A GENERATION METHOD FOR ARBITRARY SPATIALLY-VARIANT POLARIZATION

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Abstract. A new approach for the generation of arbitrary spatially-variant polarization of a light beam has been developed by manipulating vertical and horizontal field component of the optical signal individually in a Mach-Zehnder interferometric setup employing two liquid crystal spatial light modulators. The optical system has been simulated, the simulation results are presented. A first prototype has been implemented experimentally from scratch, occurring problems are discussed and possible solutions proposed.

Keywords: spatially-variant polarization, phase hologram, spatial light modulator

1. Introduction

Polarization is a fundamental property of light. The propagation of polarized light and its interaction with matter have been extensively explored in optical inspection and meteorology, display technologies, data storage, optical communications, material sciences and astronomy, as well as in biological research [1]. While past studies mainly dealt with spatially homogeneous polarization states such as linear, elliptical and circular polarization, in recent years the interest for arbitrary spatially-variant polarized (ASVP) beams has increased significantly due to their special abilities and properties compared to homogeneously polarized beams. These peculiar properties are useful to expand the capability and enhance the functionality of optical systems.

With radial polarization for instance a laser beam can be focused to generate a strong longitudinal and non-propagating electrical field at the focal plane, which results in sharper spots than achieved with a common homogeneous polarized beam [2, 3]. This leads to controlled point spread functions [4] and hence improvements in microscope resolution, increased packing density for optical storage, and finer optical lithography [5]. Furthermore it is possible to generate three dimensional polarization fields by ingenious polarization engineering [6], which is very applicable for optical trapping and particle manipulation [7]. Additional applications which can be realized or improved by spatially-variant polarization are particle acceleration by the inverse Cerenkov effect [8], single molecule imaging [10], optical Fourier processors [9] and near-field optics [11].
The generation process of ASVP is still a challenging task. While static techniques including birefringent materials, interferometric setups or sub-wavelength diffractive optical elements, don’t allow dynamic or arbitrary encoding of two dimensional ASVP patterns \cite{12} a solution can be found by means of liquid crystal spatial light modulators (LCSLM), which act as controlled optical phase retarders or amplitude modulators. Our aim was to develop an optical system to generate dynamic ASVP that has less complexity \cite{13} and higher flexibility compared to former methods \cite{13, 14} due to the ability to encode all elliptical polarization states simultaneously. In our approach this flexibility will be provided by the individual processing of $x$ (horizontal) and $y$ (vertical) component of the incident beam and subsequent recombining in a Mach-Zehnder interferometric setup. For the manipulation of the beams two LCSLMs are employed displaying specific tailored computer generated phase holograms (CGPH).

This paper is organized in four main parts. In section 2 the basic theory of polarization is elucidated followed by a short introduction of the applied LCSLMs. Afterwards the computing process of the phase holograms and the subjacent development method are described. In section 3 our new approach for the generation of arbitrary spatially-variant polarization is presented and the ideal simulation of the setup implemented. Finally in section 4 the first prototype development is depicted and occurred problems are discussed. The work finishes with a summarizing conclusion and possible improvement proposals.

2. Basic Principles

For the generation of ASVP the understanding of polarization theory and computing of phase holograms for encoding complex transmittance is necessary. Hence the following paragraphs will introduce the theoretical foundations of polarization and the process of computer generated phase holograms. Finally the functionality of the employed spatial light modulators will be explained.

2.1. Polarization

Light is the visible range of the electromagnetic radiation. Thus it possesses the properties of electromagnetic waves, i.e. light waves consist of coupled electric and magnetic fields, which are oscillating in phase and perpendicular to each other as well as to the propagation direction.

