Final project

Phase error assessment of MIRAS/SMOS by means of Redundant Space Calibration method

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<td>AP</td>
<td>Antenna Plane</td>
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<tr>
<td>BPF</td>
<td>Band-pass Filter</td>
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<td>CAS</td>
<td>Calibration System</td>
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<tr>
<td>CCU</td>
<td>Central Control Unit</td>
</tr>
<tr>
<td>CDTI</td>
<td>Centro para el Desarrollo Tecnológico Industrial</td>
</tr>
<tr>
<td>CIP</td>
<td>Calibration Internal Plane</td>
</tr>
<tr>
<td>CMN</td>
<td>Control and Monitoring Node</td>
</tr>
<tr>
<td>CMU</td>
<td>Control Monitoring Unit</td>
</tr>
<tr>
<td>CNES</td>
<td>Centre National d’Études Spatiales</td>
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<tr>
<td>DICOS</td>
<td>Digital Correlation System</td>
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<tr>
<td>ESAC</td>
<td>European Space Astronomic Centre</td>
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<tr>
<td>ESTAR</td>
<td>Electronically Steered Thinned Array Radiometer</td>
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<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
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<tr>
<td>FTR</td>
<td>Flat Target Response</td>
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<td>HAP</td>
<td>Horizonal Antenna Plane</td>
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<td>IVT</td>
<td>Image Validation Test</td>
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<td>LICEF</td>
<td>Light-weight Cost-Effective Front-end</td>
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<td>LNA</td>
<td>Low Noise Amplifier</td>
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<tr>
<td>LO</td>
<td>Local Oscillator</td>
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<td>MIRAS</td>
<td>Microwave Imaging Radiometer using Aperture Synthesis</td>
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<td>NDN</td>
<td>Noise Distribution Network</td>
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<td>NIR</td>
<td>Noise Injection Radiometer</td>
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<tr>
<td>NS</td>
<td>Noise Source</td>
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<tr>
<td>PD</td>
<td>Power Divider</td>
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<tr>
<td>PMS</td>
<td>Power Measuring System</td>
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<tr>
<td>RF</td>
<td>Radio Frequency</td>
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<td>RFI</td>
<td>Radio Frequency Interference</td>
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<td>RSC</td>
<td>Redundant Space Calibration</td>
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<td>SMOS</td>
<td>Soil Moisture and Ocean Salinity</td>
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<td>SMOS-BEC</td>
<td>SMOS Barcelona Expert Centre</td>
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<td>TPR</td>
<td>Total Power Radiometer</td>
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<td>VAP</td>
<td>Vertical Antenna Plane</td>
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Chapter 1

1 Introduction

This chapter is intended to explain the scope and the objectives of this project and its organization.

1.1 Scope and objectives of the final project

This project has been developed in the framework of the SMOS (Soil Moisture and Ocean Salinity) mission carried out by the European Space Agency (ESA) (4). The first steps were done in February 2010, three months after the SMOS satellite was launched, in the Remote Sensing Laboratory (5) of the Department of Signal Theory and Communications at the Universitat Politècnica de Catalunya.

One of the main objectives of any mission is to obtain and provide stable and accurate data. So, a well-calibrated instrument provides the basis for stable and accurate measurements. The calibration of any Earth Observation sensor is a key stage which encompasses those tasks which are necessary to convert the raw measurement data into science data. The characterization of the instrument is a requirement for the development of the calibration activities.

The aim of this project is to assess the phase error of SMOS payload, MIRAS instrument, developing a method called Redundant Space Calibration (RSC) (1)(2). This method is based on the use of the visibility samples measured by the instrument to retrieve the phase introduced by the receivers and show how the brightness temperature and the images are affected.

1.2 Organization of the final project

This project is divided in ten chapters, where two sections can be distinguished: the chapters of the first section explain the SMOS mission, MIRAS instrument and the calibration and imaging processes; while the chapters of the second section explain the phase assessment, the method used and the simulations that have been carried out.

Chapter 2 explains the basic concepts of radiometry needed to understand the working principles of MIRAS.

Chapter 3 gives an overview of the SMOS mission, describing in detail the architecture, the operating principle and the main elements and subsystems of the instrument.

Chapter 4 explains the obtaining process of the calibrated visibility and some of the amplitude and phase calibration procedures performed on-flight in the instrument.
Chapter 5 gives an overview of the equations and algorithms used to retrieve the brightness temperature, as well as the sampling and windowing processes performed to the visibility and the two inversion algorithms used.

Chapter 6 explains in detail the Redundant Space Calibration method, emphasizing in the equations used to build the system, how they are obtained and the properties of the system with a simulation.

Chapter 7 gives an overview of the phase behaviour assessment of the redundant baselines, giving special importance to the situations that must be avoided when the visibility samples are selected to perform a phase analysis.

Chapter 8 assess the quality of the RSC method by studying the pointing errors and the image degradation.

Chapter 9 retrieves the relative phases by applying the RSC method using flight data over oceans and shows a strategy to discard bad estimations of these phases.

Finally, chapter 10 explains the conclusions obtained from this project and the further work that must be done.
Chapter 2

2 Fundamentals of radiometry

All bodies with a physical temperature higher than 0 K (-273.15 °C) emit electromagnetic radiation. The field of the engineering that studies and measures this radiation is the radiometry.

MIRAS instrument, which is boarded in the SMOS satellite, is a radiometer, that is, a passive instrument that measures the electromagnetic radiation emitted by the Earth. This chapter is devoted to explain the basic concepts of radiometry needed to understand the working principles of remote sensing and MIRAS radiometer (2)(3).

2.1 Brightness and power measured by an antenna

The power emitted by a body at a solid angle per unit area [W·sr⁻¹·m⁻²] is called brightness. The definition of the brightness for an extended source of incoherent radiation area with a determined pattern is:

\[ B(\theta, \phi) = \frac{F_i(\theta, \phi)}{A_i} \]

where \( B(\theta, \phi) \) is the brightness, \( F_i(\theta, \phi) \) is the radiation pattern of the source and \( A_i \) is the effective radiating area.

Considering the case of two lossless antennas separated a distance \( R \) (large enough to consider constant power over a solid angle \( \Omega_i \)), oriented in the direction of maximum directivity, with effective areas \( A_i \) and \( A_r \) for the transmitting and the receiving antennas respectively, as shown in Figure 2.1:

![Figure 2.1: Geometry for the power received from an emitting source](image)

\[ \text{Figure 2.1: Geometry for the power received from an emitting source} \]
The power measured by the receiving antenna $P_r$ is described by equation 2.2:

$$P_r = S_t \cdot A_t$$  \hspace{1cm} 2.2$$

where $S_t$ indicates the power density, which can be defined as:

$$S_t = \frac{F_1}{R^2}$$  \hspace{1cm} 2.3$$

Replacing equations 2.1 and 2.3 in equation 2.2, the power measured by the antenna depending on the brightness is:

$$P_r = B \cdot A_t \cdot \frac{A_t}{R^2}$$  \hspace{1cm} 2.4$$

The solid angle subtended by the transmitting antenna $\Omega_t$ which is observed by the receiver antenna can be expressed as:

$$\Omega_t = \frac{A_t}{R^2}$$  \hspace{1cm} 2.5$$

Therefore, the power measured by the antenna is:

$$P_r = B \cdot A_t \cdot \Omega_t$$  \hspace{1cm} 2.6$$

If the emitting surface is not observed by the receiving antenna in the direction of maximum radiation, the radiation diagram must be included in the equation:

$$dP = A_t \cdot B(\theta, \phi) \cdot \left| F_n(\theta, \phi) \right|^2$$  \hspace{1cm} 2.7$$

If the brightness is not constant with frequency, it is defined the spectral brightness density $B_t(\theta, \phi)$ with units [W·sr⁻¹·m⁻²·Hz⁻¹]. Therefore, the total power measured by the antenna can be obtained by integrating equation 2.7 in bandwidth and space:

$$P = \frac{1}{2} A_t \int_{f}^{f+\Delta f} \int_{4\pi} \int_{4\pi} B_t(\theta, \phi) \left| F_n(\theta, \phi) \right|^2 \, d\Omega \, df$$  \hspace{1cm} 2.8$$

The term $\frac{1}{2}$ in equation 2.8 takes into account that the antenna presents a determined polarization and only half the thermal power emitted will be measured if the source emission is randomly polarized.

### 2.2 Thermal radiation

As mentioned before, all bodies at a physical temperature above 0 K emit electromagnetic radiation. According to quantum theory, each spectral line corresponds to the transition of an electron from an atomic energy level $\varepsilon_1$ to a lower energy level $\varepsilon_2$. The emitted radiation occurs at a certain frequency given by Bohr’s equation:
where the parameter $h$ is Planck’s constant ($h = 6.626070 \times 10^{-34} \text{ m}^2 \cdot \text{kg} / \text{s}$)

Atomic emission is caused by a collision with another atom or particle. The probability of emission is higher for higher atomic and kinetic energy densities. According to Kirchhoff’s law of thermodynamic equilibrium, all the energy absorbed is re-emitted.

In the case of a black body (perfectly opaque ideal body that absorbs all incident radiation at all frequencies) the radiated energy follows Planck’s law, so radiates uniformly in all directions with a spectral brightness $[\text{W} \cdot \text{sr}^{-1} \cdot \text{m}^2 \cdot \text{Hz}^{-1}]$ which corresponds to the following expression:

$$B_f = \frac{2h \cdot f^3}{c^2} \cdot \frac{1}{e^{\frac{hf}{k_B T_{\text{ph}}}} - 1}$$

where $f$ is the frequency [Hz], $k_B$ is the Boltzmann’s constant ($1.38 \cdot 10^{-23} [\text{J} \cdot \text{K}^{-1}]$), $T_{\text{ph}}$ is the absolute temperature [K] and $c$ is the velocity of light ($3 \cdot 10^8 [\text{m} \cdot \text{s}^{-1}]$).

Stefan-Boltzmann obtained another expression for the total brightness by integrating equation 2.10 on the whole spectrum, so that the brightness of a black body responds to equation 2.11:

$$B_{\text{bb}} = \int_0^\infty B_f \, df = \frac{\sigma \cdot T_{\text{ph}}^4}{\pi}$$

where $\sigma = 5.67 \cdot 10^{-8} [\text{W} \cdot \text{sr}^{-1} \cdot \text{m}^2 \cdot \text{K}^{-4}]$ is the Stefan-Boltzmann’s constant.

Figure 2.2 shows the variation of the brightness spectral density with the frequency for different physical temperatures:

![Figure 2.2: Brightness spectral density for different frequencies and temperatures](image)
Two approaches of Planck’s law are used depending on the range of frequencies, which are shown in Figure 2.3:

![Image of Planck's Law and Rayleigh-Jeans Law](image)

**Figure 2.3: Approaches of the Planck’s law at 300 K: Rayleigh-Jeans and Wien’s laws**

For high frequencies, equation 2.10 is reduced to the Wien’s law as follows:

\[
B_f = \frac{2h}{c^2} \cdot f^3 \cdot e^{\frac{h}{k_B T_{ph}}} \tag{2.12}
\]

In the case of low frequencies, the function approaches the Rayleigh-Jeans law:

\[
B_f = \frac{2f^2 \cdot k_B \cdot T_{ph}}{c^2} = \frac{2k_B \cdot T_{ph}}{\lambda^2} \tag{2.13}
\]

As shown in equation 2.13, there is a linear relationship between the spectral brightness density of the body and its physical temperature.

### 2.3 Radiation of a grey body. Brightness temperature and emissivity

A black body in thermal equilibrium radiates all the energy it has absorbed and therefore emits as much energy to a specific physical temperature. A black body is a perfect absorber.

Real materials (also called grey bodies) emit less power than a black body because they do not necessarily absorb the entire incident energy.

In the case of a grey body, the brightness emitted depends on the direction \(B(\theta, \phi)\) and can be expressed as:

\[
B(\theta, \phi) = 2 \cdot \frac{k_B}{\lambda^2} \cdot T_{B}(\theta, \phi) \cdot \Delta f \tag{2.14}
\]
where $T_b(\theta, \phi)$ is the brightness temperature and $\Delta \varphi$ is the bandwidth.

The relationship between the brightness of a material ($B(\theta, \phi)$) and the brightness of a black body ($B_{bb}$) with the same physical temperature is called emissivity ($e(\theta, \phi)$) and can be expressed as:

$$e(\theta, \phi) = \frac{B(\theta, \phi)}{B_{bb}} = \frac{T_b(\theta, \phi)}{T_{bb}} \quad 2.15$$

where $B(\theta, \phi) \leq B_{bb}$ and $0 \leq e(\theta, \phi) \leq 1$.

The brightness temperature of a grey body expresses its emission properties (angular dependent) compared with the ones of a black body. Since the brightness temperature of a grey body is less than the brightness temperature of a black body, it is always smaller or equal to the physical temperature of the body. Therefore, the emissivity has value 0 for a fully reflective material and has value 1 for a perfect absorber (black body).

### 2.4 Apparent temperature

The incident radiation upon an antenna from any specific direction may contain components originated from several different sources such as the radiation emitted by the ground ($T_b$), the radiation emitted by the atmosphere or the radiation emitted by the atmosphere that falls on the ground and that is reflected.

Apparent radiometric temperature ($T_{ap}(\theta, \phi)$) is the black body equivalent temperature distribution representing the brightness distribution ($B(\theta, \phi)$) of the energy incident upon the antenna:

$$B(\theta, \phi) = \frac{2k_b}{\lambda^2} \cdot T_{ap}(\theta, \phi) \cdot \Delta \varphi \quad 2.16$$

The brightness temperature ($T_b(\theta, \phi)$) is related to the radiation received on a surface or volume, while the apparent temperature ($T_{ap}(\theta, \phi)$) is related to the incident energy received by the antenna. Only in the case where the losses of the atmosphere can be considered negligible, the apparent temperature coincides with the brightness temperature ($T_{ap} = T_b$) since the only contribution to the apparent temperature is the radiation emitted by the surface.

As seen, the brightness's distribution of a grey body can be expressed in terms of the apparent temperature. Thus, taking into account the previous theory and equation 2.8, the power received by the antenna can be expressed as:

$$P = \frac{1}{2} \cdot A \cdot \int_{\Delta \varphi} \frac{2k}{\lambda^2} \cdot T_{ap}(\theta, \phi) \cdot \Delta \varphi \cdot F_{at}(\theta, \phi) \cdot d\Omega \quad 2.17$$
When computing the transfer function of the receiver measuring the output voltage as a function of physical temperature of a load placed at the receiver input, it is possible to obtain the noise power ($P_n$) which is proportional to the physical temperature. If the correspondence is done with the power supplied by the antenna to the receiver, it is called radiometric antenna temperature ($T_A$) such as an equivalent resistance to deliver the same power:

$$P_n = P = k \cdot T_A \cdot \Delta f$$  \hspace{1cm} (2.18)

Therefore, the antenna temperature can be expressed in terms of the normalized radiation diagram of the antenna ($F_n(\theta, \phi)$) and its effective area ($A_e$) as follows:

$$T_A = \frac{A_e}{\lambda^2} \cdot \int_4 \int_0 \int \int F_n(\theta, \phi) \cdot F_n(\theta, \phi) \cdot d\Omega$$ \hspace{1cm} (2.19)

A passive radiometer is an instrument that measures the spontaneous electromagnetic emission, which is normally associated with thermal effect: the brightness temperature.

