CRANFIELD UNIVERSITY

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BENCHMARK OF TYRE MODELS FOR MECHATRONIC APPLICATION

SCHOOL OF ENGINEERING
Automotive Product Engineering

MSc
Academic Year: 2010 - 2011

Supervisor: Prof. Francis Assadian
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(NB. This section can be removed if the award of the degree is based solely on examination of the thesis)

ABSTRACT

In this paper a comparison matrix is developed in order to examine three tyre models through nine criteria. These criteria are obtained after the requirements study of the main vehicle-dynamics mechatronic applications, such as ABS, ESP, TCS and EPAS.

The present study proposes a weight for each criterion related to its importance to the mentioned applications. These weights are obtained by taking into account both practical and theoretical judgement. The former was collected through experts' opinion and the latter from publications.

In this study an overview of tyre-models categories has been done. Pacejka's Magic Formula, TMeasy and the LuGre tyre model have been analysed in more detail and the steady-state tyre characteristics implemented through Matlab.

Subsequently, the fulfilment process of the matrix has been carried out taking into account the information from tyre-model papers and the results obtained from the model comparison.

By using the procedures mentioned, the LuGre tyre model has been chosen as the best one for mechatronic applications for vehicle-dynamics control systems. Finally, further improvement works are recommended.

Keywords:

Comparison matrix, vehicle-dynamics control systems, criteria, parameters weight.

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ACRONYMS

ABS	Anti-lock Brake System
ECU	Electronic Control Unit
EPAS	Electric Power Steering System
ESP	Electronic Stability Program
MBS	MultiBody System
TCS	Traction Control System

NOTATION

a	half of contact length
B	stiffness factor in Pacejka’s Magic Formula
c_i	model parameters in Pacejka’s Magic Formula
C	shape factor in Pacejka’s Magic Formula
C	set of acceptable friction coefficients for 2D motion
$C_{F\alpha}$	cornering stiffness
$C_{F\kappa}$	longitudinal stiffness
$C_{M\alpha}$	aligning torque stiffness
dF^0	cornering stiffness in TMeasy
D	peak factor in Pacejka’s Magic Formula
E	curvature factor in Pacejka’s Magic Formula
F^M	maximum tyre force on the parabola in TMeasy
F^S	force on the sliding area in TMeasy
F_z^N	nominal wheel load in TMeasy
F_b	longitudinal braking force
F_d	longitudinal driving force
F_n	normal force
F_x	longitudinal force
F_y	lateral force
F_z	vertical or normal force
g	Stribeck effect
L	length of the contact patch
M_k	friction coefficient in 2D motion in LuGre tyre model
M_x	overturning torque
M_y	rolling resistance moment
M_z	self-aligning torque
n	pneumatic trail
r	tyre dynamic radius
r_e	effective rolling radius
s	longitudinal slip rate in LuGre tyre model
s^0	slip on the adhesion area in TMeasy
s^M	slip where the maximum tyre force is produced in TMeasy
s^S	slip where the tyre starts sliding in TMeasy
T_x	tipping torque

T_y	rolling resistance torque
T_z	self-aligning torque
v	vehicle speed
v_r	relative velocity between the tyre and the ground
v_s	Stribeck relative velocity
V_x	longitudinal speed
V_y	lateral speed
W	width of the contact patch
Y	global coordinate in Pacejka's Magic Formula
z	bristle deflection
$\dot{z}$	brislte friction state of bristle elastic deflection

α	lateral slip angle; side slip angle
γ	camber angle (wheel inclination)
δF	differential friction force
κ	longitudinal slip
μ	coefficient of friction
μ_C	normalized Coulomb friction
μ_S	normalized Static friction
μ_k	kinetic friction coefficient
μ^*	admissible friction coefficient in LuGre tyre model
ρ	tyre radial deflection
σ_0	rubber longitudinal lumped stiffness
σ_1	rubber longitudinal lumped damping
σ_2	viscous relative damping
φ	spin slip; turnslip
ω	wheel angular velocity
Ω	wheel speed of revolution
Ω_0	initial wheel speed of revolution
ζ	origin of the axis coordinate in LuGre tyre model

1 GENERAL INTRODUCTION

1.1 Motivation and background

Nowadays, dynamic-behaviour simulations of vehicle components are becoming more important for the development and improvement of new systems such as anti-lock brake systems (ABS), the electronic stability programme (ESP) and electric power-steering systems (EPAS).

Consequently, the vehicle model has to be capable of yielding simulation results as close development, because the results obtained through it are the inputs of the other vehicle-systems models.

Advanced vehicle control systems require accurate modelling of tyre dynamics and vehicle interactions to develop better automatic controls in order to ensure vehicle tracking performance and stability. In the dynamics simulations, these systems use professional multi-body system (MBS) software that employs a tyre model with the same interface [19].

However, no complete and satisfactory theory describing tyre characteristics precisely has yet been developed, because of tyre behaviour and structural complexity [22]. The study of the mechanical behaviour of tyres has to take into account tyres' reaction to some inputs in relation to the road conditions and the wheel motion.

Finally, it is essential to differentiate between the tyre's dynamic behaviour and the steady-state performance in order to study tyre models. The first one is much more difficult to describe using mathematical equations, because it has to take into account tyre rolling and some braking/driving and steering forces [22]. The tyre is ordinarily modelled by using a static model because of its high computing efficiency and simplicity. However, modelling the dynamic tyre behaviour is important from a vehicle-dynamics control systems standpoint.

1.2 Objectives

The objective of this project is to choose the best tyre model for mechatronic applications, particularly for vehicle-dynamics control systems. In order to achieve this objective, different tyre-models categories are analysed and a comparative study of the most significant models of each category is conducted.

Pacejka's Magic Formula is the most famous existing tyre model and hence is used as a benchmark. It is an empirical model that approximates tyre behaviour using only mathematical formulae. The second model studied is Georg Rill's TMeasy. It is a simple semi-empirical tyre model that describes tyre forces by using a small quantity of physical parameters obtained from measured data.

The last model compared is the physical LuGre tyre model of Canudas-de-Wit (1999). It is a dynamic tyre model based on the brush model that has a compact mathematical structure for describing the tyre–road interaction.

The steady-state tyre characteristics of the three models are studied for the lateral and longitudinal forces and the self-aligning moment. By using Matlab, the plots of these forces versus the slip have been analysed and compared. The tyre dynamic behaviour using the LuGre model is also presented.

The other aim of this thesis is to provide an overview of mechatronics, examining its more significant applications and looking at what vehicle-dynamics control systems expect from tyre models.

Based on the insight obtained by studying a number of systems, such as ABS, ESP and EPAS, some criteria are defined and weighted according to their importance for the development of advanced vehicle control systems.

Finally, the comparison matrix is built and a total score is given for each of the three tyre models in order to know which one best satisfies the needs of mechatronic applications. Also, some recommendations are given for further studies.

The main challenges to face in this project are the large volume of papers and books that need to be covered in order to develop a good background and an understanding of tyre models and vehicle-dynamics control systems.

1.3 Methodology

The methodology employed to conduct this research was separated into three main stages. The first was the study of mechatronics and its applications in order to examine their main requirements.

The second stage was the review and study of tyre models and their categories. The most important model of each category has been analysed: Pacejka's Magic Formula, TMeasy and the LuGre tyre model.

For all the models studied, the formulation for longitudinal and lateral forces and self-aligning torque has been presented. Then, using Matlab, the formulation has been computed and the force versus slip plots depicted. Moreover, for the LuGre tyre model, the dynamic tyre characteristics have been simulated through Simulink. Finally, a comparison of the forces and self-aligning torque characteristics of Pacejka, TMeasy and LuGre has been conducted.

The last stage was the research of the best tyre model for mechatronic applications for vehicle-dynamics controls systems. The tool that has been used is a matrix comparison, because it is an effective visual aid with a very simple structure.

In order to compare the models, some criteria have been defined in detail considering the mechatronic application requirements. These criteria have been weighted according to their importance following the *multiple methods approach* [29]. As a consequence, two estimations have been employed: experts' judgement and the author's opinion representing the theoretical point of view. Lastly, the comparison matrix has been performed using the information in the documentation studied and the graphics obtained through Matlab.

To conclude this section, it is important to highlight that another important task completed through the present work development has been the poster shown in Appendix A, presented in the Poster Competition in July 2011. This poster allows someone interested in the project to gain an overview of it.

1.4 Outline of the thesis

The information in this thesis is presented in the following order. Firstly, Chapter 2 contains an overview of mechatronics: its main characteristics, the interfaces it uses in relation to actuators and sensors, and two examples of mechatronics systems. A brief review of the main mechatronics applications, as well as a detailed description of vehicle-dynamics control systems, is given in Chapter 3.

A literature review of the following tyre models is presented in Chapter 4: Pacejka's Magic Formula, TMeasy and the LuGre tyre model. In addition to studying the formulation for the definition of the forces and moment for each model, the steady-state characteristics of these forces and moment are computed using Matlab. Therefore, an analysis of the influence of the normal force increase is also given. Moreover, a comparison of the models is presented in this chapter for later use in the comparison matrix.

A brief review of the principal tyre-model applications and a definition of the nine criteria used for the comparison matrix are presented in Chapter 5. In addition, this chapter also presents the weight given to each criterion.

Chapter 6 contains the comparison matrix and the justification of all the model-criteria weights as well as the choice of the best tyre model for a vehicle-dynamics mechatronic application. Finally, concluding remarks and recommendations for further studies are given in Chapter 7.

2 MECHATRONICS

2.1 What is mechatronics?

The word mechatronics has obtained recognition in recent years because it embraces and integrates a number of technologies, including computer software, control engineering, mechanical engineering, electrical engineering and electronic software. Figure 2-1 provides a clear summary of mechatronics.

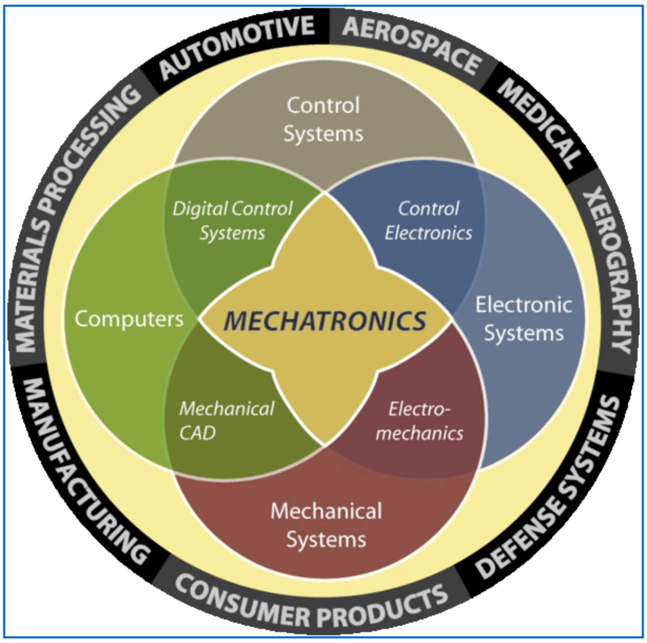

Figure 2-1 Systems and applications integrated in mechatronics (source: Craig, 2007)

Mechatronics is the application that solves mechanical-engineering problems by using computer-based digital control techniques [3]. This combination of systems that had commonly been separate disciplines has led to new approaches to achieving the performance of more complicated engineering systems [12]. Therefore, it offers the chance to have new standpoints in relation to problems. Engineers can look into a problem in terms of a range of technologies and not just from the mechanical point of view.

A mechatronic system is characterized by the following features:

- Physical and technical complexity.
- A high level of system integration.
- Substantial relocation of system functions from the mechanical to the electronic.
- Higher systems performance than compared to traditional systems.
- Use of an integrated and distributed processing architecture.

Mechatronics is neither a simple combination of mechanical and electrical systems nor an advanced control system; it is an accomplished integration of all of them. However, in order to obtain more flexible, reliable and cheap solutions, this integration has to be done at the design phase of any process [3].

During the design stage of intelligent mechatronic behaviour systems, several control strategy levels must be considered, as presented in Table 1.

Table 1 Levels of control and operation for mechatronic systems [5]

Strategic level	User production goals
Tactical level	Previous goals are analysed to establish goal
Task level	Decide tasks to be accomplished in relation to the designated goal
Action level	Separate individual tasks into a suitable sequence of actions
Trajectory level	Define the motion path required from the current position

2.2 Mechatronics interface

Figure 2-2 shows how a mechatronic system works through two differentiated domains together with a world interface. The domains are separated into energetic and information environments. Moreover, these world interfaces provide the measurements and control functions that are essential to any mechatronic system.

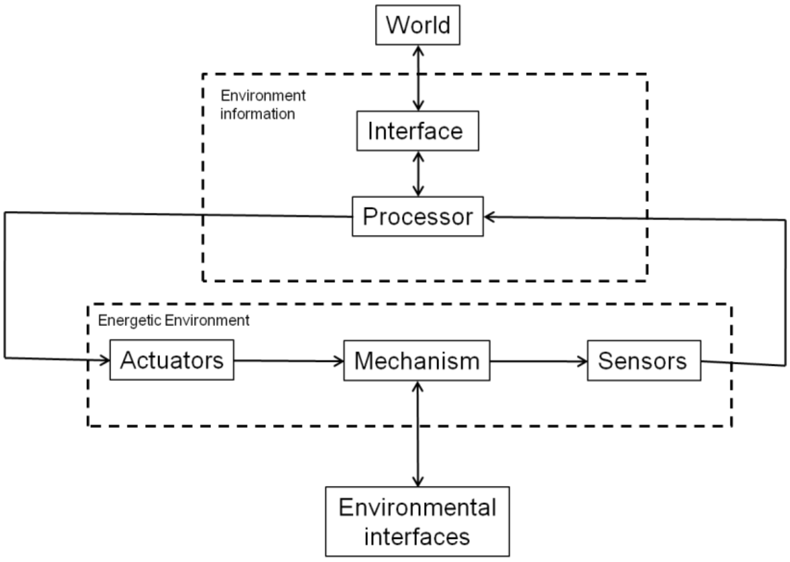

Figure 2-2 General mechatronic system [12]

In an intelligent mechatronic system, sensors are used to provide information about both system and world conditions. Sensors are important in ensuring performance integrity and system reliability and are mainly silicon-based.

On the other hand, actuators are mechanical devices used to move or control a mechanism or a system. The most common actuators are based on conventional and established technologies such as electronic motors, fluid power and mechanical drives, which are becoming increasingly available [5].

2.3 Examples of mechatronic systems

A typical mechatronic system structure can be observed in the functional diagram of a camera [12]. As a mechatronic system, each of the essential elements – the flash, lens and body – have both their actuating and their processing elements, which are coordinated by the main processor in the camera body.

Another example of a mechatronic system is the Electric Power Steering System (EPAS) system, which uses a power steering control module to regulate the amount of assist steering. Figure 2-3 shows the different components of the EPAS system and the power steering torque flowchart. To determine the amount of steering assist that is required by the electric motor mounted on the steering column, the control module needs the following inputs: knowing how much the driver turns the wheel, which is detected by the steering sensor; the amount of torque measured with the steering torque sensor mounted on the steering shaft; the vehicle speed; and other inputs from ESP or TCS. Then the control module commands the electric motor and a sensor provides feedback to it in order to monitor the motor's position.

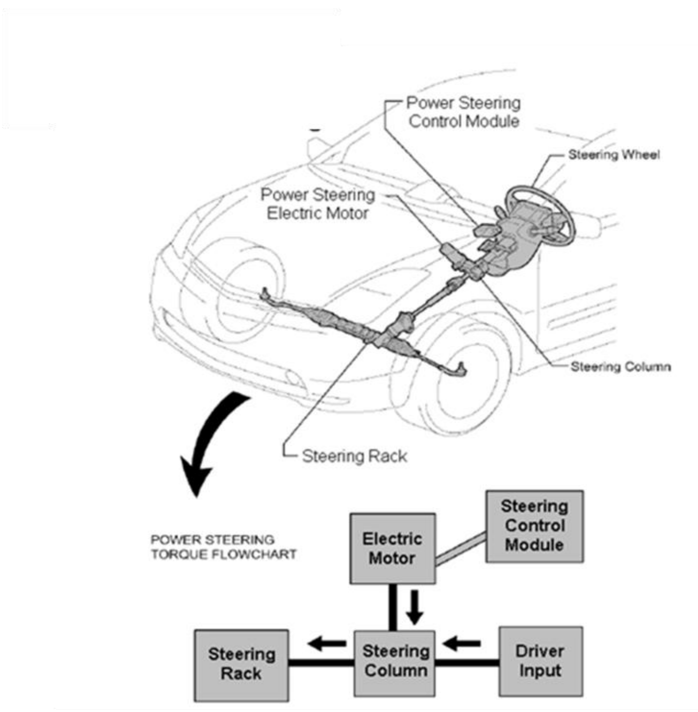

Figure 2-3 Electric Power Steering System (source: www.aa1car.com)

Likewise, a Collision Avoidance System also satisfies mechatronic features. It uses a radar system mounted in the front of the vehicle to monitor the presence of a lead vehicle. By combining the information from the radar system with the direct operation of the vehicle throttle, it is possible to control the speed of a following vehicle directly in response to changes in the speed of the lead vehicle while maintaining the appropriate distance between vehicles [3].

3 MECHATRONIC APPLICATIONS

Mechatronics gives both a title and a focus to the design, development and improvement of a wide range of engineering systems.

The main areas of automotive mechatronic development shown in Figure 3-1 are: process engineering, which is manufacturing; materials processing; and product engineering, which is vehicle systems. In all cases, the engineering discipline of mechatronics has the greatest role in improving the physics of the materials and devices and enhancing the sciences of instrumentation and measurement.

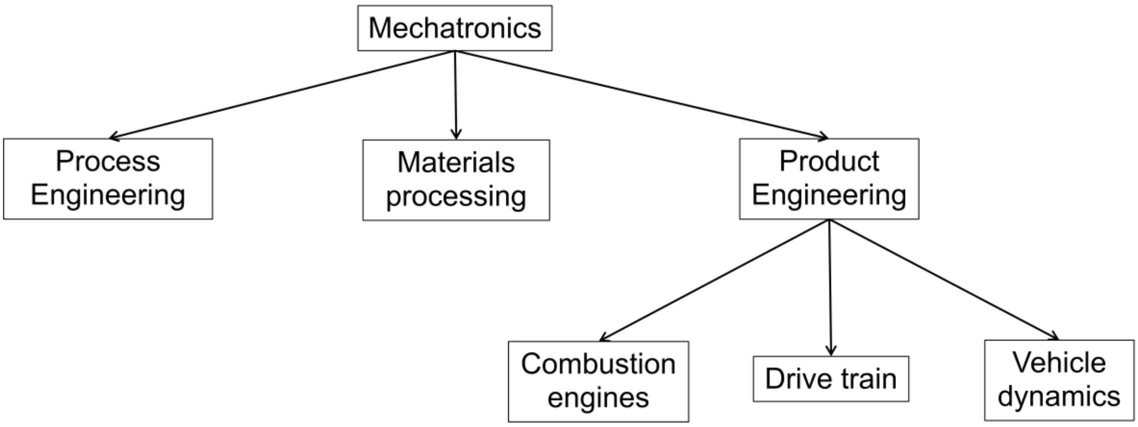

Figure 3-1 Automotive mechatronic applications

Manufacturing systems (process engineering) are considered as mechatronic systems because their operations depend on sensors and they have an associated signal processing. The different types of sensors that they have are capable of monitoring a great range of physical and dimensional parameters such as length, weight, roughness, depth, velocity, position, acceleration, time, profile, temperature, force and many others [12].

