COMPARISON OF LOW-ORDER 4D DYNAMICAL MODELS
FOR CARDIAC RESPIRATORY MOTION USING MRI

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“...però, quins són els fets?”
Abstract

Cardiovascular diseases account for more than one third of the annual mortal rate in western countries. Although AF does not directly lead to mortality is pointed as one of the possible causes of stroke. Recent developments in catheter intervention open promising surgical treatments for this disease, although the clinical and technical challenges must be overcome for it to become standard.

Movement of the heart because of cardiac and respiratory movement complicate the positioning of the catheter inside the heart. The aim of this project is to purpose and test models for automatically segmenting and tracking the heart in 4D cardiac MRI datasets, for potential use in in surgery training, planning and guidance.
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Chapter 1

Introduction

Cardiovascular diseases were in 2007 the cause of 34% of all the deaths in United States and only cerebrovascular diseases accounts for 136,000 deaths. [5]

Among all arrhythmias affecting the heart, Atrial Fibrillation (AF) is the most common in clinical practice. It is estimated that around 2.3 million adults in United States have clinically recognized AF and have sought medical attention for this condition.

Atrial Fibrillation is often diagnosed by the observation of irregular pulse, but a conclusive indicator is the absence of P waves on the Electrocardiogram (ECG). The name comes from the fibrillation of the heart muscle in the atria which does not contract with coordination. This erratic beating is not a hazard by itself but can lead to the most feared consequence of Atrial Fibrillation: stroke.

Recently developed intra-operative methods use radiofrequency catheter ablation to carry out pulmonary vein isolation [6]. This surgery is performed by inserting a catheter inside the heart entering through the vena cava, where the surgeon guides it to the tissue to be electrically isolated. Since this surgery is not carried out with an open chest, the surgeon guides the catheter through real-time X-rays (Fluoroscopy), ECG, and tracking devices on the catheter. This method present significantly promising results. Unfortunately, due to the clinical and technical challenges this intervention is used only in a small subset of patients. However, more recent developments in MRI and catheter intervention techniques show some promise towards a future with this technique becoming a standard approach for Atrial Fibrillation treatment.

As commented before, though this intervention is highly effective, there are many problems when placing the catheter inside the heart due to movement, cardiac beating and breathing. Also, the current X-ray visualization gives only a 2D vision of the position of the catheter while the surgeon needs a 3D positioning to carry out the precise ablation.

The aim of our group is to generate a 3D computational model of the heart that moves with the respiration and the cardiac beating. This was first started by G. Pons [7] in 2008 who created a 4D model of the heart beating at a breath-hold position. The following years G. Crosas (2009) [8] and M. Queralt (2010) [9] have been working on the modeling of the respiratory movement of the heart. This current project continues their work and presents a comparison of different models we have created to model this movement.

This work will be structured in the following parts:
Sections 1.1, 1.2 and 1.3 present a descriptive introduction to the heart, lungs and *Atrial Fibrillation* that will serve for us to understand the problem.
In sections 1.5 and 1.4, we will comment on the previous work done in this field and we will set our long and short term objectives.
Chapters 3 and 2 are a brief discussion about the data used and the challenges it presents and a review of some technical concepts and theory.
Sections 4.1 and 4.2 describe the segmentation process we have followed and the respiratory phase extraction.
Chapter 5 presents the details about the different 4D models studied and their respective results.
Finally, Chapters 6 and 7, discuss about the different results obtained and future work.
1.1 The Heart

The heart is one of the most important organs in the body. Its function is to keep the blood moving around the circulatory system. It is located near the center of the chest and protected by the rib cage and it is surrounded by the lungs and the diaphragm.

In this section we introduce some background about the anatomy and physiology of this vital organ. For more information refer to [5, 10–15].

1.1.1 Anatomy

The heart is mainly formed by muscular tissue which is somewhat different than the standard muscle that we would find in the arms or legs (skeletal muscle). It is called cardiac muscle and is formed by myocites.

The most important characteristic about cardiac muscle is that does not need external input for contraction. It triggers itself. This means that the heart is let independent and there is no conscious control over it, although its rate and strength can be modulated by the nervous system. Because of this, the heart continues beating after the nerves connected to it have died (e.g. in the case of a heart transplants).

The auto-stimulation produced for the contraction is an electrical impulse generated in the sinoatrial node (SA node) that expands throughout all the volume. This stimulation is observable from outside of the body using the electrocardiogram (ECG or EKG), which measures the potential differences generated by the heart transmitted to the surface of the body. An example of the typical ECG signal is in Figure 1.1

![ECG Signal](image)

Figure 1.1: Heartbeat in ECG. Figure obtained from [1]

The heart is divided into four chambers illustrated in fig. 1.2:

- The right atrium and right ventricle, separated by the tricuspid valve and connected to the circulatory system by the superior and inferior vena cava and the pulmonary artery respectively.

- The septum is between the right and left chambers of the heart and keeps them separated. It is also important for the conduction of the electrical impulse responsible for the contraction of the heart.

- The left atrium and left ventricle, separated by the mitral valve and connected to pulmonary vein and aorta.
1.1. THE HEART

Low-Order Dynamic Modeling of the Heart Motion Under Respiration

![Diagram of the heart and blood flow](image)

Figure 1.2: Parts of the heart and blood flow in the interior. Figure obtained from [1]

1.1.2 Beating

To make the blood circulate the heart pumps it with its contraction. The usual beating rate is about 60 to 100 beats per minute, each beat having 2 main stages:

- **Systole**: the heart is relaxed.
- **Diastole**: of the atria and ventricles, is the stage in which a part of the heart is contracted.

The process of a heartbeat, shown in fig. 1.3 is the following:

1. The heart starts relaxed and the atria full of blood.

2. A electrical impulse is generated in the SA node, placed on the top of the right atrium. This impulse expands throughout all the atria and makes it contract after some delay. The contraction increases the pressure in the atria and forces the tricuspid valve to open. The blood in the atria flows to the ventricles. (P wave on the ECG)

3. The atrioventricular node (AV node) slows down the conduction of the depolarization which has the effect of allowing time for the atria to pump all their blood. (Q wave)

4. The pulse is rapidly conducted through the bundle of His, inside the septum, to the distal end of the ventricles. (still Q wave)

5. Once the electrical stimulation reaches the wall of the right ventricle it spreads through both ventricles, which, after some delay, start to contract and pump blood to the arteries. (R wave) In the meanwhile, the atria start filling with new blood from the veins.
6. Once the ventricles start to relax, the aortic and pulmonary valves close, forced by the higher pressures in the arteries compared to those in the ventricles. (S wave)

7. Finally, the heart returns to relaxation start point and prepares for next contraction. (T wave)

Figure 1.3: Evolution of the heart during beating. Figure obtained from [1]
1.2 LUNGS AND RESPIRATION

The lungs are two organs, similar to bags, that fill the inside of the chest cavity on either side of the heart. Their purpose is to exchange the residual carbon dioxide $CO_2$ from the body for oxygen $O_2$ in the atmosphere.

