is more commonly described as current flow through the water from the anode (more active site) to the cathode (less active site), [9]

The annual costs related to corrosion and corrosion prevention have been estimated to constitute a significant part in the gross national product. In addition to the economic costs and technological delay, corrosion can lead to structural failures that have dramatic consequences for humans. Nowadays, most of the offshore structures, including offshore wind parks, are metallic and to study the problem deeply is required.

4.2 Corrosion treatments in offshore structures

Corrosion damage can take many shapes and forms that are often related to specific alloy, environment or operation conditions. The several forms of corrosion may be divided into three groups: (a) those identifiable by visual inspection, (b) those which are more easily discerned with specific aids, (c) those which can only be identified definitely by optical or electronic microscopy. Figure 4.1 shows the different types of corrosion inside each group.

Figure 4.1: Main forms of corrosion and methods to identify them
Uniform corrosion is the most important form of corrosion on the basis of tonnage wasted and is characterized by corrosive attack proceeding over the entire surface area that is in contact with seawater. However, uniform corrosion is relatively predictable and easily measured and thus, disastrous failures are relatively rare. In many cases, it is objectionable only from an appearance standpoint. As corrosion occurs uniformly over the entire surface, it can be duly controlled by cathodic protection or use of coatings or paints. The consequences of the uniform corrosion are that the surface become rough and red due to the formation of oxide and this can be produced by a breakdown in the protective coating system.

Localized corrosion is an intense attack on a specific part of the surface that is corroding much faster than the rest either because of an inherent property of the component material or because of some environmental effect. In some circumstances, corrosion protection breaks down locally producing that effect. If the attack starts on the free surface of a component, it is termed pitting.

Pitting is considered to be more dangerous than uniform corrosion because it is more difficult to detect, predict and design against and is the second most common problem. Fatigue and stress corrosion cracking may initiate at the base of corrosion pits. One pit in a large system can be enough to produce the catastrophic failure of that system. It is initiated by (a) localized chemical or mechanical damage to the protective oxide film; seawater high concentrations of chloride can cause breakdown of a passive film; (b) Poor application of a protective coating; (c) The presence of non-uniformities in the metal structure of the component, e.g. non-metallic inclusions.

The most common materials used for offshore structural and production applications are: (a) Carbon and low alloy steels used for structures and pipelines and production/process equipment; (b) Corrosion resistance alloys used for production and process equipments that are subjected to corrosive environments containing $CO_2$ and $H_2S$. They involve stainless steels, nickel base alloys, cobalt base alloys, nickel-copper alloys and titanium alloys; (c) Non-metals including elastomers, coatings, plastics and composites, [4, 1127-1137]

The most common corrosion protection methods used are alloy selection, metallic coatings, organic coatings and cathodic protection. Thermal spray, galvanizing, and for specific circumstances electroplating are metallic coatings used in various marine corrosion applications.

In terms of coating performance on steel, the severity of the seawater environment dictates both the durability of the coating with respect to self-integrity and the ability of the system to reduce the corrosion rate of the metal substrate. Under severe condition, corrosion of steel will
be rapid and readily visible, will disrupt unsuitable coatings and may endanger the safety of the structure. In such conditions, where severe chemical attack by polluted atmospheres or liquid contact is expected, coatings of high performances such as chlorinated rubbers, epoxies and solvent-soluble vinyl reins are frequently used.

In practice, paints can reduce the corrosion rate of steel in a number of ways: (a) through the formation of a relatively impervious surface layer on the metal, i.e. by barrier action; (b) by the provision of a supply of inhibitor through leaching of inhibitive pigments from the paint coating; (c) through the formation of an inhibitive agent by reaction of hydrolytic decomposition products from the polymer with constituents of the pigment to form a soap; by the inclusion in the paint coating of metallic powders which cathodically protect the metal substrate, [6, 91-94]

A typical anticorrosive system for highly corrosive marine environments usually consist in one or several intermediate coat and a topcoat. The primer has the function to protect the substrate and ensure good adhesion. Zinc or inhibitive pigments are often formulated into coatings applied as primers for structures located in the splash zone of the waves. The function of the intermediate coat is generally to build up the thickness of the coating system and impede transport of aggressive species to the substrate surface and moreover, ensure good adhesion between coats. The topcoat is exposed to the environment and has to provide resistance to alternating weather conditions and ultraviolet radiation due to the ultraviolet radiation, temperature and moisture will reduce the lifetime of the coating. The durability of all the system is really difficult to predict because depends of many factors such as chemical, mechanical and physical properties and environmental properties, [8]

Cathodic protection prevents corrosion by converting all of the anodic (active) sites on the metal surface to cathodic (passive) sites by supplying electrical current (or free electrons) from an alternate source. Cathodic protection can be accomplished by either using an impressed current system or by using sacrificial anode system. Magnesium, aluminium and zinc alloys are the most frequently used sacrificial anode systems.