In recent years, mechatronics has had an increasing impact on the design and functioning of modern automobiles. With the introduction of powertrain management, advanced steering systems, traction control, passenger safety systems, and emission control, vehicles are gradually becoming mechatronic in nature, establishing high demands on on-board sensors.

A vehicle will have a different number of sensors, depending on its size and nature: a basic vehicle has around 20 sensors, a mid-range vehicle around 40 and a top-of-the-range vehicle around 80 [12].

Sensors on the vehicle are distributed in three main locations: the engine, the chassis and the body. Table 2 lists some examples of sensors for each area. The majority of sensors are standard models that are high-volume and low-cost devices that require no maintenance and are reliable.

Table 2 Location of some vehicle sensors [12]

Engine	Chassis	Body
Air-fuel ratio	Wheel speed	Security systems
Oil and water temperature and pressure	Tyre pressure	Crash sensors
Engine speed and torque	Speed over ground	Ice warning sensors
Fuel Injection		
Air Mass		
Lambda control		
Idle Speed control		

However, as Figure 3-2 shows, mechatronics in automobiles are divided into five main systems: combustion engines, drive trains, suspensions, brakes and steering. The last three define vehicle-dynamics behaviour and are going to be considered as one big system.

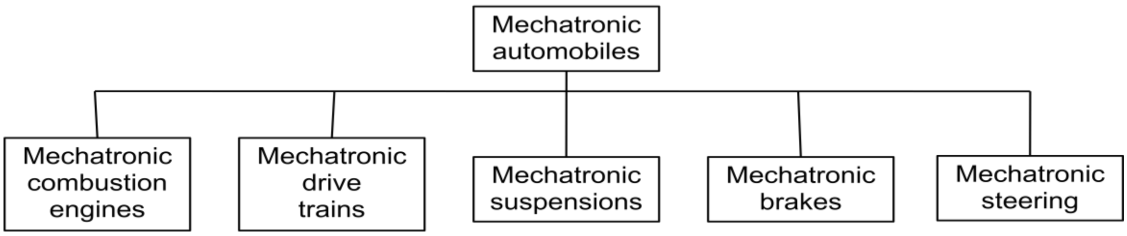

Figure 3-2 Main automotive mechatronics systems (source: presentation given by Francis Assadian, Automotive Control and Simulation, 2011)

3.1 Mechatronic combustion engines

In engine management systems, it is very important to monitor the combustion process in each cylinder separately in order to obtain data on parameters such as initiation and completion of combustion and timing, fuel/air ratio and cylinder compression. Consequently, a reduction in exhaust emissions and the development of lean-burn engines, as well as an improvement in two-stroke, four-stroke and diesel engines, can be achieved.

In the engine management field it is crucial that sensors are able to provide the essential information and to survive the extreme conditions in the engine during combustion [17].

The main controls in combustion engines are:

- Electrical throttle
- Mechatronic fuel injection
- Mechatronic valve trains
- Variable geometry turbocharger
- Emission control
- Evaporative emission control
- Electrical pumps and fans

3.2 Mechatronic drive train

At present, the main control systems for the vehicle's drive train, which consists of the parts of the torque path excluding the engine, are:

- Automatic hydrodynamic transmission
- Automatic mechanic shift transmission
- Continuously variable transmission
- Traction Control System (TCS)
- Automatic speed and distance control
- Electronic Limited-Slip Differential

All of these systems are very valuable in improving ride quality, stability and passenger comfort for all types of vehicles and roads. For instance, active suspension needs to be provisioned with some parameters such as steering angle, cornering angle, ground clearance and suspension motions, including velocity and acceleration, for the desired performance of the vehicle.

In addition, Traction Control systems decrease or eliminate excessive slipping during vehicle acceleration and consequently improve the controllability and manoeuvrability of the car [12].

3.3 Mechatronic vehicle-dynamics systems

Finally, the field known as vehicle dynamics is concerned with four aspects of the vehicle, as can be seen in Figure 3-3: refinement, ride, handling and performance. Vehicle dynamics divides the inputs into two groups: isolation and control.

Isolation is about separating the driver from disturbances generated as a result of vehicle operation. The disturbances can be divided into those generated by the vehicle itself (for example noise and engine vibration) and those that come from the external world. The disturbances in this last category are aerodynamic interaction of the vehicle with its surroundings and road undulations. The response to road disturbances is a change in the forces acting on the tyre: vertical, longitudinal and lateral.

Control is concerned largely with the behaviour of the vehicle in response to driver demands, as the driver continuously varies both path curvature and speed. In the adjustment of the path curvature controlling the stability, fidelity and linearity of the vehicle, it is important to obtain good handling conditions. Where the meaning of stability is absence of disturbances amplification, fidelity means rejection of disturbances, and linearity means the vehicle’s instinctive behaviour for the driver.

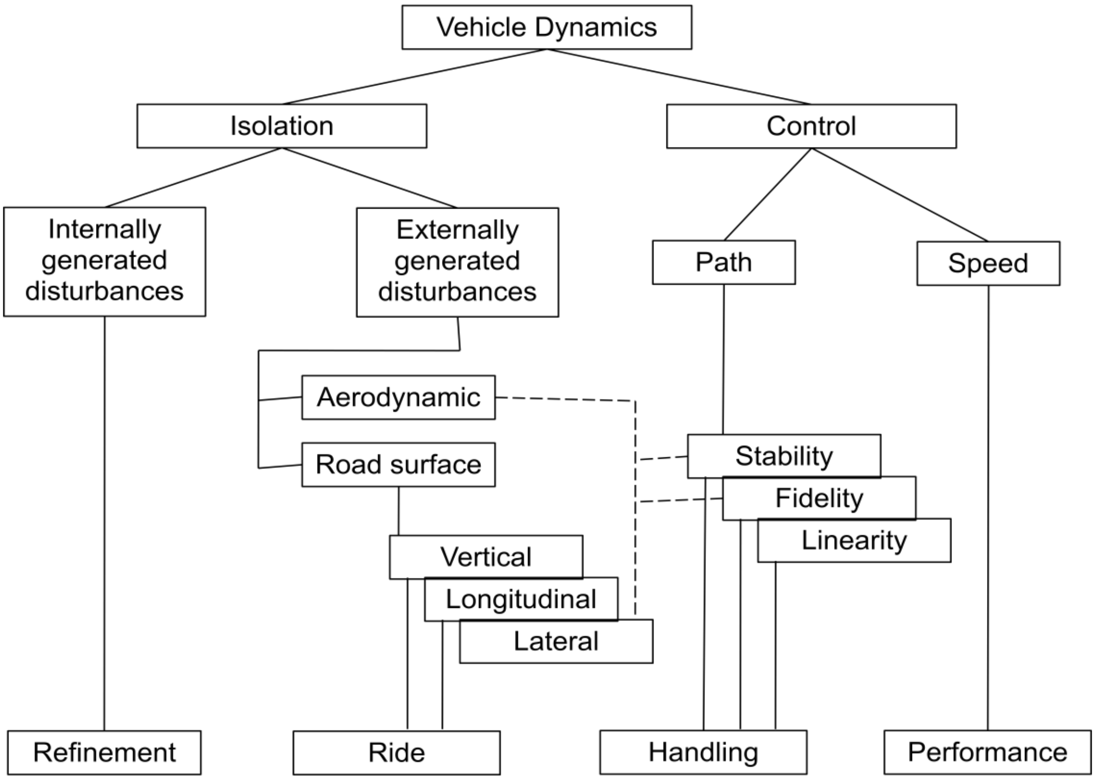

Figure 3-3 Vehicle-dynamics interactions [2]

The main vehicle-dynamics active control mechanisms that have been developed and commercialized are listed below and may be tested and optimized using vehicle simulators with proper tyre models.

Mechatronic suspensions:

- Semi-active shock-absorbers
- Active hydraulic suspension
- Active pneumatic suspension
- Active anti-roll bars (dynamic drive control or roll-control)

Mechatronic brakes:

- Hydraulic anti-lock braking system (ABS)
- Electronic stability programme (ESP)
- Electro-hydraulic brake
- Electro-mechanical brake
- Electrical parking brake

Mechatronic steering:

- Parameterizable power-assisted steering
- Electro-mechanical power-assisted steering (EPAS)
- Active front steering

All of these systems have reduced vehicle accidents and improved vehicle stability. Anti-lock braking systems prevent the wheels from locking and skidding during braking by regulating brakes' pressure and maximize the braking forces produced by the tyres by restricting the longitudinal slip ratio and preventing it from exceeding an optimal value. Consequently, anti-lock braking systems increase steerability and lateral stability, particularly in wet and icy conditions. Electric power steering provides mechanical steering assistance to the driver in order that he needs less force to turn the steering wheel. Finally, an electronic stability programme prevents automobiles from drifting out, spinning and rolling over.

These vehicle control systems consist of a number of on-board computer nodes, called electronic control units (ECU), which are interconnected via a network. They utilize on-board algorithms to calculate data from the vehicle that is provided by wheel speed sensors.

4 TYRE MODELS

In this chapter the main inputs and outputs of a tyre model and the features of the different categories of tyre models are explained. Moreover, three tyre models are studied in detail in order to be able to compare them later.

One model of each main category is going to be examined: Pacejka's Magic Formula in the empirical category, TMeasy in the semi-empirical category, and the LuGre tyre model in the physical category.

Finally, the results obtained through Matlab are going to be plotted on the same graph to compare the differences and similarities between the three models.

4.1 Introduction to tyre models

The principal requirement of tyre models is to predict the tyre forces and moments in the three directions between the tyre and the road. Figure 4-1 shows the forces and torques transmitted, and hence the model's outputs.

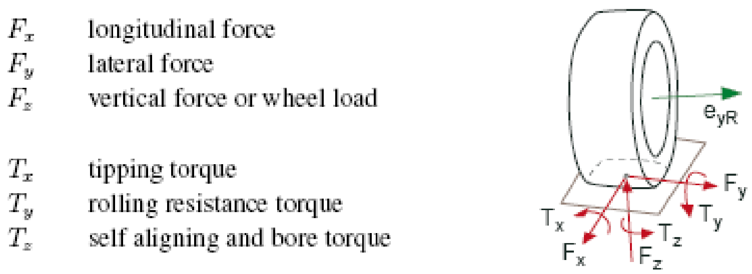

Figure 4-1 Forces and torques acting on the contact point between the tyre and the road [24]

These forces may not be exactly the same as the real ones that are acting between the tyre and the surface due to the dynamic forces acting on the tyre when it vibrates. Figure 4-2 presents the tyre model's input and output vectors. The input has the motion information of the wheel.

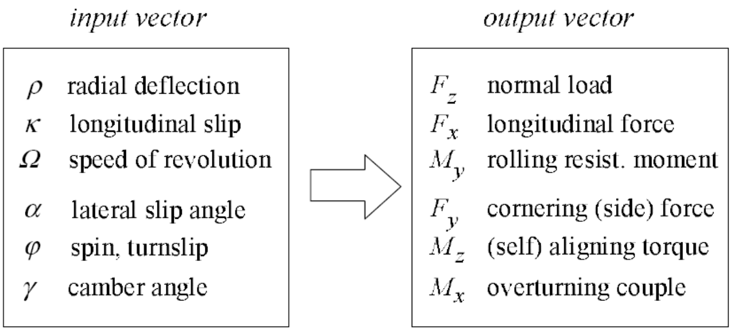

Figure 4-2 Tyre model's input and output quantities (source: Pacejka, 2006, p.62)

4.2 Tyre-model categories

Several tyre models have been developed in recent years, and they can be divided into empirical models and physical approaches [19]. Each category has a different degree of complexity and accuracy, hence they have distinct purposes. Figure 4-3 illustrates the main aspects that characterize the different tyre-model categories.

From the right to the left, models are based more on full-scale tyre experiments and less on the behaviour of the structure of the tyre. On the right, the theoretical models, which are also known as physical models, are more complex, because the tyre is described in great detail. The physical models are appropriate for studying tyre construction and simulate the tyre's behaviour.

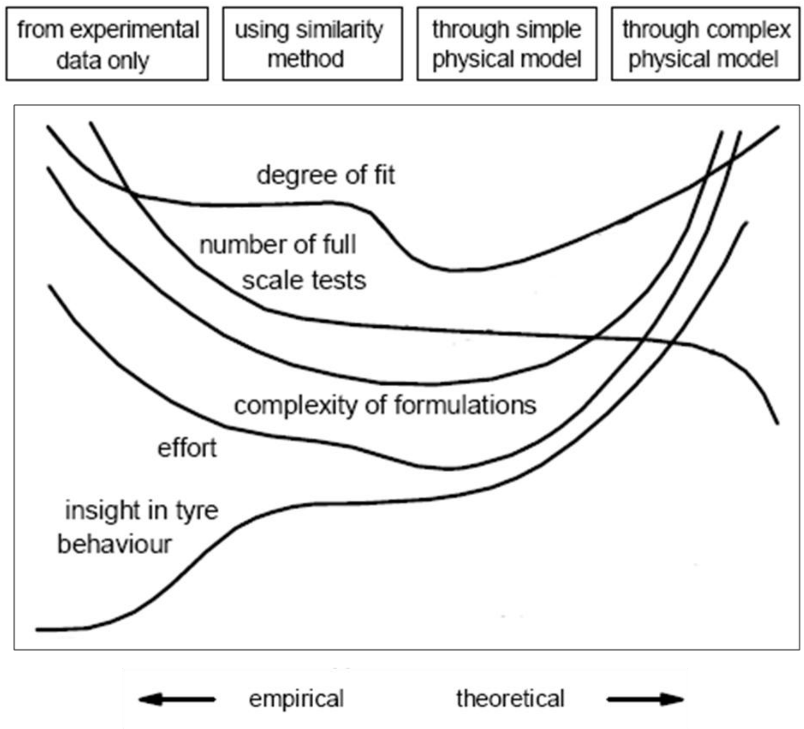

Figure 4-3 Tyre-model categories' main characteristics [22]

There are two categories of physical models. The first is complex finite element models that are geared towards more detailed analysis of the tyre. These models are able to represent carcass deflection and to determine the total forces and moment through the tread elements' motion and the integration of frictional forces. The model solves the deflection of the tread elements when they run through the contact patch using an iteration process. In addition, these models are capable of handling non-steady-state conditions. Tyre models like this have been used in [13].

The second category is that of the relatively simple physical models. These kinds of models help to gain a better understanding of tyre behaviour, since their simple equations provide accuracy. The best-known model in this category is the 'brush model' developed in [14]. The four fundamental factors that it represents are parabolic pressure distribution, the frictional properties between the tyre and the road, carcass flexibility, and the compliance of the tread rubber. The LuGre tyre model is a modification of the brush model proposed by Canudas-de-Wit. Nowadays, it is the most used tyre model in this category.

The empirical models are on the left-hand side of the figure. The first category uses the similarity method. This means that these models are able to describe tyre behaviour through distortion and rescaling and by combining basic tyre characteristics from measured data. They are also appropriate for real-time computations, despite being simple models and not totally accurate. They are also called semi-empirical models, because they have some formulation that is based on physical models and yet they use measured data. The main disadvantage these kinds of models present is that they only describe steady-state tyre behaviour.

Finally, in the second category of empirical models, there are the tyre models that describe the tyre's behaviour using only mathematical formulae that fit real test data. The best-known model is Pacejka's Magic Formula, a purely empirical model based on functions to describe the tyre forces and moment at combined slip on the steady state [22].

In this thesis three out of the four categories are going to be studied and compared. As an empirical model, Pacejka's Magic Formula is going to be examined in detail and used as a benchmark. As a semi-empirical model, the TMeasy tyre model developed by Rill is going to be studied. Lastly, as a simple physical model, the LuGre tyre model is going to be examined.

The reason why a complex physical model is not studied in this project is because it is based on computer simulation and needs very powerful software to be implemented. Moreover, little information is published about this category of tyre models, as only the tyre manufacturers use them and almost all the results and improvements are confidential.

4.3 Pacejka's Magic Formula

4.3.1 Introduction to Magic Formula

Magic Formula is an empirical tyre model used to estimate the steady-state tyre forces and moment characteristics. It was devised in 1985 by TU-Delft and Volvo, but the model was formulated from a physical point of view and the results were not accurate. In 1993 Michelin developed a purely empirical model using Magic Formula to describe the longitudinal tyre-force generation at combined slip [22].

The formula is as follows:

$$Y = Dsin[C\ arctan\{B\ x - E\ (B\ x - \arctan B\ x)\}] \qquad \textbf{(4-1)}$$

Where Y is the output variable F_x , F_y or M_z and x is the input variable: wheel side slip angle for F_y and M_z, or longitudinal wheel slip for F_x. And the coefficients that describe the curve shape are: B, the stiffness factor; C, the shape factor; D, the peak value; and E, the curvature factor.

The input variables are very important, and it is essential to understand the meaning of both of them. The longitudinal wheel slip κ is defined in Equation 4-2 such that for a positive longitudinal force F_x (braking force), the sign of κ is negative.

$$\kappa = -\frac{V_x - r_e\Omega}{V_x} \qquad \textbf{(4-2)}$$

Where V_x is the forward wheel speed, Ω is the wheel angular velocity, and the rolling radius is calculated as follows:

$$r_e = \frac{V_x}{\Omega_o} \qquad \textbf{(4-3)}$$

Where Ω_o is the initial wheel angular velocity, so $\Omega > \Omega_o$.

The lateral wheel slip corresponds to the minus tangent of the ratio of the lateral velocity and the forward velocity.

$$tan\alpha = -\frac{V_y}{V_x} \qquad \textbf{(4-4)}$$

For the lateral and longitudinal forces, the coefficients are defined using the following equations:

$$B = \frac{C_{F\alpha}}{CD} \qquad \textbf{(4-5)}$$

$$D = \mu F_z \qquad \textbf{(4-6)}$$

$$C_{F\alpha} = c_1 sin\left\{2 \arctan\left(\frac{F_z}{c_2}\right)\right\} \qquad \textbf{(4-7)}$$

This considers that the shape factors C, E as well as c_1, c_2 and μ are parameters that can be estimated using regression techniques and F_z is the vertical force applied to the tyre.

Once the inputs of the model are known and the coefficients have been obtained, the outputs are the function of the slip components and the wheel load.

$$F_x = F_x(\kappa, \alpha, \gamma, F_z)\,, \qquad F_y = F_y(\kappa, \alpha, \gamma, F_z)\,, \qquad M_x = M_x(\kappa, \alpha, \gamma, F_z) \qquad \textbf{(4-8)}$$

For this research, the camber angle γ is always equal to zero. The outputs of the functions are then obtained from measurements for a given speed of travel and road and environment conditions.

Magic Formula generates a curve that passes through the origin, goes to a maximum and tends to a horizontal asymptote. It is capable of producing tyre-force characteristics that roughly match measured curves, taking into account the effect of the vertical load and the camber angle due to the factors' dependence on them [22].

Figure 4-4 shows pure lateral ($\kappa = 0$) slip characteristics and other lateral slip characteristics for different longitudinal wheel slips, from 5% to 100%. Figure 4-5 shows the longitudinal slip characteristics for pure slip ($\alpha = 0$) and for lateral wheel slip increasing from 2° to 16°. The figures demonstrate that in combined slip situations, both forces decrease.