Lungs are of interest for the study of the heart because of their anatomical and physiological close relation. Not only they need each other to obtain and distribute oxygen, but also their overlapping position forces them to interact producing effects such as an increase of cardiac frequency during inspiration or the mainly vertical movement of the heart when breathing [16–20].

1.2.1 Anatomy

Lungs and blood vessels:

As we explained, lungs are two bags inside the chest cavity, the left one slightly smaller than the right one to allow room for the heart. The interior part of the lungs is formed by a structure of tubes that start at the bronchi and branch into thousands of thinner tubes called bronchioles ending up in groups of tiny round air sacs called alveoli. See Figure 1.4

Each of these air sacs is covered in a mesh of tiny blood vessels called capillaries. The capillaries connect with the pulmonary artery that carries the deoxygenated blood and pulmonary vein that takes the $O_2$ exchanged in the alveoli.

![Anatomy of the lungs. Figure obtained from [2]](image-url)
1.2. LUNGS AND RESPIRATION

Muscles used for breathing:
The muscles near the lungs help expand and contract the lungs to allow breathing. This muscles are:

- Diaphragm: dome-shaped muscle situated below the lungs, which separates the chest cavity from the abdominal cavity.
- Intercostal muscles: situated between the ribs.
- Abdominal muscles: situated beneath the diaphragm. Helpful when breathing fast.
- Muscles in the neck and collarbone area: help to breath when the other muscles can’t work well.

1.2.2 Respiration

The respiration process is composed of two main parts.

- **Breathing In (Inhalation)**
  Inhalation is produced when the diaphragm contracts and moves downward. At the same time, the intercostal muscles contract and move the chest cavity upwards. These movements increase the chest cavity and expand the lungs. The pressure difference between the atmosphere and inside the lungs makes the exterior air rich in oxygen flow into the alveoli. In the meanwhile, with this new air, the alveoli exchange the O$_2$ from the incoming gas for the CO$_2$ carried by the pulmonary arteries.

- **Breathing Out (Exhalation)**
  Exhalation is the opposite movement. The muscles that were contracting before now relax and the atmospheric pressure forces the chest cavity to reduce space and the lungs contract. The air, now poor in O$_2$, inside the alveoli blows outside the body through the mouth and the nose expelling all the CO$_2$ produced in the body.
1.3 ATRIAL FIBRILLATION

Atrial Fibrillation (AF) is the most common type of arrhythmia. Estimates are that about 2.3 million adults in the United States have been clinically diagnosed with AF and its prevalence rises up to 9% past the age of 80. The specific characteristics of the arrhythmia vary from patient to patient, but the main processes involved are the following:

As explained in Sec. 1.1.2, the contraction of the heart is normally triggered by the electrical stimulation generated in the SA node that spreads through the atria. Patients with AF have other stimulation points or reentrant circuits, external or in the atria substrate that impede the normal contraction of the myocardium. The most common reentrant origin is the pulmonary vein, but other veins, thoracic nerves or abnormal myocardium can also be the cause. When these secondary pulses appear, the atria start to fibrillate (disorganized fast contractions). This fibrillation leads to an inefficient pumping of blood not only because of the ineffective contractions but also due to the irregular excitation of the AV node. The stalled blood in the atria could form thrombi which can end up producing a stroke. Strokes are produced when these thrombi move with the blood flow (now called emboli) and go to the arteries. The problem occurs when these emboli get stuck in an artery too thin for them to continue and block the blood that kept the tissue further away alive. The worst case is when this artery was feeding brain tissue, yielding to the death of neurons and possible consequent physical or mental disabilities or even death.

Generally, doctors classify AF in two types: paroxymal and nonparoxymal. The first, approximately 40% of the AF cases, produce short AF cases that terminate spontaneously within 7 days, usually even less than 24 hours. Patients with paroxymal AF are usually younger and have no structural heart diseases. The causes are abnormal extra-atrial triggers such as diseases of the thoracic vein and the autonomous nervous system in which pathological triggers reinitialize AF.

The second classification, nonparoxymal AF, is related to older patients and often further classified as persistent or permanent. Both terms are used to describe AF episodes that do not finish within 7 days and cannot be consistently terminated with cardioversion. In this case the cause is primarily diseases of the atrial myocardium and is often accompanied by atrial enlargement, fibrosis and multiple reentrant circuits with even minor triggers that reinitialize and maintain AF.

More information about AF and its treatments can be found in [6, 21–26].

1.3.1 Diagnose:

AF is usually detected via these symptoms:

- palpitations.
- shortness of breath.
- chest pain.
- dizziness or fainting.
- confusion.
- hypertension.

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Although it is further diagnosed using ECG, where its effects are visible in the absence of P waves, where the disorganized impulses cancel each other, and the non-periodicity of the QRS waves, due to the arrhythmic contraction of the ventricles.

1.3.2 Causes and risk factors:

The exact causes of AF are unknown. There is a complex interplay between the triggers that initiate AF and the abnormalities of the atrial substrate that allow perpetuation. But frequent episodes of AF increase the likelihood of future, more frequent and longer episodes. “AF begets AF”. 25% of paroxymal AF cases will progress to chronic or permanent AF within 5 years.

It is important to remark that factors like obesity, alcohol use, structural heart diseases, previous heart surgery interventions, inflammation and genetics increase the chances of suffering from AF.

1.3.3 Treating Atrial Fibrillation:

Why and how to treat AF depends mainly on whether it is the likelihood of it causing a stroke.

The treatments options are broad. Many patients take drugs, alone or complimentary to other interventions, to maintain a the heartbeat rhythm, keep it at a fixed rate or to avoid blood clots. For onset AF, pharmaceutical or shock cardioversion are used to stop the atria from fibrillating and recover the normal heartbeat conditions. Further surgical interventions are only used when AF is permanent or persistent.

These interventions ablate some part of the tissue by cooling or heating techniques to stop its conduction. This way the external trigger or the abnormal myocardium that maintained and reinitialized the fibrillation are electrically isolated from the rest of healthy heart that will beat normally.

These complicated procedures are realized by the surgeon with a catheter inserted in the heart through the vena cava. The success of these interventions highly depends on the accuracy, robustness and stability of the visualization methods used. It is necessary that these methods provide precise information about the intervened area as well as avoid sudden changes to help the surgeon steer the catheter to the target tissue.