Unlike other receivers, such as radar receivers that consider the radiometric antenna temperature is a noise contribution, the radiometers obtain from this signal information on the emission characteristics of the scene being viewed.

The next sections explain the main features of two different types of radiometers: real aperture radiometers and interferometric radiometers by aperture synthesis.

### 2.5 Total power radiometer

All microwave radiometers used for Earth observation have been real aperture radiometers, being the Total Power Radiometer (TPR)(3) the more simplified version.

A TPR consists of an antenna connected to a superheterodyne receiver with bandwidth $\Delta f$ and total gain $G$, followed by a power detector and a low-pass filter (Figure 2.4). The power delivered by the antenna is usually broadband noise higher than the range of the receiver. The antenna receives the radiofrequency (RF) power emitted by the material observed and a RF low noise amplifier increases the noise power of the signal acquired. The band-pass filter selects the desired frequency band which is converted in the mixer. The signal is amplified before passing through the power detector. Finally, it is necessary to use a low pass filter to average the obtained voltage.

![Block diagram of a Total Power Radiometer](image.png)

*Figure 2.4: Block diagram of a Total Power Radiometer*
In a total power radiometer, the output voltage is proportional to the noise temperature of the system and can be written as:

\[ V_{\text{out}} = k \cdot T_{\text{sys}} \cdot \Delta f \]

where \( T_{\text{sys}} = T_A + T_R \) is the system noise temperature, \( T_A \) is the equivalent noise temperature measured by the antenna, \( T_R \) corresponds to the equivalent noise temperature of the receiver and \( \Delta f \) is the bandwidth.

In order to calibrate a total power radiometer it is only necessary to measure the output voltage corresponding to two noise temperatures at the input, \( T_C \) (cold load) and \( T_H \) (hot load), which can be seen in Figure 2.5. Therefore, a TPR requires only external calibration.

\[ \text{Figure 2.5: Total Power Radiometer calibration using a hot and a cold load} \]

Two important parameters that characterize radiometric measurements are the sensitivity or radiometric resolution and the accuracy.

The absolute accuracy is the closeness of the agreement between the result of a brightness temperature measurement and the true value, which depends on the calibration strategies and the stability of the instrument. In contrast, the sensitivity or radiometric resolution of the measure can be defined as the smallest change in temperature of antenna that can be detected at the output of the radiometer. The desired sensitivity value is typically of the order of 1 K.

The spatial resolution that can reach a radiometer is limited by the size of its antenna. Measuring geophysical parameters such as soil moisture and ocean salinity (L-band) requires high spatial resolution, and therefore the size of the antenna of a real aperture radiometer to allow such resolution is too big to be technologically viable.

### 2.6 Interferometric radiometer with aperture synthesis

As mentioned above, the radiometry is concerned with measuring the radiation power emitted by the materials. Interferometry is also addressing the measurement of the phase information of this radiation. However, the spatial resolution requirements needed by the scientific community would force to use radiometers with large antennas.
The interferometric called aperture synthesis is a technique in which the cross-correlation between signals acquired from two or more antennas are measured. Substantial reductions in the antenna aperture needed for a given spatial resolution can be achieved with this technique. As a result, aperture synthesis has been the solution that has improved spatial resolution with respect to the actual opening passive microwave remote sensing instruments in space to obtain geophysical parameters such as soil moisture and ocean salinity which require observations at long wavelengths and, therefore, large antennas.

An interferometric radiometer consists of an array of antennas. The output voltages of different pairs of antennas are correlated and return the visibility function. From the samples of this function, using image inversion algorithms, the image is reconstructed obtaining brightness temperature maps of the scene. This type of radiometers require a previous correction of the visibility samples before the external calibration as explained in total power radiometers, since the interferometric radiometer by aperture synthesis do not measure the distribution of brightness temperature but the samples of its Fourier transform.

The American hybrid real and synthetic aperture radiometer ESTAR (Electronically Steered Thinned Array Radiometer) on board an aircraft demonstrates the validity of the 1D aperture synthesis. The experiments indicate that a valid image reconstruction and calibration have been obtained for this remote sensing technique.

Nowadays, a European 2D interferometric radiometer by aperture synthesis called MIRAS (Microwave Imaging Radiometer using Aperture Synthesis) is used for the implementation of these measures within the SMOS mission.
Chapter 3

3 The SMOS mission and MIRAS instrument

This chapter is intended to explain the main aspects of the SMOS (Soil Moisture and Ocean Salinity) mission of the European Space Agency (ESA) such as its objectives, MIRAS instrument or its operating principles.

3.1 SMOS objectives

SMOS is the second of the Earth Explorer Opportunity missions that are part of the ESA Living Planet Program (4), which is formed by exploring missions oriented to increase the knowledge of the mechanisms that explain the behaviour of the Earth by using advanced observation techniques.

SMOS is the first satellite mission devoted to measure the ocean surface salinity and the soil moisture of the continental masses from space. The duration of the mission is three years, but it can be extended two years more.

The data obtained from the measurements will allow to create soil moisture and ocean salinity maps every three days and thirty days, respectively. These maps will be very useful to obtain more information about the hydrological cycle, the climate change and the meteorological phenomenons, which are close related with the oceanic currents. They will also contribute to improve the climatological and meteorological models and to put into practice this information in the agriculture and in the administration of the water resources.

The measurements will have an accuracy of a 4% with a spatial resolution of 35-50 km for soil moisture and 0.5-1.5 psu (practical salinity units) in each observation or 0.1 psu for a 30-day average.

The SMOS satellite carries a single payload, an L-band 2-D microwave interferometric radiometer using aperture synthesis called MIRAS. The satellite orbit is sun-synchronous, almost circular, with equatorial crosses at dawn and dusk in order to obtain optimum measurements. The satellite is orbiting at a mean attitude of 758 km with an inclination of 98.44°.

The SMOS payload was loaded in a standard spacecraft transport Proteus, developed by the Centre National d’Études Spatiales (CNES) and Thales Alenia Space, both of them in France, and connected to a Rockot launcher commercialized by the company Eurockot.

SMOS satellite was launched together with Proba-2 satellite (Project for On-Board Autonomy-2) the 2th of November, 2009 at 01:50 GMT (02:50 CET) from the cosmodrome of Plesetsk, located in Arkhangelsk Oblast, about 800 km north of Moscow.
The satellite sends scientific data via X-band link to the ground station of the European Space Astronomic Centre (ESAC) in Villanueva de la Cañada, Madrid, where the data is processed. This station is complemented by another one in Svalbard, Norway, that receives the processed data in almost real time. Satellite flight operations are controlled by the Proteus Control and Command Centre of the CNES in Toulouse, France, but the telecommand and telemetry information is sent via S-band link from the station in Kiruna, Sweden.

The development of the SMOS mission was led by ESA in collaboration with the CNES in France and the Centro para el Desarrollo Tecnológico Industrial (CDTI) in Spain. The theoretical design of MIRAS instrument was done by different European universities, such as Aalto University School of Science and Technology (TKK), in Helsinki (Finnland), and the Remote Sensing Laboratory Group (5) (RSLab) from the Theory and Signal Communication department (TSC) of the Universitat Politècnica de Catalunya (UPC), Spain.
The receivers have been manufactured by the company MIER Comunicaciones and the integration of the different elements has been carried out by EADS-CASA Espacio, both of them in Spain.

A consortium formed by different Spanish companies such as EADS-CASA Espacio, GMV and INDRA Espacio, and the Portuguese company Deimos Space, performs data processing and validation. European Universities and other institutions, among them the SMOS Barcelona Expert Centre (6) (SMOS-BEC), are also involved in the data processing.

### 3.2 MIRAS instrument

In this section the single payload of the SMOS mission will be characterized, explaining the architecture, the operating principle, the observation modes and the important subsystems that form part of it (9)(10)(11).

#### 3.2.1 Instrument architecture

MIRAS is a 2-D interferometric radiometer using aperture synthesis operating in L-band (1.400 MHz – 1.427 MHz), which is a band of frequencies reserved for scientific purposes where RF emissions are forbidden. It is capable of measuring the thermal radiation at the centre frequency of 1413.5 MHz.

The instrument is formed by three deployable arms separated 120° forming a Y-shape. The arms are connected to a central part called hub. Each arm is divided in three sections, having each section 6 antennas, which means that each arm has 18 antennas. The hub has 15 antennas, so that the whole instrument has 69 antennas.

Figure 3.3 shows MIRAS instrument after the assembling:

![MIRAS instrument in the premises of EADS-CASA](image)

*Figure 3.3: MIRAS instrument in the premises of EADS-CASA*
Each antenna is connected to a receiver called LICEF (Light-weight Cost-Effective Front-end). Three of the antennas placed in the hub are connected to three NIRs (Noise Injection Radiometers), which perform as two LICEFs in the same position but with different polarizations. Thus, MIRAS has 69 antennas but 72 receivers.

### 3.2.2 Operating principle

The operating principle of MIRAS is based on the cross-correlation between pairs of receivers, called baselines. Taking into account that the instrument has 72 receivers, the total number of possible baselines is:

$$\binom{72}{2} = \frac{72!}{70! \cdot 2!} = 2556 \quad \text{(3.1)}$$

The baselines can be classified according to the distance between its receivers and its direction. A baseline of distance \(n\) has two receivers separated by \(n\) units of distance.

As MIRAS is a Y-shape radiometer, the distance of a baseline can be calculated as:

$$d = \sqrt{\Delta x^2 + \Delta y^2} \quad \text{(3.2)}$$

where \(\Delta x = x_j - x_i\) and \(\Delta y = y_j - y_i\) are the difference between the \(x\) coordinates and the \(y\) coordinates of the receivers \(i\) and \(j\).

Instead of the usual \(x-y\) coordinates, \(u-v\) values (normalized antenna positions or spatial frequencies) are commonly used to describe the baselines. These new values are calculated as the difference between the positions of the two receivers of the baseline divided by the centre wavelength of operation, as shown in equation 3.3:

$$u_{ji} = \frac{x_j - x_i}{\lambda_0} \quad v_{ji} = \frac{y_j - y_i}{\lambda_0} \quad \text{(3.3)}$$
Each antenna measures the thermal radiation at L-band. The measured values are processed in the receiver and sent to a correlation matrix (Figure 3.5) where all the possible complex correlations between the signals of the receivers are calculated.

\[
F_{q,n}(\xi, \eta) \quad \text{Complex correlator} \quad F_{p,m}(\xi, \eta)
\]

**Figure 3.5: Cross-correlation between the receivers m and n**

Once all the cross-correlations are calculated, the visibility function is calculated for all baselines using the following equation:

\[
V_{k}(u, v) = \frac{1}{k_{B} \sqrt{B_{k} B_{j}}} \cdot \left\langle \frac{1}{2} b_{k} \cdot b_{j}^{*} \right\rangle
\]

where \(u, v\) correspond to the set of spatial frequencies where the visibility function is sampled, \(G_{k}, G_{j}\) are the power gains of each receiver chain, \(B_{k}, B_{j}\) correspond to the equivalent noise bandwidths and \(k_{B} = 1.38 \times 10^{-23} \text{J} / \text{K}\) is the Boltzmann constant.

The main difference between the real aperture radiometers and the aperture synthesis radiometers is that the first ones measure directly the brightness temperature while the second ones measures the samples of its Fourier transform which need image reconstruction algorithms to obtain a brightness temperature image.

The visibility function expressed in terms of the brightness temperature is:

\[
V_{k}(u, v) = \iint \frac{T_{B}(\xi, \eta) - T_{r}}{\sqrt{1 - \xi^{2} - \eta^{2}}} \cdot \frac{F_{u}(\xi, \eta) \cdot F_{v}(\xi, \eta)}{\sqrt{\Omega_{k} \cdot \Omega_{j}}} \cdot r_{k}(u, v, \eta) \cdot e^{-j2\pi(u\xi + v\eta)} d\xi d\eta
\]

where \(r_{k}(u, v)\) corresponds to the fringe washing function normalized to unity at origin (related to the spatial decorrelation errors), \(T_{B}(\xi, \eta)\) is the brightness temperature, \(T_{r} = (T_{k} + T_{j}) / 2\) is the mean of the physical temperature of the receivers \(k\) and \(j\), \(F_{u}(\xi, \eta), F_{v}(\xi, \eta)\) are the normalized voltage antenna patterns of the receivers, \(\Omega_{k}, \Omega_{j}\) correspond to the equivalent solid angle of the antennas and \((\xi, \eta) = (\sin \theta \cdot \cos \phi, \sin \theta \cdot \sin \phi)\) are the director cosines with respect to the X and Y axes, respectively.
In the ideal case where all the antenna patterns are identical, spatial decorrelation is negligible and there are not antenna position errors, the visibility can be calculated using the 2D Fast Fourier Transform (FFT):

\[ V_{ij}(u,v) = \text{FFT} \{ I_i(x,y) \} \]

3.2.3 Observation modes

MIRAS has two operating modes: the measurement mode, where the instrument performs measures that after applying reconstruction algorithms will turn into brightness temperature maps and the calibration mode, where the instrument also performs measures to adjust internal parameters in case it is needed.

The measurement mode has two operating modes: dual polarization mode and full polarization mode.

In dual polarization mode, the antennas measure in both polarizations alternatively with an integration time of 1.2 seconds (known as epoch) for each measurement. In each epoch, both antennas are measuring in the same polarization (HH or VV).

The measurement procedure is shown in Figure 3.6:

![Figure 3.6: Dual polarization measurement mode](image)

In full polarization mode, the antennas measure all possible combinations of polarizations alternatively with an integration time of one epoch for every measurement (HH, HV, VH and VV). This mode allows to obtain both polarizations at the same time, which has some scientific advantages, although the data to transmit by the satellite has the double size of the dual polarization measurements.

These four steps measurement procedure is shown in Figure 3.7:

![Figure 3.7: Full polarization measurement mode](image)
3.2.4 MIRAS description

In this section the most important subsystems that are part of the instrument will be described in detail, explaining their characteristics and their working principles.

3.2.4.1 Antennas

MIRAS has 69 antennas uniformly distributed along the three arms and the hub. The antennas are separated a distance $d = 0.875\lambda$, where $\lambda$ is the wavelength at the centre frequency ($\lambda = 21$ cm), that is $d = 18.375$ cm.

Each antenna has four probes connected in pairs in order to receive both polarizations of the signal (horizontal and vertical). The circuits of the antenna have been created using microstrip technology, being each layer for a different polarization.

The dimensions of an antenna are 165 mm of diameter and 19 mm of height, and its weight is 190 grams.

![Antenna diagram](image)

1. Carbon fiber structure
2. Patch antenna
3. Feeding discs
4. Cavity floor to patch antenna
5. Aluminium spacer
6. Feeding circuits
7. Aluminium spacer

*Figure 3.8: Different parts of the antenna*

3.2.4.2 LICEF

A LICEF (Light-weight Cost-Effective Front-end) is the name for a MIRAS receiver. Its mission is to process the signals acquired by the antenna and send the processed signal (1 bit of information) to the correlation unit.