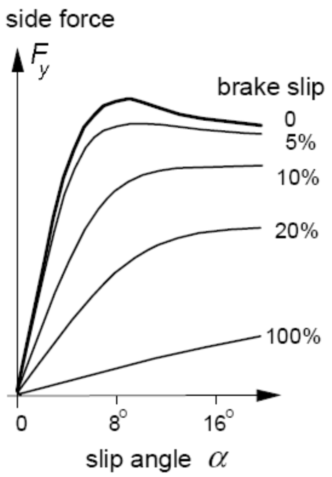

Figure 4-4 Lateral slip characteristics plot using Magic Formula [22]

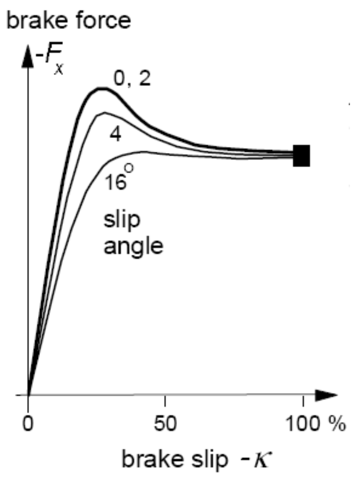

Figure 4-5 Longitudinal slip characteristics plot using Magic Formula [22]

The slope of the pure slip curves is defined by the tyre stiffness, which is the most important parameter of the tyre and essential for handling and stability performance. The cornering stiffness of the tyre is designated as $C_{F\alpha}$ and the longitudinal stiffness as $C_{F\kappa}$.

4.3.2 Magic Formula simulation using Matlab

The mathematical formula of Pacejka's model has been computed using Matlab to calculate the steady-state force and moment response to side slip and longitudinal slip. Although there is a general formula for the lateral and longitudinal force, the equation and the factors for the self-aligning torque are as follows:

$$M_z = D_z sin[C_{M\alpha}\ arctan\{B_z\ \alpha - E_z\ (B_z\ \alpha - \arctan B_z\ \alpha)\}] \qquad \textbf{(4-9)}$$

$$C_{M\alpha} = c_4 \, a \, C_{F\alpha} \tag{4-10}$$

$$C_{F\alpha} = c_1 sin\left\{2 \arctan\left(\frac{F_z}{c_2}\right)\right\} \tag{4-11}$$

$$B_z = -\frac{C_{M\alpha}}{C_z D_z} \tag{4-12}$$

$$D_z = c_3 \, a \, \mu_{M\alpha} \, F_z \tag{4-13}$$

The parameter and factor values used in the calculation are listed in Table 3.

Table 3 Parameters used in Magic Formula's plots [22]

Param.	Value	Param.	Value	Param.	Value	Param.	Value
C_x	1.5	C_y	1.3	C_z	1.3	c_1	60,000
E_x	–1	E_y	–3	E_z	–3	c_2	4,000
μ_x	1.26	μ_y	1	μ_z	0.8	c_3	0.25
a	0.08					c_4	0.5

The program used in Matlab to generate the lateral force characteristics is shown in Appendix B-1. As can be seen in Figures 4-6, 4-7 and 4-8, the results are obtained by applying a vertical force from 1,000 N to 8,000 N. The curves' slope is lower for the lateral force than for the longitudinal force and the torque.

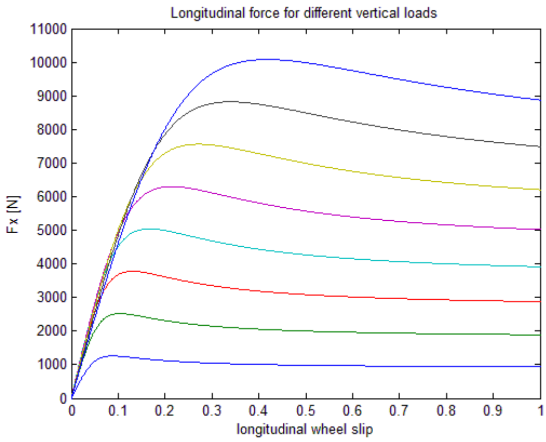

Figure 4-6 Longitudinal force vs slip for different vertical loads

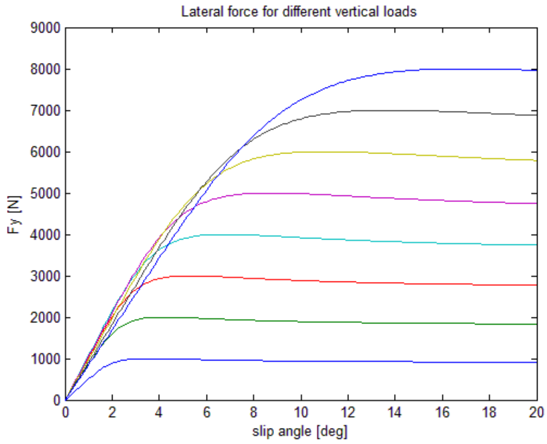

Figure 4-7 Lateral force vs slip angle for different vertical loads

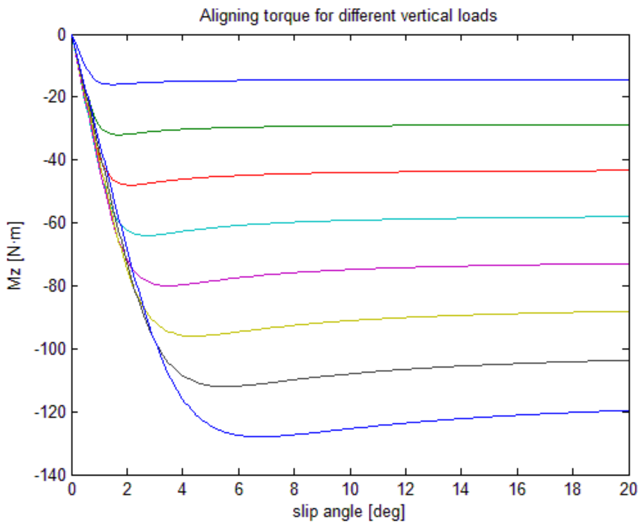

Figure 4-8 Self-aligning torque vs slip angle for different vertical loads

In addition, it can be seen that the peak value for the longitudinal force curves is higher than for the other two cases. In all the cases, as the vertical force increases, the forces studied and the torque also increase. It is also observable that the shape of the two forces after the peak is different due to the different values of the curve factors.

The meaning of some of the shape factors can be graphically demonstrated. In Figures 4-9, 4-10, 4-11, 4-12, 4-13 and 4-14, the influence of the C and E factors can be easily appreciated in the shape of the curves. The plots have been normalized by dividing Y by D. Moreover, the initial slope (B) and the curve peak value (D) have been made independent of the parameters by multiplying x by BC.

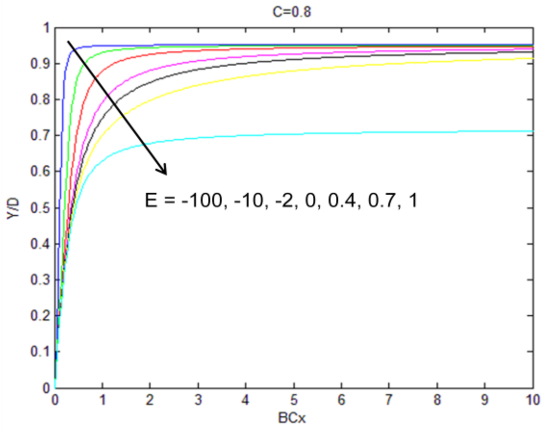

Figure 4-9 Curvature factor's influence for a shape factor of 0.8

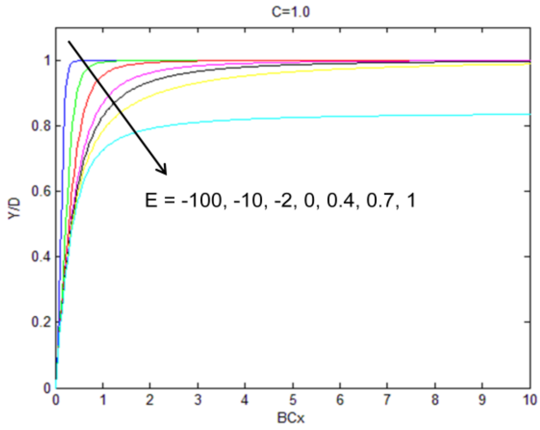

Figure 4-10 Curvature factor's influence for a shape factor of 1.0

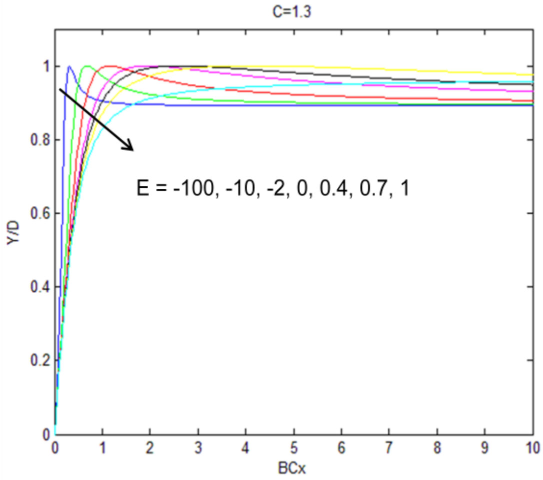

Figure 4-11 Curvature factor's influence for a shape factor of 1.3

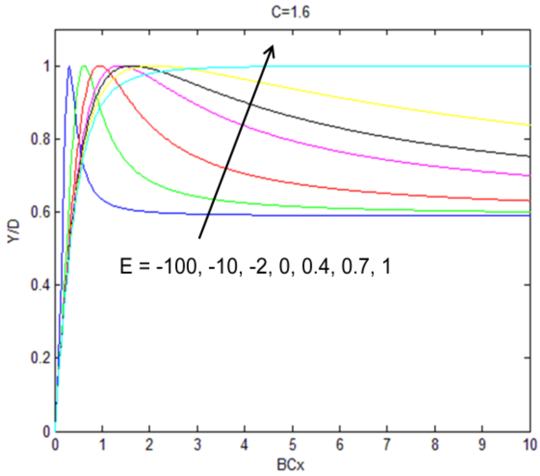

Figure 4-12 Curvature factor's influence for a shape factor of 1.6

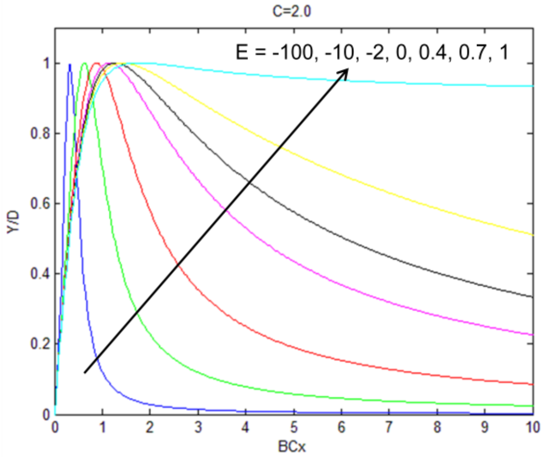

Figure 4-13 Curvature factor's influence for a shape factor of 2.0

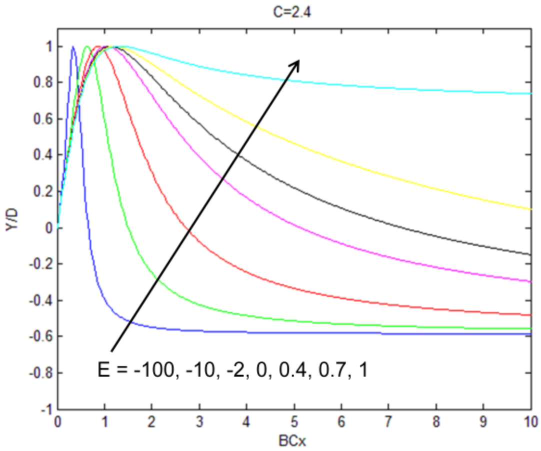

Figure 4-14 Curvature factor's influence for a shape factor of 2.4

On the one hand, the influence of the shape factor (C) can be observed in the figures above, as it determines the shape of the curve after the peak. Therefore, as the factor increases, the part of the curve after the peak decreases faster.

On the other hand, the figures also show that the curvature factor (E) influences the curvature of the curve's peak, the shape of the horizontal asymptote and the final value of the curve. It can also be observed that from –100 to –10 the curve's shape does not change as much as it does from –2 to 1. This is very important to know when choosing the best curvature factor to fit measured curves.

4.4 TMeasy tyre model

4.4.1 Introduction to TMeasy

TMeasy was developed in 1994 by Georg Rill. It is a semi-physical tyre model used in low-frequency applications for vehicle-dynamics and handling analyses. It predicts with reasonable accuracy the steady-state characteristics of the forces and the self-aligning torque between the tyre and the road.

The longitudinal and lateral contact forces are defined by several physical parameters that take into account the influence of decreasing tyre load [16]. The self-aligning torque is described by the multiplication of the pneumatic trail and the lateral force.

Figure 4-15 shows how the inputs and the outputs are defined in this model.

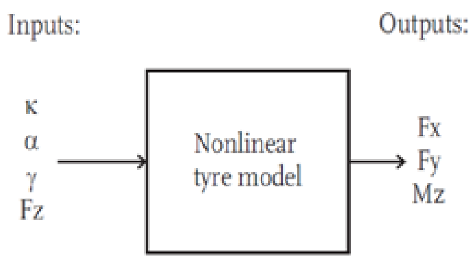

Figure 4-15 Inputs and outputs of the TMeasy tyre model [31]

As Hirschberg, Rill and Weinfurter mention in [16], TMeasy has been used within the simulation system SIMPACK for several years, and it has been integrated into the simulation system veDyna. In addition, it has been implemented successfully in the MBS system of Adams.

TMeasy is characterized by a compromise between model complexity and efficiency in computation time, user-friendliness, and precision in representation [24].

4.4.2 TMeasy approximation of tyre forces

In this model, the slip is described as follows during acceleration or deceleration situations.

$$s_x = \frac{-(v_x - r\omega)}{r|\omega|} \tag{4-14}$$

$$s_y = \frac{-v_y}{r|\omega|} \tag{4-15}$$

Where *r* is the dynamic radius of the tyre, ω is the angular velocity of the wheel, and v_x and v_y describe the contact point velocity.

The steady-state tyre force characteristic is approximated by a function. This function is different in every interval and has the following characteristic parameters: dF^0, s^M, F^M, s^S, F^S. Because of their physical meaning, these parameters can be acquired from measured data [16].

For the longitudinal force, Figure 4-16 shows the approximation of the steady-state tyre characteristics.

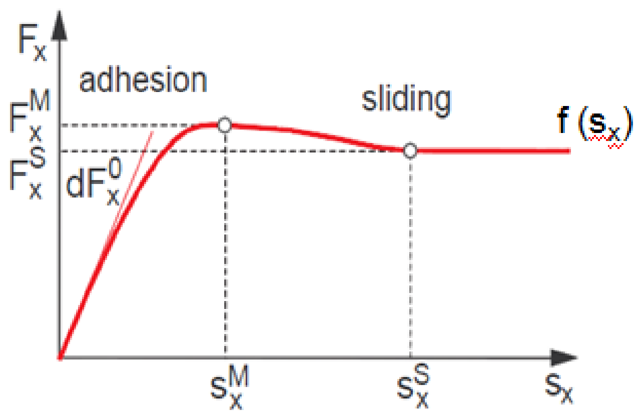

Figure 4-16 TMeasy longitudinal tyre-force characteristics [24]

Different formulas are defined depending on the region of the slip where the force wants to be predicted, as shown in Figure 4-17.

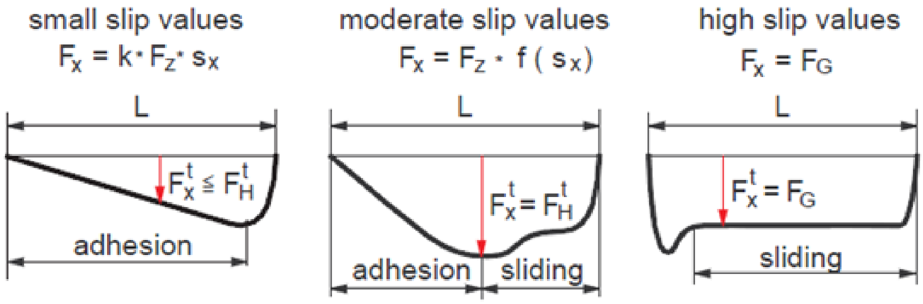

Figure 4-17 TMeasy longitudinal force definition for the whole range of slip [24]

For the steady-state lateral force, the approximation is very similar to the previous longitudinal force, as shown in Figure 4-18.

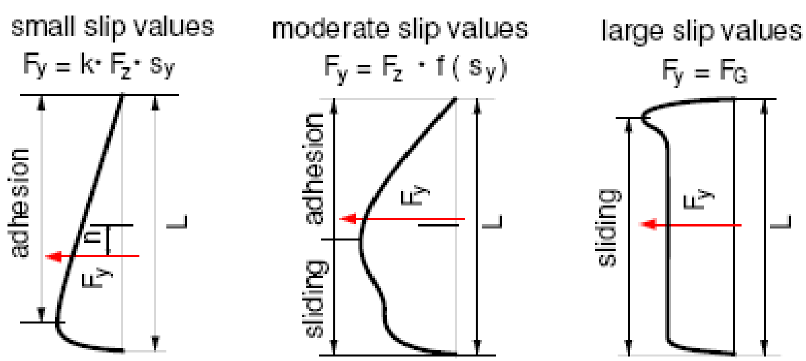

Figure 4-18 TMeasy lateral force definition for the whole range of slip [24]

Using the model parameters, it is possible to find the functions that describe the tyre characteristics. The results of this effective and simple approach correspond reasonably well to measurements [24].

4.4.3 TMeasy modelling of the lateral and longitudinal forces

TMeasy approximates the characteristics of steady-state tyre forces, dividing the curve into four different intervals as depicted in Figure 4-19. The first interval is a straight line that starts at the origin and has the slope of dF^0. The second and third intervals where the maximum tyre force is located are parabolas. Finally, the sliding area is a straight line.

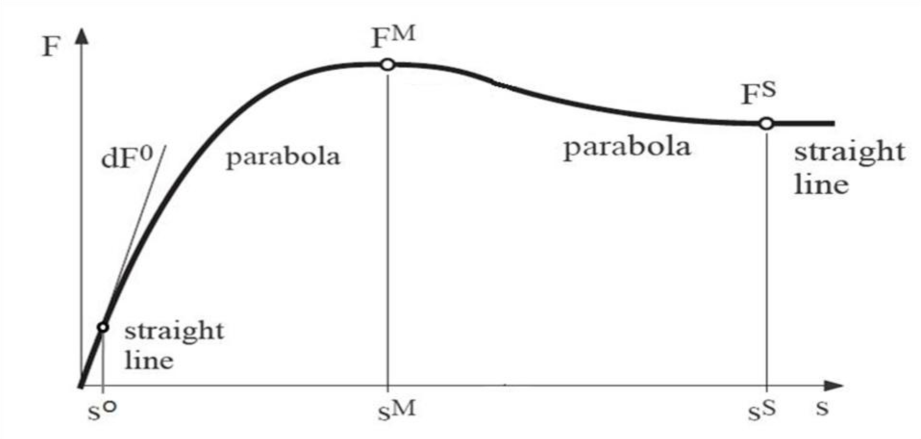

Figure 4-19 TMeasy approximation of steady-state tyre characteristics with the four intervals indicated [16]

Considering these intervals, the function that describes the force is:

$$F(s) = \begin{cases} dF^0 & 0 \le s < s^0 \\ as^2 + bs + c & s^0 \le s < s^M \\ ds^2 + es + f & s^M \le s < s^S \\ F^S & s \ge s^S \end{cases} \quad \textbf{(4-16)}$$

It is essential that this function has a solution for every slip; hence it must be a C^1 class. This means that the function is continuous and that its derivative exists and is also continuous. These kinds of functions are also called continuously differentiable.

