Chapter 5

Surface interaction

5.1 Introduction

This chapter introduces the description of the surface interaction mechanism based on the friction, wear and excavation laws.

Whenever two solids touch each other so that forces of action and reaction are brought into play, the solids may be said to undergo a surface interaction (Rabinowicz 1995). This interaction phenomenon is vital in different fields of engineering when studying the influence of the forces in the friction and wear mechanical problems. The friction effects are those that arise from the tangential forces transmitted across the interface of contact when solid surfaces are pressed together by a normal force. The wear phenomenon consists on the removal of the material from the surfaces of one of the contacting bodies, as a result of the interaction with the other contacting body. Both phenomena have a direct influence in the excavation process.

In the microscopic scale, the solid surfaces present junctions referred as regions between the surfaces at which atom-to-atom contact takes place. The sum of the areas of all the junctions constitutes the real area of contact $A_r$. The total interfacial area, consisting on both, the real area of contact and those regions that appear as if contact might have been made there, will be denoted as the apparent area of contact $A_a$. These contact points are in charge of transferring the normal force and also generates friction.
5.2 Friction

Friction is the resistance to motion that exits when a solid object is moved tangentially with respect to the surface of another that it touches, or when an attempt is made to produce such motion.

Friction phenomenon is expressed as a force, being the one exerted by either of two contacting bodies tending to oppose relative tangential displacement of the other. It is important to distinguish between two situations, the first in which the applied force is insufficient to produce motion and is related to the static friction coefficient. The second one, related to the dynamic friction coefficient appears when imminent sliding occurs. The Coulomb law is used to describe the relative tangential kinetics of solids in contact in the PFEM. Simplifying the dependencies of the friction coefficient and incorporating only dependency of the sliding velocity $\dot{g}_T$ in the expression, the friction coefficient can be expressed as

$$\mu \left( \dot{g}_T \right) = \mu_D + (\mu_S - \mu_D) e^{-c(\dot{g}_T)} \quad (5.1)$$

where $\mu_D$ and $\mu_S$ denotes the dynamic and static friction coefficient. The constitutive parameter $c$ describes how fast the static coefficient approaches the dynamic one. This constitutive parameter will be taken as $c = 0.1$ for the calibration tests presented in this work.
5.2.1 Adhesion Theory

Adhesion is defined as the force that holds substrates together in opposition to stresses exerted to pull the substrates apart.

Three quantitative relations are required to express the magnitude of the friction force as a function of: the applied load, the size of the region of contact and the sliding velocity. These three quantitative relations are described as follows:

1. The friction force $F$ is proportional to the normal force $L$.
   \[ F = L \mu \]  
   where $\mu$ denotes the friction coefficient.

2. The friction force is independent of the apparent area of contact $A_a$. Thus, large and small objects have the same coefficients of friction.

3. The friction force is independent of the sliding velocity $\dot{g}_r$.

5.2.2 Rolling Friction

Rolling friction is the resistance to motion that takes place when an object is rolled over an abutting surface.

This phenomena is described considering a situation where the force to initiate rolling movement $F_r$ is equal to $L \tan \theta$, where $L$ is the load and $\theta$ the angle between the vertical and the line joining the center of gravity of the body and the projection point where rolling is going to take place. Thus, we may define a coefficient of rolling friction $F_r$ according to the equation

\[ F_r = \frac{F_r}{L} = \tan \theta \]  

If the contact of the two bodies takes place over a flat geometry, it is possible to express the rolling friction in the form

$$Fr = \frac{\nu_s}{\nu_r} F_k$$  \hspace{1cm} (5.4)

Where $F_k$ represents the kinetic coefficient of sliding friction, $\nu_s$ is the slip velocity and $\nu_r$ the rolling velocity.

As a general summary we may say that the force of rolling friction is a very small fraction of the applied load and is caused by a number of diverse factors.

### 5.4 Wear

Wear may be defined as the removal of material from solid surfaces as a result of a mechanical action.

#### 5.4.1 Types of wear

Modern research has established that there are four main forms of wear besides a few marginal processes that are often classified as forms of wear (Burwell 1957). However the present study will describe three of them:

1. **Adhesive wear**: It occurs when two smooth bodies are sliding over each other, and fragments are pulled off from one surface and adhere to the other. This phenomenon arises from the strong adhesive forces set up whenever atoms come into intimate contact.

2. **Abrasive wear**: It occurs when a rough hard surface, or a soft surface containing hard particles, slides on a softer surface and ploughs a series of grooves on it. The material from the grooves is displaced in the form of wear particles, generally loose ones.
3. Impact wear: It occurs when two surfaces collide while having large velocities normal to their interface.