Figure 3.9 shows the image of a LICEF:

![LICEF image](image)

*Figure 3.9: LICEF*
The block diagram of a LICEF can be seen in Figure 3.10:

![Block diagram of a LICEF](image)

**Figure 3.10: Block diagram of a LICEF**

The first block of the LICEF is the switch. It has four inputs which allow to select the different sources of signal, which are:

- H (Horizontal) and V (Vertical): are connected directly to the antenna and allows to select the polarization to measure.
- C (Correlated): is connected to the NDN (Noise Distribution Network) in order to perform internal calibration using correlated noise.
- U (Uncorrelated): is connected to a 50 Ω load to perform internal calibration using uncorrelated noise. Each LICEF has a different load.

After the switch, there is an isolator and a Low Noise Amplifier (LNA) before the Band-Pass Filter (BPF) which selects the frequencies that belong to the working band (1400-1423 MHz) and rejects the rest of them. After that, there is a RF amplifier before the mixer, where the selected frequencies are converted to lower ones by using a local oscillator with a reference frequency of 55.84 MHz.

I and Q components of the signal are calculated in two different branches. After passing through a filter, an attenuator and the amplifiers, the ADC converter transforms the input signal into 1 bit, being 0 or 1 depending if the signal has a negative value or a positive value. The values of the Q branch are used for the Power Measuring System (PMS), which will be explained after.

Finally, these bits are sent to the Digital Correlation Unit (DICOS) through optical fibre (to avoid electronic disturbances) where the cross-correlations of the signals of the different baselines are calculated.

The parameters of the LICEF are temperature and age sensitive, that is why they must be internally calibrated several times in each orbit. This calibration period takes 1.2 seconds (one epoch). Moreover, an external calibration is performed each 1-2 weeks using as a reference the cosmic radiation turning the satellite towards the galaxy.
3.2.4.3 NIR

A NIR is a noise injection radiometer at the working band of the instrument (1.4 GHz). MIRAS has three NIRS located in the hub.

![Noise Injection Radiometer](image)

*Figure 3.11: Noise Injection Radiometer*

The main purposes of the NIRS are providing absolute calibration of the brightness temperature and calibrating the internal calibration system (CAS) by measuring the noise temperature level very accurately.

The Noise Injection Radiometer consists of:

- Two radiometer receivers (H/V) almost identical to a LICEF
- A controller, which incorporates an antenna identical to the one of the LICEFs to receive the target noise
- Phase stable RF cables that connect the controller to the receivers

A block diagram of a NIR is shown in Figure 3.12:

![Block diagram of a NIR](image)

*Figure 3.12: Block diagram of a NIR*
The tasks of the controller are:

- Inject reference noise into the two receivers chains
- Regulate the amount of injected noise to keep the system balanced
- Control the switches depending on the selected operation mode

![NIR controller](image)

*Figure 3.13: NIR controller*

The NIRs have three different operation modes, which are:

- NIR-A mode: used to measure the antenna temperature
- NIR-R mode: used to measure the CAS temperature
- NIR-AR mode: used to calibrate the NIR-R mode

According to the previous information, a NIR can act as two LICEF measuring at both polarizations at the same time when it is operating in NIR-A mode. The measures obtained in this mode can be used to compute the visibility of the so-called mixed baselines, which are baselines where one of the receivers is from a NIR.

### 3.2.4.4 PMS

The Power Measurement System (PMS) converts the received signal into a voltage. As explained before, the PMS is placed in the Q branch of the LICEF, before the signal is sent to the correlation matrix.

The block diagram of the PMS is shown in Figure 3.14:

![Block diagram of the PMS](image)

*Figure 3.14: Block diagram of the PMS*

The PMS is formed by an amplifier with gain $G_k$, an attenuator with attenuation $L_k$, a quadratic power detector (a diode) and an integrator (a low-pass filter). Therefore, a PMS is equivalent to a Total Power Radiometer.
At the end of the chain, the value of the voltage follows this expression:

\[ v_k = G_k \cdot T_{\text{syn}} + v_{\text{off}_k} = G_k \cdot (T_{A_k} + T_{n_k}) + v_{\text{off}_k} \]  \hspace{1cm} (3.7)

where \( v_{\text{off}_k} \) is the PMS offset [V], \( G_k \) is the PMS gain [V/K], \( T_{A_k} \) is the antenna temperature [K] and \( T_{n_k} \) is the receiver noise temperature [K].

Although the PMS gain and the PMS offset were characterized on ground, they have to be estimated in the PMS calibration because the system temperature (\( T_{\text{syn}} \)) is used in the denormalization of the visibilities needed to calculate the brightness temperature.

### 3.2.4.5 DICOS

The function of the Digital Correlator System (DICOS) is to correlate the signals produced by all the LICEFs. The DICOS is formed by several 1-bit digital correlators, which correlates two signals as shown in Figure 3.15:

![Figure 3.15: 1-bit Digital Correlator](image)

The output of the XNOR gate is 1 if the two inputs are equal and 0 if they are different. The correlation is calculated by accumulating the outputs of the XNOR gate during an integration time, defined by the 55.84 MHz clock frequency. The total number of clocks (\( N_{c_{\text{max}}} \)) is limited by the number of bits of the accumulator (\( N_{c_{\text{max}}} \) is 65437 for dual-pol mode and 43625 for full-pol mode).

The correlations calculated by the DICOS are:

- 2556 correlations between I channels of different receivers (\( I_k - I_j \))
- 2556 correlations between I and Q channels of different receivers (\( I_k - Q_j \))
- 72 correlations between I and Q channels of same receivers (\( I_k - Q_k \))
- 72 correlations \( I - 0 \)
- 72 correlations \( Q - 0 \)
- 72 correlations \( I - 1 \)
- 48 correlations \( Q - 1 \)
- 36 control correlations between 1 and 0 channels

Therefore, 5484 digital correlators are needed to perform all correlations.
3.2.4.6 CAS and NS

The calibration subsystem (CAS) uses the noise distribution network (NDN) to distribute the noise generated by the noise sources (NS) to calibrate the instrument.

The noise sources generate noise at two different levels, known as cold noise (75 K) and warm noise (1500 K). Each segment has two NS, but the second one it is only used in case of failure. The noise is distributed to all the receivers using a network of power dividers (PD), which have two inputs in order to receive the noise of two noise sources, except the last PD, that only has one input.

3.2.4.7 CMN and Local Oscillators

The Control and Monitoring Node (CMN) acts as a remote terminal of the Central Control Unit (CCU). Each of the three arms contains three segments of six receivers each one. In each segment, there is one CMN responsible for the control and monitoring of the signals. The hub is divided in three sectors with a CMN in each sector. Therefore, there are a total of 12 CMN’s in the instrument.

The main functions of the CMN’s are the reception of commands and sending them to the CCU, the acquisition of the physical temperature readings from the thermistors, the acquisition of the voltages, the control of the LICEF polarization switch, the control of the noise injection and the distribution of the thermal control actuations. Each CMN also provides power and a phased local oscillator (LO) signal controlled by a 55.84 MHz reference clock supplied by an optical link to each LICEF.

![Figure 3.16: CMN and Local Oscillator](image)

3.2.4.8 Thermal Control System

A thermal control system has been designed and implemented in MIRAS to minimize the temperature differences between all receivers in different orbits.

The different temperature sensors distributed along the instrument acquire the physical temperatures that are sent to the CMNs that decide to switch on or off the heaters. Each heater is controlled by their associated CMN. There are 12 heaters, one in each section of the three arms and three more in the hub.
Chapter 4

4 SMOS calibration

This chapter is intended to explain in detail the calibration processes performed in MIRAS instrument which will allow to fulfil the requirements of the SMOS mission.

The calibration is a key stage needed to provide stable and accurate data. Although all elements have been characterized on ground, drifts in the values of their parameters may appear due to changes in the temperature or because of an aging effect. The calibration processes correct these possible drifts by estimating the new values of the parameters.

As said before, the calculated brightness temperature is based on the so-called calibrated visibility, which has the following expression:

\[ V_{kj} = \sqrt{\frac{T_{sys_k} \cdot T_{sys_l}}{G_{kj}}} \cdot M_{kj} \tag{4.1} \]

where \( M_{kj} \) is the normalized complex correlation between the receivers \( k \) and \( j \) after the self-calibration procedure, \( G_{kj} \) is the fringe washing function at the origin and \( T_{sys} \) is the temperature of the system for the LICEF \( k \) which can be calculated as \( T_{sys} = T_A + T_N \).

In order to obtain this calibrated visibility, all values present in the equation must be also calibrated. To achieve this, the following parameters are used in the calibration to correct these values:

- Instrument outputs
  - Correlator counts \( (N_C) \)
  - PMS voltages \( (V_k) \)
  - NIR values \( (T_N) \)

- On-ground characterized parameters
  - S-parameters of the NDN path to the NIR
  - S-parameters of the LICEF switch
  - Antenna efficiency

Two different types of calibration are performed in MIRAS instrument: internal calibration and external calibration. Each type performs different calibration sequences which provide calibration for different subsystems.
Table 4.1 shows a summary of the calibration processes

<table>
<thead>
<tr>
<th>Type</th>
<th>Calibration processes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal</td>
<td>• Visibility offset</td>
</tr>
<tr>
<td></td>
<td>• Fringe washing function shape</td>
</tr>
<tr>
<td></td>
<td>• Fringe washing function at the origin</td>
</tr>
<tr>
<td></td>
<td>• PMS calibration</td>
</tr>
<tr>
<td></td>
<td>• Normalized complex correlations computation</td>
</tr>
<tr>
<td>External</td>
<td>• NIR absolute calibration</td>
</tr>
<tr>
<td></td>
<td>• FTR computation</td>
</tr>
<tr>
<td></td>
<td>• One-point PMS calibration for CAS validation</td>
</tr>
<tr>
<td></td>
<td>• G-matrix computation</td>
</tr>
</tbody>
</table>

*Table 4.1: Calibration parameters*

In this project only the following calibration processes will be explained: PMS calibration, One-point PMS calibration and fringe washing function at the origin.

There are some parameters that only need an amplitude calibration such as the PMS voltages and gains or other that need both amplitude and phase calibration, for instance, the fringe washing function.

This chapter is divided in three different sections: the first will explain the visibility function and the last two will explain the amplitude and phase calibration.

### 4.1 The visibility function

In this section, it will be explained the process to obtain the calibrated visibility from the correlator counts (12)(16).

First of all, the correlator counts $N_c$ are converted to digital correlation with the following equation:

$$Z = 2 \cdot \frac{N_c}{N_{c_{\text{max}}}} - 1 \quad 4.2$$

Thus, the digital correlation can have a minimum value of -1 (signals totally different) and a maximum value of 1 (equal signals). The normalized correlation of two Gaussian signals is related to the digital correlation (supposing zero offset) with equation 4.3:

$$\mu = \sin\left(\frac{\pi}{2} Z\right) \quad 4.3$$

If the offset is not zero, but it is small, it can be corrected if $\mu$ is computed solving a non-linear equation using an initial solution.
Once the correction is applied, the normalized correlation can be expressed as:

\[
\mu_{ij}^{\alpha \beta} = \frac{\text{Re} \left[ e^{i \Delta \phi_{ij}} \cdot b_i^{\alpha} \cdot b_j^{\beta} \right]}{\sqrt{|b_i^{\alpha}(0)|^2 |b_j^{\beta}(0)|^2}}
\]

4.4

where \( \alpha, \beta = i \) (phase component) or \( \alpha, \beta = q \) (quadrature component), \( b_k(t) \) is the analytic signal of the receiver \( k \) and \( \Delta \phi_{ij} \) is the local oscillator phase (ideally, 0 if \( \alpha = \beta \) or \( \pi \) if \( \alpha \beta = q_i \)).

The previous equation can be also written as:

\[
\mu_{ij}^{\alpha \beta} = \frac{\text{Re} \left[ e^{i \Delta \phi_{ij}} r_{ij}^{\alpha \beta}(0) \cdot V_{ij} \right]}{\sqrt{T_{sys_k} T_{sys_j}}}
\]

4.5

where \( T_{sys_k} = T_{A_k} + T_{n_k} \) is the system temperature of the receiver \( k \), \( T_{A_k} \) is the antenna temperature of the receiver \( k \), \( T_{n_k} \) is the equivalent noise temperature of the receiver \( k \), \( r_{ij}^{\alpha \beta}(0) \) is the fringe washing function at the origin and \( V_{ij} \) is the calibrated visibility.

The fringe washing function at the origin, which is related with the transfer functions of the channels, follows equation 4.6:

\[
r_{ij}^{\alpha \beta}(0) = \frac{1}{\sqrt{B_k^{\alpha} B_j^{\beta} G_k^{\alpha} G_j^{\beta}}} \int_0^\infty H_k^{\alpha}(f) \cdot H_j^{\beta}(f)^* \, df
\]

4.6

where \( B_k^{\alpha}, B_j^{\beta} \) are the bandwidths of the channels, \( G_k^{\alpha}, G_j^{\beta} \) the gains of the channels and \( H_k^{\alpha}(f), H_j^{\beta}(f) \) the transfer functions of the channels.

The previous equation can be expressed using equation 4.7, depending if the cross-correlation is between \( i-j \) channels or \( i-q \) branches:

\[
r_{ij}^{\alpha \beta}(0) = G_{kj} \cdot e^{-\frac{\theta_{ij} - \theta_{ij}^{\alpha \beta}}{2}}
\]

\[
r_{ij}^{\alpha \beta}(0) = G_{kj} \cdot e^{-\frac{\theta_{ij} + \theta_{ij}^{\alpha \beta}}{2}}
\]

4.7

where \( G_{kj} \) is the common term of the fringe washing function (which will be explained in the phase calibration) and \( \theta_{ij} \) is the quadrature error for receiver \( k \) which value is:

\[
\theta_{ij} = -\arcsin(\mu_{ij}^{\alpha \beta})
\]

4.8

After the quadrature error is calculated, the calibrated visibility is obtained from equation 4.9:
Phase error assessment of MIRAS/SMOS by means of Redundant Space Calibration method

\[ V_{kj} = \sqrt{\frac{T_{sys_k'} \cdot T_{sys_j'}}{G_{kj}}} \cdot M_{kj} \]  

4.9

where \( T_{sys_k} \) is the system temperature of the receiver \( k \), \( G_{kj} \) is the common part of the fringe washing function between receivers \( k \) and \( j \) and \( M_{kj} \) is the quadrature-corrected normalized correlation that can be expressed as:

\[ M_{kj} = \frac{1}{\cos(\theta_{qk})} \cdot (\text{Re} [M_1 \cdot \mu_{qk}] + j \cdot \text{Im} [M_2 \cdot \mu_{qk}]) \]  

4.10

where \( \mu_{qk} = \mu_{qk}^{(i)} + j\mu_{qk}^{(j)} \) is the normalized complex correlation and \( M_1, M_2 \) verify equation 4.11:

\[ M_1 = \cos(\theta_{k}) + j \cdot \sin(\theta_{k}) \]
\[ M_2 = \cos(\theta_{k'}) + j \cdot \sin(\theta_{k'}) \]  

4.11

and \( \theta_{k}, \theta_{k'} \) are:

\[ \theta_{kj} = \frac{\theta_{qj} - \theta_{qk}}{2} \]
\[ \theta_{kj} = \frac{\theta_{qj} + \theta_{qk}}{2} \]  

4.12

4.2 The amplitude calibration

In the case of the PMS, the amplitude calibration refers to the calibration of the system temperature \( (T_{sys}) \), which is calculated from the values of the gain and the offset, and in the case of the fringe washing function it refers to the complex modulus of the fringe washing term \( (G_{kj}) \).