Another important part is the methodology and tests performed to provide information about the degradation of the materials due to corrosion. Corrosion testing can be divided into two categories: electrochemical and nonelectrochemical.
The main variables that are measured in an electrochemical test are the voltage and the current translating this information into a corrosion rate or some other information that describes the corrosion process. Measurement of current or current density is the most common output of electrochemical corrosion tests.

The most common method of the nonelectrochemical test is the corrosion rate from mass loss of a metal of known dimensions immersed in a fluid for a known amount of time. The weight of the specimen is obtained before and after exposure. The corrosion rate is obtaining by dividing the expose area, the time and the density, [1, 59-64]

The first and second most common surface analysis techniques are Auger Electron Spectroscopy (AES) and X-Ray Photoelectron Spectroscopy (XPS). They are electron spectroscopies that quantify the energy of the electrons that are emitted by the surface during analysis, [2, 76-80]

4.3 Case study: the inertial plate

The inertial plate of the structure has to be studied separately because changing his porosity the range of high motions can be displaced as was demonstrated in section 3.3. The solution proposed in this chapter has been found with the help of CeNTI during a stage in their installations. CeNTI is a centre of nanotechnology and smart materials located in Vila Nova de Famalicão, Portugal.

The porosity of a material is determined by measuring the amount of void spaces of the total volume. Another important consideration is the shape and size of the void spaces in the material and the level of interconnection between them. Obviously, if the spaces are connected, the flow is going to be higher.

The porosity of a material is related to the ductility and fragility. Thus, an increase of the porosity has the effect to decrease the ductility. This can be dangerous to the plate causing a crack due to the low wide. The material used for the spar structure should be resistant and ductile. Composites probably are going to replace steel applications in wind offshore towers in the future due to the better properties that offer. Nowadays, some research studies propose composite designs with competitive properties in front of steel application.

The best properties of concrete are:
• **Low maintenance:** Concrete is a durable material that can keep its properties under extreme conditions and it’s not affected as steel with corrosion problems.

• **Design and construction flexibility:** No restrictions on height or size enable a big versatility of design solutions.

• **Material flexibility:** Concrete mix designs can be finely tuned to optimize key parameters such as strength, stiffness, density and environmental impact.

• **Whole life performance:** Concrete can deliver durable, large diameter pylons of unlimited height to providing higher levels of power generation.

• **Environmental impact:** Concrete constructions are fully recyclables and this contributes to reduce levels of embodied energy and \( CO_2 \) in comparison to other methods.

• **Upgradeable:** Concrete can provide long life wind tower solutions capable of accommodating to future wind turbine models.

Therefore, concrete seems to have better properties than steel for sea applications and can be a good candidate material for the body of the spar.

Another requirement is the porosity variation. That property, in the case of the concrete, is function of the temperature: as the temperature increases the % of porosity augments too. Nonetheless, when the porosity increases the strength and ductility decrease, which is not a requirement of the spar. For this reason, the concrete is discarded as a possible candidate for the inertial plate. Moreover, increasing the temperature of the material in an offshore application to control just the porosity is a waste of energy in turn that other important properties of the materials are damaged.

The next option is try to find a smart material able to change the porosity and with the strength and corrosion requirements for an offshore application. Nowadays, the porosity of some materials can be controlled when they are manufactured. This control is at microscopic level. The inertial plate needs a variation of the porosity from 0% until 60%. With 0 % of porosity the material will be ductile and on the other hand, with 60 % of porosity will be fragile. Therefore, the increase of porosity of the material detriment the ductility property and the resistance so there isn’t any existent material with the requirements specified.

The proposed solution consists in a mechanical plate with holes that simulate the voids of the material but in a macroscopic scale. The mechanism will have two parts: one fixed and another mobile, composed like a “sandwich”. The mobile part is a plate with 72 holes with a radius of 60cm distributed like is shown in figure 4.2. The plate has a radius of 11m and a height of 20cm.
The area of all the holes is:

\[ A_h = 72\pi r_h = 81.43m^2 \]  

(4.5)

and represents 20% of the total porosity. Therefore, the variation of unstable frequencies that can offer the proposed solution goes approximately from 0.3 to 0.4 rad/s taking that values from the simulation in figure 3.13 of the plate with the radius of 10m and 11m.

![Diagram of mobile part of the inertial plate with 72 holes](image)

Figure 4.2: Mobile part of the inertial plate with 72 holes of \( r_h = 0.6m \). Dimensions in meters.

The fixed part of the design has two equidistant and concentric plates with the same number of holes and distribution of the mobile one and is fixed to the bottom of the external cylinder as figure 4.3 present. The radius of the plate is 11m with a height of 20cm and they are separated by 25cm. To avoid the maximum entrance of water inside that part the external diameter is closed. Thus, the water can only enter across de holes.