5.4.2 Quantitative Laws of Adhesive Wear

It is possible to write the laws of adhesive wear as follows:

1. The amount of wear is generally directly proportional to the load $L$.
2. The amount of wear is generally proportional to the distance of sliding $x$.
3. The amount of wear is generally inversely proportional to the hardness $H$ of the surface being worn away.

We can also write the volume worn away in the form (Holm 1946)

$$V = \frac{c L x}{H}$$  \hspace{1cm} (5.5)

where $c$ is a no dimensional constant dependent on the materials in the contact and their exact degree of cleanliness.

Archard (1953) has presented a very plausible model of the sliding process that enables the definition of the constant $c$. Archard assumes that when two asperities come into contact and form a junction there is a constant probability $k$ that an adhesive fragment will be formed. Each fragment is assumed to be a hemisphere of diameter $d$ equal to the junction diameter.

![Figure V-4. Hypothetical model of a hemispherical wear particle](image)

The probability that any junction leads to the formation of a transferred fragment has been postulated to be equal to $k$, and, on the assumption that such a fragment is a hemisphere of diameter $d$ the volume $V$ of wear per distance $X$ of sliding is given by the relation
This expression may be considered the fundamental law of adhesive wear, and $k$ the dimensionless coefficient of wear.

The derivation of the Holm-Archard equation is straightforward. The equation itself is appealing because it is based on a simple energy relationship. The relation is linear. Assuming a constant wear coefficient, the more energy we introduce into the interface $L \times x$, the greater is the wear volume.

Adhesive wear coefficient values for lubricated metals

$0.00003 < k < 0.005$

5.4.3 Quantitative expression for abrasive wear

To derive a quantitative expression for abrasive wear, we may consider a simple model in which the asperities on the hard surface are conical.

![Figure V-5. Simplified abrasive wear model](image)

In this way we may express the adhesive wear in the form

$$V = \frac{k L x}{H}$$

(5.6)

In this equation the term $\frac{\tan \theta}{\pi}$ represents the adhesive wear constant $k$.

For both abrasive and adhesive wear, it is convenient to select the simplest form of the wear equation even if its physical significance becomes less apparent. Thus we write

$$V = \frac{kabr L x}{H}$$

(5.8)

where $kabr$ represents the abrasive wear coefficient.
Abrasive wear coefficient values for 2-body cases
\[ 0.006 < k_{abr} < 0.06 \]

Abrasive wear coefficient values for 3-body cases
\[ 0.0003 < k_{abr} < 0.003 \]

5.4.4 Impact wear

Impact wear is the wear that occurs when two surfaces collide while having large velocities normal to their interface.

As a quantitative measure of the wear, we may compute a simple no dimensional parameter, based on an Archard-type wear coefficient. Starting with the relationship

\[ V = \frac{K L n}{H} \quad (5.9) \]

where \( V \) represents the energy dissipated in sliding \( \frac{L}{f} \times \) product of the load \( L \), the sliding distance \( x \) and the friction coefficient \( f \); \( K \) is the non-dimensional impact wear coefficient \( \frac{k}{f} \alpha = K \) and \( \alpha \) is the porportion of the total impact energy expended in the interfacial slip.

In general only about 0.5 of the energy is no elastic, and only a relative small proportion of that would be expended in shear at the interface, expecting \( \alpha \) to be around 10%. For unlubricated surfaces, \( f \) is generally in the range of 0.5-1.0. Based on these values it is possible to define a range for the impact wear coefficient as follow

Impact wear coefficient values
\[ 0.000001 < K < 0.0005 \]

The general conclusions are that impact wear is not itself a form of wear but merely describes one way in which the normal force and shearing action required to produce friction and wear can be applied.
5.5 Excavation

The mechanism of cutting and digging on the ground is not so much different compared with wear a solid surface. The most significant difference is the scale where the process takes place. The wear of materials usually is associated with the micro-scale of the solid, but on the other hand when concerning the erosion of a fluid on a solid surface the scale starts to change. The same happens when a solid surface digs onto another. From that we resolve that excavation processes can be described with the same physical variables and laws of wear.

The most common mechanical processes in an excavation are the material fracture and crush. By means of these two mechanisms the cutting tool removes the material of the surface. The distribution of forces in the contact surface and the properties of the material govern these processes. With the coupling of the constitutive behavior from crushing and fracturing a rate of removed volume can be defined. The volume loss rate depends on the relative displacement of the cutting tool and the hardness of the material. A rate function is used for the description of the excavation as was done with wear.