In heart surgery, these pre-requisites are not trivial. The beating of the heart and the breathing motion produce a effect of deformation and translation of the heart inside the body; this movements translate to instabilities and jumps in the intervention visualization that need to be corrected.
1.4 OBJECTIVES

Because of the difficulties presented by the motion of the heart, motion correction is needed for the proper visualization of the operation.

1.4.1 Long term

Our solution to this problem is to create a model of the heart that, with a few coefficients, characterizes the shape and the movement generated by the beating and the breathing processes. This model will be a pre-procedure estimation of the real movement, so it will need in-vivo data for real-time corrections. Information from the heartbeat obtained with the ECG, breathing phase and an image of the heart from a CT or MRI should be enough to adapt the model to the actual situation of the patient.

To achieve this goal, we have divided the whole problem into two separate models that will be later joined into one sole model. This separation is done according to the two main movements present, which can be characterized by separate reference signals. The first uses ECG to follow the heartbeat and the second can take advantage of the position of the liver, stomach, chest, among other options to determine the respiratory phase.

1.4.2 Short term

Currently, our short term objective is to model the breathing movement of the heart and test our models validity. This work describes three different approaches, explaining their strong and weak points, and tries to establish the next steps to find a definitive model that characterizes the translation, rotation and shape changes of the heart due to respiration.
1.5 PREVIOUS WORK

For years, respiration motion has been subject to study, yet there is still much to be understood. 40 years ago, Dougherty started the study of the effects of respiration on electrocardiogram and noted that the heart undergoes anatomic rotation in the frontal plain when breathing [16, 17]. Miller et al. continued studying it by giving the first quantitative analysis of the motion of the heart [18]. He observed that the superior-inferior motion of the diaphragm averaged 15mm during normal respiration.

15 years later, Wang concluded that the primary motion of the heart is translation in superior-inferior direction, based on 2D MRI at multiple breath-hold positions. This movement, at the level of the coronary ostia, was between 0.6 to 0.7 times the displacement of the diaphragm [19].

Entered the 2000s McLeish et al. conducted studies on the movement of the heart due to respiration. He published a 3D rigid body motion analysis of the heart [20], in his study imaged the whole heart with MRI at multiple breath-hold positions and he acknowledged rotations and translations for different patients and volunteers, but the 6mm slice thickness and the 180ms temporal resolution were insufficient for isolating the effects of cardiac motion.

Due to the recent improvement in catheter intervention technologies, there has been an increasing interest in motion compensation systems. In 2008, Richa et al. took advantage of the pseudo-periodic nature of the movement of the Heart to create a autoregressive and a Fourier models whose parameters were estimated with Kalman filter. This model, presented in [27], used only visual feedback provided from the laparoscope to compensate for the movement.

One year later, Bachta et al. added information from the cardiac and respiratory phases to solve for the compensation problem [28]. A study that they carried out on pigs showed that the spectrum of the cardiac movement is similar to the one produced by an amplitude modulation where the carrier frequency corresponds to the heartbeat frequency and the modulating signal has the same frequency as the respiratory component. This previous knowledge allowed them to create a motion compensation model based on this characteristic.

More recently, in 2011, Savill et al. carried out a study ([29]) of the predictive accuracy of respiratory motion models. In this study they showed that the accuracy is dependent not only on the signals of reference, but also on the breathing pattern of the subject. They classified the breathing patterns into 3 possible (normal, fast and deep) and compared different combination of signals (diaphragm, chest, abdominal, etc.) for each of them. Their results indicate that the appropriate signals for normal and fast breathing are diaphragm and abdominal and diaphragm plus lateral wall of the heart for deep breathing.

The same year, Ma et al. presented 3 methods to correct for the respiration motion. The first two use image intensity in a region of interest to track the diaphragm and the heart border respectively and the third detects the tracheal bifurcation by means of the generalized Hough transform and a pre-operative 3D model [30].

Finally, our research group has worked previously on cardiac motion compensation. The final objective of our group is to create a pre-operative 3D model of the heart that moves because of the heartbeats and the respiration. First G. Pons et al. started with 4D cardiac segmentation of the beating heart [7]. His method segments a set of pre-operative MR images of the heart and takes advantage of the quasi-periodicity of the heartbeat to interpolate the position of the heart at each phase.

Next, G. Crosas [8] continued developing a models of the movement due to respiration, first looking at different parts of the body in ECG gated MR images and later extracting the information from the navigator signals obtained during the imaging.

One year later, M. Queralt [9] focused his work on non ECG gated data to construct a 3D model of the heart.
moving due to respiration based on the navigator signals.
From this last two works, this paper [31] was presented at ISBI on 2011.
Chapter 2

Background Theory

2.1 LMS Fitting

Least squares is an approximate solution for overdetermined problems commonly used in data fitting. It finds the solution of the overdetermined problem that minimizes the sum of the squared error of the data.

Data fitting, which is our usage, can be understood as minimizing the $l_2$ norm of the curve to fit with the data.

$$\min(\epsilon) = \min_k \| x - f(x, \beta) \|^2$$

Where $\epsilon$ is the error, $x$ is the data and $f(x, \beta)$ is the fitted curve, $\beta$ being the set of parameters to adjust.

Depending on $f(x, \beta)$ we will talk about linear least squares (LLSQ) or non-linear least squares (NLLSQ). In the first, the function is a linear combination of the parameters $\beta$, the later describes any other type of function, i.e. $\beta^2, e^{\beta x}$. NLLSQ do not generally have closed term solutions and are a broad field of study, but in this case, we focus on LLSQ.

As discussed above, LLSQ presents linear combination of the unknowns, this means that $f(x, \beta)$ is:

$$f(x, \beta) = \sum_j \beta_j \phi_j(x)$$

with $\phi_j(x)$ are any function. Written in vectors the problem equation results:

$$\min(\epsilon) = \min_x \| x - \beta M \|^2$$

where

$$M = [m_{ji}] = [\phi_j(x_i)]$$

$j, i$ are row and column indexes of $M$ and has the solution:

$$\hat{\beta} = (M^T M)^{-1} M^T x$$

Characteristics of LLSQ:

- Is a convex problem $\rightarrow$ exists a unique $f(x, \beta)$
- Errors must only be in the dependent variable, errors in the independent variable are assumed to be 0 or negligible.
- The error of the data are assumed to be uncorrelated and with Normal distribution with zero mean; $N(0, \sigma)$.
- Unequal distribution of the data leads to over-weighting.
2.2 Interpolation

Interpolation is a method used to construct new data points in positions within the range of a set of discrete known data. Also, interpolation can be used to obtain simplified functions that fit the data from a more complicated distribution. Examples of this usages are finding new points between already existing data to have an equal and fair distribution of the samples or to smooth data we already had in order to eliminate noise. The election of the adequate interpolation method depends on the data. So if for example the data can be explained by a periodic function and the purpose is to smooth the data, curve fitting on Fourier harmonics presents a good option. Instead, if the objective is to find new intermediate points and the data behaves like a polynomial function, splines can be used.