The recommended material for seawater systems that work in temperatures below 15°C are 6Mo and 25Cr duplex stainless steels. For services greater than 15°C the recommended materials are titanium, Ti, or fibre-reinforced polymer (FRP), [4, 1127-1137]

Therefore, the proposed material for the mechanical inertial plate is the 6Mo and 25Cr duplex stainless steel with a barrier coating typified by an inert pigmentation, typically titanium dioxide. The degree of protection offered by the barrier depends on the thickness of the coating system as well as the generic type and nature of the binder system but in general offers good strength and abrasion resistance as well as good resistance towards UV-radiation. The surface should have a cyclic corrosion testing based on standards. In Europe, ISO 12944 is the most accepted
test of anticorrosive properties.

Figure 4.3: Fixed part with a cut in the front plate that shows the two concentric plates spaced 0.25m.

The inertial plate is sustained by the motor shaft and welded to the spar body. The place where the motor should be and the dimensions of it are not determined in this project. Inside the external cylinder there is enough space to put one or more motors to rotate the plate. The rotation of the motor has to be adapted with the frequency of the incident wave, the control parameter.

The proposed solution is presented in figure 4.4. Some chamfers has been added to help to sustain the plate as the welding between the plate and the external cylinder may have problems due to the water corrosion. The dimensions and materials used are summarized in the following table 4.1.

<table>
<thead>
<tr>
<th>Part</th>
<th>Dimensions</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed plate</td>
<td>$r_p = 11m$, $r_h = 0.6m$, draft= 0.65m, plate height= 0.2m</td>
<td>6Mo and 25Cr duplex stainless steel with titanium dioxide barrier coat.</td>
</tr>
<tr>
<td>Mobile plate</td>
<td>$r_p = 11m$, $r_h = 0.6m$, height= 0.2m</td>
<td>6Mo and 25Cr duplex stainless steel with titanium dioxide barrier coating.</td>
</tr>
<tr>
<td>Inner cylinder</td>
<td>$r_1 = 5m$, $z_1 = 5m$</td>
<td>6Mo and 25Cr duplex stainless steel with titanium dioxide barrier coating or concrete.</td>
</tr>
<tr>
<td>External cylinder</td>
<td>$r_2 = 6m$, $L = 29.617m$</td>
<td>6Mo and 25Cr duplex stainless steel with titanium dioxide barrier coating.</td>
</tr>
</tbody>
</table>

Table 4.1: Final dimensions and materials used for each part of the spar.
The design presented in figure 4.4 is just a possible solution. To optimize the result the area of the holes has to be maximized taking into account that there should be enough surface to cover the holes with the mobile plate. This restriction causes that the variation of the porosity can not be as high as one wanted initially (from 0% to 60 %).

Another problem of the proposed solution is corrosion. The seawater can enter between the fixed and mobile plates through the holes causing problems of corrosion and damaging the structure and thus, the service life. Moreover, the cost of the solution is high due to the expensive price of the anticorrosive coats and the maintenance systems.

Finally, the analysis done in section 3.3 is too simple for the proposed solution. Another analysis without viscosity simplifications should be done to know how act the new spar in the reality.
4.4 Candidate materials and processes for the inner cylinder

To change the radius of the inner cylinder can be considered as a particular case of the plate if one considers that the variations represent a porosity change. Therefore, the study carried out in section 4.3 can be applied in this case. Nonetheless, the solution is really difficult to apply in the case of the cylinder. To create a system of concentric walls around the cylinder able to open and close holes increases too much the difficulty of the design and the cost.

Moreover, another possibility is to find an elastic material able to change the shape but the scale required complicates too much that option in the design. Thus, no good solution has been found for this problem.

4.5 Conclusions

Marine atmospheres are generally considered to be one of the more aggressive atmospheric corrosion environments. Some factors that affect corrosion rates in marine atmospheres are: temperature, airborne contaminants, location and biological organisms.

Anticorrosive coatings (metallic and organic) and cathodic protection are commonly used methods to protect offshore structures. This methods need maintenance consisting in standard control test specified in ISO 12944.

The case of the inertial plate has been studied deeply. Firstly, one has studied the materials used nowadays for offshore structures and the candidate materials that are in development and have not been applied yet. Steels and alloys are the most used nowadays and research groups are considering the good and competitive properties of concrete to be used in the future.

The porosity variation has been the most difficult requirement to satisfy. Although concrete can vary this property as a function of the temperature another properties like the fragility and the strength fatigue are affected to the detriment. Any existing material is able to change his porosity without losing other required properties. Thus, another solution has been found.

Secondly, as the research of materials has not been successful, a mechanical solution has been designed. The solution consists in three concentric plates with holes working like a “sandwich”, the one in the middle mobile and the others fixed. Depending on the alignment of the mobile plate versus the fix one the holes are open or closed simulating the variation of the poros-
ity.