A plane, monochromatic wave propagating in $z$-direction of the Cartesian coordinate system can be expressed by

$$\vec{E} = E_x \vec{e}_x + E_y \vec{e}_y,$$

with

$$E_x = E_{x0} \cos(kz - \omega t),$$

$$E_y = E_{y0} \cos(kz - \omega t + \psi).$$

(1)

where $E_{x0}$ und $E_{y0}$ are the amplitudes of the electric field components and $k = \frac{2\pi}{\lambda}$ the absolute value of the wave vector $\vec{k}$. $\omega = 2\pi f$ stands for the angular frequency of the
light wave with the oscillating frequency $f$. $(kz - \omega t)$ describes the phase at time $t$ and point $z$, $\psi$ is the phase difference between both components [16].

The orientation of this oscillation observed in the $x$- and $y$-plane is called **polarization**. Amplitude of and phase between both components determine the type of polarization. If the changes of the oscillation direction occur arbitrary the wave is denoted as unpolarized light [16]. On the contrary if the oscillation vector follows a well defined regularity the light is called polarized. One distinguishes between the three main polarization types **linear**, **circular** and **elliptical** polarized light.

### 2.2. Polarization Types

#### 2.2.1. Linear polarized light

Oscillates in a spatially fixed direction while absolute value and algebraic sign of the oscillation are changing periodically. This is the case if $\psi$ in (1) is equal to 0 or a multiple of $\pi$. Further the polarization angle $\varphi$ in relation to the $x$-axis can be determined by the amplitude ratio $\tan \varphi = \frac{E_y}{E_x}$.

#### 2.2.2. Circular polarized light

Occurs, if the phase difference between both oscillating components $\Delta \psi = \left(2n - 1\right)\frac{\pi}{2}$ with $n = \pm 1, \pm 2, \pm 3, etc.$ and their amplitude is of the same value $E_{x0} = E_{y0} = E_0$. With $\cos(\alpha \pm \pi/2) = \mp \sin \alpha$ and $\sin^2 \alpha = 1 - \cos^2 \alpha$ follows by [16] that

$$E_x^2 + E_y^2 = E_0^2. \quad (2)$$

Because of the constant absolute value of $\vec{E}$ at every time $t$ and every point $z$ the peak of the field vector apparently moves on the circle given by (2) with the radius $E_0$. Dependent on the phase difference one distinguishes between right circular polarized wave for $\psi = \frac{\pi}{2}, -\frac{3\pi}{2}, \frac{5\pi}{2}, etc.$ and left circular polarized wave for $\psi = -\frac{\pi}{2}, \frac{3\pi}{2}, -\frac{5\pi}{2}, etc.$

#### 2.2.3. Elliptical polarized light

Denotes every further well oriented combination of amplitude and phase of the oscillating wave components with $E_{x0} \neq E_{y0}$ and $\psi \neq n\pi$ and $\psi \neq 0$. With (1) and the general case $\psi \neq \left(2n - 1\right)\frac{\pi}{2}$, arises that

$$\frac{E_x^2}{E_{x0}^2} + \frac{E_y^2}{E_{y0}^2} - 2\frac{E_x E_y}{E_{x0} E_{y0}} \cos \psi = \sin^2 \psi. \quad (3)$$

Linear and circular polarized light are special cases of elliptical polarized light.

### 2.3. Arbitrary Spatially-Variant Polarization

A polarized light beam, for instance a laser beam, is declared spatially-variant polarized if its polarization type or direction differs in different points of the observed polarization plane. Any arbitrary combination of the in paragraph 2.2 mentioned polarization types can be involved in a ASVP beam. Figure 1 shows three different ASVP patterns with (a) radial, (b) azimutal and star and (c) a combination of linear, circular and elliptical...
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**Figure 1:** Samples of ASVP patterns with analyzed intensity distribution in the background: radial (a), azimuthal and star (b), combination of linear, circular and elliptical polarization (c).

polarization. The background of all patterns displays the intensity distribution of the polarized beam with a homogeneous amplitude when analyzed by a linear polarizer in horizontal direction.