Consequently, to ensure that the function is C^1, and in addition to find the function unknowns, continuity must first be imposed using the following equations:

$$a(s^0)^2 + b(s^0) + c = dF^0 \cdot s^0 \quad \textbf{(4-17)}$$

$$a(s^M)^2 + b(s^M) + c = F^M \quad \textbf{(4-18)}$$

$$d(s^M)^2 + e(s^M) + f = F^M \quad \textbf{(4-19)}$$

$$d(s^S)^2 + e(s^S) + f = F^S \quad \textbf{(4-20)}$$

The derivative function’s continuity must then be determined using the subsequent equations:

$$2a(s^M) + b = 2d(s^M) + e \tag{4-21}$$

$$2d(s^S) + e = 0 \tag{4-22}$$

As a consequence, the final matrix to be solved in order to find the six unknowns is:

$$\begin{bmatrix} (s^M)^2 & s^M & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & (s^M)^2 & s^M & 1 \\ 0 & 0 & 0 & (s^S)^2 & s^S & 1 \\ (s^0)^2 & s^0 & 1 & 0 & 0 & 0 \\ 2s^M & 1 & 0 & -2s^M & -1 & 0 \\ 0 & 0 & 0 & 2s^S & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} F^M \\ F^M \\ F^S \\ dF^O \\ 0 \\ 0 \end{bmatrix} \tag{4-23}$$

The function unknowns can easily be found by resolving the following equations using Matlab.

$$A * Y = B \tag{4-24}$$

$$Y = inv(A) * B \tag{4-25}$$

Once the function is completely defined, it is essential to have characteristic tyre data to implement it. Table 4 presents characteristic data for two nominal wheel loads, $F_z^N = 4$ and $2F_z^N = 8$ kN.

Table 4 Characteristic tyre data with wheel load influence [25]

Longitudinal force F_x		Lateral force F_y	
F_z = 4 kN	**F_z = 8 kN**	**F_z = 4 kN**	**F_z = 8 kN**
s_x^0 = 0.015	s_x^0 = 0.015	s_y^0 = 0.015	s_y^0 = 0.015
dF_x^0 = 120 kN	dF_x^0 = 200 kN	dF_y^0 = 55 kN	dF_y^0 = 80 kN
s_x^M = 0.11	s_x^M = 0.1	s_y^M = 0.2	s_y^M = 0.22
F_x^M = 4.4 kN	F_x^M = 8.7 kN	F_y^M = 4.2 kN	F_y^M = 7.5 kN
s_x^S = 0.5	s_x^S = 0.8	s_y^S = 0.8	s_y^S = 1.0
F_x^S = 4.25 kN	F_x^S = 7.60 kN	F_y^S = 4.15 kN	F_y^S = 7.40 kN

In order to calculate six model parameters for any vertical load F_z, the quadratic function shown in Equation 4-26 and the linear function shown in Equation 4-27 are used. The program to calculate them is in Appendix B-2.

$$F^M(F_z) = \frac{F_z}{F_z^N}\left[2F^M(F_z^N) - \frac{1}{2}F^M(2F_z^N) - (F^M(F_z^N) - \frac{1}{2}F^M(2F_z^N))\frac{F_z}{F_z^N}\right] \quad \textbf{(4-26)}$$

$$s^M(F_z) = s^M(F_z^N) + \left(s^M(2F_z^N) - s^M(F_z^N)\right)\left(\frac{F_z}{F_z^N} - 1\right) \quad \textbf{(4-27)}$$

The parameters obtained are used to generate the steady-state tyre characteristics for different vertical loads, which are shown in Figures 4-20 and 4-21. The program to obtain these plots in Matlab is presented in Appendix B-2.

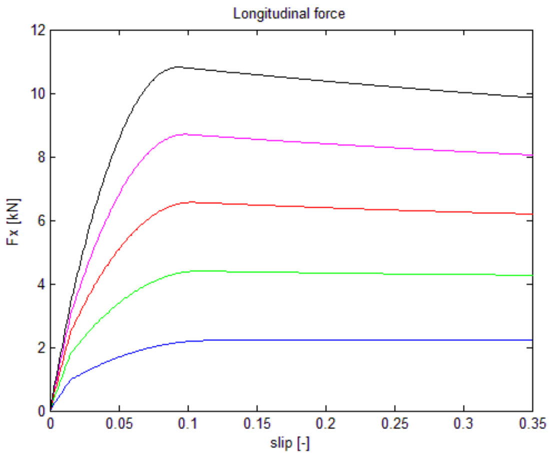

Figure 4-20 TMeasy longitudinal steady-state tyre characteristics

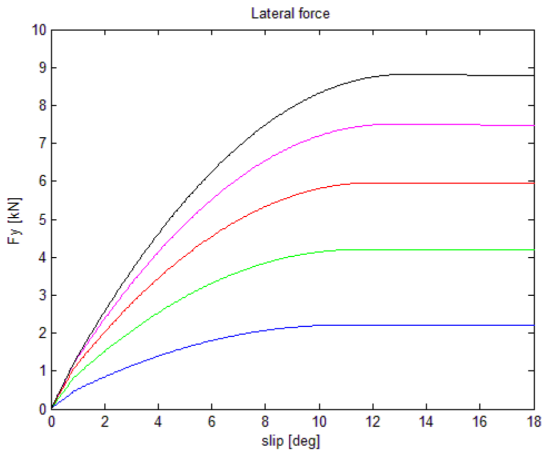

Figure 4-21 TMeasy lateral steady-state tyre characteristics

Different vertical loads have been applied to the previous plots of force characteristics. The blue line represents a normal force of 2 kN, the green line 4 kN, the red line 6 kN, the magenta line 8 kN and the black line 10 kN.

As the figures suggest, the shape of the lateral and longitudinal forces can be quite different for the same vertical force. The reason for this is that the longitudinal force is significantly larger than the lateral force.

4.4.4 TMeasy self-aligning torque modelling

The steady-state self-aligning moment is generated through the resulting lateral force and the pneumatic trail. The pneumatic trail, also called steady-state tyre offset, is the distance between the resulting point of lateral force application and the centre of the contact patch.

The application point of the resultant lateral force is defined by the distribution of the lateral forces over the contact patch. This point is not always situated at the same place; its position depends on the slip between the tyre and the road. For small slip values, this point is behind the centre of the contact patch. However, for higher slip values, it moves forward.

To study the pneumatic trail, it has been normalized by dividing it by the length of the contact patch, L.

The pneumatic trail is defined by three parameters, as depicted in Figure 4-22. They are also obtained by linear extrapolation between two nominal wheel loads, $F_z^N = 4$ and $2F_z^N = 8$ kN. The values of these parameters for each nominal load are listed in Table 5.

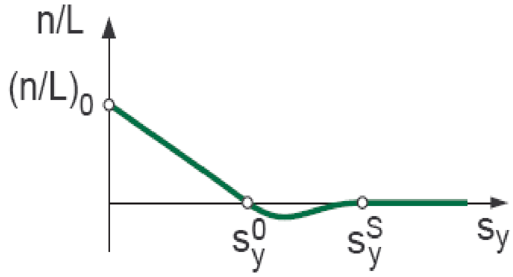

Figure 4-22 Normalized pneumatic trail characteristics [16]

Table 5 Tyre data to describe the pneumatic trail [25]

Pneumatic trail data	
F_z = 4 kN	**F_z = 8 kN**
$\left(\frac{n}{L}\right)_0$ = 0.178	$\left(\frac{n}{L}\right)_0$ = 0.190
s_y^0 = 0.200	s_y^0 = 0.225
s_y^S = 0.350	s_y^S = 0.375

In this case, the function has three parts: the first is a straight line between two points, the second is a parabola and the third is a constant. Consequently, the function's equation is defined as follows:

$$n(s) = \begin{cases} -s\,\dfrac{\left(\frac{n}{L}\right)_0}{s_y^0} + \left(\dfrac{n}{L}\right)_0 & 0 \le s < s_y^0 \\ as^2 + bs + c & s_y^0 \le s < s_y^S \\ 0 & s \ge s_y^S \end{cases} \tag{4-28}$$

The function unknowns are found through Matlab by applying the C^1 class function conditions, as has been done previously for the forces. The plot obtained for the pneumatic trail's function is rendered in Figure 4-23.

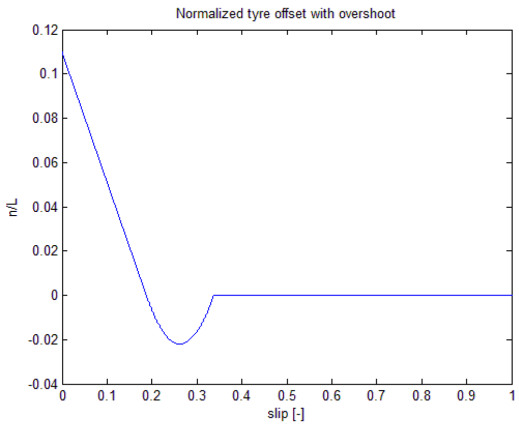

Figure 4-23 Normalized pneumatic trail plot obtained using Matlab

Once the pneumatic trail has been totally defined, the steady-state self-aligning torque can be given by the following equation:

$$T_z = n \cdot F_y \tag{4-29}$$

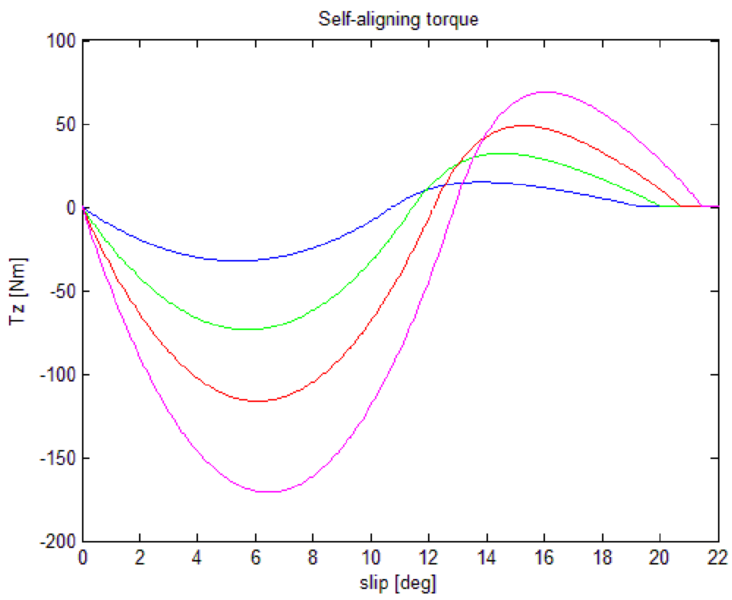

Figure 4-24 TMeasy self-aligning torque characteristics

Figure 4-24 presents the steady-state self-aligning torque for different vertical loads. The blue line represents a normal force of 2 kN, the green line 4 kN, the red line 6 kN, and the magenta line 8 kN. It can be observed that as the vertical force increases, the torque also increases.

The first region of the plot, until approximately 12° depending on the vertical force, is the adhesion region, where the torque is negative in case of braking. The second one is the sliding region, where it has positive value, and, finally, in the full sliding region, the self-aligning torque is zero.

4.4.5 TMeasy tyre-model conclusions

TMeasy is a simplified tyre model that analytically describes the tyre contact forces and torque characteristics. It is relatively easy to use because it only needs a small number of model parameters.

The physical meaning of these parameters helps the user in the fitting process. Thus, it is possible to offer a proper technical compromise between modelling accuracy on the one hand and user-friendliness on the other.

The TMeasy tyre model is focused on the practical requirements in vehicle-dynamics analysis, and it is capable of being linked to any multi-body simulation system (MBS). Although the model is at present restricted to steady-state conditions, it is intended that the tyre model will be extended to include internal tyre dynamics.

4.5 The LuGre tyre model

4.5.1 Introduction to the LuGre model

The LuGre friction model comes as a result of adding the Stribeck effect to the Dahl model [7]. It can be formulated as a lumped model or as a distributed model depending on the shape of the friction contact between surfaces [6]. This dynamic friction model predicts the transient behaviour of the tyre-road forces under varying velocity circumstances [7].

The LuGre dynamic model presents a concise form that is very appropriate in control analyses. It has been advantageously utilized in vehicle state estimation problems and in tyre slip control design. The LuGre model captures crucial aspects of friction, such as stiction, the Stribeck effect, stick slip, zero slip displacement and hysteresis. One of the most important advantages of the LuGre tyre model is its ability to reflect the surface conditions, the effects of tyre vertical force and the effects of the vehicle speed on the friction force [19].

There is an interaction between the lateral force and the longitudinal force acting on the tyre, but in order to understand the LuGre model, the longitudinal and lateral motion are analysed first.

The longitudinal slip occurs because of the decrease in the effective circumference of the tyre when the rubber deforms [7]. The slip rate is presented in Equation 4-30 and its interval is [0,1]. There is no sliding when $s = 0$ and full sliding or skidding when $|s| = 1$.

$$s = \begin{cases} s_b = \dfrac{r\omega}{v} - 1 & \text{if } v > r\omega, \qquad v \neq 0 \text{ for braking} \\ s_d = 1 - \dfrac{v}{r\omega} & \text{if } v < r\omega, \qquad \omega \neq 0 \text{ for driving} \end{cases} \tag{4-30}$$

In this model the relative velocity between the tyre and the ground is defined as $v_r = r\omega - v$. Consequently, the slip coefficient is negative for braking and positive for driving [7].

In this section three forms of LuGre tyre model are going to be presented: the lumped form, the average lumped form and the distributed form. Afterwards, the former and the latter are compared. In addition, the steady-state characteristics and the dynamic tyre behaviour for combined lateral and longitudinal motion are also examined to study the distributed form in more depth. Finally, Matlab/Simulink is used to implement the static equations for the distributed model and the dynamic equations for the lumped model.

All the equations that are going to be introduced are valid for both lateral and longitudinal motion; the only thing that changes is the fitting parameters values, which are different due to the anisotropy of the friction characteristics.

4.5.2 Lumped LuGre model

Punctual tyre-road friction contact is assumed in lumped friction models. The deflections of the bristles are modelled by the following formulas, and the representation of the wheel with lumped friction is depicted in Figure 4-25.

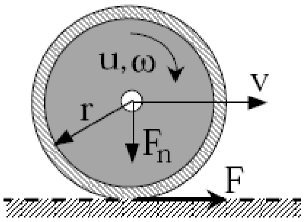

Figure 4-25 Lumped form of LuGre tyre model with punctual contact [6]

$$\dot{z} = v_r - \frac{\sigma_0 |v_r|}{g(v_r)} z \tag{4-31}$$

$$F = (\sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v_r)\, F_n \tag{4-32}$$

$$g(v_r) = \mu_C + (\mu_S - \mu_C)\, e^{-\left|\frac{v_r}{v_s}\right|^{1/2}} \tag{4-33}$$

Where z is the internal friction state of bristle elastic deflection, μ_C is the normalized Coulomb friction, μ_S is the normalized static friction, v_r is the relative velocity between sliding bodies, v_s is the Stribeck relative velocity, F_n is the normal force, σ_0 is the rubber longitudinal lumped stiffness, σ_1 is the rubber longitudinal lumped damping and σ_2 is the viscous relative damping [6].

4.5.3 Average lumped LuGre model

The average lumped form of the LuGre tyre model includes the average bristle deflection for simplicity of tyre-dynamics analysis and computational efficiency. Thus, it has only one internal state, the average deflection [9]:

$$\tilde{z}(t) = \frac{1}{L}\int_0^L z(\zeta, t)\, d\zeta \approx \frac{1}{N}\sum_{i=1}^{N} z_i(t) \tag{4-34}$$

Where z_i is i^{th} bristle deflection variable.

The bristle deflection can then be modelled with Equation 4-35 and the final equations of the average lumped LuGre tyre model are:

$$\frac{d\tilde{z}(t)}{dt} = \dot{\tilde{z}} = v_r - \frac{\sigma_0 |v_r|}{g(v_r)} \tilde{z} - \frac{k}{L} |r\omega| \tilde{z} \quad \textbf{(4-35)}$$

$$F = \left(\sigma_0 \, \tilde{z} + \sigma_1 \, \dot{\tilde{z}} \, + \sigma_0 \, v_r\right) F_n \quad \textbf{(4-36)}$$

$$g(v_r) = \mu_C + (\mu_S - \mu_C) \, e^{-\left|\frac{v_r}{v_s}\right|^{1/2}} \quad \textbf{(4-37)}$$

Where the factor k = 1.2 provides a quite accurate approximation.

4.5.4 Distributed LuGre model

The presence of a contact area between the tyre and the road is assumed in the distributed form of the LuGre tyre model. The projection of the small part of the tyre which is in contact with the surface represents the patch [6]. The length of this contact patch is L and the axis coordinate is ζ (see Figure 4-26). Using this assumption, a first-order nonlinear differential equation is obtained.

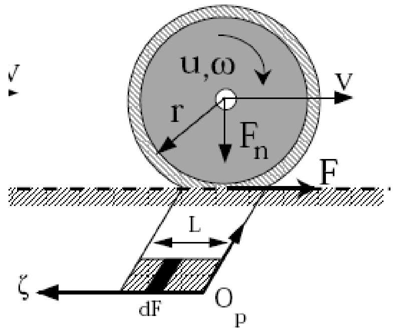

Figure 4-26 Distributed form of LuGre tyre model with contact area [6]

$$\frac{d\,z}{dt}(\zeta, t) = v_r(t) - \frac{\sigma_0 |v_r(t)|}{g(v_r)} z(\zeta, t) \quad \textbf{(4-38)}$$

$$F = \int_0^L dF(\zeta, t) d\zeta \quad \textbf{(4-39)}$$

$$dF(t, \zeta) = (\sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v_r) \, dF_n \quad \textbf{(4-40)}$$

Where z is the differential friction state called deflection, δF is the differential friction force, $\delta F_n = F_n/L$ is the differential vertical force, $v_r = (r\omega - v)$ is the relative velocity, and $g(v_r)$ is the Stribeck tyre-road sliding friction force.

Considering that the model's assumptions are that the vertical force is uniformly distributed along the patch $f_n(\zeta)$ (force per unit length), as shown in Figure 4-27, and that each differential state element has a contact velocity equal to v_r [6], the total friction force is:

$$F(t) = \int_0^L \left(\sigma_0 \, z(\zeta,t) + \sigma_1 \frac{\partial z}{\partial t}(\zeta,t) + \sigma_2 v_r\right) f_n(\zeta) d\,\zeta \qquad \textbf{(4-41)}$$

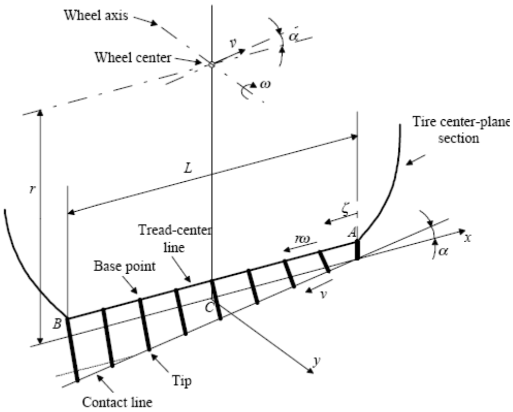

Figure 4-27 LuGre tyre model diagram of uniform normal pressure distribution [10]

It is assumed that the origin of the axis coordinate ζ changes location depending on the direction of the wheel motion. Consequently, it is assumed that $\dot{\zeta} = |r\omega|$, because $\dot{\zeta} = r\omega$ for $\omega > 0$ and $\dot{\zeta} = -r\omega$, for $\omega < 0$ [7].