Next, the material chosen for the inertial plate is $6Mo$ and $25Cr$ duplex stainless steel with titanium dioxide barrier coating. As the structure may have problems with corrosion the control tests and the maintenance of the coating are essential to prolong the service life.

Finally, the problems of the proposed solution have been explained. The study of the vertical motions done in section 3.3 is not totally realistic because the viscosity must be considered as an important factor in the plate. Thus, another analysis considering this factor is required to study the real response of the new design of the spar. Moreover, the proposed design can be optimized maximizing the area of the holes and calculating the best radius and distribution.
Chapter 5

Conclusions and future work

This thesis is focused on the analysis of the vertical loads and motions of a spar concept for a floating platform of wind offshore turbine. The concept is, in essence, a structure composed by two concentric vertical cylinders where the narrow one is on top, crossing the free surface. This configuration has two levels of horizontal surfaces (the cross section between the two cylinders and the structure). Therefore, the cylinders diameters (or the areas of the horizontal surfaces) and the lengths were defined to minimize the vertical loads, which is zero for a particular frequency (when the load on the highest surface balance the load in the bottom).

The first objective was to optimize the dimensions of the spar in order to minimize the vertical forces. The assumptions of the linear theory were surmised for the mathematical formulation and some parameters were fixed initially like the radius of the inner cylinder, $r_1$, and its draft, $z_1$. Both variables took as a first iteration the value of 5m, in accordance to other solutions already proposed and under development nowadays. Nonetheless, the value chosen for $z_1$ is quite close to the free surface of the sea and this can cause problems for waves with amplitudes of 5m that are quite usual. Hence, it may be interesting to realize another future study taking a high value of the parameter.

Once the dimensions of the spar were selected the next step was the analysis of the spar vertical motions, under the linear theory assumptions. Within the range of frequencies between 0.4 and 0.45 rad/s the spar vertical motions were too high due to resonance. However, the displacement values in that interval are unrealistic because non-linear viscous effects (which play an important rule for high velocities) are not considered in the model (based on linear theory).
For the particular density spectrum considered (which characterize the more common sea state of the north of Portugal) was verified that the high motions bandwidth coincide with the more relevant spectral frequencies. Thus, the bandwidth and the resonance frequency must be modified and the initial hypothesis to do it was the addition of an inertial plate on the bottom of the spar to increase the added mass and decrease the resonance frequency. Varying the porosity of the plate, the resonance frequency of the spar may be modified in order to deviate it from the most relevant frequencies of the sea state. This method revealed to be promising and might be useful to control the dynamics of wind offshore platforms.

Moreover, the design with two concentric cylinders has been justified comparing the vertical motions and resonance frequencies with with a unique cylinder with the same volume. Thus, the design with one cylinder seems to be less appropriate, as the resonance could be, in this case, within the typical sea state range of frequencies, which is an undesired situation.

On the other hand, the variation of the inner cylinder radius $r_1$ produces a displacement of the resonance frequency due to the strong effect of the hydrostatic coefficient $G$. This effect was quantified and seems to be a likely alternative to control the dynamics of the platform.

In chapter 4 the problem gets focused to find a material for the inertial plate able to vary his porosity. First of all, characteristics for the seawater materials has been studied obtaining the result that the most used material for offshore structures are made of steel or alloys which have big problems with corrosion. The techniques more used nowadays to solve the corrosion problems are based on anticorrosive coatings that are expensive and require control maintenance.

Concrete offshore structures are in development phase and some studies show that this material has better properties than the steel. The most important characteristics are that concrete does not have corrosion problems, is totally recyclable and offer a big flexibility of design and construction structures. Unfortunately, the variation of the porosity of the material is function of the temperature. As the temperature increases, the porosity increases too, reducing the resistance and the ductility of the inertial plate. Therefore, change the porosity of the concrete affects important required properties of the structure. However, is recommended to make a future study of concrete applied to the system.

The proposed solution is based on a $6M_o$ and $25C_r$ duplex stainless steel with titanium dioxide barrier coating spar and inertial plate. The inertial plate contains three plates with holes acting like a “sandwich”. Two are fixed and the one in the middle can rotate while the holes get open or
close because of the superposition of the planes. Thus, the effect simulates the porosity variation. The dimensions used are resumed in table 4.1 and the 3D spar is presented in figure 4.4.

The proposed design can be improved in the future making a optimization study that maximize the surface of the holes taking on account that should be enough area on the plate to cover them and calculating the optimum distribution around the surface.

The inertial plate will rotate with the help of a motor that should be turned on depending on the frequency of the incident wave. The calculus of the optimum angular displacement for each state of the sea is an important future project. The design of all the control system and electronic equipment as well as the position of all the components of the motor have not been studied and may be done in the future projects.