There is an inexorable declaration of the theory of dimensions which says that there is no other way in which the key variables of load, distance of sliding and hardness can be arranged. Thus, the excavation model will be similar to an Archard-type equation. Taking as a reference the equation (5.7), the evaluation of excavated volume is given by:

\[ V_d = K_d \frac{L_N g_T}{H_d} \]  \hspace{1cm} (5.10)

where \( V_d \) is the volume loss of material along the contact surface, \( g_T \) is the sliding distance of the cutting tool, \( L_N \) is the normal force to the contact surface and \( H_d \) is the equivalent hardness of the material for an excavation process. The constant \( K_d \) is the non-dimensional digging coefficient, which depends on the relative contribution of body grooving abrasion and fracture process operating.

It is important to notice that equation (5.10) is a very general one. For its features it would be useful for indenters and drag bits, but not for other excavation tools like discs. In this case, a particular model is formulated.

5.5.1 Boring machine

The proposed excavation model, equation (5.10) must be adapted to the tunnel boring machine under analysis. The model is specially determined by the sum of cutting tools employed in each case. The critical conditions for wear and excavation will appear in hard materials, where the interaction between the solid ground and the boring
machine is harder. For the excavation of these materials two types of machines are used:

- Tunnel Boring Machine (TBM’s) for a more general excavations inside of a certain range of rocks hardness and
- Roadheaders, which allows more punctual attacks usually in softer rocks.

The characteristic cutting tools for TBM’s are discs and for Roadheaders are drag picks.

Figure V-6.Tunnel Boring Machine (TBM’s)

Figure V-7.Integrated Road Header.
5.5.2 TBM’s

Some basics concepts related with the design of TBM’s are presented here in order to formulate an excavation model for cutting discs. The objective is to define a boring law, similar to equation (5.10) to predict the amount of material that is excavated during the interaction of a TBM with a solid ground. Therefore, some characterizing parameters are introduced.

The TBM thrust, or normal force, usually is distributed along the discs of the machine head. For each cutting disc two force variables are defined: The normal force $F_n$ and the disc rolling force $F_r$.

![Diagram of a TBM head with forces $F_n$ and $F_r$.]

The cutting rate is defined as $\frac{s}{p}$, where $s$ are the disc spacing and $p$ the penetration of the disc due to a machine head revolution. Usually $p$ is expressed by the index $PRev = p$, which means the penetration per revolution. $PRev$ is controlled primarily by $F_n$. The ratio $F_n$ and $F_r$ is defined as the cutting coefficient $C_c$, this is:

$$C_c = \frac{F_n}{F_r}$$ (5.11)

The penetration of the disc in the solid material $p$ is a nonlinear function of the disc normal and rolling forces. The interaction, in the case of rocks is presented in the next figure.
Figure V-9. Disc cutter force variation with penetration for a high and low strength rocks

The changing slope corresponds to a transition in the dominance between crushing and chip formation and has been called the critical thrust. Unless a force of this magnitude can be applied, chipping between grooves will not occur. The critical thrust is directly related to rock strength and increases with cutter spacing $s$ and disc edge width $w$.

Using that information, the prediction of the excavated volume can follow the simple model of Figure V-10. The cutting tool has a known section. The part of the tool that will penetrate on the solid domain can be considered that has a sectional area of $p.w$. Thus when the disc moves through a distance $dx$ it will remove a volume $dV$ which is given by

$$dV = p.w.\,dx \quad (5.12)$$

The positioning of discs on a cutting head of a TBM is the factor that performs the excavation. The chips formed between discs must be also considered as removed volume, which gives

$$dV = p.w.\,dx + p.s.\,dx \equiv PRev.\,(s + w)dx \quad (5.13)$$

Equation (5.12) accounts the kerf interaction and chip formation which basically depends on the disc geometry and penetration index. There are some empirical equations developed from data on rock testing that can be used to determine $PRev = PRev\,F_n$. The most important are presented as follow
For sedimentary rocks, Farmer and Glossop

$$P_{Rev} = 624 \frac{F_n}{\sigma}$$  \hspace{1cm} (5.14)

where $\sigma$ is the Brazilian tensile strength.

For hard rocks (UCS 140 to 200 MPa), Graham

$$P_{Rev} = 3940 \frac{F_n}{UCS}$$  \hspace{1cm} (5.15)

where $UCS$ is the uniaxial compressive strength.