In the scope of this project, only the PMS calibration will be explained, leaving the fringe washing function only for phase calibration.

In order to understand the subsystems that are part of the calibration procedure, Figure 4.1 shows a block diagram of one baseline involving receivers \( k \) and \( j \), the NIRs, the noise sources and the NDN:
Three planes have been defined to express clearly where the values of the variables are referred to. These planes are Calibration Internal Plane (CIP), Antenna Plane (AP) and NIR Plane.

To calibrate the PMS, two different procedures have been designed: PMS internal calibration and PMS external calibration, with a variant called one-point calibration.

### 4.2.1 PMS internal calibration

The first calibration method is called PMS internal calibration, or four-point calibration method (12)(13)(15). It consists of injecting correlated noise at two different temperatures in the C input of the switch of the LICEF.

From the previous chapter, it is known that the PMS voltage follows equation 4.13:

\[
v_k = G_k^C \cdot T_{sys}^C + v_{off_k} \tag{4.13}
\]

where \( v_{off_k} \) is the PMS offset, \( G_k^C \) is the PMS gain and \( T_{sys}^C \) is the system temperature (values referred to CIP).

To correct the visibilities, \( T_{sys}^C \) must be obtained with:

\[
T_{sys}^C = \frac{v_k - v_{off_k}}{G_k^C} \tag{4.14}
\]

which means that \( G_k^C \) and \( v_{off_k} \) must be calibrated to calculate the system temperature.
Since the values of the PMS gain and the PMS offset are needed, first of all they must be calculated. To do this, two known external noise signals at different temperatures \( T_{S_1} < T_{S_2} \) are generated. The signal with lower temperature \( T_{S_1} \) is known as cold noise and the signal with higher temperature \( T_{S_2} \) is known as warm noise. The attenuator of the PMS is switched on and off for each noise signal, which will allow to obtain one extra equation for each noise temperature.

With this procedure, the following equations are obtained:

\[
\begin{align*}
\text{Cold noise, Attenuator OFF} & \quad v_{l_k} = v_{off_k} + G^C_k \cdot (T_{S_k}^C + T_k^C) \\
\text{Warm noise, Attenuator OFF} & \quad v_{2_k} = v_{off_k} + G^C_k \cdot (T_{S_k}^C + T_k^C) \\
\text{Cold noise, Attenuator ON} & \quad v_{3_k} = v_{off_k} + \frac{G^C_k}{L_k} \cdot (T_{S_k}^C + T_k^C) \\
\text{Warm noise, Attenuator ON} & \quad v_{4_k} = v_{off_k} + \frac{G^C_k}{L_k} \cdot (T_{S_k}^C + T_k^C)
\end{align*}
\]

Therefore, \( G^C_k \) and \( v_{off_k} \) can be computed as:

\[
\begin{align*}
G^C_k &= \frac{v_{2_k} - v_{l_k}}{T_{S_k}^C - T_{S_1}^C} \\
v_{off_k} &= \frac{v_{2_k} \cdot v_{3_k} - v_{l_k} \cdot v_{4_k}}{v_{2_k} - v_{4_k} - v_{l_k} + v_{3_k}}
\end{align*}
\]

The two noise temperatures \( T_{S_1} \) and \( T_{S_2} \) are generated by the same noise source, which is placed at the port 0 of the NDN. The equivalent noise temperatures present in the equations of the PMS are referred to the calibration plane, but can be expressed in terms of the noise temperatures of the noise source with the following equations:

\[
\begin{align*}
T_{S_k}^C &= |S_{k0}|^2 \cdot T_{S_1} + (1 - |S_{k0}|^2) \cdot T_{pk} \\
T_k^C &= |S_{k0}|^2 \cdot T_{S_2} + (1 - |S_{k0}|^2) \cdot T_{pk}
\end{align*}
\]

where \( S_{k0} \) is the S-parameter of the NDN between the input port 0 and the output port \( k \) and \( T_{pk} \) is the physical temperature of the system.

The equations for the LICEF \( j \) at CIP plane are the same, only changing the parameter \( S_{k0} \) for the parameter \( S_{j0} \).

The noise temperatures are measured by the NIRs and can be expressed at NIR plane as follows:
\[ T^N_{s_1} = \left| S_{s_1} \right|^2 \cdot T_{s_1} + (1 - \left| S_{s_1} \right|^2) \cdot T_{ph} \]  \hspace{1cm} 4.23

\[ T^N_{s_2} = \left| S_{s_2} \right|^2 \cdot T_{s_2} + (1 - \left| S_{s_2} \right|^2) \cdot T_{ph} \]  \hspace{1cm} 4.24

where \( S_{s_1} \) is the S-parameter of the NDN between the input port 0 (noise source) and the output port 1 (NIR).

If the difference between the equivalent noise temperatures at CIP is calculated, it expression follows equation 4.25:

\[ T^C_{S_{s_1}} - T^C_{S_{s_2}} = \frac{\left| S_{k0} \right|^2}{\left| S_{s_1} \right|^2} \cdot (T^N_{S_{s_2}} - T^N_{S_{s_1}}) \]  \hspace{1cm} 4.25

If equations 4.14, 4.19 and 4.25 are combined, the system temperature is obtained:

\[ T^C_{synk} = \frac{v_k - v_{off_k}}{v_k - v_{k}} \cdot \frac{\left| S_{k0} \right|^2}{\left| S_{s_1} \right|^2} \cdot (T^N_{S_{s_2}} - T^N_{S_{s_1}}) \]  \hspace{1cm} 4.26

In order to obtain the system temperature at the antenna plane, a plane transformation must be applied by adding the S parameters of the switch of the LICEF and the antenna efficiency.

Therefore, the system temperature at the antenna plane can be computed as:

\[ T^H_{synk} = T^C_{synk} \cdot \frac{\left| S_{LH_k} \right|^2}{\left| S_{LH_k} \right|^2 \cdot \eta_{H_k}} \]  \hspace{1cm} 4.27

\[ T^V_{synk} = T^C_{synk} \cdot \frac{\left| S_{LV_k} \right|^2}{\left| S_{LV_k} \right|^2 \cdot \eta_{V_k}} \]  \hspace{1cm} 4.28

where \( \eta_{H_k} \) and \( \eta_{V_k} \) are the antenna efficiencies in horizontal and vertical of the LICEF \( k \) and \( S_{LH_k}, S_{LV_k}, S_{LC_k} \) are the S-parameters of the LICEF for the H, V and C inputs.

This PMS internal calibration is currently performed every 8 weeks approximately.

### 4.2.2 PMS external calibration

The external calibration, or absolute calibration, is performed when the satellite is pointing to the sky. It is known that the brightness temperature of the galaxy is very constant with an approximately temperature of 3.6 K. The PMS external calibration is also called one-point PMS calibration (12)(13)(15).
Every time an external calibration has to be done, the satellite rotates and positions its receivers towards the target instead of looking to the Earth, while keeping it in inertial pointing. Figure 4.2 shows the manoeuvres of the satellite when performing an external calibration:

![Manoeuvres of the satellite to perform an external calibration](image)

*Figure 4.2: Manoeuvres of the satellite to perform an external calibration*

When the satellite is looking to the deep sky, the LICEFs switch between the H/V antenna inputs (cold noise) and U input (warm noise), where it is placed a 50 ohm load which temperature is known.

If the front-end has a constant temperature, switching the input port to an internal load is equivalent to place an external target with the same physical temperature, because in both cases the spectral power of the noise at the input port can be calculated as $K \cdot T_{ph}$.

Figure 4.3 shows the process of a one-point PMS calibration:

![Switch positions when performing a one-point PMS calibration](image)

*Figure 4.3: Switch positions when performing a one-point PMS calibration*
When the U input is selected (warm noise), the system temperature at CIP plane can be computed using the Friis formula as follows:

\[
T_{C_{\text{sysload}}}^C = T_{ph_1} + \frac{1 - |S_{LU}|^2}{|S_{LU}|^2} \cdot T_{ph_1} + T_{rec} = T_{ph_1} + T_{rec}
\]

where \(T_{ph_1}\) is the physical temperature at the U port, \(S_{LU}\) is the S-parameter of the switch between the U input and the output and \(T_{rec}\) is the noise temperature of the receiver at LICEF plane.

To calculate the system temperature at antenna plane, a plane transformation must be performed taking into account the antenna efficiency and the S-parameter of the switch between the A input and the output:

\[
T_{A_{\text{sysload}}}^A = T_{C_{\text{sysload}}}^C \cdot \frac{|S_{LU}|^2}{|S_{LA}|^2 \cdot \eta_A} = T_{ph_1} + T_{rec}
\]

where \(T_{ph_1}\) is the physical temperature at the U port, \(S_{LA}\) is the S-parameter of the switch between the A input and the output, \(T_{rec}\) is the noise temperature of the receiver at LICEF plane and \(\eta_A\) is the antenna efficiency.

A similar analysis can be performed when the A input is selected. The system temperature at antenna plane can be expressed as:

\[
T_{A_{\text{sky}}}^A = T_{sky} + \frac{1 - \eta_A}{\eta_A} \cdot T_{ph_2} + \frac{1 - |S_{LA}|^2}{|S_{LA}|^2 \cdot \eta_A} \cdot T_{ph_1} + \frac{T_{rec}}{\eta_A \cdot |S_{LA}|^2}
\]

Remembering that the output voltage of the PMS verifies equation 4.32:

\[
v_k = G_k^A \cdot T_{sys}^A + v_{df_k}
\]

The PMS gain \(G_k^A\) can be computed as:

\[
G_k^A = \frac{V_{LOAD}^A - V_{SKY}^A}{T_{sys}^A - T_{sky}^A}
\]

If the difference of the system temperatures is calculated, it can be seen that depends on the physical temperature of the A input \((T_{ph_2})\), the physical temperature of the U input \((T_{ph_1})\), the temperature of the sky \((T_{sky})\) and the antenna efficiency \((\eta_A)\):
Phase error assessment of MIRAS/SMOS by means of Redundant Space Calibration method

\[ T_{\text{sys,LOAD}}^A - T_{\text{sys,SKY}}^A = \frac{T_{\text{ph1}} - T_{\text{ph2}}}{\eta_A} + T_{\text{ph2}} - T_{\text{sky}} \quad 4.34 \]

If the front-end temperature is assumed to be constant, that is \( T_{\text{ph1}} = T_{\text{ph2}} = T_{\text{ph}} \), equation 4.34 is reduced to:

\[ T_{\text{sys,LOAD}}^A - T_{\text{sys,SKY}}^A = T_{\text{ph}} - T_{\text{sky}} \quad 4.35 \]

Therefore, PMS gain \( G_k^A \) and the receiver noise temperature \( T_R^A \) can be expressed as:

\[ G_k^A = \frac{V_{\text{LOAD}} - V_{\text{SKY}}}{T_{\text{ph}} - T_{\text{sky}}} \quad 4.36 \]

\[ T_{R}^A = \frac{V_{\text{SKY}} \cdot T_{\text{ph}} - V_{\text{LOAD}} \cdot T_{\text{sky}}}{V_{\text{LOAD}} - V_{\text{SKY}}} \quad 4.37 \]

However, if the physical temperature of the front-end is not constant, an equivalent temperature can be defined which depends on physical temperature of the antenna input \( T_{\text{ph}_i} \), the physical temperature of the U input \( T_{\text{ph}1} \) and the antenna efficiency \( \eta_A \) as follows:

\[ T_{\text{eq}}^A = \frac{T_{\text{ph1}} - T_{\text{ph}_i}}{\eta_A} + T_{\text{ph}_i} \quad 4.38 \]

Thus, the gain and the receiver noise temperature are:

\[ G_k^A = \frac{V_{\text{LOAD}} - V_{\text{SKY}}}{T_{\text{ph}_i} - T_{\text{sky}}} \quad 4.39 \]

\[ T_{R}^A = \frac{V_{\text{SKY}} \cdot T_{\text{ph}_i} - V_{\text{LOAD}} \cdot T_{\text{sky}}}{V_{\text{LOAD}} - V_{\text{SKY}}} \quad 4.40 \]

As it can be seen, the equations 4.36 and 4.39 are equal if the constant physical temperature \( T_{\text{ph}} \) is substituted for the equivalent physical temperature \( T_{\text{ph}_i} \). In the case of equations 4.37 and 4.40, happens exactly the same.

The one-point PMS external calibration is currently performed every 1-2 weeks.

Since performing an external calibration is difficult for the manoeuvres of the satellite and brightness temperatures measures of the soil moisture and ocean salinity are lost while the instrument is calibrating, a variation of the one-point method has been developed in which the satellite does not have to look at the sky.
This new method consists of using the receiver noise temperature ($T_{R}^{A}$) computed from a one-point external calibration, the PMS offset of an internal calibration and the PMS voltage when the switch has selected the U input.

Therefore, the new gain (known as $G_{IPK}^{A}$) can be computed as:

$$G_{IPK}^{A} = \frac{V_{Uk} - V_{offk}}{T_{Rk}^{A} (T_{pk}) + T_{pk}}$$

where $V_{Uk}$ is the PMS voltage when the U port of the switch is selected, $V_{offk}$ is the offset from an internal calibration (with temperature correction), $T_{Rk}^{A} (T_{pk})$ is the receiver noise temperature in the antenna plane (also corrected in temperature) and $T_{pk}$ is the physical temperature of the receiver.

### 4.3 The phase calibration

The two parameters of equation 4.1 that need a phase calibration are $M_{kj}$ (normalized complex correlation) and $G_{kj}$ (related to the fringe washing function at the origin). In this project, only $G_{kj}$ will be studied and its application to the LO phase track.

It will be also explained the on-ground characterization of the inter-element phase of each receiver during the Image Validation Test (IVT).

#### 4.3.1 Fringe washing term and LO phase track

The fringe washing function at the origin is related with the response of the channels involved in a baseline (12)(13)(16). To measure it, two correlated noise signals at two different temperatures $T_1$, $T_2$ are injected to the baselines that are formed by two receivers which share a common noise source. For the rest of the baselines, an estimation must be calculated (12).

The correlation temperature $T_{C_{kj}}$ of a baseline depends on the power of the noise source, the physical temperature of the NDN and the S-parameters of the NDN. However, if the difference between the two temperatures is calculated, it only depends on the S-parameters and the system temperature measured by the NIRs:

$$T_{C_{kj}}^{T_{kj}^{T_{kj}}} - T_{C_{kj}}^{T_{kj}} = \frac{S_{kj}}{S_{NO}} \cdot (T_{sys}^{T_{kj}} - T_{sys}^{T_{kj}})$$

where $T_{C_{kj}}^{T_{kj}}$, $T_{C_{kj}}^{T_{kj}}$ are the correlation temperatures for the baseline of receivers $k$ and $j$, $S_{kj}$ is the S-parameter of the NDN between the noise source and the output $k$, $S_{kj}$ is the
conjugated S-parameter of the NDN between the noise source and the output \( j \), \( S_{N0} \) is the S-parameter of the NDN between the noise source and the NIR and \( T_{\text{sys}}^{T_1, C}, T_{\text{sys}}^{T_2, C} \) are the system temperatures of the NIRs for the two levels of correlated noise.