Finally, the study of vertical motions done in chapter is not totally realistic due to the simplifications of the linear theory. It is recommended to study the motion of the inertial plate taking on account viscosity effects to study the movement of the spar with a more realistic model.
Bibliography

Appendix A

Appendix: Matlab code

A.1 Preliminary dimension design

The code used to simulate the area ratio \( r = A_2/A_1 \) as a function of \( kL \) for \( F_v = 0 \) is:

```matlab
clear all
close all

g=9.81;  % Gravity
j=0;     % Vector position

for w=0.25:0.025:2.5  % Angular velocity
j=1+j;               % Vector position
i=0;                 % Vector position
for L=5:2:25          % Length of the outer cylinder
i=1+i;               % Vector position
wn=w^2/g;            % Wave Number
ka(j,i)=wn*L;        % Dimensionless Diameter
r(j,i)=exp(wn*L);    % Area ratio
end

plot(kl,r)
xlabel('kl');
ylabel('A_2/A_1');
```

Moreover, the code used to plot the dimensionless force as a function of \( T \) for different candidate dimensions is:
clear all
close all

%%%%%% VARIABLES %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
ro=1025; % Water density
g=9.81; % Gravity
H=2; % Wave width
r1=5; % Radius internal cylinder
z1=-5; % Depth to face A1

%%%%% CURVE 1 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L=6.689; % Length of the outer cylinder
rel=1.3; % Area ratio
r2=10.408; % Radius outer cylinder
A1=pi*(r2^2-r1^2); % Area 1
A2=pi*r2^2; % Area 2
j=0 % Vector position
for w=0.25:0.025:2.5 % Angular velocity
  j=j+1; % Vector position
  z2=-L+z1; % Depth to face A2
  F(j)=abs((ro*g*H/2*(exp(z1*w^2/g)*(A1-exp(-L*w^2/g)*A2))/(ro*g*(A2-A1)))); % Force
  T(j)=2*pi/w; % Period
end

%%%%% CURVE 2 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L=10.401; % Length of the outer cylinder
rel=1.503; % Area ratio
r2=8.643; % Radius outer cylinder
A1=pi*(r2^2-r1^2); % Area 1
A2=pi*r2^2; % Area 2
j=0 % Vector position
for w=0.25:0.025:2.5 % Angular velocity
  j=j+1; % Vector position
  z2=-L+z1; % Depth to face A2
  F2(j)=abs((ro*g*H/2*(exp(z1*w^2/g)*(A1-exp(-L*w^2/g)*A2))/(ro*g*(A2-A1)))); % Force
  T(j)=2*pi/w; % Period
end

%%%%% CURVE 3 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L=14.668; % Length of the outer cylinder
rel=1.777; % Area ratio
r2=7.561; % Radius outer cylinder
A1=pi*(r2^2-r1^2); % Area 1
A2=pi*r2^2;  % Area 2
j=0  % Vector position

for w=0.25:0.025:2.5  % Angular velocity
    j=j+1;  % Vector position
    z2=-L+z1;  % Depth to face A2
    F3(j)=abs((ro*g*H/2*(exp(z1*w^2/g)*(A1-exp(-L*w^2/g)*A2)))/(ro*g*(A2-A1)));  % Force
    T(j)=2*pi/w;  % Period
end

%%%%% CURVE 4 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L=17.668;  % Length of the outer cylinder
rel=2.019;  % Area ratio
r2=7.038;  % Radius outer cylinder
A1=pi*(r2^2-r1^2);  % Area 1
A2=pi*r2^2;  % Area 2
j=0  % Vector position

for w=0.25:0.025:2.5  % Angular velocity
    j=j+1;  % Vector position
    z2=-L+z1;  % Depth to face A2
    F4(j)=abs((ro*g*H/2*(exp(z1*w^2/g)*(A1-exp(-L*w^2/g)*A2)))/(ro*g*(A2-A1)));  % Force
    T(j)=2*pi/w;  % Period
end

%%%%% CURVE 5 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L=21.495;  % Length of the outer cylinder
rel=2.322;  % Area ratio
r2=6.627;  % Radius outer cylinder
A1=pi*(r2^2-r1^2);  % Area 1
A2=pi*r2^2;  % Area 2
j=0  % Vector position

for w=0.25:0.025:2.5  % Angular velocity
    j=j+1;  % Vector position
    z2=-L+z1;  % Depth to face A2
    F5(j)=abs((ro*g*H/2*(exp(z1*w^2/g)*(A1-exp(-L*w^2/g)*A2)))/(ro*g*(A2-A1)));  % Force
    T(j)=2*pi/w;  % Period
end

%%%%% CURVE 6 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L=23.403;  % Length of the outer cylinder
rel=2.503;  % Area ratio
r2=6.452;  % Radius outer cylinder
A1=pi*(r2^2-r1^2);  % Area 1
A2=pi*r2^2; % Area 2
j=0 % Vector position

for w=0.25:0.025:2.5 % Angular velocity
  j=j+1; % Vector position
  z2=L+z1; % Depth to face A2
  F6(j)=abs((ro*g*H/2*(exp(z1*w^2/g)*(A1-exp(-L*w^2/g)*A2)))/(ro*g*(A2-A1))); % Force
  T(j)=2*pi/w; % Period
end