Therefore, the differential of the deflection variable z given in Equation 4-42 can be resolved in both time and space.

$$\frac{d\,z}{dt}(\zeta,t) = \frac{\partial z}{\partial \zeta} \cdot \frac{\partial \zeta}{\partial t} + \frac{\partial z}{\partial t} \qquad \textbf{(4-42)}$$

$$\frac{d\,z}{dt}(\zeta,t) = \frac{\partial z}{\partial \zeta}(\zeta,t)|r\omega| + \frac{\partial z}{\partial t}(\zeta,t) = v_r - \frac{\sigma_0 |v_r|}{g(v_r)} z \qquad \textbf{(4-43)}$$

Rearranging Equation 4-43, it produces the final partial differential equation that follows:

$$\frac{\partial z}{\partial t}(\zeta,t) = v_r - \frac{\sigma_0|v_r|}{g(v_r)}z - |r\omega|\frac{\partial z}{\partial \zeta}(\zeta,t) \tag{4-44}$$

Consequently, the equations for the distributed LuGre tyre friction model are:

$$\frac{\partial z}{\partial t} + \frac{\partial z}{\partial \zeta}|r\omega| = v_r - \frac{\sigma_0|v_r|}{g(v_r)}z \tag{4-45}$$

$$F(t) = \int_0^L \left(\sigma_0\, z + \sigma_1\frac{\partial z}{\partial t} + \sigma_2 v_r\right) f_n(\zeta)d\,\zeta \tag{4-46}$$

$$g(v_r) = \mu_C + (\mu_S - \mu_C)\, e^{-\left|\frac{v_r}{v_s}\right|^{1/2}} \tag{4-47}$$

4.5.5 Lumped versus distributed LuGre tyre model

The lumped form of the LuGre tyre model is an ordinary differential equation that can be resolved by time integration. Nevertheless, the distributed form assumes normal pressure distribution and is formulated using a partial differential equation which should be resolved in both time and space [7]. The steady-state distributed model is used in vehicle-dynamics analysis for parameters-fitting purposes, while the lumped model is used for the development of control strategy [19].

As demonstrated in [7], the lumped LuGre model is a good approximation of the distributed LuGre model, as they have similar steady-state and dynamic behaviour.

4.5.6 Steady-state LuGre distributed model

Steady-state characteristics of the distributed model occur when $\frac{\partial z}{\partial t}(\zeta, t) = 0$ and the velocities v and ω are constant (hence also s and v_r) [7]. It results in the following bristle deflection:

$$\frac{\partial z}{\partial \zeta}(\zeta, t) = \frac{1}{|r\omega|}\left(v_r - \frac{\sigma_0 |v_r|}{g(v_r)} z(\zeta, t)\right) \quad \textbf{(4-48)}$$

The above equation should be integrated along the patch using $\delta z(0, t) = 0$.

$$z_{ss}(\zeta) = sgn(v_r)\frac{g(v_r)}{\sigma_0}\left(1 - e^{-\frac{\sigma_0}{g(v_r)}\left|\frac{v_r}{\omega r}\right|\zeta}\right) \quad \textbf{(4-49)}$$

Subsequently, the friction force can be calculated as shown in the next equation:

$$F_{ss} = \int_0^L (\sigma_0 z_{ss}(\zeta) + \sigma_2 v_r)\, \delta F_n d\zeta \quad \textbf{(4-50)}$$

Considering that the vertical uniform force distribution is $\delta F_n = F_n / L$, the steady-state longitudinal force as well as the steady-state lateral force can be obtained with the following equation:

$$F_{ss} = sgn(v_r)\, F_n\; g(v_r)\left[1 - \left|\frac{\omega r}{v_r}\right| \frac{g(v_r)}{\sigma_0 L}\left(1 - e^{-\frac{L\,\sigma_0}{g(v_r)}\left|\frac{v_r}{\omega r}\right|}\right)\right] + \sigma_2 v_r F_n \quad \textbf{(4-51)}$$

For the longitudinal driving case $v < r\omega$ the slip rate can be formulated as $s_d = \frac{r\omega - v}{v} = \frac{v_r}{v}$ and the force is given by Equation 4-52.

$$F_d(s) = sgn(v_r)\, F_n\; g(s)\left[1 + \frac{g(s)}{\sigma_0 L|s|}\left(e^{-\frac{L\,\sigma_0|s|}{g(s)}} - 1\right)\right] + \sigma_2 F_n r\omega s \quad \textbf{(4-52)}$$

With $g(s) = \mu_C + (\mu_S - \mu_C)\, e^{-\left|\frac{r\omega s}{v_s}\right|^{1/2}}$ and assuming constant ω and $s = s_d$.

For the longitudinal braking case $v > r\omega$ the slip rate can be formulated as $s_b = \frac{r\omega - v}{r\omega} = \frac{v_r}{r\omega}$ and the force is given by the next equation:

$$F_b(s) = sgn(v_r)\, F_n\, g(s) \left[1 + \frac{g(s)|1+s|}{\sigma_0 L|s|}\left(e^{-\frac{L\,\sigma_0|s|}{g(s)|1+s|}} - 1\right)\right] + \sigma_2 F_n v s \qquad \textbf{(4-53)}$$

With $g(s) = \mu_C + (\mu_S - \mu_C)\, e^{-\left|\frac{vs}{v_s}\right|^{1/2}}$ and assuming constant v and $s = s_b$ [7].

As shown in the above equations, longitudinal force is dependent not only on the slip, but also on car speed v for the driving case and on wheel velocity ω for the braking case.

4.5.7 Combined longitudinal and lateral motion for the distributed LuGre tyre model

In [10], a dynamic tyre model for combined lateral and longitudinal motion is established based on the LuGre friction model. Moreover, it includes the calculation of the self-aligning torque. Finally, the steady-state model has been validated with respect to the Pacejka static tyre model, which uses experimental data.

Considering the tyre coordinate system as shown in Figure 4-28, the dimensionless longitudinal slip can be defined as follows taking into account the combined longitudinal and lateral motion.

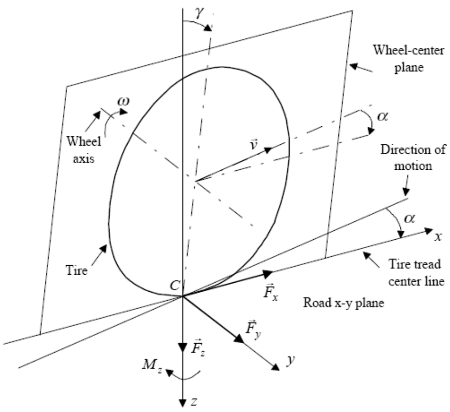

Figure 4-28 Tyre's coordinate system [10]

$$s = \begin{cases} s_b = \dfrac{vcos\alpha - r\omega}{vcos\alpha} & \text{if } vcos\alpha > r\omega, \quad v \neq 0 \text{ for braking} \\ s_d = \dfrac{r\omega - vcos\alpha}{r\omega} & \text{if } vcos\alpha \leq r\omega, \quad \omega \neq 0 \text{ for driving} \end{cases} \quad \textbf{(4-54)}$$

The relative velocity between the road contact point and the bristle base point is presented in Equation 4-57.

$$v_{rx} = r\omega - vcos\alpha \quad \textbf{(4-55)}$$

$$v_{ry} = v\ sin\alpha \quad \textbf{(4-56)}$$

$$v_r = \sqrt{v_{rx}^2 + v_{ry}^2} \quad \textbf{(4-57)}$$

In the distributed dynamic model, the bristles are deflected in both lateral (y) and longitudinal (x) directions. Consequently, the deflection equation can be written as:

$$\frac{\partial z_{x,y}}{\partial t}(\zeta, t) = v_{r\ x,y} - \frac{\sigma_{0\ x,y}\left|v_{r\ x,y}\right|}{g_{x,y}(v_r, v_{r\ x,y})} z_{x,y} - |r\omega| \frac{\partial z_{x,y}}{\partial \zeta}(\zeta, t) \quad \textbf{(4-58)}$$

Where the corresponding directions are pointed using the subscripts x and y. The stiffness coefficient $\sigma_{0\ x,y}$ depends on the direction because of the anisotropic nature of the tyre.

The tyre-road sliding friction force is expressed as shown in the next equation:

$$g_{x,y}(v_r, v_{r\ x,y}) = \left|\frac{v_{r\ x,y}}{v_r}\right| g(v_r) \quad \textbf{(4-59)}$$

So rearranging the previous equation yields the following bristle deflection equation:

$$\frac{\partial z_{x,y}}{\partial t}(\zeta, t) = v_{r\ x,y} - \frac{\sigma_{0\ x,y}|v_r|}{g(v_r)} z_{x,y} - |r\omega| \frac{\partial z_{x,y}}{\partial \zeta}(\zeta, t) \quad \textbf{(4-60)}$$

The contribution force of a bristle at the position ζ in the central plane of the tyre is:

$$\varphi_{x,y}(\zeta, t) = \frac{1}{L\ W}\left[\sigma_{0\ x,y}\ z_{x,y} + \sigma_{1\ x,y} \frac{\partial z_{x,y}}{\partial t} + \sigma_2 v_{r\ x,y}\right] \quad \textbf{(4-61)}$$

Where W is the width of the tyre contact patch.

The total force can then be obtained in both directions considering uniform normal pressure distribution.

$$F_{x,y}(t) = \int_{-W/2}^{W/2} \int_0^L \varphi_{x,y}(\zeta,t) d\zeta dy = W \int_0^L \varphi_{x,y}(\zeta,t) d\zeta \tag{4-62}$$

$$F_{x,y}(t) = \frac{1}{L} \int_0^L \left[\sigma_{0\ x,y}\ z_{x,y} + \sigma_{1\ x,y} \frac{\partial z_{x,y}}{\partial t} + \sigma_2 v_{r\ x,y} \right] d\zeta \tag{4-63}$$

For the case of pure cornering, the self-aligning torque is:

$$M_z = W \int_0^L \varphi_y(t) \left(\frac{L}{2} - \zeta \right) d\zeta \tag{4-64}$$

4.5.7.1 Steady-state equations for the combined longitudinal and lateral motion of the distributed LuGre tyre model

In the steady-state operation $\frac{\partial z}{\partial t}(\zeta,t) = 0$ is imposed and the following ordinary differential equation is obtained [10].

$$z_{ss\ x,y}(\zeta) = sgn(v_{r\ x,y}) \left| \frac{v_{r\ x,y}}{v_r} \right| \frac{g(v_r)}{\sigma_{0\ x,y}} \left(1 - e^{-\frac{\sigma_{0\ x,y}}{g(v_r)} \left| \frac{v_r}{\omega r} \right| \zeta} \right) \tag{4-65}$$

The expression for the lateral and longitudinal steady-state tyre forces F_x and F_y is:

$$F_{ss\ x,y} = sgn\left(v_{r\ x,y}\right) \left| \frac{v_{r\ x,y}}{v_r} \right| F_n\ g(v_r) \left[1 - \left| \frac{\omega r}{v_r} \right| \frac{g(v_r)}{\sigma_{0\ x,y}\ L} \left(1 - e^{-\frac{L\ \sigma_{0\ x,y}}{g(v_r)} \left| \frac{v_r}{\omega r} \right|} \right) \right] + \sigma_2 v_{r\ x,y} F_n \tag{4-66}$$

And the expression for the self-aligning moment is:

$$M_z = -sgn(v_{ry}) \left|\frac{v_{ry}}{v_r}\right| F_n\, g(v_r) \left|\frac{\omega r}{v_r}\right| \frac{g(v_r)}{\sigma_{0y}\, L} \left[\frac{1}{2} - \left|\frac{\omega r}{v_r}\right| \frac{g(v_r)}{\sigma_{0y}\, L} + \left(\frac{1}{2} + \left|\frac{\omega r}{v_r}\right| \frac{g(v_r)}{\sigma_{0y}\, L}\right) e^{-\frac{L\,\sigma_{0y}}{g(v_r)}\left|\frac{v_r}{\omega r}\right|}\right] \quad \textbf{(4-67)}$$

4.5.7.2 Friction coefficient bounds for combined longitudinal and lateral motion

The methodology to derive both dynamic and static friction models for 2D motion is presented in [32]. Initially, the paper describes the Coulomb friction force coefficient for 1D motion, as shown in Equation 4-68.

$$\mu(v) = \begin{cases} \mu_k & for\ v > 0 \\ [-\mu_s\ , \mu_s] & for\ v = 0 \\ -\mu_k & for\ v < 0 \end{cases} \quad \textbf{(4-68)}$$

Where μ_s is the static and μ_k is the kinetic friction coefficient.

The friction coefficient is then considered in 2D motion and formulated as presented in Equation 4-69.

$$M_k = \begin{bmatrix} \mu_{kx} & 0 \\ 0 & \mu_{ky} \end{bmatrix} \quad \textbf{(4-69)}$$

However, the friction coefficients in both directions, x and y, are bounded by set C, which provides the coupling between friction forces in both directions. Set C is shown in Figure 4-29 and formulated as follows:

$$C = \{\mu \in \mathbb{R}^2 :\ \|M_k^{-1}\mu\| \leq 1\} \quad \textbf{(4-70)}$$

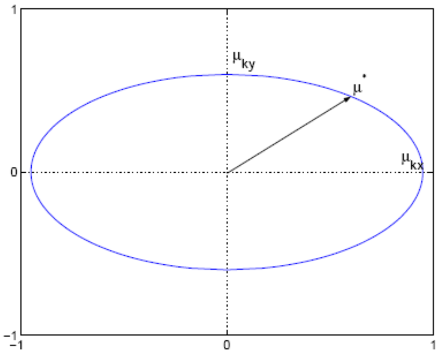

Figure 4-29 Set C of acceptable friction coefficients for 2D motion [32]

Consequently, the friction coefficient that gives an admissible friction force is:

$$\mu^* = -\frac{M_k^2 v}{\|M_k v\|} \tag{4-71}$$

Finally, this paper presents the equations of the LuGre friction model in 2D, written as a vector as presented in Equation 4-72. However, these equations are not going to be studied in this thesis.

$$\begin{bmatrix}\mu_x \\ \mu_y\end{bmatrix} = \begin{bmatrix}-\sigma_{ox}\ z_x - \sigma_{1x}\ \dot{z}_x - \sigma_{2x}\ v_{rx} \\ -\sigma_{oy}\ z_y - \sigma_{1y}\ \dot{z}_y - \sigma_{2y}\ v_{ry}\end{bmatrix} \tag{4-72}$$

4.5.8 Static LuGre distributed model implementation using Matlab/Simulink

The steady state of the distributed LuGre model for the braking case, explained in Subsection 4.5.6, is shown in Figure 4-30. The Matlab program to achieve the tyre characteristic using the following equations is presented in Appendix B-3.

$$F_b(s) = sgn(v_r)\ F_n\ g(s)\left[1 + \frac{g(s)|1+s|}{\sigma_0 L|s|}\left(e^{-\frac{L\ \sigma_0|s|}{g(s)|1+s|}} - 1\right)\right] + \sigma_2 F_n vs \tag{4-73}$$

$$g(s) = \mu_C + (\mu_S - \mu_C)\ e^{-\left|\frac{vs}{v_s}\right|^{1/2}} \tag{4-74}$$

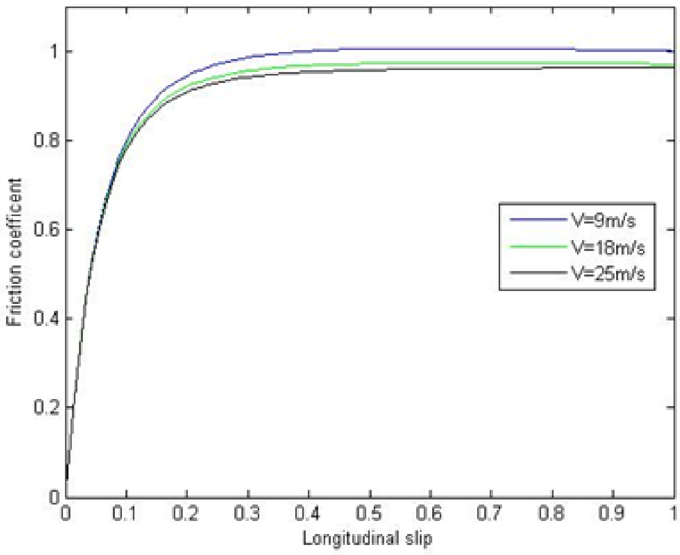

Figure 4-30 Steady-state distributed LuGre model with uniform vertical force distribution under different values of V

The longitudinal force over the normal force can be plotted as follows:

$$Friction\ coefficient = \frac{F_b(s)}{F_n} \qquad \textbf{(4-75)}$$

The steady-state dependence on the vehicle velocity can then be observed.

4.5.9 Dynamic LuGre lumped model implementation using Matlab/Simulink

To represent the dynamic behaviour of the tyre, the following equations for the LuGre lumped model, explained in Subsection 4.5.2, have been simulated using Simulink.

$$\dot{z} = v_r - \frac{\sigma_0 |v_r|}{g(v_r)} z \tag{4-76}$$

$$F = (\sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v_r)\, F_n \tag{4-77}$$

$$g(v_r) = \mu_C + (\mu_S - \mu_C)\, e^{-\left|\frac{v_r}{v_s}\right|^{1/2}} \tag{4-78}$$

The Simulink program is shown in Figure 4-31 and the Matlab program used to simulate it is presented in Appendix B-3.

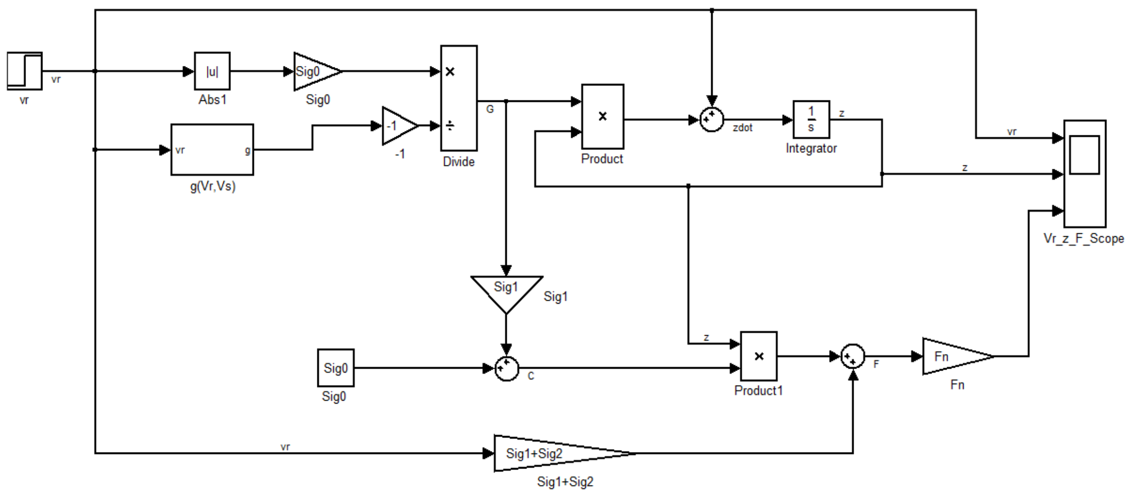

Figure 4-31 Simulink representation of the lumped LuGre model

For a step input of v_r, the value of z is recorded for 0.1 seconds and then plotted point by point versus v_r, (see Figure 4-32). Figure 4-33 depicts the dynamic friction coefficient (F/F_n) and the Stribeck effect ($g(v_r)$) recorded for 0.1 seconds and then plotted point by point versus v_r. The parameters of the model are given in [19] and shown in Table 6.