2.2.1 Spline Interpolation

The name of this interpolation method comes from the interpolant used: the spline. The splines are piece-wise polynomials defined within intervals, usually between known data, which can be uniform or not. So that:

\[ S : [a, b] \rightarrow \mathbb{R} \]

\([a, b]\) is divided in subintervals \([t_{i-1}, t_i]\) so that:

\[ a = t_0 < t_1 < \cdots < t_{k-1} < t_k = b \]

where the polynomials are defined:

\[ P_i : [t_{i-1}, t_i] \rightarrow \mathbb{R} \]

The polynomials are chosen in a way that guarantees sufficient smoothness; This means that, for a spline of order \(n\) (maximum degree of the polynomial) it is required to be \(n-1\) times differentiable at the interior points \(t_i\), \(i = 1, \cdots, k - 1\), \(0 \leq j \leq n - 1\),

\[ P_i^{(j)}(t_i) = P_{i+1}^{(j)}(t_i) \]

Cubic splines which are commonly used for interpolation, have degree 3, and must be second order differentiable. So, knowing a set of interior points \(t_i = (x_i, y_i)\), \(i = 0, \ldots, N - 1\) we will have \(4(n - 1)\) unknowns, 4 for each polynomial \(q_i(x)\), and \(4n - 6\) equations:

\[ y_i = q_i(x_i) = a_i + b_i x_i + c_i x_i^2 + d_i x_i^3 \]

\[ y_{i+1} = q_i(x_{i+1}) = a_i + b_i x_{i+1} + c_i x_{i+1}^2 + d_i x_{i+1}^3 \]

\[ q'_i(x_i) = b_i + 2c_i x_i + 3d_i x_i^2 = q'_{i+1}(x_i) = b_{i+1} + 2c_{i+1} x_i + 3d_{i+1} x_i^2 \]

\[ q''_i(x_i) = 2c_i + 6d_i x_i = q''_{i+1}(x_i) = 2c_{i+1} + 6d_{i+1} x_i \]

Unfortunately there are 2 more unknowns than equations, so further assumptions must be made in order to solve the problem. The most common assumption made is enforcing \(q'_i(x_0) = q''_{N-1}(x_N) = 0\).

2.2.2 Curve Fitting

Another way of interpolating is assuming that the set of given points lie on a parameterizable curve. The type of which can be learned from the data or inferred by previous information about the problem and can be any type of curve, the most common are Fourier and polynomial expansions.

The idea behind this interpolation method is, once defined this set of base functions, the data is fitted by Least Squares (Sec. 2.1). As previously commented, for LLSQ we need a set of base functions \(\phi_j(x)\), which in Fourier and Polynomial cases are:
2.3. SPHERICAL HARMONICS

- **Polynomial:**
  \[ \phi_j = x^j; \quad 0 \leq j \leq N - 1 \]
  which, in matricial representation is:
  \[ M_{ji} = x^j; \quad M = 1 + x + \cdots + x^{N-1} \]

- **Fourier:**
  \[ \phi_j = e^{-i2\pi j}; \quad 0 \leq j \leq N - 1 \]
  again, in matricial representation:
  \[ M_{ji} = e^{-i2\pi j}; \quad M = 1 + e^{-i2\pi 1} + \cdots + e^{-i2\pi(N-1)} \]

This base functions form the \( M \) that is inverted to find the solution.

### 2.3 Spherical Harmonics

Spherical harmonics are defined as eigenfunctions of the Laplace operator. [32–34]

\[
\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}
\]

After solving the equation, have the expression:

\[ Y_{lm}^m(\theta, \varphi) = Ne^{im\varphi} P_l^m(\cos \theta) \]

where \( N \) is the normalization factor, \( P_l^m \) is the associated Legendre polynomial and \( l \) and \( m \) are the degree and order of the spherical harmonic.

Depending on the application there are several normalization factors, but in our case we will use the normalization factor that make the functions orthonormal:

\[ N = \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}} \]

Finally, this is the expression of the spherical harmonics we use as base functions:

\[ Y_{lm}^m(\theta, \varphi) = \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}} e^{im\varphi} P_l^m(\cos \theta) \]

\[ 0 \leq \phi \leq 2\pi, 0 \leq \theta \leq \frac{\pi}{2}, l,m = 1,2,3,4,\ldots \]

The spherical harmonics form a complete set of orthonormal functions, thus any square-integrable function on the unit sphere can be explained as a linear combination of these:

\[ f(\theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} f_l^m Y_{lm}^m(\theta, \varphi) \]

where \( f_l^m \) is the coefficient for each \( Y_{lm}^m \).

Nodal lines represent a good way to visually understand how each spherical harmonic base function represents the unity sphere. These nodal lines, shown in Fig.2.1, represent the points where \( Y_{lm}^m \) vanishes. The number of nodal lines is determined by the degree and order of the spherical harmonic: the number of latitudinal nodes is \( l - |m| \) while the number of longitudinal nodes is \( |m| \) and both combined give sets of base functions with the shape shown in Fig.2.2:
2.3. **SPHERICAL HARMONICS**

Low-Order Dynamic Modeling of the Heart Motion Under Respiration

Figure 2.1: Representation of the nodal lines. Figure obtained from [4]

Figure 2.2: Shape of the first spherical harmonics. Figure obtained from [4]
Chapter 3

The Data

The data set used to generate the model of the moving heart is composed by a set of 2D MR sagittal images accompanied by their corresponding navigator signals. The images were obtained with a 3 Tesla MR machine from the chest of a volunteer at the University of Utah. The person was imaged with sagittal cuts. The images obtained were of 264x416 pixels of resolution, separated 6mm, covering the chest from the Sternum to the ribs at the left side of the thoracic cavity (Fig.3.1). Each of these slices has been sampled from 70 to 80 times in order to obtain as many snapshots of the breathing cycle as possible.

Figure 3.1: Set of Sagittal images from slices 1, 4, 7, 10, 15 and 20 taken at different respiratory phases.
The navigator signals are composed images of low resolution snapshots. These snapshots are presented in the form of columns in the image and show an absolute position inside the body at the time each sagittal image was taken. Each column has a marker indicating the position of the top of the liver, so that its height can be obtained. Due to the sampling process, the images are taken without considering the respiratory phase, implying that

- There is no prior knowledge nor control over the respiratory phases that are sampled.
- Since the phases sampled cannot be controlled, there is no guarantee that the data set contains a full heart, a image per each slice, for any of the sampled phases.