For mining in coal, Hughes suggested

$$P_{Rev} = 1.667 \left( \frac{F_n}{UCS} \right)^{1/2} \left( \frac{2}{D} \right)^{0.6}$$  \hspace{1cm} (5.16)

Where $D$ is the disc diameter in millimeter assuming that only one disc tracks in each kerf groove, the normal practice in TBM design.

Expressing the variable in SI units, the following expression is written

$$V_d = K_p \frac{F_n g_T}{UCS}$$  \hspace{1cm} (5.17)

where $F_n$ represents the total normal force, $g_T$ is the relative sliding distance and $K_p$ is the boreability coefficient, that physically represents the weighted average of the values of spacing $s$ and the disc width $w$. 

Figure V-10. A simplified excavation model for a removal of a solid material by means of a cutting disc.
\[ K_d = \frac{k_p (w+s)}{10^3} \]  \hspace{1cm} (5.18)

Depending on the selected equation for the determination of \( PRev \) different expressions of \( K_d \) can be formulated. That means that \( K_d \) is mainly a parameter that has to be calibrated using the properties of the excavated material and the geometrical properties of the TBM’s cutter head: disc distribution and disc geometry.

Equation (5.17) depends on the normal force applied to the excavation front \( F_N \). This is only one component of the contact forces that the head of the TBM receives. There are also the rolling forces on the discs of the TBM that have the tangential direction. From equation (5.11) the relation between rolling forces and normal forces is defined by the cutting coefficient \( C_C \). The inverse of this coefficient can be seen as an analog friction coefficient for the macroscopic scale \( \mu_{exc} = \frac{1}{C_C} \). Therefore the tangential forces on the cutting head of the TBM are defined as

\[ F_T = \mu F_N \]  \hspace{1cm} (5.19)

Usually \( F_T \) is defined as the cutting force and \( F_N \) as the thrust force of the tunneling machine.

The cutting coefficient \( C_C \) can be predicted as a function of \( PRev \) and the disc diameter only, with the influence of rocks strength implicit in the achieved \( PRev \).

Equation (5.17) is only a simple excavation law for the performance of TBM machines. The accuracy must be tested in future with a calibration and a disciplined investigation on the field.

There are other geomechanical characteristics with big influence in the solid (rock) properties for excavation: the rock quality design (classified by the index RQD), the spacing of the most influential joints of the rock masses (classified by the Join Spacing index), the groundwater situation, the orientation of the joint sets respect to the engineering work. These parameters are used to classify the Rock Mass Rating (RMR) which gives a description of the rock mass. That information could be used and included in a more precise excavation model.
5.5.3 Road Headers

Road headers are another type of boring machines used for ripping and milling rocks.

![Figure V-11.Milling and ripping cutting systems.](image)

Usually roadheaders are composed by drag picks installed on a rotation head at the end of a strong mechanical arm. The excavation system is performed by the pick cutting sequence. The first pick that points out will let a free face for the successive picks. There are two main types of picks: the conic bit and the chisel.

![Figure V-12.Types of picks.](image)

Usually chisels are more efficient than other types of picks in soft rocks where the conic bit is the least efficient because it concentrates smaller forces during the cut. But for abrasive rocks and hard rocks, conic bits will be more robust.

The cutting scheme for each individual pick corresponds to the most basic model presented for the particular case of drag picks as

\[
V_{dp} = K_{dp} \frac{F_N g T}{H_d} \quad (5.20)
\]
where \( K_{dp} \) is a measure of the boreability or excavability that depends on extra variables like the clearance angle in the cutting direction \( \beta \) and the rake angle of a bit \( \alpha \).

\[
K_{dp} = \frac{\tan \theta}{\pi} \quad (5.21)
\]

where \( \theta = \beta + \alpha \), is the sum of the average clearance angles and the average rake angle for the bits of the Road header.

5.6 Excavation with the PFEM

The goal of using a particle based method is to have the capacity of modeling rapidly changing geometries. When a hard solid domain digs on a softer one the geometry of the surface continuously changes. Some part of the domain is removed from the excavation solid. This is how the phenomenon occurs and how it is modeled with the PFEM.

In the PFEM every particle represents a part of the domain volume. The surface of a body is composed by particles that have the superficial volume of the same body associated to them. By means of the contact model, the interaction between two solid domains can be quantified. The result is the normal force \( F_N \) associated to the surface particles, and the relative velocities or sliding distance. For this reason, for each particle in the surface, the volume loss of material can be estimated in time by the excavation model

\[
V_d^{t+\Delta t} = V_d^t + K_d \frac{F_N}{H_d} (v_t \cdot \Delta t) \quad (5.22)
\]
where \( v_t \) is the relative tangent velocity between the contact surfaces and \( \Delta t \) is the time step. The volume loss of material \( V_d \) can be compared now with the volume associated to each contact particle. There are two ways to apply the volume loss prediction \( V_d \) to the geometry shaping.