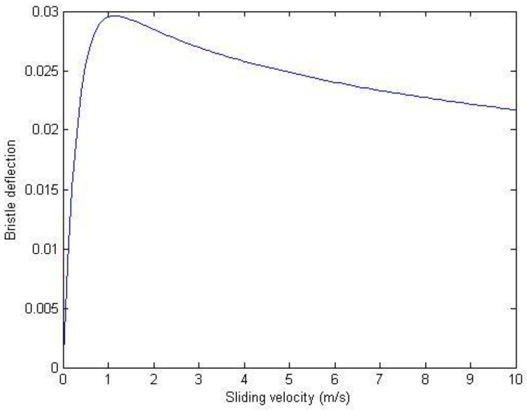

Figure 4-32 Bristle deflection versus v_r

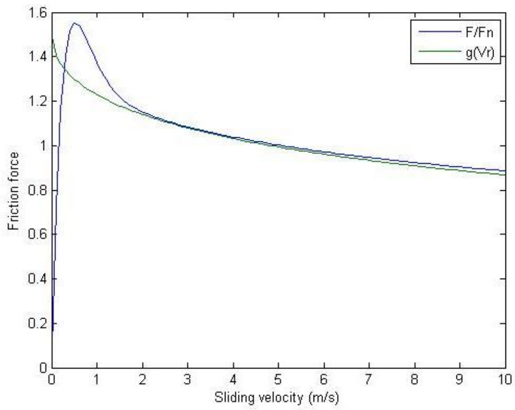

Figure 4-33 Friction force versus v_r

Table 6 LuGre tyre model parameters [19]

Longitudinal force					
σ_0	σ_1	σ_2	μ_c	μ_s	V_s
150	4.95	0.002	0.5	1.7	10

Looking at the results, it can be concluded that for very low sliding velocities, both z and force friction are pretty small. Since the deflection rate $\dot{z}$ is determined by v_r, when v_r is small the transient response will be slow, and when v_r is large the response will be faster.

Figure 4-33 shows that the ideal Stribeck effect only depends on v_r, while the dynamic friction coefficient depends on v_r and time, and during the transient response it is higher than the steady-state g(v_r) [19].

4.6 Comparison between models

When the TMeasy tyre model is compared with Pacejka's Magic Formula, the curves for the longitudinal and lateral forces have very similar shapes, as Figures 4-34 and 4-35 show.

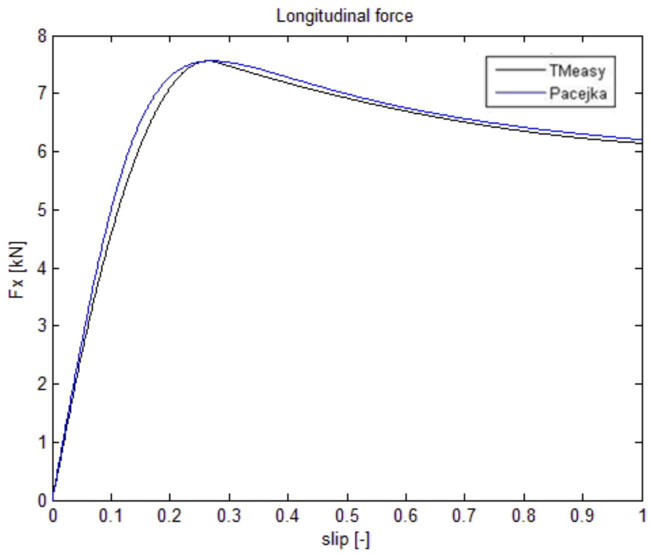

Figure 4-34 Longitudinal force comparison between the TMeasy and Pacejka models

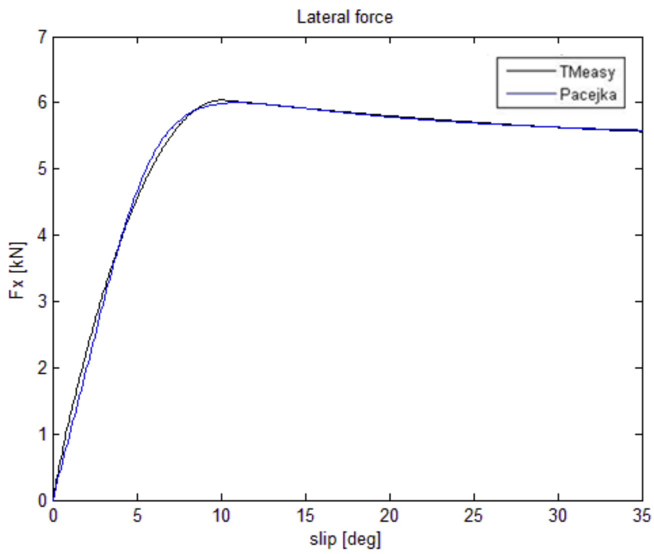

Figure 4-35 Lateral force comparison between the TMeasy and Pacejka models

However, in the case of the self-aligning moment, they diverge a little bit, as the TMeasy model is not able to approximate the moment in the same way as Pacejka (see Figure 4-36).

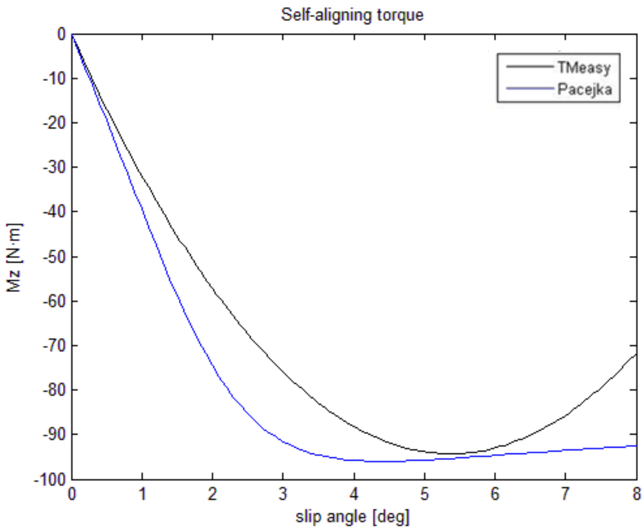

Figure 4-36 Self-aligning torque comparison between the TMeasy and Pacejka models

When the steady-state longitudinal force of the three models is plotted in the same graph, certain parameters are used to fit Pacejka's Magic Formula as closely as possible. These parameters are given in Table 7.

Table 7 Set of parameters used to fit TMeasy with Pacejka's tyre characteristics

Longitudinal force F_x	
F_z = 6 kN	
Magic Formula	**TMeasy**
C_x= 1.5	s_x^0 = 0.015
E_x= -1	dF_x^0 = 71.25 kN
μ_x = 1.26	s_x^M = 0.18
c_1 = 60 kN	F_x^M = 6.04 kN
c_2 = 4 kN	s_x^S = 1.0
	F_x^S = 5.48 kN

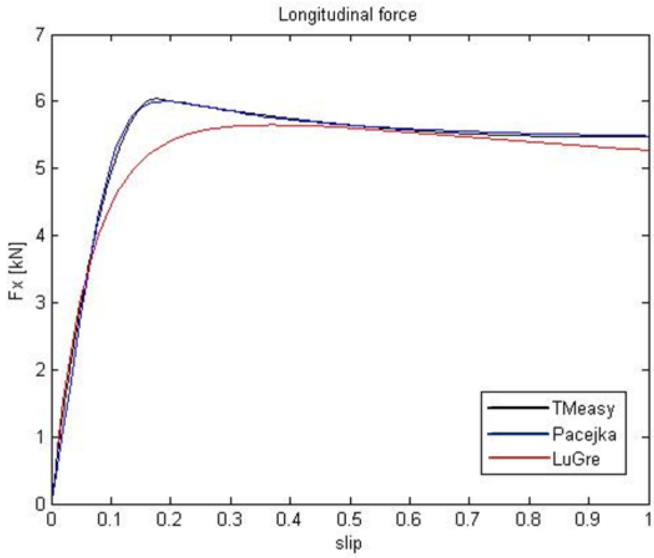

Figure 4-37 Steady-state longitudinal force comparison between the three models studied

Figure 4-37 shows that TMeasy fits Pacejka's curve better than LuGre. The reason for this is that the TMeasy model parameters are able to adjust the resulting curve but have no physical meaning, while the LuGre model parameters have physical meaning but are not useful in fitting the curve.

These parameters are given in Table 8.

Table 8 Set of parameters used to fit TMeasy and Lugre with Pacejka's tyre characteristics

Longitudinal force F_x		
F_z = 6 kN		
Magic Formula	**TMeasy**	**LuGre**
C_x= 1.3	s_x^0 = 0.015	σ_0 = 150
E_x= -3	dF_x^0 = 71.25 kN	σ_2 = 0.002
μ_x = 1	s_x^M = 0.18	μ_c = 0.5
c_1 = 60 kN	F_x^M = 6.04 kN	μ_s = 1.7
c_2 = 4 kN	s_x^S = 0.9	v_s = 7 m/s
	F_x^S = 5.48 kN	L = 0.2 m

4.7 Tyre-model conclusions

The main conclusions that can be drawn from the study are summarized below in terms of the strengths and weaknesses of the different models.

The strengths of Pacejka's Magic Formula are that:

- The model best describes steady-state tyre behaviour.
- There is a lot of information about this model, and many extensions have been done.

On the other hand, the weaknesses of this model are that:

- It does not describe dynamic tyre behaviour.
- The model parameters do not have a physical meaning, since it describes only the curve shape.

Regarding the TMeasy tyre model, its strengths are that:

- The equations describing it are easy to implement.
- The model parameters have a physical meaning.

The weaknesses, on the other hand, are that:

- It does not describe dynamic tyre behaviour.
- Since it is a very novel model, there is little information about it. As a consequence, the literature does not describe how to determine the model parameters from experimental data.

The LuGre tyre model's strength is that:

- It describes both steady-state and dynamic tyre behaviour.

Finally, its weakness is that:

- Many improvements are needed in order to extend its application range.

All the tyre-model results and comparisons establish that the tyre behaviour must be described very carefully in order to describe the reality efficiently. This will ensure that the effect of the tyre behaviour in vehicle-dynamics simulations is acceptable.

5 Tyre-model requirements for vehicle-dynamics control systems applications

5.1 Tyre-model applications

Many researchers have developed different kinds of mathematical models that describe pneumatic tyre behaviour. These models have been designed for various purposes in the vehicle-dynamics field. The complexity of them can vary from a straightforward two-degree-of-freedom model to a finite element representation of tyre behaviour. The requirements of a tyre model are totally dependent on its application [11].

The main applications for tyre modelling in vehicle development are:

- Durability studies.
- Suspension design.
- Handling specification.
- Control systems development.
- Ride comfort analyses.
- Steering systems design.

The demand on the tyre model differs considerably, depending on the application. For instance, ride comfort analyses require tyre models that cover a frequency range up to 80 Hz, and vehicle handling analyses need tyre models that describe the tyre slip behaviour very accurately up to 5 Hz [26]. Figure 5-1 shows the frequency required for different vehicle design stages and analyses.

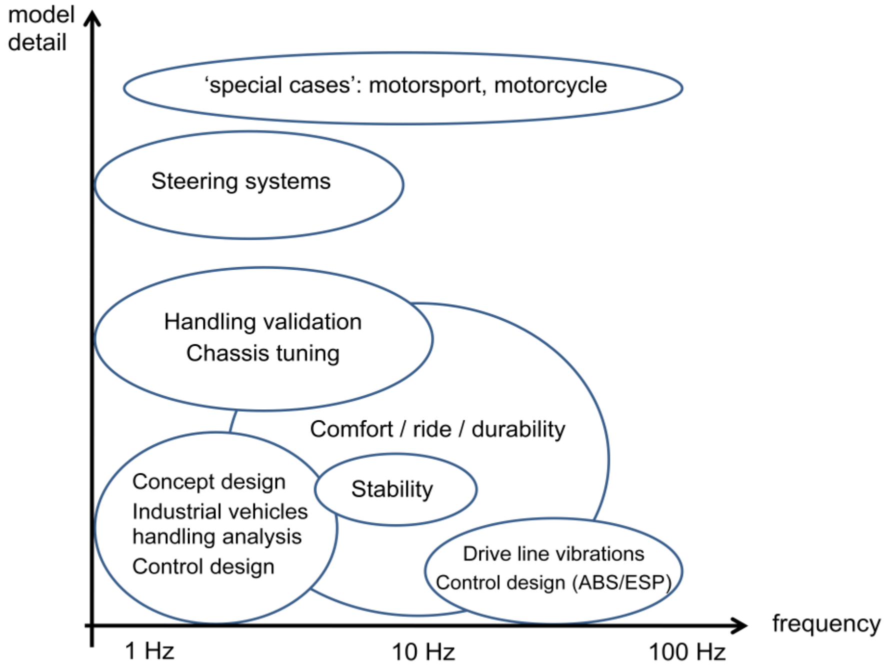

Figure 5-1 Range of frequencies needed for vehicle-systems analyses [30]

5.2 Definition of criteria

Of the mechatronic applications explained previously, this thesis is going to focus on vehicle-dynamics control systems. This application needs tyre models in order to know the forces produced between the road and the tyre, because the pneumatic tyre is the most influential component of the vehicle, and all the forces required to control the vehicle's motion are transmitted through the tyres.

Three different tyre-model categories are going to be compared: Pacejka's Magic Formula, TMeasy and the LuGre tyre model. In order to know which one is best for mechatronic applications and more specifically for vehicle-dynamics control systems, they are examined in terms of the criteria outlined below.

These criteria have been defined explicitly after reading and understanding what systems such as ABS, ESP, TCS and EPAS expect from tyre models. The nine most important criteria are described in detail below and are then weighted according to their importance.

5.2.1 Real-time capability

For a real-time system, it is important to know in which moment of the simulation the output has been produced. This aspect is important partly because the vehicle-dynamics simulators need to be able to stop and start again [34]. Real-time models provide the opportunity to monitor the change in tyre performance with wear and age [33]. Since vehicle-dynamics systems control constantly changing environments, the model must support real-time properties. The real-time simulations allow control-systems solutions to be designed and developed in much more detail. A central requirement of real-time systems is predictability, which means that the system may be constructed so that its behaviour is always predictable [21]. In a real-time system, it should always be feasible to access and manipulate data within a certain period of time.

5.2.2 Ability to describe dynamic tyre behaviour

This criterion is the ability to characterize dynamic tyre behaviour in order to describe the pure longitudinal and lateral slip and also the combined slip [26]. It is important to highlight that tyre in-plane dynamics are associated with vibrations in longitudinal and vertical directions and rotational vibrations about the wheel spindle. Consequently, tyre models should describe the forces and moments transmitted from the road to the wheel spindle during vehicle motion. Moreover, steady-state tyre models are not valid when there are variations in time on the wheel’s motion, because the slip variations are not followed by horizontal tyre deformation [34].

5.2.3 Accuracy

This is the ability to predict precisely the forces and moments transmitted between the tyre and the road [26]. Results from accurate models can match experimental data much better [23]. It is important to have a comprehensive and detailed description of the forces generated at the tyre-road contact patch. The range where the tyre model is accurate also has to be considered, as it can be from slow steady-state conditions to conditions where the vehicle is nearly skidding [11]. Taking into account the assumptions that every model uses would be helpful in order to know which one describes the real tyre behaviour accurately.

5.2.4 Computing efficiency

The model should spend as short time and use as small amount of memory and space as possible while running the simulation [26]. The model should be compact and should have the equations positioned together in a tidy way, using very little memory space. The data management mechanisms utilized in vehicle control systems need to be sufficiently efficient, with respect to both memory requirement and CPU usage, in order that they are suitable in case memory capacity, processor performance or both are limited. Freshness is crucial for vehicle control systems data, since they are used in environments that are rapidly changing. Consequently, it is important to find a good balance in computational resource usage. The resources-demand in vehicle control systems is kept as low as possible, since they have to be implemented in a larger multi-body system [21].

5.2.5 Easily measured tyre input parameters

The number of parameters that tyre models need, derived from measured data before the simulation, may vary from 10 to 50. Models that require a low number of parameters are going to be more straightforward to use. Depending on the physical meaning of the parameters, they could be quicker and easier to derive from measured data [15]. It is also important that the user has the ability to generate the set of parameters required for each tyre model.

5.2.6 Availability of analysis tools

The tyre model should be implemented in computer software available to a great number of people. The most common software is Matlab/Simulink; most companies and universities have access to it because the licence is affordable. The more powerful the software needs to be, the more expensive the licence is going to be [2]. The language in which the tyre model is implemented is also very important, because if a standard language is used, a wider range of software could implement and simulate the model.

5.2.7 Numerically simple and meaningful equations

The equations used in the model are easy to understand, brief and pithy, because they have a physical meaning and it is not difficult to find the output. The tyre model should be practical and simple for people to use, since it is easy to follow the development of the model equations. If the model formulation is kept as general as possible, it will be compatible with both simple and complicated vehicle-dynamics systems [11].

5.2.8 Widely applicable

This criterion is applicable to tyre models that can be used in different applications, for high and low velocities and frequencies [34]. Models are widely applicable if they can be utilized in a range of applications, from the specific component design area to the development of an integrated vehicle-dynamics control system. Moreover, a tyre model is extensively used when it is implemented in main simulation software such as ADAMS, dSPACE, CarMaker or PC-Crash.

5.2.9 Popularity

The popularity of the tyre model depends on how much information has been published on it – whether there are examples of its results and whether there are some improvements published in order to enhance the initial solution. It is also important where it is used [2]. For instance, if it is utilized at universities, some papers on it may already have been published; if it is a company's innovative model, many years may pass before information on it is published.

All these criteria have the objective of describing the tyre model in terms of cost. This means for example that if a model is efficient on computational time and uses widely available analytical tools such as Matlab/Simulink, its utilization will have a low cost.

If a tyre model is popular and widely applicable, it will probably have a low initial price. However, if it describes the dynamic tyre behaviour and in addition describes it accurately, the tyre model may be much more expensive.

5.3 Weights of each criterion

The requirements that tyre models must satisfy for vehicle-dynamics analysis are described using the criteria detailed above. Following the article [29], a *multiple methods approach* is going to be used to compare the three tyre models in order to determine which is best for mechatronic applications.

When using a *multiple methods approach*, at least two estimators are necessary. For instance, the most common methods used to generate estimates are experimentation, observation, surveys and expert judgement. In this thesis, experts' judgement and the author's opinion representing the theoretical proposals in articles and papers are considered.

The first method takes into account the opinions of several experts in vehicle dynamics – Francis Assadian, David Purdy and Bob Williams – and a number of automotive experts: James Marco, Nicholas Vaughan and Amir Soltani.