2.4. Liquid Crystal Spatial Light Modulator (LCSLM)

Spatial light modulators (SLM) play a significant role in optical computing and wavefront encoding. As the name implies the SLMs are used to manipulate the properties of light at different spatial points. This is realized by converting data in electronic form (modulation function) into spatially modulated coherent optical signals [17]. By exploiting the principles of liquid crystals (LC) due to an applied voltage, liquid crystal spatial light modulators (LCSLM) can be constructed to obtain a dynamic light modulating device. LCSLMs have pixel structures that are transmissive or reflective. Either amplitude or phase or amplitude and phase of an incident light beam can be modified in each pixel individually by displaying a modulation function onto the LCSLM. In this prototype two phase only LCSLMs with parallel aligned liquid crystal molecules will be employed to encode ASVP.

2.5. Computer Generated Phase Holograms

By a convenient optical setup such the one is discussed in paragraph 3 and by displaying a well defined modulation function on the SLM an arbitrary complex optical output signal can be created. This modulation function is called computer generated phase hologram (CGPH) and can be expressed for a SLM with squared pixel size and pitch of $a$ as

$$h(x, y) = \sum_{n,m} e^{i\beta_{nm}} w(x - na, y - ma),$$  \hspace{1cm} (4)

where $w(x, y) = \text{rect}(x/a)\text{rect}(y/a)$ and $\beta_{nm}$ is the discrete CGPH phase modulation [18]. In 2003 Victor Arrizón developed an on-axis reconstruction algorithm for encoding complex optical signals with arbitrary amplitude and phase distribution with a phase only SLM by employing this CGPH $h(x, y)$. A complex optical output is fundamental for
our generation method of ASVP and thus we use Arrizón’s method for our purpose. The derivation in detail of the CGPH generation process can be inspected in the published paper [18], a rough overview is given in the following part.

2.5.1. Derivation The required discrete complex modulation \( c(x, y) \) is denoted as

\[
c(x, y) = \sum_{n,m} c_{nm} w(x - na, y - ma),
\]

whose \((n, m)\)th pixel has complex transmittance \( c_{nm} = |c_{nm}| \exp(i \epsilon_{nm}) \) following the condition that \( |c_{nm}| \leq 1 \). To encode the complex modulation (5) by the CGPH in (4) and reconstruct the signal on-axis the CGPH modulation error function \( e(x, y) \) has to be found, which fulfills the expression \( h(x, y) = c(x, y) + e(x, y) \). With \( e(x, y) = b(x, y)g(x, y) \), \( b(x, y) = \sum_{n,m} b_{nm} w(x - na, y - ma) \) a binary grating with the discrete modulation \( b_{nm} = (-1)^{n+m} \) and \( g(x, y) = \sum_{n,m} g_{nm} w(x - na, y - ma) \) a centered function with minimum possible bandwidth the noise field \( E(u,v) \) is given by four off-axis replicas of the function \( G(u,v) \) as shown in figure 2. \( E(u,v) \) and \( G(u,v) \) are the Fourier transforms of \( e(x,y) \) and \( g(x,y) \). These noise fields can simply be filtered by a spatial pupil filter in the Fourier domain of the reconstruction arrangement so that only the Fourier transform \( C(u,v) \) of the required complex signal \( c(x,y) \) remains.

By ingenious application of algebraic conversions the identity \( g_{nm} = |g_{nm}| \exp(i \tau_{nm}) \) with \( |g_{nm}| = (1 - |c_{nm}|^2)^{1/2} \) and phase \( \tau_{nm} = \epsilon_{nm} + d_{nm} (\pi/2) \) is obtained. For a high signal to noise ratio and a preferably narrow bandwidth of \( g(x,y) \) the binary factor \( d_{nm} \) must be determined for each pixel independently by a convenient method [18]. Finally \( \beta_{nm} \) of \( h(x,y) \) can be expressed as

\[
\beta_{nm} = \epsilon_{nm} + (-1)^{n+m} d_{nm} \cos^{-1}(|c_{nm}|).
\]
3. Conceptual design and ideal simulation

Our aim was to develop an optical setup for the generation of ASVP that has nominal complexity but high flexibility due to the ability to encode all elliptical polarization states dynamically and simultaneously. Compared to former methods our solution meets these demands by processing horizontal and vertical component of an incident electromagnetic field independently. The realization is based on two phase only LCSLMs displaying specific tailored CGPHs. Our approach for the optical system as well as an ideal simulation of the setup are given in the following paragraphs.