Knowing that the correlation temperature can be computed as:

\[
T_{C_{kj}}^C = \sqrt{\frac{T_{\text{sys}}^{T_1, C} \cdot T_{\text{sys}}^{T_2, C}}{G_{kj}^C \cdot M_{kj}^C}} \quad 4.43
\]

where \( T_{C_{kj}}^C \) is the correlation temperatures for the baseline of receivers \( k \) an \( j \), \( T_{\text{sys}}^{T_1, C}, T_{\text{sys}}^{T_2, C} \) are the system temperatures of the receivers \( k \) and \( j \) at CIP, \( G_{kj}^C \) is the fringe washing term and \( M_{kj}^C \) is the quadrature-corrected normalized correlation.

The fringe washing term can be expressed as:

\[
G_{kj}^C = \sqrt{\frac{T_{\text{sys}}^{T_1, C} \cdot T_{\text{sys}}^{T_2, C} \cdot M_{kj}^T}{T_{\text{sys}}^{T_1, C} - T_{\text{sys}}^{T_2, C} \cdot M_{kj}^T}} \cdot \frac{|S_{N0}|}{S_{k0} \cdot S_{j0}} \quad 4.44
\]

where \( T_{\text{sys}}^{T_1, C}, T_{\text{sys}}^{T_2, C} \) and \( T_{\text{sys}}^{T_2, C}, T_{\text{sys}}^{T_1, C} \) are the system temperatures measured at CIP by receivers \( k \) and \( j \) for the two different correlated noise signals, \( M_{kj}^T \) are the quadrature-corrected normalized correlations for the baseline of receivers \( k \) an \( j \) for the two different correlated noise signals, \( T_{\text{sys}}^{T_1, C}, T_{\text{sys}}^{T_2, C} \) are the system temperatures measured by the NIRs at CIP for the two different correlated noises, \( S_{k0} \) is the S-parameter of the NDN between the noise source and the output \( k \), \( S_{j0}^* \) is the conjugated S-parameter of the NDN between the noise source and the output \( j \) and \( |S_{N0}| \) is the modulus of the S-parameter of the NDN between the noise source and the NIR.

The previous equation can be also expressed in terms of the voltages measured by the PMS as follows:

\[
G_{kj}^C = \frac{M_{kj}^T \sqrt{(v_{2_k} - v_{\text{off}_k})(v_{2_j} - v_{\text{off}_j})} - M_{kj}^T \sqrt{(v_{k} - v_{\text{off}_k})(v_{j} - v_{\text{off}_j})}}{\sqrt{(v_{2_k} - v_{k}) \cdot (v_{2_j} - v_{j})}} \cdot \frac{|S_{N0}|}{S_{k0} \cdot S_{j0}^*} \quad 4.45
\]

where \( v_{k}, v_{j} \) are the PMS voltages of the receivers \( k \) and \( j \) while injecting noise at temperature \( T_1 \), \( v_{2_k}, v_{2_j} \) are the PMS voltages of the receivers \( k \) and \( j \) while injecting noise at temperature \( T_2 \), \( v_{\text{off}_k}, v_{\text{off}_j} \) are the PMS offsets of the receivers \( k \) and \( j \).
where \( M_{k,j}^T, M_{k,j}^F \) are the quadrature-corrected complex correlations of the baseline of receivers \( k \) and \( j \), and \( S_{k|i}^S, S_{j|i}^S \) are the S-parameters of the NDN between the noise source and the outputs \( k \) and \( j \).

To obtain the fringe washing term at antenna plane, equation 4.46 is used:

\[
G_{kj}^A = G_{kj}^C \cdot \frac{S_{LA}^*}{S_{LC_k}^*} \cdot \frac{S_{LA}^*}{S_{LC_k}^*}  \tag{4.46}
\]

where \( S_{LA}^*, S_{LA}^* \) are the normalized S-parameters between the antenna input and the output of the switch and \( S_{LC_k}^*, S_{LC_k}^* \) are the normalized S-parameters between the CIP input and the output of the switch.

These normalized parameters follow equation 4.47:

\[
\begin{align*}
S_{LA_k} &= \frac{S_{LA_k}}{S_{LA_k}} \\
S_{LA_k}^* &= \frac{S_{LA_k}^*}{S_{LA_k}^*} \\
S_{LC_k} &= \frac{S_{LC_k}}{S_{LC_k}} \\
S_{LC_k}^* &= \frac{S_{LC_k}^*}{S_{LC_k}^*} \tag{4.47}
\end{align*}
\]

Assuming that the \( G_{kj} \) term of the fringe washing function can be separated in two products which depend on each receiver of the baseline as:

\[
G_{kj} \approx g_k e^{i\alpha_k} g_j e^{-i\alpha_j} \tag{4.48}
\]

where \( g_k, g_j \) and \( e^{i\alpha_k}, e^{-i\alpha_j} \) are the amplitude and the phase terms for receivers \( k \) and \( j \), respectively.

The phase for each receiver can be retrieved by solving the following system:

\[
\begin{align*}
\alpha_{21} &= \alpha_2 - \alpha_1 \\
\alpha_{31} &= \alpha_3 - \alpha_1 \\
&\vdots \\
\alpha_{7271} &= \alpha_7 - \alpha_1 \\
\end{align*}
\implies
\begin{pmatrix}
\alpha_{21} \\
\alpha_{31} \\
\vdots \\
\alpha_{7271}
\end{pmatrix} = \begin{pmatrix}
-1 & 1 & 0 & \cdots & 0 \\
-1 & 0 & 1 & 0 & \cdots & 0 \\
-1 & 0 & 0 & 1 & \cdots & 0 \\
\vdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & \cdots & -1 & 1
\end{pmatrix} \cdot \begin{pmatrix}
\alpha_1 \\
\alpha_2 \\
\vdots \\
\alpha_{72}
\end{pmatrix} \tag{4.49}
\]

where \( \alpha_{kj} = \arg(G_{kj}) \) is the phase of the fringe washing term.

The number of rows of the matrix of the system is 612 because only the baselines formed by receivers with the same noise source are used to compute these phases and
72 columns, one for each receiver. However, the rank of the matrix is 71 due to the fact that one constant phase can be added to the phase terms.

Once the phases assigned to each receiver are known, the baseline phases can be estimated as:

\[ \alpha_{kj} = \alpha_k - \alpha_j \]

The calibration of the \( G_{kj} \) is used to correct the LO phase drift which affects the phase of the correlation. This LO phase drift is known to have certain temperature dependence (23)(24), because the tests performed in the Maxwell chamber the phase of the correlation was very stable due to the fact that the LICEF temperatures were also very stable. This drift is thought to be related with the power divider temperature drift.

In the operational calibration procedure, the fringe washing function is calibrated every 10 minutes (using the LO Phase Tracking Sequence) and it is interpolated between calibrations using a spline approach.

### 4.3.2 Inter-element phase estimation

Due to the fact that the brightness temperature is computed as the inverse Fourier transform of the visibility, all visibilities must be calibrated in order to obtain the most accurate results. To do this, the phase difference between the two receivers of all pairs of antennas (baselines) must be known.

During the SMOS Image Validation Tests and ground characterization performed on-ground in the ESTEC (European Space Research and Technology Centre) of the ESA in Nederland, these differences were measured. The method used consisted on placing MIRAS instrument and a probe in a known location in the Maxwell chamber and measure the phase difference of the elements for four different positions and power emission of the probe (14).

The instrument and the probe location are shown in Figure 4.4 while the scheme of the phases is explained in Figure 4.5:

![Figure 4.4: MIRAS instrument and the probe in Maxwell chamber](image-url)
Therefore, the phase difference between the inter-element phases of receivers $k$ and $j$ is:

$$
\alpha_{kj} = \alpha_i(k) - \alpha_i(j) = \Phi_{kj} - \left(\alpha_p(k) - \alpha_p(j)\right) - \left(\alpha_G(k) - \alpha_G(j)\right) - \left(\alpha_L(k) - \alpha_L(j)\right) \tag{4.51}
$$

where $\Phi_{kj}$ is the phase of the correlation measured between the receivers $k$ and $j$, $\alpha_p$ is the phase of probe antenna pattern referred to zero in boresight, $\alpha_G = -kr$ is the geometrical phase (where $k$ is the wave-number), $\alpha_L$ is the phase of element antenna pattern referred to zero in boresight and $\alpha_i$ is the inter-element phase (to be retrieved).

If the individual phases of each element are wanted, they can be obtained by just solving the following system:

$$
\begin{bmatrix}
\alpha_{1,2} \\
\vdots \\
\alpha_{71,72}
\end{bmatrix} =
\begin{bmatrix}
1 & -1 & 0 & \ldots & \ldots & 0 \\
1 & 0 & -1 & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \ldots & 0 & 1 & 0 & -1 \\
0 & \ldots & 0 & 1 & -1
\end{bmatrix}
\begin{bmatrix}
\alpha_i(1) \\
\vdots \\
\alpha_i(N)
\end{bmatrix} \tag{4.52}
$$

These inter-element phases are applied when calculating the visibility function in the level 1A of the MIRAS Testing Software.
Chapter 5

5 SMOS image reconstruction

This chapter is devoted to explain the theoretical background of the brightness temperature retrieval performed by presenting the equations involved, the spatial sampling and windowing processes as well as the two inversion algorithms that can be used (7)(8)(17)(18).

5.1 Brightness temperature equations

The equation that links the visibility function with the brightness temperature is:

\[
V_{kj}(u_{kj}, v_{kj}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} T_{\delta_{kj}}(\xi, \eta) \cdot e^{-i2\pi(u_{kj}\xi + v_{kj}\eta)} \, d\xi \, d\eta
\]

where \( T_{\delta_{kj}}(\xi, \eta) \) follows equation 5.2:

\[
T_{\delta_{kj}}(\xi, \eta) = \frac{T_B(\xi, \eta)}{\sqrt{1 - \xi^2 - \eta^2}} \cdot \frac{F_{\infty}(\xi, \eta) \cdot F'_{\infty}(\xi, \eta)}{\sqrt{\Omega_k \cdot \Omega_j}} \cdot r_{kj} \left( \frac{-u_{kj}\xi + v_{kj}\eta}{f_0} \right), \quad \text{if } \xi^2 + \eta^2 < 1
\]

\[
T_{\delta_{kj}}(\xi, \eta) = 0, \quad \text{if } \xi^2 + \eta^2 > 1
\]

where \( r_{kj}(\cdot) \) corresponds to the fringe washing function normalized to unity at origin, \( T_B(\xi, \eta) \) is the brightness temperature, \( T_{\delta_{kj}} \) is the mean of the physical temperature of the receivers \( i \) and \( j \), \( F_{\infty}(\xi, \eta), F'_{\infty}(\xi, \eta) \) are the normalized voltage antenna patterns of the receivers, \( \Omega_k, \Omega_j \) correspond to the equivalent solid angles of the antennas, \( (\xi, \eta) \) are the director cosines and \( \sqrt{1 - \xi^2 - \eta^2} \) is the obliquity factor.

The values of the visibility \( V_{kj} \) needed to complete the \( u-v \) plane must verify the following property:

\[
V_{kj}(u_{kj}, v_{kj}) = V^*_{jk}(u_{jk}, v_{jk})
\]

5.3

The baselines that have the same spatial frequency are measuring the same values of the samples of the visibility. Therefore, the value of \( V(u, v) \) will be the mean of the values measured by all the baselines with the same spatial frequency \( (u, v) \).

The centre value \( V(0,0) \) is called zero baseline, and its value corresponds to the antenna temperature which is measured by the NIRs. Since there are three of them
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located in the hub, the zero baseline value will be the mean of the three values given by the three of NIRs.

However, V(0,0) can achieve different values if the consistency of equations 5.1 and 5.2 is preserved. Therefore, the zero baseline value must be equal to the inverting function at the origin.

A figure of merit is defined, the Flat Target Response (FTR), which expression is shown in equation 5.4:

\[
FTR(k,j) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\sqrt{1 - \xi^2 - \eta^2}} \cdot \frac{F_{\nu_k}^\nu_r(\xi, \eta) \cdot F_{\nu_r}^\nu_k(\xi, \eta)}{\sqrt{\Omega_k \cdot \Omega_j}} \cdot \tau_k \left( -\frac{u_{k_j} \xi + v_{k_j} \eta}{f_{0_j}} \right) 
\]

5.4

The FTR can be measured by pointing to a flat target. The properties of a flat target must be:

- Completely unpolarised
- Same brightness temperature in all directions
- Constant brightness temperature over the time

A good approximation of a flat target is an anechoic chamber or the cosmic background radiation.

When the instrument is pointing to a flat target, the FTR can be computed as follows:

\[
FTR(k,j) = \frac{V_{k_j}^{\nu_k}(u_{k_j}, v_{k_j})}{T_B - T_{k_j}} 
\]

5.5

To compute the brightness temperature, different approaches can be used depending on the value assigned to the zero baseline or if the FTR is used or not.

Table 5.1 shows the values of the visibility and the brightness temperature used in each approach:

<table>
<thead>
<tr>
<th>Approach</th>
<th>V(u,v)</th>
<th>V(0,0)</th>
<th>T(\xi,\eta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>V_{k_j}(u_{k_j}, v_{k_j})</td>
<td>T_{A_k} - T_{R_k}</td>
<td>T_B(\xi, \eta) - T_{R}</td>
</tr>
<tr>
<td>2</td>
<td>V_{k_j}(u_{k_j}, v_{k_j}) + T_{R_k} \cdot FTR(k,j)</td>
<td>T_{R_k}</td>
<td>T_B(\xi, \eta)</td>
</tr>
<tr>
<td>3</td>
<td>V_{k_j}(u_{k_j}, v_{k_j}) - (T_{A_k} - T_{R_k}) \cdot FTR(k,j)</td>
<td>0</td>
<td>T_B(\xi, \eta) - T_{A}</td>
</tr>
<tr>
<td>4</td>
<td>V_{k_j}(u_{k_j}, v_{k_j})</td>
<td>T_{A_k}</td>
<td>T_B(\xi, \eta)</td>
</tr>
<tr>
<td>5</td>
<td>V_{k_j}(u_{k_j}, v_{k_j})</td>
<td>0</td>
<td>T_B(\xi, \eta) - T_{A}</td>
</tr>
</tbody>
</table>

*Table 5.1: Different approaches to compute brightness temperature*
5.2 Spatial sampling and aliasing

As explained before, each baseline has associated a pair of $u$-$v$ values (also known as spatial sampling frequencies). If all possible $u$-$v$ values are represented, including the hermitic ones, the following picture is obtained, known as MIRAS star:

![Figure 5.1: Spatial sampling frequencies](image1)

As it can be seen, the spatial frequencies have discrete values. Every discrete sampling produces spatial periodicity (replicas). The case of MIRAS is shown in Figure 5.2, where the fundamental period and the replicas can be distinguished:

![Figure 5.2: Spatial periodicity due to discrete sampling](image2)

If the separation between antennas is higher than $\lambda/\sqrt{3} = 0.577\lambda$ (Nyquist criterion), aliasing (overlapping between replicas) is produced. In MIRAS, the distance between antennas is $d = 0.875\lambda$, therefore there is aliasing, as it can be seen in Figure 5.3:
The \((u,v)\) points of the star belong to a hexagonal grid (Figure 5.4). These points can be computed mathematically using the following equations:

\[
\begin{align*}
    u &= d \cdot \left( \frac{k_1 - k_2}{2} \right) \\
    v &= k_2 \cdot \sqrt{3} \cdot \frac{d}{2}
\end{align*}
\]

where \(d\) is the distance between antennas (in MIRAS, \(d = 0.875\lambda\)) and \(k_1, k_2\) are integer values.