A questionnaire shown in Appendix C has been completed by the experts mentioned above. They were asked to assign a number from 1 to 5 to each criterion, indicating the importance of the criterion when choosing a tyre model for vehicle-dynamics control systems. The results obtained from the questionnaires are presented in Table 9.

Table 9 Experts' responses to the questionnaire

F. Assadian	
Importance	**Parameters**
5	Real-time capability
5	Ability to describe dynamic tyre behaviour
4	Accuracy
4	Computing efficiency
5	Easily measured tyre input parameters
4	Availability of analysis tools
3	Numerically simple and meaningful equations
3	Widely applicable
2	Popularity

D. Purdy	
Importance	**Parameters**
3	Real-time capability
5	Ability to describe dynamic tyre behaviour
4	Accuracy
5	Computing efficiency
4	Easily measured tyre input parameters
5	Availability of analysis tools
4	Numerically simple and meaningful equations
3	Widely applicable
3	Popularity

B. Williams	
Importance	**Parameters**
5	Real-time capability
4	Ability to describe dynamic tyre behaviour
3	Accuracy
3	Computing efficiency
5	Easily measured tyre input parameters
4	Availability of analysis tools
4	Numerically simple and meaningful equations
5	Widely applicable
3	Popularity

N. Vaughan	
Importance	**Parameters**
3	Real-time capable
5	Ability to describe dynamic tyre behaviour
5	Accuracy
2	Computing efficiency
5	Easily measured tyre input parameters
3	Availability of analysis tools
3	Numerically simple and meaningful equations
4	Widely applicable
2	Popularity

J. Marco	
Importance	**Parameters**
4	Real-time capability
4	Ability to describe dynamic tyre behaviour
3	Accuracy
3	Computing efficiency
5	Easily measured tyre input parameters
4	Availability of analysis tools
3	Numerically simple and meaningful equations
3	Widely applicable
3	Popularity

A. Soltani	
Importance	**Parameters**
5	Real-time capability
3	Ability to describe dynamic tyre behaviour
4	Accuracy
3	Computing efficiency
3	Easily measured tyre input parameters
2	Availability of analysis tools
3	Numerically simple and meaningful equations
4	Widely applicable
3	Popularity

The criteria that have 1 or 2 denote that they are not very relevant when choosing a tyre model, while the criteria that have scored 5 denote that they are important aspects that a tyre model must satisfy.

The summary of all the answers is depicted in Figure 5-2. The most important criteria according to these six automotive experts are "Easily measured tyre input parameters", "Real-time capability" and "Ability to describe dynamic behaviour of the tyre".

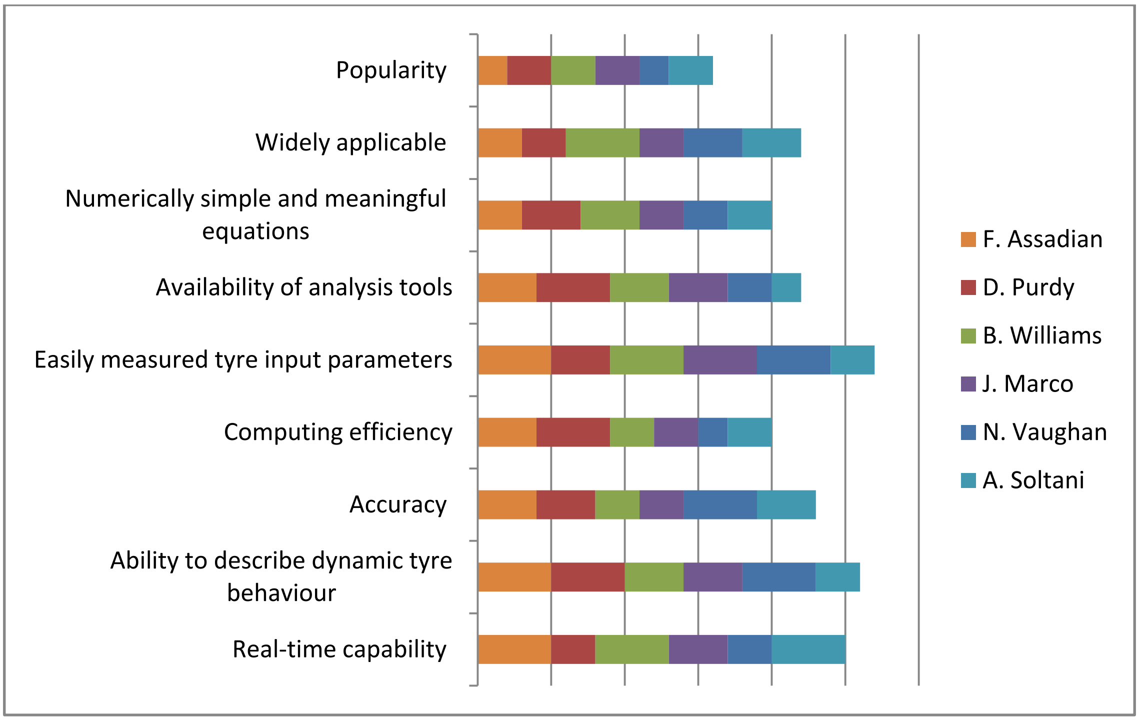

Figure 5-2 Results from the questionnaires

The weight for each criterion has been calculated as a percentage, summing the experts' score for each criterion and dividing by the total score of all the criteria. Table 10 shows the weights for each criterion based on the experts' judgement about the importance of the criteria in describing a tyre model's suitability for vehicle-dynamics control system applications.

Table 10 Weighted criteria obtained from the questionnaires' results

Criterion	Weight
Real-time capability	14.4
Ability to describe dynamic tyre behaviour	12.9
Accuracy	11.4
Computing efficiency	10.0
Easily measured tyre input parameters	13.4
Availability of analysis tools	10.9
Numerically simple and meaningful equations	10.0
Widely applicable	10.9
Popularity	8.0
TOTAL	100.0

The second method to estimate the weight of the criteria is from the author's opinion representing the theoretical point of view. The author of this thesis has done a weighting of the same criteria for the same application. In this case, the author has read several papers on ABS, ESP, TCS and EPAS systems and has proposed the weights that are shown in Table 11.

Table 11 Weighted criteria from the theoretical point of view

Criterion	Weight
Real-time capability	15.0
Ability to describe dynamic tyre behaviour	17.0
Accuracy	10.0
Computing efficiency	13.0
Easily measured tyre input parameters	18.0
Availability of analysis tools	9.0
Numerically simple and meaningful equations	7.0
Widely applicable	6.0
Popularity	5.0
TOTAL	100.0

From the theoretical point of view, the most important criterion is "Easily measured tyre input parameters", because in order to simulate the model, it is essential to have this data. As the behaviour of the tyre is non-linear, the "Ability to describe dynamic tyre behaviour" has a weight of 17%, because it is very important that the tyre model describes the wheel motion correctly. "Real-time capability" also has a high weight, because vehicle-dynamics control systems are always used in changeable environments and need to constantly predict the tyre behaviour.

Similarly, "Computing efficiency" is a criterion with a high weight, because it defines whether a tyre model is able to be implemented in hardware with a short memory and a small amount of space, which is directly related to the cost of using the model. Moreover, the criterion "Accuracy" has a weight of 10%, because it is important, but not crucial, that the output of the model matches the reality as closely as possible.

Finally, the last four criteria are considered to be the least influential when choosing a tyre model for mechatronic applications. The analysis tools to simulate a model are not a problem for companies or universities anymore, because nowadays they can afford the price of most software licences. “Numerically simple and meaningful equations” is also a criterion that has little importance, because an advanced user of a control system will not have many problems in understanding and using the model. Additionally, the criteria of “Widely applicable” and “Popularity” are generic model requirements for any application, and hence they have the lowest weight.

Returning to the utilization of the *multiple methods approach*, the two estimates explained above – expert judgement and the author’s opinion representing the theoretical point of view of the articles and papers – are going to be used to determine the weight of each criterion. The two sets of results do not differ by much but have some small variations. If the two methods produce different results, it is possible to reconcile or combine the estimations in some way to arrive at a single final result [29].

Reconciling and combining the results from different measures can be achieved with a quantitative model. In [20] a mathematical rule is recommended, which is an arithmetic average of estimates.

Thus, Table 12 presents a first set of results that have been obtained by giving 50% of the weight to the experts’ opinion and the other 50% to the theoretical point of view.

Table 12 Weights of the criteria considering estimations to be 50/50%

Criterion	Weight
Real-time capability	13.7
Ability to describe dynamic tyre behaviour	15.0
Accuracy	10.7
Computing efficiency	11.5
Easily measured tyre input parameters	15.7
Availability of analysis tools	10.0
Numerically simple and meaningful equations	8.5
Widely applicable	8.5
Popularity	6.5
TOTAL	100.0

However, in [20] it is also said that if it is considered that one estimate has more weight than another, the percentages can change. Accordingly, it has been estimated that the experienced point of view should be of 60% importance and the theoretical point of view of 40% importance. Consequently, the final weight of each criterion is presented in Table 13 and plotted in Figure 5-3.

Table 13 Weights of the criteria considering estimations to be 60/40%

Criterion	Weight
Real-time capability	13.5
Ability to describe dynamic tyre behaviour	14.6
Accuracy	10.9
Computing efficiency	11.2
Easily measured tyre input parameters	15.3
Availability of analysis tools	10.2
Numerically simple and meaningful equations	8.8
Widely applicable	9.0
Popularity	6.8
TOTAL	100.0

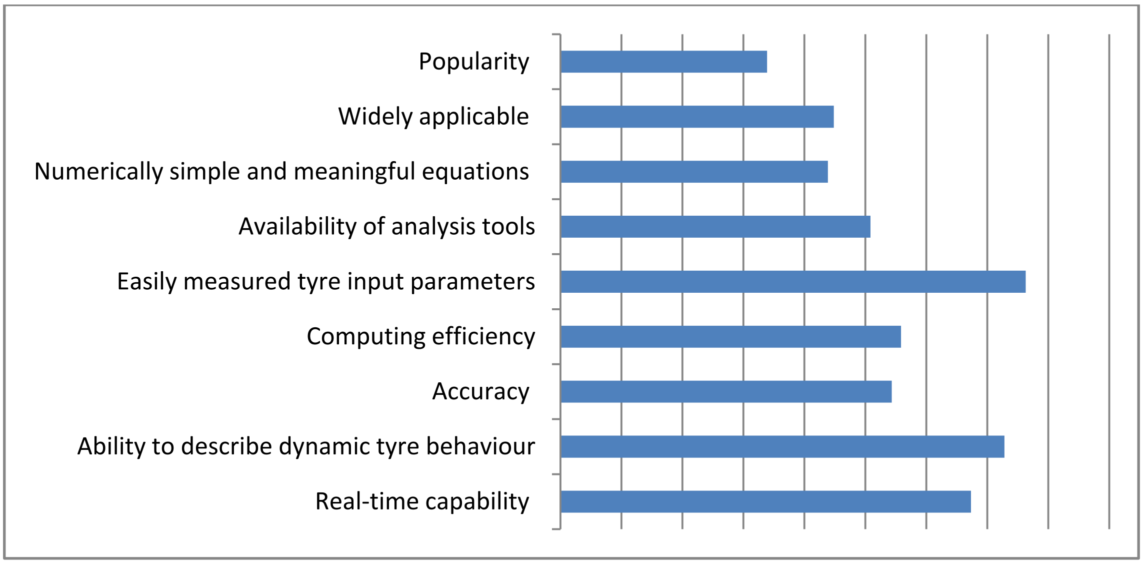

Figure 5-3 Criteria importance plot considering estimations to be 60/40%

The reason why a weight of 60/40 has been considered acceptable is because the author believes that the experience of the automotive professionals is more important than the papers, as they have used tyre models in real problems. This gives the experts a high level of knowledge about the most important features in a tyre model when it is used in vehicle-dynamics control systems.

Finally, it should be highlighted that these criteria could be used to compare tyre models for any other mechatronic application. The only necessary thing to do would be to study the application requirements in order to give a new weight to the criteria according to their importance in the new application.

6 Comparison matrix

Having ascertained the weight of each criterion for mechatronic applications and looked at the equations and plots for each model studied, this section compares the three tyre models using the examined criteria.

As an efficient analytic tool, a comparison matrix serves to determine the basic characteristics of a tyre model. The utilization of a comparison matrix outlines the most typical features of the tyre model without drawing a conclusion directly, but by making easy the process of analysis.

Used as a tool for educational purposes, a comparison matrix is an effective visual aid, with a simple and exact structure. Utilizing a comparison matrix, it is possible to make in-depth correlations and to compare multiple models and their aspects. Consequently, the use of this visual method enhances the analytical study.

In order to make a comparison matrix, it is important to make sure that all comparison criteria have been explained; otherwise it will be difficult to outline the criteria characteristics [4].

For each criterion, tyre models have a weight in a range from 0 to 1. However, considering that the weights are a comparison tool to measure the importance of each model's criteria, three kinds of weight are allocated: low, medium and high.

A low weight of 0.2 is allocated if the model does not fulfil the requirement or only fulfils it slightly. A medium weight of 0.6 is assigned when the model satisfies the criterion quite well. Finally, a high weight of 1 is allocated if the model satisfies the criterion perfectly.

These three specific weights have been chosen because the distance between them is the same, 0.4, and also because it makes the comparison task easier.

Table 14 shows the final matrix comparison. Below the table is the justification of the weight of each criterion for each model, taking into account the models' references used previously and the knowledge acquired from the simulations and the graphical comparison.

Table 14 Matrix comparison with allocated weights

Criterion	Pacejka's Magic Formula	TMeasy	LuGre tyre model
Real-time capability	0.6	1	1
Ability to describe dynamic tyre behaviour	0.2	0.2	1
Accuracy	1	0.6	0.6
Computing efficiency	0.6	1	0.6
Easily measured tyre input parameters	0.2	0.6	1
Availability of analysis tools	1	1	1
Numerically simple and meaningful equations	0.2	0.6	0.6
Widely applicable	1	0.6	0.2
Popularity	1	0.2	0.6

6.1 Pacejka's Magic Formula

Pacejka's Magic Formula is one of the most popular tyre models currently used by both industry and universities; hence it has a weight of 1 in the criterion "Popularity" [11]. It is an empirical tyre model that is very valuable for calculating steady-state tyre forces and moments characteristics for vehicle-dynamics studies. It is a mathematical model that describes measured tyre characteristics through mathematical formulae. The formulation is able to produce characteristics that closely match measured curves for lateral and longitudinal force; hence in "Accuracy" it has a weight of 1 [22].

This model is based on a sin(arctan) formula, which can not only be simulated using not very powerful software, but also has coefficients that have relationships with typical shape and magnitude factors of the curves to be fitted. Therefore, it has a weight of 1 in “Availability of analysis tools” and 0.2 in “Numerically simple and meaningful equations”, as the meaning of the coefficients is about the resulting curve and not about the physical features of the tyre [22].

Magic Formula is very useful for applications in vehicle simulation models that need real-time computations. However, the computing requirements are slightly high, as the model needs powerful hardware because sinusoidal equation characteristics require a large amount of work to be accomplished using a small amount of computer resources; hence “Computing efficiency” has a weight of 0.6, as does “Real-time capability” [18].

Although it is possible to develop a tyre model for non-steady-state conditions using purely empirical means, transient and dynamic tyre models are based on the physical features of the tyre. Magic Formula describes the steady-state friction forces for given sliding velocities and excludes the transient state of the tread deflection; hence it has a weight of 0.2 in “Ability to describe dynamic tyre behaviour” [19].

Magic Formula is a complex model that needs to know a large number of parameters, which have been determined from experimental tyre measurements [15]. Such measurements require advanced testing equipment; hence the weight of “Easily measured tyre input parameters” is 0.2.

Pacejka’s Magic Formula is implemented in several available simulation packages such as MADYMO, ADAMS, DADS and SIMPACK. Consequently, it has a weight of 1 in the criterion “Widely applicable” [22].

6.2 TMeasy

TMeasy is a semi-physical tyre model used for low-frequency purposes in vehicle-dynamics and handling analyses. For years MAN Nutzfahrzeuge AG has been utilizing it in the SIMPACK system for the study of truck safety and dynamics. Moreover, TMeasy is the tyre model used in veDyna [28] for online simulations of road vehicles, and it is also implemented in the MBS Adams. Consequently, the weight of the criterion “Widely applicable” is 0.6.

The number of model parameters is fairly low according to the low availability and accuracy of the basic experimental data. Furthermore, the model parameters have direct physical meaning that allows them to be recognised in the case of doubtful or incomplete measurement data sets. For instance, the plot of lateral force versus slip only needs a set of five parameters to be determined: initial slip, initial curve inclination (tyre cornering stiffness), location in terms of slip and magnitude of the maximum, location of the beginning of full sliding, and the sliding force [16]. Thus, the weight of “Easily measured tyre input parameters” is 0.6 and the weight of “Numerically simple and meaningful equations” is 0.6, because the model approximates the forces and the torque on the tyre using mathematical functions.

The first version of TMeasy was published in 1994 by Georg Rill. Since then, some model derivatives have been developed and used by dSPACE in the Vehicle Dynamics Simulation Package and by PC-Crash, a software for accident reconstruction [16]. Although many simulations, tests and improvements have been done using this model, not many papers have been published on it due to all the work that is done by private companies. For this reason, the weight of the criterion “Popularity” is 0.2.

The steady-state answer of the model is published in [16], and the modelling of the dynamic tyre behaviour is presented in [24]. The dynamics of the tyre deflections are predicted via the steady-state tyre characteristics; and, accordingly, the dynamics of the tyre forces depend on the vertical tyre load and the longitudinal and lateral slip. Consequently, the weight for the criterion "Ability to describe dynamic tyre behaviour" is only 0.2 because further work must be done in this area.

The model can be implemented easily with Matlab/Simulink; hence the weight of the criterion "Availability of analysis tools" is 1.

The TMeasy tyre model is real-time capable because it aims to yield the results at a prescribed point of time. When constant step size integrators are utilized, the maximum step size is restricted by the numerical stability and the accuracy, which are directly affected by tyre stiffness. The results of this model correspond quite well to the measurements, although a compromise exists between model-complexity and efficiency in computation time [24]. Consequently, the model has a weight of 1 in "Computing efficiency", 0.6 in "Accuracy" and 1 in "Real-time capability". The description of forces and torques also relies on measured and observed force-slip characteristics in contrast to the purely physical tyre models.

6.3 The LuGre tyre model

The LuGre friction model is a physical tyre model that is suitable for any vehicle-motion situation, such as tyre slip control design and vehicle state estimation problems [9]; hence, as it is used in few applications, it has a weight of 0.2 in the criterion "Widely applicable".

In previous studies, the LuGre tyre model has been studied and well discussed. In [19] the LuGre model was used to successfully solve vehicle state estimation problems. Numerical simulations show that the model has a good estimation of the transient dynamic of the tyres.

In addition, a comparative analysis done in [32] shows that for a sufficiently high tyre stiffness, the LuGre tyre model reproduces the tyre's dynamics behaviour pretty accurately. The tyre-model simulation consisted of linearly decreasing angular rate ω (from 32 rad/s to 0 rad/s in 2 sec) and maintaining the velocity constant at 8 m/s.

Consequently, the weight of "Ability to describe dynamic tyre behaviour" is 1, as it is of interest to control studies when tracking and stability issues involve control of the car under large variations of its states.