3.1. Setup description

Figure 3 represents the experimental scheme in a Mach-Zehnder interferometric setup. A widened, by LP\textsubscript{1} linear polarized laser beam with a planar wave front is split into two beams by the beam splitter BS\textsubscript{1}. Reflected by mirror M\textsubscript{1} and M\textsubscript{2} both beams are now transmitted through the transmissive LCSLMs SLM\textsubscript{1} and SLM\textsubscript{2} where the desired phase retardation will be applied by the CGPHs. To separate horizontal and vertical component from each other the oscillation direction of beam one is rotated 90° degrees by passing a λ/2-wave plate. Subsequently the components are recombined by the beam splitter BS\textsubscript{2} and led into the on-axis reconstruction system consisting of a 4f-Fourier lens system with FL\textsubscript{1} with focal length \( f_1 \) and FL\textsubscript{2} with focal length \( f_2 \), respectively. As discussed in paragraph 2.5 the spatial circular pupil filter in the Fourier plane of FL\textsubscript{1} is necessary for the signal isolation and noise reduction. In our setup both individually processed beams constitute the \( x \) and \( y \) component of the final beam. The induced polarization can either be evaluated by the analyzer LP\textsubscript{2} and the CCD camera or used for intended applications. Note that the Mach-Zehnder arrangement is convenient.
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Figure 4: Simulation Processing Chart.

for transmissive SLMs. For reflective SLMs a Michelson interferometric setup can be applied.

3.2. Simulation

The simulation of the experimental setup considers ideal properties of all used devices. Both LCSLMs have equal dimensions of 512 pixels in horizontal and vertical direction, each pixel has a $20\,\mu m \times 20\,\mu m$ size. The incident field amplitude is conveniently chosen to obtain a maximum output amplitude $A = 1\,V/m$. In this first prototype the ASVP states are determined by the angular distribution $\delta_{\text{amp}} = k\theta + \theta_{\text{amp}}$ and the phase shift $\delta_{\text{ph}} = l\theta + \theta_{\text{ph}}$ between $x$ and $y$ component of the processed electromagnetic field. $\theta$ is here the azimuth of the polar coordinates and $k, l$ and $\theta_{\text{amp}}, \theta_{\text{ph}}$ the topological charges and the initial angle and phase shift, respectively. In the special case that $\theta_{\text{ph}} = 0$ and $\theta_{\text{ph}} = \pm \pi$ the output field is linear polarized. Note that both angular and phase distribution are quasi-continuous, which means that the function is continuous except possible phase jumps of multiples of $2\pi$. To achieve these polarization states $x$ and $y$ component of the processed field must be of the complex form

$$A_{x,y}e^{i\theta_{x,y}}$$

where $A_{x,y}$ and $\theta_{x,y}$ are the amplitude and the phase shift of each component. This is realized by the CGPHs discussed in paragraph 2.5. Figure 4 displays the simulation
processing chart. Starting from the desired polarization state (a) the required complex output signals according to (7) are computed (b). These are matching the conditions $A = \sqrt{A_x^2 + A_y^2}$ and $\delta_{\text{amp}} = \arctan \frac{A_y}{A_x}$ with respect to the processed quadrant to obtain a plane and homogeneous output amplitude. It follows that $A_x = A(\sqrt{\tan^2 \delta_{\text{amp}} + 1})^{-1}$ and $A_y = A \tan \delta_{\text{amp}}(\sqrt{\tan^2 \delta_{\text{amp}} + 1})^{-1}$. Note that only the $x$ component is shown in the process chart since the $y$ component undergoes the same procedure.