%%%%% CURVE 7 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L=25.395; % Length of the outer cylinder
rel=2.706; % Area ratio
r2=6.297; % Radius outer cylinder
A1=pi*(r2^2-r1^2); % Area 1
A2=pi*r2^2; % Area 2
j=0 % Vector position

for w=0.25:0.025:2.5 % Angular velocity
  j=j+1; % Vector position
  z2=L+z1; % Depth to face A2
  F7(j)=abs((ro*g*H/2*(exp(z1*w^2/g)*(A1-exp(-L*w^2/g)*A2)))/(ro*g*(A2-A1))); % Force
  T(j)=2*pi/w; % Period
end

%%%%% CURVE 8 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L=27.474; % Length of the outer cylinder
rel=2.935; % Area ratio
r2=6.158; % Radius outer cylinder
A1=pi*(r2^2-r1^2); % Area 1
A2=pi*r2^2; % Area 2
j=0 % Vector position

for w=0.25:0.025:2.5 % Angular velocity
  j=j+1; % Vector position
  z2=L+z1; % Depth to face A2
  F8(j)=abs((ro*g*H/2*(exp(z1*w^2/g)*(A1-exp(-L*w^2/g)*A2)))/(ro*g*(A2-A1))); % Force
  T(j)=2*pi/w; % Period
end

%%%%% CURVE 9 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L=29.617; % Length of the outer cylinder
rel=3.194; % Area ratio
r2=6.033; % Radius outer cylinder
A1=pi*(r2^2-r1^2); % Area 1
A2 = \pi r^2 \cdot 2; \quad \% \text{Area 2}

j = 0 \quad \% \text{Vector position}

\text{for } w = 0.25:0.025:2.5 \quad \% \text{Angular velocity}
\quad j = j + 1; \quad \% \text{Vector position}
\quad z2 = -L + z1; \quad \% \text{Depth to face A2}
\quad F9(j) = \text{abs}(\frac{\rho g H}{2} \cdot (\exp(z1 \cdot w^2/g) \cdot (A1 - \exp(-L \cdot w^2/g) \cdot A2)))/(\rho g \cdot (A2 - A1)); \quad \% \text{Force}
\quad T(j) = 2\pi/w; \quad \% \text{Period}
\text{end}

%%%%% PLOT %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
plot(T, F9)
xlabel('T');
ylabel('F_v/\rho g (A_2-A_1)');

\textbf{A.2 \ Motions equation}

The Matlab code used to simulate the equation of motion for the spar is:

\begin{verbatim}
clear all
close all

%Dimensions spar
r2 = 6;
r1 = 5;
z1 = 5;
z2 = 29.6;
L = 1; \quad \% Reference

%Variables
h = 1E4; \quad \% Water Deepness
d = 1025; \quad \% Water Densit
\quad g = 9.81; \quad \% Gravitional Acceleration
\quad A = 1.0; \quad \% Incident Wave Amplitude

V = \pi (r2^2 \cdot z2 + r1^2 \cdot z1); \quad \% Volume
m = V \cdot d; \quad \% Mass of the structure

coefs = 'spar.1'
forcesa = 'spar.2';

%%%%% LOAD WAMIT OUTPUT %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
a = load(coefs); \quad \% Hydrodynamic Coefficients
b = load(forcesa); \quad \% Pressure Integration
\end{verbatim}
pre=zeros(1,length(a(:,1)));  % Length of vector or largest array dimension
w=pre; M=pre; B=pre;
iFz=pre; rFz=pre; mFz=pre;
for nt=1:length(a(:,1))
    w(nt)=(2*pi())/(a(nt,1));  % Frequency [rad/s]
    M(nt)=a(nt,4)*d*L^3;  % Added Mass Coefficients
    B(nt)=a((nt-1)+1:nt,5)*d*w(nt)*L^3;  % Damping Coefficients
    iFz(nt)=b((nt-1)+1:nt,7)*d*A*L^2*g;  % Imaginary part of the Exc. Forces
    rFz(nt)=b((nt-1)+1:nt,6)*d*A*L^2*g;  % Real part of the Exc. Forces
    mFz(nt)=b((nt-1)+1:nt,4);  % Modulus of the Exc. Forces
% phz(nt)=b((nt-1)+1:nt,5);  % Phases of the Exc. Forces
end

%%%%% HIDROSTATIC COEFFICIENTS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
S=pi*r1^2;
Hs=S*d*g;

%%%%% EQUATION OF MOTION %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Z=(rFz+1i*iFz)./((-w.^2).*(m+M)+Hs+1i.*w.*B)

where spar.1 and spar.2 are files containing the Wamit results.