Moreover, it can be implemented in a simple language that Matlab/Simulink can understand and simulate properly; hence the weight of "Availability of analysis tools" is 1 [6].

In the steady state, when comparing the fitted LuGre model with Magic Formula for different vertical loads, at lower slip ratios the LuGre model shows a good fit to Magic Formula. However, at higher slip ranges, the difference is larger. Nevertheless, experimental results in [7] validate the accuracy of the LuGre model in predicting the friction forces during transient vehicle motion. Taking into account both operation cases, given that one is much more precise than the other, the criterion "Accuracy" has a weight of 0.6, because it is the midpoint between both situations.

The LuGre friction model has some formulation complexity despite its good performance. The input parameters of the model have a physical meaning that allows the designer to tune the model parameters by utilizing experimental data; hence the weight of the criterion “Easily measured tyre input parameters” is 1 [7]. As this model is based on a dynamic friction model, it takes into account three different effects on the friction force – elastic, viscous and damping effects – and all the equations have a physical meaning. Therefore, 0.6 is the weight of the criterion “Numerically simple and meaningful equations”.

Although this dynamic tyre model has low computing efficiency [9], it is capable of being simulated in real-time studies. Subsequently, it has a weight of 0.6 for the criterion “Computing efficiency” and of 1 for “Real-time capability”.

The LuGre model is based on the Dahl dynamic friction model [8], a simple dynamic model used extensively in simulation studies, with two new ideas added: the Stribeck effect and the stick-slip phenomenon. The first is the effect of the relative slip velocity on the steady-state friction forces [19], and the second is caused by the surfaces alternating between sticking to each other and sliding over each other, with an equivalent change in the friction forces [1].

In 1995, Canudas-de-Wit, Olsson and Aström published a paper to present their new tyre model: the LuGre friction model. Since then, there have been many modifications and new developments of this model. Consequently, the weight for the criterion “Popularity” is 0.6.

6.4 Best tyre model for mechatronic applications

Once the weighting of the criteria has been done according to each criterion's importance and the model-criteria have also been weighted, it is possible to calculate the final score for each model studied.

Every established model-criterion weight should be multiplied by its correspondent criterion-relevance weight in order to achieve this objective. For example, in Equation 6-1, the calculation for the real-time capability criterion of Pacejka's Magic Formula is shown, where 0.6 is the model-criterion's weight and 0.135 is the criterion's relevance weight.

$$0.6 \times \frac{13.5}{100} = 0.08 \quad \textbf{(6-1)}$$

The total score has been obtained by summing all the resultant multiplied weights. Table 15 presents the values of the multiplied weights and the total score for each model.

Table 15 Comparison matrix with the total score for each model

Criterion	Pacejka's Magic Formula	TMeasy	LuGre tyre model
Real-time capability	0.08	0.13	0.13
Ability to describe dynamic tyre behaviour	0.03	0.03	0.15
Accuracy	0.11	0.07	0.07
Computing efficiency	0.07	0.11	0.07
Easily measured tyre input parameters	0.03	0.09	0.15
Availability of analysis tools	0.10	0.10	0.10
Numerically simple and meaningful equations	0.02	0.05	0.05
Widely applicable	0.09	0.05	0.02
Popularity	0.07	0.01	0.04
TOTAL SCORE	**0.59**	**0.65**	**0.78**

As the resulting comparison matrix shows, the LuGre tyre model is the one that has the highest total score and hence it is the best model for mechatronic applications for vehicle-dynamics control systems.

By analysing the comparison matrix in more detail, it is possible to establish that within the LuGre tyre model, the two criteria that have the highest weight are the "Ability to describe dynamic tyre behaviour" and "Easily measured tyre input parameters".

These criteria make the difference when a multi-body system (MBS) uses the LuGre tyre model instead of the others, because, by using this model, tyre-force characteristics are better described, since the model predicts the dynamic behaviour, and its implementation is cheaper, as the information necessary to implement it is easier to obtain.

The results achieved do not suggest that Pacejka's Magic Formula or TMeasy are not good tyre models. They only recommend using the LuGre tyre model for vehicle-dynamics controls systems if the designer wants to spend less money, time and resources on the tyre-model simulations.

7 CONCLUSIONS

This chapter explains the conclusions that can be drawn from the overall comparison matrix, bringing together the results from the implementation of the models and the comparison of the criteria for each model.

The objective of this project was to do a benchmarking of three different tyre models in order to choose the most appropriate for mechatronic applications for vehicle-dynamics control systems. The selected tyre models were studied in detail and compared with each other.

The starting point of the analysis was the study of mechatronics and its applications. It can be concluded that automotive technologies have always been at the forefront of mechatronics, with features such as engine management systems, continuously variable transmission and vehicle-dynamics systems. Mechatronics has had and will continue to have a major impact on the design and development of many engineering systems used for vehicle improvement.

7.1 Tyre-model conclusions

Firstly, all the available documentation needed to fully understand the three different tyre-model categories was collected.

Considering Pacejka’s Magic Formula, TMeasy and the LuGre friction model, it can be concluded that the best model to study the steady-state behaviour of the tyre is Magic Formula, because the curves fit very accurately to the real data experiments. However, it is not the best model for vehicle-dynamics control systems, because they need to know the characteristics of dynamic tyre forces.

Moreover, it can be concluded that the LuGre tyre model cannot achieve the same accuracy when describing the steady state as the empirical tyre models. In the physical tyre models, some assumptions are made to decrease the computational effort, for instance the poor approximation of the normal pressure distribution or the rectangular shape of the contact patch.

The LuGre tyre model is the only model that predicts dynamic tyre behaviour. However, many new proposals and extensions of this model need to be done in order that it will take into consideration some of the aspects that it currently neglects, such as rotation of the wheel rim or the anisotropy of friction characteristics. It would then be the model that best and most easily predicts transient tyre behaviour.

Finally, due to its efficiency in computation and handling, the TMeasy tyre model covers a wide range of practical demands in vehicle-dynamics simulation. In particular, it allows at least sufficient approximations of the resulting force and torque characteristics, even in the case of incomplete or missing measurement data. In contrast, Pacejka's Magic Formula actually enables a high degree of modelling accuracy while requiring extensive sets of testing data in any application.

7.2 Comparison matrix conclusions

The most important challenge accomplished in Chapter 5 was the definition of nine criteria. These tyre requirements were defined explicitly from vehicle-dynamics control systems such as ABS, ESP, EPAS and TCS. Moreover, a weight was assigned to each criterion, taking into account experts' judgement and a more theoretical point of view.

Subsequently, the models comparison was done, filling the matrix with the appropriate weight that best describes how the models satisfy each criterion. To make the comparison easier, the matrix could only be filled with three values: low, medium and high.

The total scores for each model were 0.59 for Pacejka's Magic Formula, 0.65 for TMeasy and 0.78 for the LuGre tyre model. The LuGre tyre model has the highest score and hence is the best model for mechatronic applications for vehicle-dynamics control systems.

The final weight that each criterion had on the matrix comparison in this thesis was assigned to compare the tyre models in terms of mechatronic applications for vehicle-dynamics control systems. If it was necessary to study any other mechatronic application, a similar comparison matrix could be used, changing the weights of the criteria according to their importance in this new application.

7.3 Further works

Different recommendations for further works can be given in order to continue the achievements obtained by this thesis.

Firstly, regarding the matrix comparison, the next step to be done is to extend the matrix by adding a tyre model belonging to the category that was not studied: complex finite element models. These models predict dynamic tyre behaviour in more detail and might be of interest for mechatronic applications if they could be run in real-time simulations.

Secondly, another area where the work could be further deepened is that of the detailed description of the model chosen for mechatronic applications. Thus, if the LuGre tyre model could be totally developed and defined, the successful integration of it into a multi-body system (MBS) would be possible. This would allow Cranfield's Automotive Mechatronic Centre to use a reliable dynamic tyre model in their simulations.

The final recommendation would be to implement the chosen tyre model in some of the vehicle-dynamics systems applications. This would be the best way to demonstrate that the tyre model works properly in a bigger system and that the tyre-model outputs' characteristics meet the requirements that are expected.

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APPENDICES

Appendix A Thesis' poster

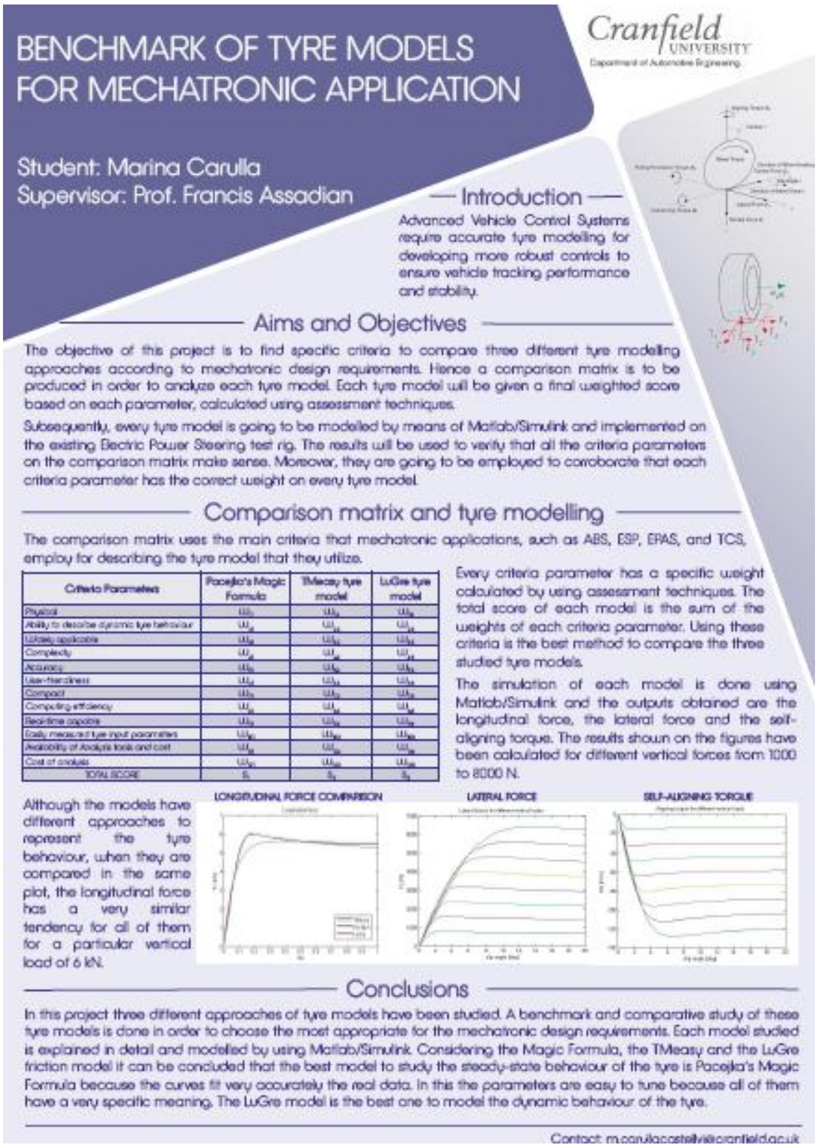

Figure A-1 Thesis's poster designed for the Poster Competition

Appendix B Matlab programs

B.1 Pacejka's Magic Formula

```
% LATERAL FORCE

alpha=0:0.1*(pi/180):60*(pi/180);
Fz=1000:1000:8000;

% Factors
C=1.3;
E=-3;
mu=0.8;
c1=60000; %N/rad
c2=4000; %N
D=mu.*Fz; % D represents the peak value
C_Falpha=c1*sin(2*atan(Fz./c2)); % Cornering Stiffness
B=C_Falpha./(C.*D);

% Side force
for x=1:length(Fz)
    Fy(:,x)=D(x)*sin(C*atan(B(x).*alpha-E*(B(x).*alpha-atan(B(x).*alpha))));
    plot(alpha*(180/pi), Fy)
    title('Lateral force for different vertical loads')
    xlabel('slip angle [deg]')
    ylabel('Fy [N]')
    hold on
end
```

Figure B-1 Magic Formula equations implemented with Matlab

B.2 TMeasy

```
Fz_N=4;
Fz_2N=8;

% dFy_0
dFy_0_Fz_N= 55;
dFy_0_2Fz_N= 80;
for x=1:length(Fz)
    dFy_0_Fz(x)=(Fz(x)/Fz_N)*(2*dFy_0_Fz_N-0.5*dFy_0_2Fz_N-(dFy_0_Fz_N-0.5*dFy_0_2Fz_N)*(Fz(x)/Fz_N));
end

% Fy_M
Fy_M_Fz_N= 4.2;
Fy_M_2Fz_N= 7.5;
for x=1:length(Fz)
    Fy_M_Fz(x)=(Fz(x)/Fz_N)*(2*Fy_M_Fz_N-0.5*Fy_M_2Fz_N-(Fy_M_Fz_N-0.5*Fy_M_2Fz_N)*(Fz(x)/Fz_N));
end

% Fy_S
Fy_S_Fz_N= 4.15;
Fy_S_2Fz_N= 7.4;
for x=1:length(Fz)
    Fy_S_Fz(x)=(Fz(x)/Fz_N)*(2*Fy_S_Fz_N-0.5*Fy_S_2Fz_N-(Fy_S_Fz_N-0.5*Fy_S_2Fz_N)*(Fz(x)/Fz_N));
end

% Sy_M
Sy_M_Fz_N=0.2;
Sy_M_2Fz_N=0.22;
for x=1:length(Fz)
    Sy_M_Fz(x)=Sy_M_Fz_N+(Sy_M_2Fz_N-Sy_M_Fz_N)*((Fz(x)/Fz_N)-1);
end

% Sy_S
Sy_S_Fz_N=0.8;
Sy_S_2Fz_N=1;
for x=1:length(Fz)
    Sy_S_Fz(x)=Sy_S_Fz_N+(Sy_S_2Fz_N-Sy_S_Fz_N)*((Fz(x)/Fz_N)-1);
```

Figure B-2 Interpolation of the main parameters of the TMeasy tyre model

```
% Limits
for x=1:length(Fz)
x1=0.015;
x2=Sy_M_Fz(x);
x3=Sy_S_Fz(x);
s1=[0:0.001:x1];
s2=[x1:0.001:x2];
s3=[x2:0.001:x3];
s4=[x3:0.1:x3+0.2];

% Parameters resolution
A=[x2^2 x2 1 0 0 0; 0 0 0 x2^2 x2 1; 0 0 0 x3^2 x3 1; x1^2 x1 1 0 0 0; 2*x2 1 0 -2*x2 -1 0; 0 0 0 2*x3 1 0];
B=[Fy_M_Fz(x); Fy_M_Fz(x); Fy_S_Fz(x); dFy_0_Fz(x)*x1; 0; 0];
Y=inv(A)*B;

% Equations
F1=dFy_0_Fz(x)*s1;
F2=Y(1)*s2.^2+Y(2)*s2+Y(3);
F3=Y(4)*s3.^2+Y(5)*s3+Y(6);
F4=[Fy_S_Fz(x),Fy_S_Fz(x),Fy_S_Fz(x)];

% Plot
plot(s1*(180/pi),F1,'k', s2*(180/pi),F2,'k', s3*(180/pi),F3,'k')
title('Lateral force')
xlabel('slip [deg]')
ylabel('Fy [kN]')
legend('6kN','7kN','8kN')
hold on
```

Figure B-3 TMeasy model using Matlab

B.3 LuGre tyre model

```
Sig0=181.54;
Sig2=0.0018;
muC=0.8;
muS=1.55;
Vs=6.57; %m/s
Fn=1; %N
L=0.2; %m
alpha=0.5;  %constant to capture the steady-state friction/slip characteristic
s=0:0.001:1;

%***************************************************
v=9;
for x=1:length(s),
    g(x)=muC+(muS-muC)*exp(-abs(v*s(x)/Vs)^alpha);
    temp(x)=g(x)*abs(1+s(x))/(Sig0*L*s(x));
    Fb(x)=Fn*g(x)*(1+temp(x)*(exp(-1/temp(x))-1))+Fn*Sig2*v*s(x);
    mu(x)=Fb(x)/Fn;
end

plot(s,mu,'b')
hold on
```

Figure B-4 Static Lugre tyre model using Matlab

```
Sig0=40;
Sig1=4.95;
Sig2=0.0018;
muC=0.5;
muS=1.5;
Vs=10;
Fn=1000; %N

% Bristle deflection vs Sliding Velocity (m/s)
it=0;
Vr_tmp=0:0.1:10;
for Vr=Vr_tmp
    it=it+1;
    sim('LuGre_sajjad',0.1)
    Z(it)=Vr_z_F_Scope.signals(2).values(end);
end

figure
plot(Vr_tmp,Z)  %Z at t=0.1 is CORRECT now!
xlabel('Sliding velocity (m/s)')
ylabel('Bristle deflection')

% F/Fn and g(Vr) vs Sliding Velocity (m/s)
it=0;
Vr_tmp=0:0.1:10;
for Vr=Vr_tmp
    it=it+1;
    sim('LuGre_sajjad',0.1)
    Y(it)=Vr_z_F_Scope.signals(3).values(end);
    g_Vr(it)=g_Scope.signals(1).values(end);
end

figure
plot(Vr_tmp,Y/Fn,Vr_tmp,g_Vr)
xlabel('Sliding velocity (m/s)')
ylabel('Friction force')
```

Figure B-5 Dynamic Lugre tyre model using Matlab

Appendix C Questionnaire

TYRE MODELS COMPARISON

The aim of this questionnaire is to know the relevance of the main parameters that describe tyre models characteristics according to Expert Automotive Engineers. Could you give a number from 1 to 5 to each parameter showing the importance you think it has in choosing a tyre model for Vehicle Dynamics Control Systems?

Consider the following parameters being from 1: little importance to 5: extremely importance

		1		2		3		4		5
1.	Real-time capable									
2.	Ability to describe dynamic tyre behaviour									
3.	Accuracy									
4.	Computing efficiency									
5.	Easily measured tyre input parameters									
6.	Availability of analysis tools									
7.	Numerically simple (user-friendliness)									
8.	Widely applicable									
9.	Popularity									

Being:

1. Real-time capable: The tyre model can be implemented in real time simulations that allow designing and developing control systems solutions.

2. Ability to describe dynamic tyre behaviour: Is the capability to characterize the dynamic tyre behaviour in order to describe the pure longitudinal and lateral slip and also the combined slip.

3. Accuracy: Is the ability to predict precisely the forces and moments transmitted between the tyre and the road. Results from accurate models can match experimental data well.

4. Computing efficiency: Practical to use. It spends short time, memory and space while running the simulation. It is compact and have the equations positioned together in a tidy way.

5. Tyre input parameters: The number of parameters that the tyre model needs derived from measured data before the simulation can vary from 10 to 50. The models that require a low number of parameters are going to be more straightforward to use.

6. Availability of analysis tools: Having the choice to simulate the tyre model in common software is very useful for the user because he does not need to pay for an expensive license of powerful software.

7. Numerically simple (user-friendliness): The equations used in the model are easy to understand because they have a physical meaning. It is not difficult to find the output since they have not got many parts. The tyre model should be practical and simple to use for people since it is simple to follow the development of the model's formulation equations.

8. Widely applicable: Describe tyre models that can be used in different applications, for high and low velocities and frequencies. Models are widely applicable if they can be utilized from the specific component design area to the development of an integrated vehicle dynamics control system.

9. Popularity: It depends on how much information has been published about its results and improvements.

Figure C-1 Tyre models comparison questionnaire