With Arrizón’s method the hologram (c) is calculated and displayed on the LCSLM. The incident laser beam (d) is modulated and Fourier transformed (e) by the Fourier lens FL$_1$ [17]. The spatial filter (f) eliminates the unwanted off-axis noise bands (g) and the second Fourier lens FL$_2$ performs the back transform into the spatial domain (h). Superposition of $x$ and $y$ component lead to the resulting amplitude distribution with the desired polarization (i). The chosen polarization pattern in the simulation chart combines elliptical and linear polarization spatially-variant. The quality of the reconstructed beam depends on low noise influence of the error function within a large spectrum band around the zero frequency position and the opening size of the pupil of the spatial filter.

4. Prototype development

In the following paragraphs the development from scratch of a first experimental prototype to implement the theory for ASVP generation is presented. Due to the fact that only one LCSLM was available at the actual state, the experiment could not be fully developed. However the occurring problems are discussed and different compensation methods for employing non ideal SLMs are proposed.

4.1. Setup configuration

The prototype was realized with the optical beam handling system of the German company OWIS. The arrangement is pictured in figure 5 hitting the sketch of figure 3. The employed fiber laser has a continuous wave output of $\lambda = 633$ nm with a Gauss profile and a power of 25 mW. To decrease the laser beam intensity a set of optical filters is integrated behind the collimating lens $F_c$ with the focal length $f_c = 40$ mm. By $F_c$ the beam propagates parallel with a diameter $d = 12$ mm which is limited by the beam splitters and the mirrors due to their border length of 15 mm. The 4$f$-Fourier lens image reconstruction part contains the lenses FL$_1$ with $f_1 = 100$ mm and FL$_2$ with $f_2 = 50$ mm. The final analyzed beam is recorded by the CCD camera Marlin F-145B2 by Allied Vision Technologies with a resolution of 656x494 pixel. Only one twisted nematic (TN) LCSLM SLM$_1$ was available for this prototype. It was removed from a commercial available VGA Epson video projector EMP-3000 and has a resolution of 640x480 pixel. The amplitude modulation of SLM$_1$ is not constant, the phase modulation only reaches from 0 to $1.6\pi$ [20]. The camera as well as the LCSLM are controlled by a specially implemented LabView environment to obtain user-defined ASVPs.
4.2. Problem discussion

The first difficulty was caused by the Mach-Zehnder interferometric setup. Since there are no capabilities of vernier adjustment for mirrors and beam splitters in the optical beam handling system, the tuning of the interferometer could only be realized very roughly. Figure 6 (a) shows the occurring interference pattern if both beams are recombined after the second beam splitter remaining the same polarization direction. Even if it is possible to avoid the interference by rotating the polarization axis of the second beam as shown in figure 6 (b) the phase difference between the components will still lead to unwanted elliptical polarization. In the further development the Mach-Zehnder interferometer has to be calibrated accurately. The most problematic issue however was the application of the twisted nematic LCSLM. While non-constant amplitude and non-full range of phase modulation could have been neglected for first principle results, the twisted nematic (TN) property of the liquid crystal is a bigger challenge. Light passing through a phase modulation TN cell does not only experience a phase retardation but also a polarization rotation dependent on the applied voltage. Hence for a convenient employment of a TN LCSLM in the Mach-Zehnder setup to encode a desired complex modulation for each beam component individually a suitable compensation method included in the hologram generation has to be found in further works.

4.3. Conclusion and Outlook

The presented approach shows that it is possible to create ASVP in a simple optical Mach-Zehnder interferometric setup. While the simulation results already promise the generation of useful polarization patterns, the experimental realization still needs to be improved. Since working with a Mach-Zehnder interferometer is well known in optical science, the integration of vernier adjustment elements will provide a fast solution of the unwanted interference. One task for further works on this topic will be the implementation of the also by Arrizón et al. proposed compensation method for phase-mostly LCSLMs to deal with non-constant amplitude modulation [19]. On the other
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Figure 6: Final beam with (a) and without interference (b) due to in-built half wave plate.

hand the employment of TN LCSLMs has to be examined and a compensation method of the induced polarization rotation has to be found to avoid the very cost-intensive application of parallel aligned LCSLMs.

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References