The following columns are an example of the file bodymaria.1:

<table>
<thead>
<tr>
<th>T</th>
<th>Mode</th>
<th>M</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.26E+03</td>
<td>3</td>
<td>3</td>
<td>4.99E+02</td>
</tr>
<tr>
<td>6.28E+02</td>
<td>3</td>
<td>3</td>
<td>4.99E+02</td>
</tr>
<tr>
<td>4.19E+02</td>
<td>3</td>
<td>3</td>
<td>4.99E+02</td>
</tr>
<tr>
<td>3.14E+02</td>
<td>3</td>
<td>3</td>
<td>4.99E+02</td>
</tr>
<tr>
<td>2.51E+02</td>
<td>3</td>
<td>3</td>
<td>5.00E+02</td>
</tr>
<tr>
<td>2.09E+02</td>
<td>3</td>
<td>3</td>
<td>5.00E+02</td>
</tr>
<tr>
<td>1.80E+02</td>
<td>3</td>
<td>3</td>
<td>5.00E+02</td>
</tr>
<tr>
<td>1.57E+02</td>
<td>3</td>
<td>3</td>
<td>5.00E+02</td>
</tr>
<tr>
<td>1.40E+02</td>
<td>3</td>
<td>3</td>
<td>5.00E+02</td>
</tr>
<tr>
<td>1.26E+02</td>
<td>3</td>
<td>3</td>
<td>5.00E+02</td>
</tr>
<tr>
<td>1.14E+02</td>
<td>3</td>
<td>3</td>
<td>5.00E+02</td>
</tr>
<tr>
<td>1.05E+02</td>
<td>3</td>
<td>3</td>
<td>5.00E+02</td>
</tr>
<tr>
<td>9.67E+01</td>
<td>3</td>
<td>3</td>
<td>5.00E+02</td>
</tr>
<tr>
<td>8.98E+01</td>
<td>3</td>
<td>3</td>
<td>5.00E+02</td>
</tr>
<tr>
<td>8.38E+01</td>
<td>3</td>
<td>3</td>
<td>5.00E+02</td>
</tr>
<tr>
<td>7.85E+01</td>
<td>3</td>
<td>3</td>
<td>5.00E+02</td>
</tr>
</tbody>
</table>

where the first column is the period $T$, the second and third column is the oscillation mode (in that case the vertical mode), the fourth column is the dimensionless added mass coefficient $M$.
and the last column is the dimensionless damping coefficient $B$.

Furthermore, some of the results of the file *spar.2* are:

<table>
<thead>
<tr>
<th>$T$</th>
<th>$\theta$</th>
<th>$\nu$</th>
<th>$F$</th>
<th>$\phi$</th>
<th>$R$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.26E+03</td>
<td>0.00E+00</td>
<td>3</td>
<td>7.85E+01</td>
<td>1.46E-08</td>
<td>7.85E+01</td>
<td>2.00E-08</td>
</tr>
<tr>
<td>6.28E+02</td>
<td>0.00E+00</td>
<td>3</td>
<td>7.85E+01</td>
<td>2.34E-07</td>
<td>7.85E+01</td>
<td>3.20E-07</td>
</tr>
<tr>
<td>4.19E+02</td>
<td>0.00E+00</td>
<td>3</td>
<td>7.85E+01</td>
<td>1.18E-06</td>
<td>7.85E+01</td>
<td>1.62E-06</td>
</tr>
<tr>
<td>3.14E+02</td>
<td>0.00E+00</td>
<td>3</td>
<td>7.84E+01</td>
<td>3.74E-06</td>
<td>7.84E+01</td>
<td>5.11E-06</td>
</tr>
<tr>
<td>2.51E+02</td>
<td>0.00E+00</td>
<td>3</td>
<td>7.83E+01</td>
<td>9.11E-06</td>
<td>7.83E+01</td>
<td>1.24E-05</td>
</tr>
<tr>
<td>2.09E+02</td>
<td>0.00E+00</td>
<td>3</td>
<td>7.82E+01</td>
<td>1.89E-05</td>
<td>7.82E+01</td>
<td>2.57E-05</td>
</tr>
<tr>
<td>1.80E+02</td>
<td>0.00E+00</td>
<td>3</td>
<td>7.80E+01</td>
<td>3.49E-05</td>
<td>7.80E+01</td>
<td>4.75E-05</td>
</tr>
<tr>
<td>1.57E+02</td>
<td>0.00E+00</td>
<td>3</td>
<td>7.79E+01</td>
<td>5.94E-05</td>
<td>7.79E+01</td>
<td>8.07E-05</td>
</tr>
<tr>
<td>1.40E+02</td>
<td>0.00E+00</td>
<td>3</td>
<td>7.77E+01</td>
<td>9.49E-05</td>
<td>7.77E+01</td>
<td>1.29E-04</td>
</tr>
<tr>
<td>1.26E+02</td>
<td>0.00E+00</td>
<td>3</td>
<td>7.75E+01</td>
<td>1.44E-04</td>
<td>7.75E+01</td>
<td>1.95E-04</td>
</tr>
<tr>
<td>1.14E+02</td>
<td>0.00E+00</td>
<td>3</td>
<td>7.72E+01</td>
<td>2.11E-04</td>
<td>7.72E+01</td>
<td>2.84E-04</td>
</tr>
<tr>
<td>1.05E+02</td>
<td>0.00E+00</td>
<td>3</td>
<td>7.70E+01</td>
<td>2.97E-04</td>
<td>7.70E+01</td>
<td>3.99E-04</td>
</tr>
<tr>
<td>9.67E+01</td>
<td>0.00E+00</td>
<td>3</td>
<td>7.67E+01</td>
<td>4.08E-04</td>
<td>7.67E+01</td>
<td>5.46E-04</td>
</tr>
</tbody>
</table>