Also, the movement of the heart is not limited only to respiration. The beating of the heart produces changes of its shape that are uncorrelated to the breathing movement and are going to distort the interpolated model. In order to tackle this issue, there are two different approaches:

- Ignore it during the sampling process and correct it in the post processing stage. In this case, the sampling time is constant and, because it only depends on the machine used, is going to be fast ($T \approx 0.3s$). The main handicap is that the samples are affected by the beating of the heart, so the precision of the algorithm diminishes.
- Use ECG gating. The sampling time depends on the ECG signal of the patient that triggers the MR at a predefined ECG phase. The advantage of ECG gating is that the samples are always taken at the same phase in the heartbeat, so there is no deformation on the images because of it [35]. The disadvantages are that the sampling rate is not constant because it depends on the non-periodic ECG signal. Also, the time between samples is bigger than the the ungated case, since the time between pulses is about 1.3 seconds.

![Figure 3.2: Navigator Levels images for ECG gated and non-ECG gated data.](image-url)
Chapter 4

Segmentation

4.1 Image Segmentation

The first step of the algorithm is to extract the information of the heart boundaries from the sagittal images. In the data at hand, the heart appears as a white closed shape close to the center of the image, so the objective of segmentation algorithm is to find a set of points lying on the contours of this shape. Here we introduce the semi-automatic process that was used to extract these boundaries.

The algorithm is composed mainly by three parts. First it uses snakes method finds the boundaries of the heart with some user intervention. Second, it interpolates between the boundary points found. And third it applies smoothing, both in space and in respiratory phase dimension.

1. Manual mask:
   The user manually creates a search area mask to facilitate the subsequent automatic algorithm. This mask, one for each image, surrounds the heart and eliminates areas were the snakes algorithm could get stuck. To create this mask, we use Seg3D software, from the center for Integrative Biomedical Computing (CIBC) at the Scientific Computing and Imaging Institute at the University of Utah [36].

2. Initial search points:
   The initial search points of the snakes algorithm are then manually selected by the user.

3. Snakes:
   The snakes algorithm, based on Terzopoulos active contours [37], searches for the sharpest edges of the image. Starting from selected initial points, the Terzopoulos algorithm iteratively moves them to the closest higher contrast changes in the image, i.e. the boundary of the white heart against the dark lungs.

4. Interpolation:
   As seen in Section 2.1, the distribution of the samples has to be fair in order to avoid over-weighting of some regions. This is not compatible with the position of the points already obtained that are arbitrarily distributed over the boundary of the heart. Splines interpolation [38] is now used to redistribute these points around the curve in order to have the desired fair sampling.
5. **Spatial smoothing:**
   By simple observation of a normal heart, it is seen that its boundary is smooth. Based on this characteristic, it is assumed that any excessive sharpness on the segmented curves has to be product of noise from the imaging technique or the contour extraction, so smoothing of contours acts as a denoising filtering. Figure 4.1 shows the segmented and spatially filtered curves superimposed on the MR images.
   In this step of the segmentation algorithm, we apply a low pass filtering in the transformed Fourier space of the radius, so that any sharp spike or jagged part of the curve is smoothed.

6. **Smoothing in respiratory phase dimension:**
   Due to the smoothness of the heart, huge variations between curve shapes in different frames of the same slice are considered noise and thus filtered.
   This filtering is now applied to the ensemble of all frames in the slice. Every curve is moved to an equal arbitrary center, radii normalized, an average curve subtracted and normalized by its length and, finally, the set of resulting curves is then SVD filtered (eigenvalues truncated after a user defined number). The result after the de-normalizations, re-adding the mean curve and re-centering is a set of curves whose shape does not change much.

7. **Add 3rd dimension:**
   Since the model is in 3D plus phase, the position of the curves must be in the 3D space. This is achieved by simply adding the coronal position of the image plane retrieved from the DICOM heather.

![Figure 4.1: Stack of frames from the same slice with the detected boundaries.](image-url)
4.2 Respiratory Phase

Modeling of the movement of the heart due to respiration strongly depends on the metric used to determine where in the respiration cycle the images are taken. In this project, the system used was the navigator signals that, as commented in Chap. 3, determine the height of a fixed point of the liver. This information comes in the form of a set of images. Each of these containing the navigator signals of 24 sagittal images of the same slice. The structure of all the images is the same for the entire data set: 24 bars, containing the low resolution images, plus numeration of the bars and height, in lab coordinates, in meters. This predetermined structure allows us to create a method to extract the information of each sagittal image individually:

The algorithm:

1. **Select the search limits:** In order to avoid complications with other information on the image, the user manually selects a top and bottom search level that the algorithm establishes as an upper and lower limits for the height.

2. **High pass filtering:** The position of the liver in each bar is highlighted by a marker or a sharp intensity transition. To capture both indicators a high pass filter is run in the vertical direction.

3. **Boundary search:** Taking advantage of the previously explained structure, the algorithm searches in 24 pixel columns, one per frame, within the range manually indicated, for the highest value of the filtered image, which is taken as the position of the liver. The row of the pixel indicates the height in image coordinates, which we call *navigator level*.

![Figure 4.2: Navigator Image with the selected levels (red dots) and the search boundaries (green lines).](image)

4. **Sampling time:** To obtain the sampling time of the frames, we retrieve it from the DICOM headers of each sagittal image. It tells us the period with which the samples were taken and is then joined with the *navigator level*.

5. **Normalization:** We do not have any information about how deep the subject breaths nor when the lungs are completely empty. To avoid this ambiguity, we consider the highest and lowest values of the navigator observed in the whole data set as the maximum level of expiration (empty lungs) and inspiration (full lungs) respectively. Once the range of the respiration is determined, all navigator levels were normalized from 0 (max. inspiration) to 1 (max. expiration). In the sequel we use these *normalized navigator levels*.
6. **Elimination of outliers:** Depending on the respiration of the patient during the imaging, the values of the normalized navigator levels close to 0 or 1 might have too few samples for the convergence of the subsequent model. In this cases, these samples must be considered as outliers and not used further. Since, once the outlier samples were eliminated, the current range of the normalized navigator levels changed and re-normalization was required. Figure 4.3 shows the histogram of the absolute value of the normalized respiratory phases. The red bar indicates the threshold we chose below which the phases were eliminated.

![Histogram of the phases.](image)

7. **Sign detection:** The relative height of the diaphragm is not enough to determine respiration. It has been shown that the movement of the heart imposed by the respiration process changes depending on whether the patient is in the expiration or inspiration phase of the cycle. To capture this behavior, we need an indicator of the direction of the respiration.