The points of the \((\xi, \eta)\) grid where the brightness temperature is obtained verify equations 5.7 and are also represented in Figure 5.4:

\[
\begin{align*}
    \xi &= \frac{n_2}{d \cdot N_T} \\
    \eta &= \frac{2 \cdot n_1 + n_2}{\sqrt{3} \cdot d \cdot N_T}
\end{align*}
\]

where \(N_T\) is the total number of antennas, \(n_1, n_2\) are integer values and \((\xi, \eta)\) are the director cosines that verify equations 5.8:

\[
\begin{align*}
    \xi &= \sin \theta \cdot \cos \phi \\
    \eta &= \sin \theta \cdot \sin \phi
\end{align*}
\]

Figure 5.3: Aliasing between replicas in MIRAS
Figure 5.4: Hexagonal grid

The geometry involved in the calculus of $(\xi, \eta)$ can be shown in Figure 5.5:

Figure 5.5: Geometry of $\xi$-$\eta$

The election of the $(\xi, \eta)$ grid should be irrelevant, but in order to use an efficient method of inversion, it is the reciprocal of the $(u, v)$ grid. This is achieved by making a change of coordinate from $(u, v)$ to $(k_1, k_2)$ and from $(\xi, \eta)$ to $(n_1, n_2)$, in which both $(k_1, k_2)$ and $(n_1, n_2)$ are identical regular grids of $N_1^2$ points, where $N_1$ is the number of non-redundant $(u, v)$ points.

In the case of a Y-shaped instrument such as MIRAS, this is also the total number of points in the smallest hexagon containing the star of measured points.

5.3 Windowing

Due to the fact that the visibility is only available in the star, a window or weighting function must be used in order to force the visibilities which are outside the star to 0. This window will have a maximum value of 1 and it will decrease if the distance of the sample of the visibility increases with respect to the centre of the $u$-$v$ plane.

The windows available in MIRAS Testing Software are Rectangular, Blackman, Hamming, Hanning and Triangular, which expressions are shown in Table 5.2:
**Table 5.2: Different windows used in MTS**

<table>
<thead>
<tr>
<th>Window</th>
<th>( W(u,v) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>( 1 )</td>
</tr>
<tr>
<td>Blackman</td>
<td>( 0.42 + 0.5 \cdot \cos(\pi \cdot \frac{\rho}{\rho_{\text{max}}}) + 0.08 \cdot \cos(2\pi \cdot \frac{\rho}{\rho_{\text{max}}}) )</td>
</tr>
<tr>
<td>Hamming</td>
<td>( 0.54 + 0.46 \cdot \cos(\pi \cdot \frac{\rho}{\rho_{\text{max}}}) )</td>
</tr>
<tr>
<td>Hanning</td>
<td>( 0.5 + 0.5 \cdot \cos(\pi \cdot \frac{\rho}{\rho_{\text{max}}}) )</td>
</tr>
<tr>
<td>Triangular</td>
<td>( 1 - \frac{\rho}{\rho_{\text{max}}} )</td>
</tr>
</tbody>
</table>

where \( \rho = \sqrt{u^2 + v^2} \) and \( \rho_{\text{max}} = \sqrt{u_{\text{max}}^2 + v_{\text{max}}^2} \) is its maximum value.

Each window has a different spatial resolution and radiometric accuracy. The election of one window is a trade-off solution because it will vary depending on the requirements needed in the image.

The aspect of the different windows used in the software can be seen in Figure 5.6:

![Figure 5.6: Different windows used in MTS](image)

In order to see the effect in the brightness temperature and the image when the visibility is windowed, Figure 5.7 shows the same snapshot processed with the same inversion approach and the different windows available:
5.4 Inversion algorithms

In the process of the brightness temperature retrieval, two different algorithms can be used: the Inverse Fourier transform and the G-matrix transform.

5.4.1 Inverse Fourier transform

The brightness temperature can be calculated using the Fourier transform if the following premises are verified:

- Only one antenna pattern is considered for all the antennas
- Spatial decorrelation (fringe washing function) is neglected
- The redundant visibilities are averaged
- The physical temperatures of the receivers are averaged
Before calculating the brightness temperature, the visibility must be discretized, as shown in equation 5.9:

\[
V(k_1, k_2) = \frac{2}{\sqrt{3} \cdot N_T^2 \cdot d^2} \sum_{N_{\text{min}}}^{N_{\text{max}}} \sum_{N_{\text{min}}}^{N_{\text{max}}} T_B(n_2, n_1) e^{-j2\pi n_2 n_1 \lambda n_1}
\]  \hspace{1cm} 5.9

Therefore, the brightness temperature is computed as:

\[
T_B'(n_2, n_1) = \frac{\sqrt{3}}{2} d^2 \sum_{N_{\text{min}}}^{N_{\text{max}}} \sum_{N_{\text{min}}}^{N_{\text{max}}} V(k_1, k_2) e^{j2\pi n_2 n_1 \lambda n_1}
\]  \hspace{1cm} 5.10

where \(d\) is the distance between antennas and \(N_T\) is the total number of antennas.

Figure 5.8 shows a snapshot processed with the FFT inversion algorithm and the different approaches explained in 5.1:

![Figure 5.8: Snapshot processed with FFT, rectangular window and approaches 1 to 5](image)

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If the number of points of the Fourier transform is a power of 2, the algorithm is faster because the Fast Fourier Transform (FFT) is used. MIRAS Testing Software implements this algorithm but allows to select the number of points used. If a higher number of points are used, the brightness temperature values are more exact and the image becomes smoother.

Figure 5.9 shows a snapshot processed with the same parameters but selecting two different numbers of points in the FFT:

![Image](image.png)

**Figure 5.9: Snapshot processed with FFT, rectangular window, approach #1, 64 and 1024 points respectively**

### 5.4.2 G-matrix transform

The second algorithm used to retrieve the brightness temperature is called G-matrix transform. The discretized visibility can be expressed as a system of equations as follows:

\[
\begin{bmatrix}
V_1 \\
\vdots \\
V_N
\end{bmatrix} = 
\begin{bmatrix}
G_{11} & G_{12} & \cdots & G_{1M} \\
\vdots & \vdots & \ddots & \vdots \\
G_{N1} & G_{N2} & \cdots & G_{NM}
\end{bmatrix}
\begin{bmatrix}
T_1 \\
T_2 \\
\vdots \\
T_M
\end{bmatrix}
\]

where \( N \) is the number of visibility samples and \( M \) is the number of \((\xi, \eta)\) points.

The number of visibility samples \( N \) can be the total number of measured visibilities plus their hermitics or the total number of measured visibilities averaging the redundant ones. The number of points \( M \) of the \((\xi, \eta)\) grid is always equal to the total number of points in the hexagon.

The elements \( G_{ln} \) are calculated as:

\[
G_{ln}(k,j) = \frac{2}{\sqrt{3} \cdot N_T^2 \cdot d^2} \frac{F_{nk}(n_2,n_1) \cdot F^*_n(n_2,n_1)}{\Omega_k \Omega_l \sqrt{1 - \xi^2 - \eta^2}} \cdot \frac{n_x^2 + k_x n_y}{f_0}
\]

5.12
Therefore, the brightness temperature matrix can be calculated using:

\[
[T] = [G]^* \cdot [V] \quad 5.13
\]

where \([G]^*\) is the Moore-Penrose pseudoinverse matrix of \(G\) and is calculated as

\[
\]

Figure 5.10 shows a snapshot processed with the G-matrix inversion algorithm and the different approaches explained in 5.1:

Figure 5.10: Snapshot processed with G-matrix and approaches 1 to 5
Chapter 6

6 The Redundant Space Calibration method

This chapter is intended to explain the Redundant Space Calibration method (1), which is used to assess the phase error in MIRAS instrument. This method uses the visibility values of the redundant baselines to compute the inter-element phase of each receiver.

In the next sections, the concepts and equations that are the basis of this method will be detailed.

6.1 Redundant baselines

Two baselines are said to be redundant (RB) one with the other when the distance and the direction of the two receivers involved in each baseline are the same. Because of the Y-shape of MIRAS instrument, only the baselines that are in the same arm can be redundant between them.

Figure 6.1 shows three different pairs of redundant baselines, each one in a different arm:

Figure 6.1: Examples of redundant baselines

Ideally, the visibility samples measured by a set of redundant baselines are the same. In practice, this does not happen due to the presence of instrumental errors (extra phases added by the receivers), propagation errors, differences in antenna patterns or in the fringe washing function.
6.2 System of equations

Performing a phase calibration in the instrument will correct any possible phases that the receivers might be adding to the measures. To determine the value of these phases, the measured visibility samples are used.

The phase of the visibility measured by a baseline is:

\[
\phi_{ji} = \varphi_j - \varphi_i + \varphi_{\text{scene,}ji} + \Delta \varphi_{ji}
\]

where \( \phi_{ji} \) is the phase of the visibility of the baseline formed by receivers \( i \) and \( j \), \( \varphi_j \) is the phase introduced by the receiver \( j \), \( \varphi_i \) is the phase introduced by the receiver \( i \), \( \varphi_{\text{scene,}ji} \) is the phase of the current scene seen by the instrument and \( \Delta \varphi_{ji} \) is a non-separable phase term that in this analysis will be neglected because its value is very low.

Each equation has 3 unknowns. The phase of the scene is different for redundant baselines of different type, so there will be as many different \( \varphi_{\text{scene}} \) as types of redundant baselines are. Having the phase of the scene in the equation adds an unknown that is not necessary to solve which increases the complexity and the dimensions of the system.

In order to eliminate the phase of the scene of the equation, baseline phase differences will be used instead of baseline phases. If another equation of a redundant baseline of the same distance is built and subtracted the previous equation to the new one, the unknown of the phase of the scene will disappear thanks to the fact that for redundant baselines of the same distance and direction the phase of the scene is the same:

\[
\begin{align*}
\phi_{ji} &= \varphi_j - \varphi_i + \varphi_{\text{scene,}ji} \\
\phi_{kj} &= \varphi_k - \varphi_j + \varphi_{\text{scene,}kj}
\end{align*}
\]

\[
\begin{align*}
\phi_{kj} - \phi_{ji} &= \varphi_k - \varphi_j + \varphi_{\text{scene,}kj} - (\varphi_j - \varphi_i + \varphi_{\text{scene,}ji}) = \\
\varphi_k - 2\varphi_j + \varphi_i + (\varphi_{\text{scene,}ji} - \varphi_{\text{scene,}kj}) &= \varphi_k - 2\varphi_j + \varphi_i
\end{align*}
\]

The system of equations is built by gathering all possible equations that are linearly independent with the rest. The equations of the system can be classified in three different types (the numbering of the receivers is shown in Appendix 1: Receivers nomenclature):

- **Type 1**: Equations involving baselines of distance 1 formed by receivers that are neither NIRs nor numbers 1, 25 and 49.
There are 57 equations of this type (the same for horizontal and vertical polarizations) which are shown below:

\[
\begin{align*}
\text{Arm A:} & \quad \phi_{0,5} - \phi_{5,4} = \phi_{2} - 2\phi_{3} + \phi_{4} \\
& \quad \ldots \\
\phi_{24,23} - \phi_{23,22} = \phi_{24} - 2\phi_{23} + \phi_{22} \\
\text{Arm B:} & \quad \phi_{30,29} - \phi_{29,28} = \phi_{30} - 2\phi_{29} + \phi_{28} \\
& \quad \ldots \\
\phi_{48,47} - \phi_{47,46} = \phi_{48} - 2\phi_{47} + \phi_{46} \\
\text{Arm C:} & \quad \phi_{54,53} - \phi_{53,52} = \phi_{54} - 2\phi_{53} + \phi_{52} \\
& \quad \ldots \\
\phi_{72,71} - \phi_{71,70} = \phi_{72} - 2\phi_{71} + \phi_{70}
\end{align*}
\]

Figure 6.2 shows some examples of baselines used to build the previous equations. The direction of the arrow indicates the direction of the baseline and each pair of arrows with the same colour is used to build a different equation:

![Figure 6.2: Examples of baselines used in equations of type 1](image)

- **Type 2**: Equations involving baselines of distance 1 formed by receivers that are NIRs and receivers inside the hub. Due to the fact that the number of the receiver of the NIR is different in H or V, different equations must be written depending on the polarization.

There are 5 equations of this type for each polarization, which are:
Phase error assessment of MIRAS/SMOS by means of Redundant Space Calibration method

Horizontal:

\[
\begin{align*}
\phi_{5,4} - \phi_{25,50} &= \phi_5 - \phi_4 + \phi_{52} - \phi_{50} \\
\phi_{29,28} - \phi_{52,26} &= \phi_{29} - \phi_{28} + \phi_{52} - \phi_{26} \\
\phi_{29,28} - \phi_{4,2} &= \phi_{29} - \phi_{28} + \phi_{4} - \phi_{2} \\
\phi_{53,52} - \phi_{30,4} &= \phi_{53} - \phi_{52} - \phi_{30} + \phi_{4}
\end{align*}
\]

6.5

Vertical:

\[
\begin{align*}
\phi_{5,4} - \phi_{25,51} &= \phi_5 - \phi_4 + \phi_{52} - \phi_{51} \\
\phi_{29,28} - \phi_{52,27} &= \phi_{29} - \phi_{28} + \phi_{52} - \phi_{27} \\
\phi_{29,28} - \phi_{4,3} &= \phi_{29} - \phi_{28} + \phi_{4} - \phi_{3} \\
\phi_{53,52} - \phi_{51,4} &= \phi_{53} - \phi_{52} - \phi_{51} + \phi_{4}
\end{align*}
\]

The last two phases of the receivers of some equations have their sign changed. This is because the two baselines involved have different directions, which is reflected with a change of sign when building the equation.

Figure 6.3 shows some examples of baselines used in the previous equations, where each pair of arrows with a different colour is used to build a different equation. Note that there is one pair of baselines with the arrows pointing to a different direction, which means that the equation has the signs changed.

![Diagram of MIRAS/SMOS setup](attachment:diagram.png)

*Figure 6.3: Examples of baselines used in equations of type 2*

- **Type 3**: Equations involving baselines of distance 2 formed by receivers 1, 25 and 49, NIRs and receivers in the direction of the arm. It also must be separated horizontal and vertical equations. For the same reason of some equations of type 2, there is one equation of horizontal polarization that has the signs changed.
There are 4 equations of this type for each polarization, which are:

**Horizontal:**

\[
\begin{align*}
\phi_6 - \phi_{26,25} &= \varphi_6 - \varphi_4 - \varphi_{26} + \varphi_{25} \\
\phi_{54,52} - \phi_{2,1} &= \varphi_{54} - \varphi_{52} - \varphi_2 + \varphi_1 \\
\phi_{50,28} - \phi_{50,49} &= \varphi_{50} - \varphi_{28} - \varphi_{50} + \varphi_{49} \\
\phi_{50,28} - \phi_{50,49} &= 2\varphi_{50} - \varphi_{28} - \varphi_{49}
\end{align*}
\]

**Vertical:**

\[
\begin{align*}
\phi_{6} - \phi_{27,25} &= \varphi_6 - \varphi_4 - \varphi_{27} + \varphi_{25} \\
\phi_{54,52} - \phi_{3,1} &= \varphi_{54} - \varphi_{52} - \varphi_3 + \varphi_1 \\
\phi_{50,28} - \phi_{51,49} &= \varphi_{50} - \varphi_{28} - \varphi_{51} + \varphi_{49} \\
\phi_{52,3} - \phi_{3,1} &= \varphi_{52} - 2\varphi_3 + \varphi_1
\end{align*}
\]

Figure 6.4 shows some examples of baselines of type 3 used in the previous equations. The direction of the arrow indicates the direction of the baseline and each pair of arrows with the same colour is used to build a different equation. As explained before, there are some receivers with their sign changed:

![Figure 6.4: Examples of baselines used in equations of type 3](image)

All the other possible equations that can be built are linearly dependent, no matter the distance or the receivers involved. Although these equations will be useful to build an overdetermined system and reduce the noise in the solutions, they are not included in the system because they are known to be noisier than the others and the complexity of the system will increase. In order to reduce the noise in the retrieved phases, it is performed a time filtering to the visibility samples used in the equations.