where the first column is the period $T$, the second is the angle of the incident wave (in that case always $0^\circ$), the third is the oscillation mode, the forth is the modulus of the excitation force, the fifth is the phase, and the penultimate and last column are respectively the real and imaginary part of the excitation force.
List of Figures

1.1 Flux average in kW/m of wave front in the world (left) and in Europe (right). Source: European Wave Energy Atlas. 2
1.2 Wind offshore source in the world. Source: European Atlas of Wind Offshore. 3
1.3 Examples of wind technologies depending on the depth of the sea on the left and its corresponding cost as a function of sea depth. Source: National Renewable Energy Laboratory, 2010. 4
1.4 Schematic representations of the spar structure with the variables of study. 5
1.5 Wind float platform and turbine. Source: Principle Power. 6

3.1 Area ratio \( r = A_2/A_1 \) as a function of \( k \cdot L \) for \( F_v = 0 \) 18
3.2 Spectral density distribution of the north Portuguese coast in winter. Peak frequency \( \omega_p = 0.62 \text{ rad/s} \) 19
3.3 Dimensionless force as a function of \( T \) for different candidate dimensions and for \( r_1 = 5m \) and \( z_1 = -5m \). 20
3.4 Dimensionless force as a function of \( T \) for different values of \( z_1 \) for the dimensions \( r = 3, 144, L = 29, 617m, r_1 = 5m \) and \( r_2 = 6.033m \) 21
3.5 Spar with the final dimensions. 22
3.6 Mesh of the spar structure used to run \( WAMIT \). 23
3.7 Spar vertical displacement as a function of \( \omega \) for the selected dimensions. 24
3.8 Limiting spar vertical displacement when \( \omega \to 0 \). 25
3.9 Effect on the vertical forces of the body when \( \omega \to 0 \) (on the left) and \( \omega \to \infty \) (on the right). 26
3.10 Representation of the spar motion amplitude as a function of \( \omega \) for the dimensions of the two cylinders for the most important frequencies of the spectrum of density. 27
3.11 Vertical motions as a function of the frequency, \( \omega \), for a spar (with volume 3779.25 \( m^3 \)) composed by two or a single cylinder. 27
3.12 Mesh of the spar structure with an inertial plate \( (r_p = 11m) \) used to run \( WAMIT \). 28
3.13 Vertical motion amplitudes as a function of $\omega$ for several radius of the plate considered.

3.14 Vertical motion amplitudes as a function of $\omega$ with a variation of $r$ until 50%.

3.15 Spar motion amplitudes for the configurations: (a) Case 1: $r_1 = 3,7m$ with plate of $r_p = 11m$; (b) Case 2: $r_1 = 3,7m$ without plate; (c) Case 3: $r_1 = 5m$ with plate of $r_p = 11m$; (d) Case 4: $r_1 = 5m$ without plate; (e) Case 5: $r_1 = r_2 = 6,033m$ with plate of $r_p = 11m$; (f) Case 6: $r_1 = r_2 = 6,033m$ without plate.

4.1 Main forms of corrosion and methods to identify them

4.2 Mobile part of the inertial plate with 72 holes of $r_h = 0,6m$. Dimensions in meters.

4.3 Fixed part with a cut in the front plate that shows the two concentric plates spaced 0,25m.

4.4 Final design of the proposed solution with the dimensions in meters.
List of Tables

3.1 Results of $L$ and $r_2$ for different area ratios $r = \frac{A_2}{A_1}$ for the peak frequency $\omega_p = 0.62$ of the spectral distribution presented in figure 3.2. ................................................. 20
3.2 Porosity variation (%) for each plate radius, $r_p$. .......................................................... 29
3.3 % Area ratio variation and its corresponding $r_1$. ............................................................ 30
3.4 Final dimensions and variables of study of the spar without the inertia plate. .......... 32

4.1 Final dimensions and materials used for each part of the spar. ................................. 42