   Simply comparing the levels around one is usually not enough to determine the direction, the interval between samples might be too big and the cycle might have changed direction. To overcome this aliasing problem, we rely on the model assumption that the breathing cycle is locally harmonic. Thus, a local fitting of harmonic curves gives an approximation of the real evolution of the breathing cycle, and the sign of its derivative determines the direction of the movement.

   This final respiratory phases, or just phases, are the Normalized Navigator Levels with the sampling time and the sign information.

   Figure 4.4 shows the respiratory phases with the harmonic expansion fitted on them. From this fitted curve we obtained the direction of the respiratory phase, indicated in magenta for expiration and black for inspiration.

   - **Smoothing:** Optionally, a smoothing of the data can be applied using the values of the previously fitted harmonic curves. Since the resolution of the navigator levels is not very high and there is room for a lot of noise, a smoothing of the data is recommended for filtering purposes. The problem comes with data with
Figure 4.4: Harmonic expansion fitting and sign detection for non-ECG gated data. The green line is the harmonic expansion fitted and the dots indicate the sampled phases. Magenta for expiration, black for inspiration.

too large sampling intervals, in these cases, although the harmonic fitting is good enough to provide sign information, it might introduce even more error if considering also the position. That is the case, is better to use the raw values. For example, with the ECG-gate data we worked with, smoothing is not a good option and we did not use it.

The respiratory phases thus obtained are a set of values ranging from -1 to 1 that can be introduced to a harmonic model where they accomplish the specification of forming a closed cycle in the model, meaning that full expiration (-1) is equal to the start of inspiration (+1) and full inspiration (0) equals the start of expiration (0).
Chapter 5

The Models

The two previous blocks of the algorithm convert the raw data obtained from MRI into a form that we can actually use to construct our models. Next comes the real aim of the project, the construction of different approaches to model the heart movement due to respiration.

Due to the specifications of the particular problem we are trying to solve, the model needs to be light in terms of computation and memory needs. Also we do not require it to be highly detailed in terms of shape precision, since we are focused on the global movement of the heart. These initial pre-requisites lead us to low-order approaches.

The following flow diagram (Fig. 5.1) shows the connection between the three models that we now introduce:

![Diagram of the whole algorithm connecting the models.](image)

Figure 5.1: Diagram of the whole algorithm connecting the models.
5.1 4D Model

The first objective set by our group was to create a 4 dimensional model able to generate a heart in 3D for any respiratory phase. These 3 dimensions in space plus phase form the 4D space.

5.1.1 Description

To create the model, we need a set of base functions that when appropriately linearly combined, define the heart at any given phase. Since the heart is a closed and convex shape, spherical coordinates are useful to describe the 3D space, so that we can characterize the radial functions of θ and φ then we add a parameter α that describes the phase dimension.

- The Base

First, from the literature on modeling a static heart in 3D, it has been shown in [8] that a low order spherical harmonic expansion presents a good approximation for it. So, with spherical harmonics of degree $L$ and order $M$, the representation in space is a description of the radius given $q$ and $f$.

$$ r(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} f_l^m Y_l^m(\theta, \phi) $$

Now we focus on the phase description. The assumption adopted about the respiration is that it locally behaves as an harmonic function. Therefore the evolution of the respiratory phases can be modeled by a $n_h$ degree harmonic expansion function of $\alpha$.

$$ f(\alpha) = 1 + \sum_{n=1}^{n_h} a_n \cos(\pi n \alpha) + b_n \sin(\pi n \alpha) $$

At this point we have a set of $L$ by $M$ spatial base functions that need to be evaluated for each one of the $n_h$ phase base functions in order to obtain the desired 4D model.

$$ r(\theta, \phi, \alpha) = \left( \sum_{l=0}^{\infty} \sum_{m=-l}^{l} f_l^m Y_l^m(\theta, \phi) \right) \cdot \left( a_0 + \sum_{n=1}^{n_h} a_n \cos(\pi n \alpha) + b_n \sin(\pi n \alpha) \right) $$

- The Coefficients

Once the basis that describes our hearts is defined, is necessary to know how to determine the coefficients of this model for a particular moving heart. This we do fitting the base functions to the data we have with a least squares fitting; in other words, finding the combination that best describes the data acquired in terms of squared error.

If this base functions can properly describe any heart, the previous equation with the adequate $f_l^m$, $a_n$ and $b_n$ represents the imaged heart, too. This way we evaluate all the base functions for each of the data points of the data set and compare to the extracted value of the corresponding radius. Written in matrix form is:

$$ \mathbf{r} = \mathbf{M} \cdot \mathbf{a} $$

In which $\mathbf{r}$ is a vector with the radii of all the data points over all phases, so it is of length $\text{numP}$. $\mathbf{M}$ is the matrix of the base functions evaluated of the $\theta$, $\phi$ and $\alpha$ of each data point, its size is $\text{numP} \times \text{LxMxn}_h$. And
\( \mathbf{a} \) is the vector of the linear combination coefficients, its length is \( L \times M \times n_b \).

This equation would be exact if the model was a perfect representation of the real world and the data had no noise, but in reality the equation is something like this:

\[
\mathbf{r} = M \cdot \mathbf{a} + \mathbf{e}
\]

Where \( \mathbf{e} \) represents the error of the model. We minimize the error in the \( l_2 \) norm, so least squares finds the optimal solution of the error:

\[
\| \mathbf{r} - M \cdot \mathbf{a} \|_2 = \| \mathbf{e} \|
\]

The later equation results after evaluation:

\[
(M \cdot M^T) \mathbf{a} = M^T \cdot \mathbf{r}
\]

- **The Interpolated Hearts**

Once the vector \( \mathbf{a} \) has been solved, to obtain a full heart at an arbitrary phase, is only necessary to evaluate the base functions at the desired \( \alpha \) for any grid of angles in \( \theta \) and \( \phi \).
5.1. 4D MODEL

Low-Order Dynamic Modeling of the Heart Motion Under Respiration

Figure 5.2: Reconstructed heart with 4D model.
5.1.2 Results

5.1.3 Positive

The first good result noticeable is that, indeed, we can generate a heart that moves with the respiration. Moreover, this movement fits the description in [20] where it is described as a main vertical movement inside the thoracic cavity with a little displacement frontwards and backwards creating a global ellipsoidal movement. With the quite shallow breathing cycle that we have in the data, the heart moves about 5cm in vertical direction and up to 1mm in the horizontal.

Second, the interpolated hearts are closed and smooth. They are not exactly the same shape as the actual heart, but a smoothed representation of it due to the low order approach. The error between the model and the data is relatively low for highly sampled slices, close to the center.