After gathering the 66 linearly independent equations and taking into account that there are only 69 phases to solve for each polarization (because the NIRs of the other
polarization do not appear in the equations), a system of equations can be built with the following aspect:

\[
A \phi_{\text{receivers}} = \phi_{\text{phase differences}} \quad 6.7
\]

\[
\begin{pmatrix}
0 & 0 & 1 & -2 & 1 & 0 & \ldots & \ldots & 0 \\
0 & 0 & 0 & 1 & -2 & 1 & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & \ldots & \ldots & \ldots & \ldots & 0 & 1 & -2 & 1 \\
\vdots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\ldots & -1 & 1 & \ldots & -1 & 1 & \ldots & \ldots & \ldots \\
\ldots & -1 & \ldots & 1 & \ldots & -1 & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\end{pmatrix}
\]

\[
\phi_{\text{receivers}} = \phi_{\text{phase differences}} \quad 6.8
\]

where \( A \) is the matrix containing the phase coefficients (66 rows, 69 columns), \( \phi_{\text{receivers}} \) is the column vector containing the phases to retrieve (69 rows, 1 column) and \( \phi_{\text{phase differences}} \) is the column vector containing the phase differences (66 rows, 1 column).

The maximum rank of the matrix that can be achieved is 66 (using the previous equations), and it will not increase if more equations are added. The three degrees of freedom of the system are due to three unknown phases which have physical meanings, as explained below:

- **Common path delay**

A common path delay, for instance, an increment of the attitude of the satellite (Figure 6.5) does not affect the visibility samples used in the system.

![Figure 6.5: Common path delay](image-url)
• **Steering angle**

If the satellite measures with different steering angles (Figure 6.6), $T_B(\xi, \eta)$ and $\varphi_{\text{sense}}$ are different in each case but the set of the equations obtained are the same in both cases.

![Image](image_url)

*Figure 6.6: Different steering angles*

• **Tilt angle**

If the satellite measures with different tilt angles (Figure 6.7), the result is a compression of the image seen by the satellite. However, the set of the equations obtained in both cases are the same.

![Image](image_url)

*Figure 6.7: Different tilt angles*

As it can be seen in 6.8, the matrix $A$ is non-squared, which implies that the system cannot be solved using a normal inverted matrix. To solve the system, it is used the Moore-Penrose pseudoinverse matrix of $A$ (denoted as $A^+$), which is calculated as:

$$A^+ = (A^T A)^{-1} A^T$$  \hspace{1cm} 6.9$$

where $A^+$ is the Moore-Penrose pseudoinverse matrix of $A$, $A^T$ is the transposed matrix of $A$ and $X^{-1}$ is the inverted matrix of $X$. 

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The pseudoinverse matrix $A^+$ has an important property: the solution retrieved by the system is non-unique (because it is an underdetermined system), but it is the one with the minimum Euclidean norm among all the possible solutions.

In order to study the behaviour of the system, some simulations have been performed following these steps:

1. Generate 69 zero mean random phases using a normal distribution and a certain standard deviation

2. Calculate the independent term of the system (phase differences) by applying the matrix $A$ to the column vector containing the values of the random phases:

$$ \phi_{\text{phase differences}} = A \phi_{\text{receivers, random}} \tag{6.10} $$

3. Calculate the retrieved phases by applying the pseudoinverse matrix $A^+$ to the column vector containing the values of the phase differences:

$$ \phi_{\text{receivers, retrieved}} = A^+ \cdot \phi_{\text{phase differences}} \tag{6.11} $$

4. Calculate the error committed in the phase retrieval as:

$$ \phi_{\text{receivers, error}} = \phi_{\text{receivers, random}} - \phi_{\text{receivers, retrieved}} \tag{6.12} $$

As an example, a simulation with a standard deviation $\sigma = 10$ has been performed, obtaining the following results (in H polarization, for instance):
As it can be observed, the error when retrieving the phases is always linear in each arm, according to the position of the receiver in the arm \((x_i, y_i)\), having each arm a different slope that can be positive or negative. This fact is a key point in this study, because it implies that the phase error is also linear in the sampling domain \((u, v)\).

To prove this, the following procedure has been followed:

1. Calculate the real baseline phases using the random set of phases of the receivers used for the simulation

2. Calculate the retrieved baseline phases using the retrieved set of phases of the receivers of the simulation
3. Calculate the baseline phase error, computed as the difference between the real baseline phases and the retrieved baseline phases:

$$\varphi_{\text{error,bl}N} = \varphi_{\text{rad,bl}N} - \varphi_{\text{retrieved,bl}N}$$  \hspace{1cm} 6.13

Assuming that this error is linear with u and v, it must obey a linear equation as:

$$\varphi_{\text{error,bl}N} = a_u u_{\text{bl}N} + b_v v_{\text{bl}N}$$  \hspace{1cm} 6.14

where $u_{\text{bl}N}$ is the u value of the baseline N, $v_{\text{bl}N}$ is the v value of the baseline N and a, b are the linear factors needed.

4. For two chosen baselines, the parameters a and b can be computed by solving the system in 6.15 for each polarization (in this case, only for horizontal):

$$\begin{bmatrix} a_H & b_H \end{bmatrix} = \begin{bmatrix} \varphi_{\text{errorH,bl}1} & \varphi_{\text{errorH,bl}2} \end{bmatrix} \begin{bmatrix} u_{\text{bl}1} & u_{\text{bl}2} \end{bmatrix}^{-1}$$  \hspace{1cm} 6.15

where $\varphi_{\text{errorH,bl}1}$, $\varphi_{\text{errorH,bl}2}$ are the errors computed from 6.13, $u_{\text{bl}N}$ is the u value of the baseline N and $v_{\text{bl}N}$ is the v value of the baseline N.

5. Calculate the linearized error by applying the retrieved $a_H$, $b_H$ values to the system:

$$\begin{bmatrix} \varphi_{\text{erlinH,bl}1} & \cdots & \varphi_{\text{erlinH,bl}2556} \end{bmatrix} = \begin{bmatrix} a_H & b_H \end{bmatrix} \begin{bmatrix} u_{\text{bl}1} & \cdots & u_{\text{bl}2556} \\ v_{\text{bl}1} & \cdots & v_{\text{bl}2556} \end{bmatrix}$$  \hspace{1cm} 6.16

where $\varphi_{\text{erlinH,bl}N}$ is the linearized error of the baseline N, $u_{\text{bl}N}$ is the u value of the baseline N, $v_{\text{bl}N}$ is the v value of the baseline N and $a_H$, $b_H$ are the linear coefficients for this polarization.

6. Calculate the difference between the real error and the linearized error:

$$\varphi_{\text{diffH,bl}N} = \varphi_{\text{errorH,bl}N} - \varphi_{\text{erlinH,bl}N}$$  \hspace{1cm} 6.17

If the error is really linear, this difference should be very small.

After calculating all the values, the results are:
As it can be seen in Figure 6.9, the difference is almost negligible. In vertical polarization, very similar results have been obtained. Therefore, it has been proved that the RSC retrieval phase error is linear with \( u \) and \( v \).
Chapter 7

7 Phase behaviour assessment of redundant baselines

The assessment of MIRAS phase error is going to be studied from the results obtained using the system of equations described in the previous chapter. The input data of the system is the phase differences of the visibility of the redundant baselines shown before.

In order to be sure that the results are reliable, the samples of the visibility used to build the independent term of the system must have the best quality possible. Although these samples are averaged in time, there are some situations where the redundant baselines are known not to work well, so these situations must be avoided when selecting data for the phase retrieval.

7.1 Phase noise due to low visibility amplitude

For any scene depending on the geometry of the brightness temperature distribution, some of the visibility samples may have a very low amplitude (below 1 K). In this case, the phases of these baselines may have unwanted variations or jumps that might be masking the real value of these phases.

This problem is solved by averaging long series of snapshots to reduce the noise to estimate the standard deviation of the phase of the redundant baselines.

In general, the measured standard deviation of visibility phases is large. This yields large errors in the retrieval of phase errors. The phase jumps due to low amplitude of the visibility are solved by avoiding time series of visibility measurements, including low visibility amplitude.

In addition, this problem affects the receivers yielding baselines with a specific orientation. This forces the need to use scenes with different TB patterns to use high amplitude visibility measurements containing all directions.

Some examples of phase noise due to low visibility amplitude are shown in Figure 7.1:

Example 1
Phase error assessment of MIRAS/SMOS by means of Redundant Space Calibration method
Example 2
Figure 7.1: Examples of phase noise due to low amplitude

7.2 Phase bias due to fast scene changes

The visibilities of the baselines are computed from the correlations. For each snapshot, these correlations are read sequentially, so the values are not obtained at the same time. The time difference between the first and the last reading of correlation is estimated in 300 ms in order to have all the correlations every 1.2 s.

If the scene is over the land or the ocean, the effect of the sequential reading is negligible but it turns important in some situations such as land-ocean transitions or small islands in the middle of the ocean, because, for instance, the visibility sample of the first baseline may be from the ocean while the visibility sample of the last baseline might be from the land.
The effect of these fast scene changes comes into a phase bias, which means that there are differences in the values of the phases for redundant baselines because each baseline is seeing a scene slightly different than the others.

Figure 7.2 shows some examples of phase bias due to land-ocean transitions:

**Example 1**
Example 2
Phase behaviour assessment of redundant baselines

**Horizontal Amplitude Visibility - Baselines Arm A**

Test: 24/05/2010

**Vertical Amplitude Visibility - Baselines Arm A**

Test: 24/05/2010

**Horizontal Phase Visibility - Baselines Arm A**

Test: 24/05/2010

**Vertical Phase Visibility - Baselines Arm A**

Test: 24/05/2010

**Horizontal Amplitude Visibility - Baselines Arm B**

Test: 24/05/2010

**Vertical Amplitude Visibility - Baselines Arm B**

Test: 24/05/2010

**Horizontal Phase Visibility - Baselines Arm B**

Test: 24/05/2010

**Vertical Phase Visibility - Baselines Arm B**

Test: 24/05/2010
7.3 Phase errors due to RFI

Although MIRAS works in a protected band reserved for scientific purposes, there are interferences that spoil the values obtained by the instrument. These interferences come from radio transmitters and they must be avoided because the phase and the amplitude measured in presence of an RFI are contaminated, spoiling the RSC method.

As seen in Figure 7.3, there are jumps in the phase samples of the visibility corresponding to jumps in the amplitude at the same time due to the presence of RFI.

**Example 1**
Phase behaviour assessment of redundant baselines
Example 2
7.4 Searching for good quality sets of redundant baselines

To perform a good time filtering, an area where the phases of the visibility samples are as flat as possible must be searched, in order to reduce the errors in the instrumental phase retrievals by averaging.

The samples of the visibility used for the phase retrieval must not be in any of the three situations mentioned before. The best option to avoid these situations is to choose orbits that cross an ocean (for instance, the Pacific, the Atlantic or the Indian) and select a filtering area where the visibility phases are the flattest possible.

In the filtering area, the visibility phases must be flat in the three arms at the same time, in order to avoid errors in the retrieval. Moreover, different filtering areas for each arm cannot be used to retrieve a solution because the independent term of the
system will not be consistent. However, the selected areas can be different for each polarization because the system to solve is different in horizontal than in vertical.

Some examples of good phase visibility sets of redundant baselines are shown in Figure 7.4. The blue line represents the orbit and the red line is the selected area of the orbit:

**Example 1**
Example 2
7.5 Conclusions

As a conclusion, the Redundant Space Calibration cannot be used to track fast phase drifts, such as LO phase drift, due to the need for long averaging series and redundant baselines filtering to reject noisy, scene biased or RFI contaminated sets of values from the system of equations.

However, it can be very useful when trying to assess the fix residual relative phase error (CIP to HAP/VAP) and to correct it if required to comply with the phase error requirements in the error budget (25).
Chapter 8

8 Quality assessment of the RSC method

The RSC method, in a first approximation, is assessed by measuring the degradation of two parameters:

- Point Source Primary to Secondary Lobe Ratio
- Half-Power Beam Width (HPBW)

One must keep in mind that the ultimate parameter is the reduction of pixel bias (radiometric spatial error). However, radiometric spatial error measurement form real images are contaminated not only by residual phase errors, but for a bunch of additional contributions such as amplitude errors, antenna errors, RFI, image roughness...

This is why the first approximation to assess the impact of phase errors is through point source response degradation.

When implementing the Redundant Space Calibration method to retrieve the phases of the receivers and simulate their effect in the brightness temperature, there are some parameters that are important because they will determine the impact of these phases in the results.

The values of the inversion parameters of MIRAS Testing Software used for the simulation are:

- Number of points of the FFT: 1024
- Approach: #1
  - $V(u, v) = V_{kl}(u_{kl}, v_{kl})$
  - $V(0, 0) = T_A - T_R$
  - $T(\xi, \eta) = T_B(\xi, \eta) - T_R$
- Window: Rectangular

8.1 Simulation of the residual phase error retrieval

In order to study the effect of phase errors in the brightness temperature, a simulation with an ideal point source will be performed. This point source will be placed in a known position of the $\xi - \eta$ grid.

The procedure followed for the simulation is (only performed for one polarization):

1. Generate the ideal visibility of a point source of temperature $T_{ps}$ placed in the position $(\xi_{ps}, \eta_{ps})$: 77
\[ V_{\text{ideal},kj} = T_p e^{j2\pi(\nu_k \tau_p + \eta_p \tau_k)} \]

2. Compute the brightness temperature using this visibility and generate a snapshot

3. Generate 69 zero mean random values for the phases of the receivers and calculate the phases of the baselines with these values

4. Simulate the uncalibrated visibility by adding the random phases of the baselines to the ideal visibility:
\[ V_{\text{meas},kj} = V_{\text{ideal},kj} \cdot e^{j\phi_{kj,\text{random}}} \]

where \( V_{\text{ideal},kj} = T_p e^{j2\pi(\nu_k \tau_p + \eta_p \tau_k)} \) and \( \phi_{kj,\text{random}} = \phi_{k,\text{random}} - \phi_{j,\text{random}} \) is the random phase of the baseline, which is the difference between the random phases of the receivers.