5.6.1 Particle liberation

The first approach for excavation with the PFEM is the Particle liberation. First, each particle of the domain has an associated volume \( V_i \). During the contact interaction the excavation model (5.22) is evaluated and the loss of volume for each particle is computed \( V_i^f \). That volume is subtracted from the volume of the particle \( V_i \). The particle will be liberated from the body domain when its associated volume is null.

The readaptation of the geometry is done by means of the domain remeshing, the liberated particles are not going to be included anymore in the body description. The geometrical changes are applied intrinsically with the method not affecting the future contact interactions.

![Figure V-14. Particle liberation excavation strategy.](image)

5.6.2 Surface shaping

The second excavation shaping scheme developed for the PFEM is the Surface shaping. It is a more precise alternative than the particle liberation when the geometrical changes due to volume loss have to be applied. It consists in a reduction of the domain volume by means of moving surface particles inwards the domain. The movement of the surface particles changes the volume of the body coinciding with the computed volume loss. This operation shapes the boundaries and improves the geometrical adaptation to the excavation. When a gradual loss of volume is performed the error induced by volume overlapping is minimized.
The scheme to shape the surface is not based in the comparison between volume loss and volume assigned to particles. It is based in the volume reduction from the current mesh. The proper distance to apply in the shaping is computed comparing the volume loss $V_d$ and the volume that needs to be reduced. In 2D this volume belongs to a triangle formed with the contour neighbor particles. In 3D it fits to the pyramid formed with the contour neighbor particles.

The process is carried out within the remeshing scheme of the PFEM. The new position of the particles establishes the new boundaries automatically by means of the $\alpha$-shape concept.

The strategy of shaping the solid surface also liberates particles when that particles are going to overlap the inside domain. In that case the criterion of the particle liberation scheme is applied. Thus a combined method is implemented in order to update the geometry with the changes in volume produced by the excavation.

During excavation, the cutting tools are worn out due to the operation. The surface that plays the role of cutting digs into the other surface. At the same time the excavating surface of the cutting tools are worn out by the abrasive excavated ground. Therefore an excavation and wear model must be included in the problem.

### 5.6.3 Justification for the conservation of mass

When the particles of the domain are set free due to the wear and excavation they are released from the model. Hence the global volume of the problem changes as the number of particles is reduced. The historical values of the variables in these particles are lost as them do not contribute to the system anymore. The mass of the domain decreases with the volume reduction.
In fact the mass conservation is fulfilled. During the mechanical computation, inside of each time step, all balance laws are preserved. Thus, the equilibrium of the physical system is reached satisfying all fundamental laws.

5.6.4 Constitutive models and excavation

Rocks and hard soils represent the most common sort of materials concerning excavation problems. The standard behavior for these materials is brittle. Hence they damage and fracture quickly. In order to model this behavior a Damage model is a good option.

Damage which fractures the ground due to excavation is considered within the excavation model. The boreability coefficient $K_d$ includes this phenomenon (5.18). This coefficient defines the material properties that quantify the damage produced on the surface of the geomaterial, but only in the cutting region. It is important to note that the damage from the stresses generated in the excavation can be extended inwards and affect other parts of the excavated material. The damage not directly produced by the excavation crush is described by the constitutive model but not by the excavation one.

When a region of the ground is damaged, automatically is considered with a modification of the boreability coefficient $K_d$ for this area. Material properties change and are easier to dig on it. Therefore the damage model affects the excavation model by means of this coupling. The modification on the boreability coefficient is applied using the damage variable $d$ as:

$$
\tilde{K}_d = \frac{K_d}{1-d} \quad \text{with} \quad d \in [0,1] 
$$

(5.22)

Where $\tilde{K}_d$ includes the damage influence in the volume loss rate. Introducing the damage influence in the general equation:

$$
V_{dp} = \left(\frac{1}{1-d}\right) K_{dp} \frac{F_{N}}{H_d} 
$$

(5.22)

The layer of elements on the contact surface is not described by the geomaterial constitutive law. Instead of that an elastic layer is used in order to avoid the local effects that contact forces produces in contact surfaces.

As occur with worn particles, totally damaged particles have no more contribution to the mechanical behavior of the domain. Thus when particles on the body surface reach the maximum damage ($d = 1$) they are removed from the analysis domain.