It is also worth mentioning that the low-order solution allows a quick interpolation a curve at any given plane for any given respiratory phase.

5.1.4 Not so positive

As mentioned before, the interpolated heart is a smoothed version of the original, which means that in parts where the original curves present high curvature, or in slices towards the top and bottom of the heart, there is high error. A possible explanation of the latter result is that the spherical harmonics might tend be too rounded in comparison to the more elongated heart.

The interpolation has also problems in following the heart for phases close to 0 or 1. The lack of data for some imaged slices of the heart at those phases explains this problem. Since the intermediate phases have more samples, there might be over-weighting of these in detriment of the extreme ones. This lack of data is also the cause of variations of the shape for different respiratory phases. Although it is true that the respiration has an effect of deformation and not only translation, the interpolation inflates too much the upper side of the heart when approaching $\alpha = 0$; at this phase, there are few or no data for this part of the heart.

All of this problems have an inherent cause underneath, the lack of data or, more precisely, the bad distribution of data across slices and phase.
5.2 3 Coefficients Model

Moving from the first model experiment, we wanted to correct the shape changes and simplify the model basing it on the three main characteristics that we want to extract: shape, position and rotation.

5.2.1 Description of the model

In this case, the training data is the output of the 4D model, which can generate a full heart at any desired respiratory phase. This source of data makes us become data rich, as now we can have any desired data points distribution in any of the 4 dimensions of the space with any desired density of samples. In the experiments, for example, we have used a set of 21 full hearts in phases equally spaced from $\alpha = -1$ to 1.

- **Center extraction:** First we obtain the center from each heart of the given hearts. Then we calculate the linear combination of harmonic functions of phase that best fits the centroids so that a center can be obtained for any $\alpha$.

- **Axis extraction:** We interpolate the axes. This way we know both the position and the direction of the heart for all the desired phases.

- **Shape extraction:** For the shape, the data hearts must be centered and rotated to the same direction so that they all are aligned. Then, the shape descriptors are another linear combination of spherical harmonics that fit the data, again found using least squares.

- **Heart interpolation**

  To obtain a heart at any respiratory phase, the necessary process is the inverse of the previous: generate the cloud of points with the shape using the spherical harmonics expansion calculated, move this shape to the interpolated centroid and rotate it according to the corresponding axis.

  From this point, to interpolate the heart we only need to move and rotate the pre-calculated points to the specific centroid and direction of the particular phase.
Figure 5.3: Reconstructed heart with 3 Coefficients model.
5.2.3 Positive

Like in the 4D model, 3 Coefficients model creates a heart that moves according to expectations, but this time without changes in the shape. This heart is still smooth in shape and is created by low-order methods. This time, even faster in generating a whole heart or just a cut from it at any given phase and plane. The modular system created also allows us to readjust the position of the heart if the interpolated model deviates too much from hypothetical real-time data in an application context.

Despite all the errors that we will now discuss, this model achieves its goal, that is, describing the movement of the heart due to respiration, and does it with a system that is extremely fast to interpolate and to adjust with real-time data.

5.2.4 Not so positive

Among the not so good results, the first is somehow contradictory; the specification of no shape changes forced in this model is not realistic since the heart actually deforms due to respiration.

Again, here we inherit some of the errors produced in the previous model. The shape is still too rounded and the error of the slices towards the ends of the heart is very high compared to the relatively low error in the middle slices. Also, the errors in axis extraction discussed in the previous model, produce rotation errors that increase the error in extreme slices for phases close to $\alpha = 0$.

Then again, the main problem we are facing here is that, although now the training data can be distributed at will, the 4D model that feeds this algorithm is highly dependent on a fair sampling in its 4 dimensions and the current data we are working with is far from optimal in this regard.
5.3 Slice By Slice Model

Realizing that with the data we currently have there are too many problems when applying least squares, we tried to move a step backward and try a different approach for the whole modeling. This time the interpolation is done in each slice plane, without considering the other slices. In this case the performance of each slice is independent of the good or bad sampling of its neighbors which gives a lot of freedom for them while loosing some information given by the adjacent slices. The objective of this model is to be used to feed the 4D model to avoid the respiratory phase fair sampling problems.

5.3.1 Description of the model

The reader must remember that this time we work with the each slice separately, so during this section we work on 2D (slice plane) plus phase space instead of the previous 3D plus phase. Also will notice some similarity with the method used for center and axis interpolation of the 3 Coefficients model:

1. The slices must be converted to polar coordinates using the average center of the curves as origin.
2. The radii, a function of angle $\phi$ of the curves, is then transformed to Fourier space.
3. Finally, a harmonic expansion function of $\alpha$ is fitted to the Fourier coefficients of all the curves in the slice, like we did in the 3 Coefficients model for the centroids.

- **Hearts interpolation**
  To recover a full heart from this interpolation method, each slice’s harmonic expansion must be evaluated at the desired phase and the 3rd spatial dimension, from the DICOM headers, added.
Figure 5.4: Reconstructed heart with Slice By Slice model.
5.3.2 Results

5.3.3 Positive

This time, the interpolated curves present very low error for all the slices when compared to the original data. The model can be used to feed the 4D model and avoid the instabilities generated by the bad distribution of samples over phase space.

5.3.4 Not so positive

The error comes when joining various slices, and this appears strongest, as usual, in phases close to $\alpha = 0$. When the slices move, a lack of coupling in the movement between adjacent slices becomes obvious for phases where more training data samples are needed. For these phases, low order fitting has to be used in order to avoid over-fitting that causes loops and other “funny” shapes in some curves. Also, this model does not include smoothness constraints outside the image plane, so the results have some jaggedness between slices.

Another negative point from this solution is that, in order to interpolate a full heart or simply an arbitrary slice, the full set of slices has to be generated independently in the original planes. This process is not as fast as we would desire for a model that needs to be readjusted real-time as the problem specifications require.

We are limited again by the same original problem, the data. The lack of data does not affect slices that are well sampled, and this allows us to achieve very good error rates for these. The problems come for slices lacking samples, this time worsening by the fact that these cannot use information from their better sampled neighbors.
Chapter 6

Discussion

In the previous section we have shown the results of the three models. These are good and promising, but there is still room for improvement. The aim of this section is to point out and discuss the problems that we have isolated and we consider that did not let us obtain better results.

The main obstacles we currently have are the lack of good data to work with, the problem of finding good metrics for the respiration, and the deformation caused by the heartbeat. In the following sections we discuss all three of them, but first we quickly review the results we have achieved.

6.1 Results Review

The first important result obtained with our models is that we can generate hearts that are moving with the evolution of the respiratory phase. This encourages us to improve the performance of the models, especially the 4D model and 3 Coefficients model.

Also, an important remark is that both models allow us to interpolate hearts and cuts from it given only the desired respiratory phase extremely quickly. allowing us to, in a future, work real-time with in-vivo data.

For now there is a lot of improvement to do on reducing error between the interpolation and the original MR data, especially in slices close to the ends, superior and inferior, of the heart.

6.2 Data Curse

We are data poor. This is the biggest pitfall we have had while working in this project. The problem is not that we have few samples per slice or not enough slices, more the distribution of the samples over the phase dimension. The MR imaging takes a lot of time to capture a full data set; it takes about 40 minutes from the first frame of the first slice to the last image, this is a lot of time for the subject to be inside the MR machine and to keep the breathing steady and not without strong variations in respiratory depth or rate. Because of breathing cycle changes the distribution of samples along the respiratory phases vary a lot from slice to slice yielding situations like having a lot of samples for some slices close to $\alpha = 1$ while being poor at $\alpha = 0$ and having the opposite situation for other slices.

Figure 6.1 shows the distribution of samples along slices and phases The vertical axis indicates the slice number
and the horizontal axis the histogram bin, 1 being $\alpha = 0$ and 51 $\alpha = 1$. The color indicates the number of samples per each cell according to the lateral colormap. The deep blue color that covers the left side of the image indicates no samples. This means that from $\alpha = 0.6$ to $\alpha = 0$ there is almost no data. This is the reason why elimination of outliers is applied during phase extraction, and forces us to discard part of the breathing cycle as an outlier. Clearly with this step we lose information and the model fails to cover the whole cycle of the respiratory motion. Nonetheless having a model of shallow breathing is preferable than a unstable model due to lack of data.

However, the problem of the non-existence of a sampling pattern in phase goes further. If the attentive reader

![Sample distribution histogram. Vertical axis are slices and horizontal phases from the lowest (0) to the highest (50). The colors show the number of samples.](image)

looks at the more populated phases, zoomed in Fig.6.2, will notice that the distribution of data over the slices is not as homogeneous as one would prefer it to be; instead slices near the center of the chest (slices ~ 1) have more sampling density in phases towards $\alpha = 1$ than $\alpha = 0$. The opposite happens for slices close to the ribs at the other side of the heart (slices ~ 25). These data gaps are the cause of the deformities appearing in the 4D model and the difficulties of the slice by slice model to follow the movement of some slices.

These data problems should be addressed by preparing the imaged volunteer. The subject must try to maintain a regular breathing pattern and avoid deep breaths or considerable variations in rate. This regulation does not have to be an absolute control of the breath making it perfectly sinusoidal, just to be reasonably constant.
6.3 Respiration Metrics

Another weak point of these studies is the characterization of the respiration. We conclude that the 1D measure of the respiration cycle, based only on the navigator signals, is a too simplistic and also yields a too low resolution metric.

There is not much precision of the measurement due to the coarse pixel resolution the navigator images present. Also, the liver is not a monolithic structure, it is flexible and it is not completely attached to the diaphragm. As consequence, its movement is not like a block and it takes time for the whole surface of the liver to react to the pressure of the diaphragm. Given that, the position of the whole surface of the liver would give a more accurate measure of the breathing cycle.

Another consideration about respiration is that there are multiple modes of respiration. Diaphragm contraction and relaxation is the main movement involved, but chest expansion or stomach movement are involved. With the 1D measure of the navigator signals, we gather information from the diaphragm only. There is much information lost that could be retrieved by optical markers on the chest and stomach that would enrich the information we have about the breathing cycle and increase the precision of our metric.

All these considerations fall in the same issue: the characterization of the breathing. Different patterns such as deep, shallow, regular or irregular should be distinguished and studies of the movement of the heart affected by different types should be addressed. This characterization also includes the definition of the range of the breath, which is not always equal, and raises questions about where to put the start and end of its travel or what happens when the patient stops the breath at a middle point of the expiration and continues inspiring (see figure 6.3).
6.4. HEARTBEAT DEFORMATION

Figure 6.3: Example of a broken cycle. Blue line shows the originally expected cycle, the red line a possible change in it.

6.4 Heartbeat Deformation

Another issue that causes difficulty for the performance of the algorithms is the deformation of the heart walls due to beating of the heart. A good solution to this problem, already used, is to apply ECG-gating when imaging, so that the images are always taken at the same point of the heartbeat. This solution indeed avoids deformation, but the time between samples triples so that without ECG-gating in earlier work we had a average of 12 samples per respiratory cycle and with the gating only about 4 samples per cycle. This increase of the sampling time affects the tracking of the navigator signals, introducing uncertainty about how the cycle really behaved, and also increases the total imaging time, so that there are more chances for the patient to change the breathing pattern.

For now the best solution for the breathing movement modeling is to use ECG-gating to avoid the deformation of the heartbeats, but in the future this model would be joined with the beating model and then the imaging would take place without the gating.
Chapter 7

Future Work

After identifying the problems we have faced, now it is time to discuss the next objectives we have to set in order to improve our methods.

7.1 Short Term Objectives

As commented in 6.2, data is the biggest barrier we have right now to advance. We need to obtain new data sets with better distribution of the samples over the phase dimension. For this, it would only be necessary to image another volunteer, this time asking the subject to breathe regularly within a controlled depth of inspiration and rate. Also it is desirable to combine this imaging with new breathing metrics, a feasible measure is, through optical systems, measure the position of the stomach and the chest during the imaging. This method has much higher sampling rate than MRI and allows us to follow the whole breathing cycle with no need of interpolations while giving the added value of new complimentary measures of respiration.

Second, the respiration process must be well defined in order to acquire the proper data for the model. The different types of respiration must be distinguished if we want to ask the patient to do one or the other while inside of the machine. The characterization of the breathing helps, too, to find new measurements that explain where in the respiratory cycle the images are. The work from [29] presents a good starting point for respiration characterization.

Another issue to tackle is the extraction of the heart main axis. At this point the SVD solution we currently have is too dependent on the data that does not have deformations on the shape. Another approach that keeps the direction of the axis without being so affected by these issues would be preferable for the 3 Coefficients model.

Finally, but not least important, we need a better definition of the error. Right now we are using simple absolute distance between the points of the interpolation versus the points extracted from the image.
7.2 Long Term Objectives

In the long run, there are three main objectives:

- Combine the movement due to breathing model with the heart beating model and create the 5D model that describes the whole movement of the heart inside the thoracic chamber.

- Create a correction algorithm of this 5D model to adapt it to reality with in-vivo, real-time measurements such as ECG, breathing controls or CT/MRI.

- Find a new shape basis for the heart. If not using new mathematical functions, create it based on learning it from real data.
Bibliography