5. Compute the brightness temperature using this new visibility and generate a snapshot

6. Calculate the retrieved phases by applying the RSC method and calculate the retrieved baseline phases. In this case, there is no need for averaging since the uncalibrated visibility is noise free

7. Simulate the calibrated visibility by adding the retrieved phases of the baselines (changing their sign, to correct) to the uncalibrated visibility:
\[ V_{\text{cal},kj} = V_{\text{meas}} \cdot e^{-j\phi_{kj,\text{retrieved}}} \]

where \( \phi_{kj,\text{retrieved}} = \phi_{k,\text{retrieved}} - \phi_{j,\text{retrieved}} \) is the retrieved phase of the baseline, which is the difference between the retrieved phases of the receivers.

Note that for a perfect phase retrieval (error free), equation 8.4 must be verified:
\[ V_{\text{cal},kj} = V_{\text{ideal}} \cdot e^{j\phi_{kj,\text{random}}} \cdot e^{-j\phi_{kj,\text{retrieved}}} \]

8. Compute the brightness temperature using this new visibility and generate a snapshot

Some interesting parameters have been calculated to study the behaviour of the point source when adding the random phases and after applying the correction.
These parameters are the following:

- Position in the $\xi - \eta$ grid:
  - Point source at nominal position (ideal case)
  - Uncalibrated point source
  - Calibrated point source
- Primary to Secondary Lobe Ratio

A simulation has been performed using a large standard deviation for the random phases ($\sigma = 25$) in order to better see the effect of the phases in the point source.

Figure 8.1 shows the result of the simulation when applying the RSC method to the random phases previously generated, as well as the distribution of the phase error in the baselines and the difference between the error and the linearized error:
Figure 8.1: Example of simulated relative phase errors in each LICEF: a) Random phases, b) Retrieved phases by the RSC method, c) Phase error showing linear dependence with the antenna position, d) Baseline phase error histogram showing some baselines with large errors, and e) Difference between the real error and the linearized error.
As it can be seen, the error committed in the retrieval of the phases is linear along the three arms, as proved before. The baseline error histogram shows large phase errors for some baselines, but these errors are systematic.

Figure 8.2 shows the resultant snapshots from the ideal, uncalibrated and calibrated visibilities. The impact of the phases in the uncalibrated visibility is image blurring, which is solved when applying the phase correction in the calibrated visibility.

![Figure 8.2: Simulation of a point source: a) Ideal Point Source, b) Uncalibrated Point Source and c) Calibrated Point Source](image-url)
Figure 8.3: Simulation of the point source with zoom: a) Ideal Point Source, b) Uncalibrated Point Source and c) Calibrated Point Source

The presence of errors in the phase retrieval turns also into a shift of the point source. This shift is small and it can only be seen when it is performed a zoom near the nominal position of the point source (Figure 8.3).
The linear coefficients \((a, b)\) which make the retrieval phase errors linear with \((u, v)\) allow to calculate this shift theoretically, as will be explained in 8.4. However, this shift is affected by the rounding effect of the brightness temperature retrieval due to the fact that the grid used only has a certain number of points. This effect depends on the grid step, and for a large grid steps, the rounding effect is important.

The grid step is also related to the spatial resolution. Figure 8.4 shows the geometry involved in the spatial resolution:

![Figure 8.4: Geometry of the spatial resolution](image)

In SMOS, \(\beta\) is 32.5° (tilt angle), \(h_{\text{sat}}\) is 755 km, \(r\) is the distance to the point the satellite is looking at and \(R_T\) is the Earth radius (6371 km). With these values, some parameters can be calculated using the following equations:

\[
\sin(\gamma) = \frac{R_T + h_{\text{sat}}}{R_T} \sin(\beta)
\]

\[
\alpha = \pi - \beta - \gamma
\]

\[
r = \frac{\sin(\alpha)}{\sin(\beta)} \cdot R_T
\]

\[
i = \pi - \gamma
\]

\[
\gamma = 143.06°
\]

\[
\alpha = 4.43°
\]

\[
r = 918 \text{ km}
\]

\[
i = 36.94°
\]

where \(r\) is the distance between the satellite and the area the satellite is looking at.

Therefore, the spatial resolution can be computed as:

\[
dL (\text{km}) = \frac{r \cdot \Delta \theta}{\cos(i)}
\]

From equation 8.6, the spatial resolution in SMOS at boresight can be calculated taking into account that \(\Delta \theta_i = 2.1°\):
\[ \Delta L_{\text{SMOS}} = 42 \text{ km} \]

8.7

Table 8.1 shows the grid steps for all the possible number of points that can be chosen in MIRAS Testing Software as well as the spatial resolution of each grid (\( \Delta \theta = \Delta \xi \)):

<table>
<thead>
<tr>
<th>FFT points</th>
<th>( \Delta \xi )</th>
<th>( \Delta \eta )</th>
<th>( \Delta L ) (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>64</td>
<td>0.01786</td>
<td>0.01031</td>
<td>20.46</td>
</tr>
<tr>
<td>128</td>
<td>0.00893</td>
<td>0.00515</td>
<td>10.23</td>
</tr>
<tr>
<td>256</td>
<td>0.00446</td>
<td>0.00258</td>
<td>5.12</td>
</tr>
<tr>
<td>512</td>
<td>0.00223</td>
<td>0.00129</td>
<td>2.56</td>
</tr>
<tr>
<td>1024</td>
<td>0.00112</td>
<td>0.00064</td>
<td>1.28</td>
</tr>
</tbody>
</table>

Table 8.1: Grid steps and spatial resolution for different FFT points

8.2 Point source degradation due to phase errors

Taking a look at the uncalibrated snapshot, it can be seen that the random phases have deteriorated it, becoming blurry. This degradation has impact in the spatial resolution of the image and the decreasing of the Primary to Secondary Lobe Ratio.

8.2.1 Spatial resolution degradations

When adding the random phases, the spatial resolution of the image (HPBW) becomes worse. After the correction, this parameter returns almost to its previous value.

8.2.2 Secondary lobes increase

When adding the random phases, the image becomes blurry with a slight change in the scale of colours. This visual effect has a direct impact in the Primary to Secondary Lobe Ratio because the primary lobe is decreasing its value at the same time the secondary lobes are increasing.

Table 8.2 shows the comparison between the three cases of the point source for the simulation performed in 8.1:

<table>
<thead>
<tr>
<th>Case</th>
<th>Primary to Secondary Lobe Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal Point Source</td>
<td>17.0788 dB</td>
</tr>
<tr>
<td>Uncalibrated Point Source</td>
<td>14.2241 dB</td>
</tr>
<tr>
<td>Calibrated Point Source</td>
<td>17.0704 dB</td>
</tr>
</tbody>
</table>

Table 8.2: Primary to Secondary Lobe Ratio

8.3 Effect of underdetermination

It has been proved that the retrieved phases correct the distortion of the image, but due to the fact that they have a systematic error because of the underdetermination of the system, the image recovered is not the same as the ideal one.
Having a look at the position of the point source in the $\xi - \eta$ plane in the three cases, the retrieved phases do not correct the shift caused by the random phases: there is a pointing error.

### 8.4 Pointing error retrieval by means of a point source

The systematic error that includes the retrieved phases can be assigned to a pointing error. Before, it was proved that the error between the real phases and the retrieved ones was linear in $u-v$ and it can be written as:

$$\varphi_{\text{error}} = a\cdot u + b\cdot v$$  \hspace{1cm} 8.8

The brightness temperature is computed from the visibility using the following formula:

$$T_{B_{\text{obs}}} (\xi, \eta) = \int \int_{-\infty}^{\infty} V(u,v) \cdot e^{j2\pi(u\cdot \xi + v\cdot \eta)} \cdot dudv$$  \hspace{1cm} 8.9

If the brightness temperature formula is rewritten taking into account the presence of this error in the visibility:

$$T_{B_{\text{obs}}} (\xi, \eta) = \int \int_{-\infty}^{\infty} V(u,v) \cdot e^{j2\pi(u\cdot \xi + v\cdot \eta)} \cdot e^{-j2\pi \left( \frac{1}{2\pi} u\cdot a + \frac{1}{2\pi} b\cdot v \right)} \cdot dudv$$  \hspace{1cm} 8.10

Reorganizing terms:

$$T_{B_{\text{obs}}} (\xi, \eta) = \int \int_{-\infty}^{\infty} V(u,v) \cdot e^{j2\pi(u\cdot \xi + v\cdot \eta)} \cdot e^{j2\pi \left( \frac{1}{2\pi} u\cdot a + \frac{1}{2\pi} b\cdot v \right)} \cdot \cdot \cdot dudv$$  \hspace{1cm} 8.11

Therefore, according to the duality property of the Fourier transform:

$$T_{B_{\text{obs}}} (\xi, \eta) = T_{B_{\text{obs}}} (\xi - \frac{a}{2\pi}, \eta - \frac{b}{2\pi})$$  \hspace{1cm} 8.12

Thus, the new position of the point source will be $(\xi_{ps}', \eta_{ps}')$ instead of $(\xi_{ps}, \eta_{ps})$:

$$\xi_{ps}' = \xi_{ps} - \frac{a}{2\pi}$$  \hspace{1cm} 8.13

$$\eta_{ps}' = \eta_{ps} - \frac{b}{2\pi}$$

Knowing the position of the ideal point source and the position of the corrected point source, the shift can be easily calculated by subtracting $(\xi_{ps}, \eta_{ps})$ to $(\xi_{ps}', \eta_{ps}')$:

$$\xi_{\text{shift}} = -\frac{a}{2\pi}$$  \hspace{1cm} 8.14

$$\eta_{\text{shift}} = -\frac{b}{2\pi}$$
Due to limitations of the grid used to compute the brightness temperature, the real shift seen in the image may be different from the theoretical one. That is because the shift must be multiple of the step of the grid. Therefore, the simulated shift will be the theoretical one rounded to the closest value multiple of the step of the grid.

In the following tables, the positions as well as the theoretical and simulated shifts have been calculated for the simulation in 8.1:

<table>
<thead>
<tr>
<th>Case</th>
<th>$\xi$ position</th>
<th>$\eta$ position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal Point Source</td>
<td>-0.299107143</td>
<td>0.150781209</td>
</tr>
<tr>
<td>Uncalibrated Point Source</td>
<td>-0.299107143</td>
<td>0.153358665</td>
</tr>
<tr>
<td>Calibrated Point Source</td>
<td>-0.299107143</td>
<td>0.153358665</td>
</tr>
</tbody>
</table>

*Table 8.3: ($\xi, \eta$) positions*

<table>
<thead>
<tr>
<th>Theoretical Shift</th>
<th>Simulated Shift</th>
<th>Theoretical Grid Steps</th>
<th>Simulated Grid Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000501444</td>
<td>0</td>
<td>0.449294176</td>
<td>0</td>
</tr>
</tbody>
</table>

*Table 8.4: $\xi$ shift and $\xi$ steps*

<table>
<thead>
<tr>
<th>Theoretical Shift</th>
<th>Simulated Shift</th>
<th>Theoretical Grid Steps</th>
<th>Simulated Grid Steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.002900539</td>
<td>0.002577457</td>
<td>4.501397699</td>
<td>4</td>
</tr>
</tbody>
</table>

*Table 8.5: $\eta$ shift and $\eta$ steps*

As it can be seen, when the random phases are applied, the point source position changes but the point source remains at the same place after applying the correction.

If the applied phases are known, the theoretical shifts can be computed using 8.14. To calculate the real shift, it is only needed to round the theoretical shift to the closest value which is multiple of the grid step.

In the case of this simulation, it has been used a grid of 1024 x 1024 points with the following grid steps:

$$
\xi_{\text{grid}} = 0.00112 \\
\eta_{\text{grid}} = 0.00064
$$

*8.15*

8.5 Shift simulation

If the random phases are zero mean, the shift (pointing error) has also zero mean. However, each single case (SMOS is a single case) presents a small pointing error. To study the effect of the random phases in the shift, a simulation has been performed consisting on the following steps:
1. Generate 2000 sets (1000 for each polarization) of 69 zero mean normal random phases with a certain standard deviation

2. Calculate the theoretical $\xi - \eta$ shift from the parameters $(a, b)$ for each set of random phases

3. Generate an ideal visibility of a point source placed in a known nominal position and add the random phases of the baselines calculated from the random phases of the receivers generated in the first step for each set

4. Compute the brightness temperature from the previous visibility for each set

5. Calculate the simulated $\xi - \eta$ shift from the brightness temperature and the nominal position of the point source

6. Repeat the whole process with different standard deviations

After the simulation, the shift $\Delta r$ is computed as:

$$\Delta r = \sqrt{\Delta \xi^2 + \Delta \eta^2}$$

8.16

The theoretical $\Delta r$ has a continuous distribution following a Rayleigh distribution. To follow a Rayleigh distribution, $\Delta \xi$ and $\Delta \eta$ must verify the following premises:

- Normal random variables: Kolmogorov-Smirnov test passed successfully

- Zero mean and equal std: The mean value and the standard deviation for each variable has been calculated:

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\Delta \xi$ Horizontal</th>
<th>$\Delta \eta$ Horizontal</th>
<th>$\Delta \xi$ Vertical</th>
<th>$\Delta \eta$ Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>-0.00001349</td>
<td>0.00000488</td>
<td>0.00001574</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.00000795</td>
<td>0.00005592</td>
<td>-0.0001180</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.00000113</td>
<td>-0.00000812</td>
<td>0.00003582</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.00000338</td>
<td>0.00001050</td>
<td>-0.0002368</td>
<td></td>
</tr>
</tbody>
</table>

**Table 8.6: Mean of $\Delta \xi$ and $\Delta \eta$**

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$\Delta \xi$ Horizontal</th>
<th>$\Delta \eta$ Horizontal</th>
<th>$\Delta \xi$ Vertical</th>
<th>$\Delta \eta$ Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.00044712</td>
<td>0.00087020</td>
<td>0.00133176</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>0.00045090</td>
<td>0.00090163</td>
<td>0.00134537</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.00045050</td>
<td>0.00090849</td>
<td>0.00135525</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.00044840</td>
<td>0.00086872</td>
<td>0.00132551</td>
<td></td>
</tr>
</tbody>
</table>

**Table 8.7: Std of $\Delta \xi$ and $\Delta \eta$**

- Independent: The correlation coefficient between the two variables has been calculated for each polarization
Due to the fact that the Kolmogorov-Smirnov test has been passed successfully, that
the mean value and the correlation coefficients are very small and that the standard
deviation of $\Delta \xi$ and $\Delta \eta$ are almost equal, $\Delta \gamma$ can be approximated to a Rayleigh
distribution; which mean value and the standard deviation are computed as:

$$
\mu_{\text{rayleigh}} = \sigma_x \sqrt{\frac{\pi}{2}}
$$

$$
\sigma_{\text{rayleigh}} = \sigma_x \sqrt{\frac{2 - \pi}{2}}
$$

where $\sigma_x$ is the standard deviation of $\Delta \xi$ and $\Delta \eta$.

In this case the standard deviation for both variables is slightly different but very close,
so $\sigma_x$ has been computed as:

$$
\sigma_x = \frac{\sigma_{\Delta \xi} + \sigma_{\Delta \eta}}{2}
$$

The percentage of values that are inside a circle of radius $R$, multiple of the mean value
of the Rayleigh variable, can be seen in Table 8.8:

<table>
<thead>
<tr>
<th>Percentage of points inside circle of radius $R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R = m_r$</td>
</tr>
<tr>
<td>$R = 2m_r$</td>
</tr>
<tr>
<td>$R = 3m_r$</td>
</tr>
</tbody>
</table>

*Table 8.8: Percentage of points inside the circle*

The results of the shift from the previous simulation are shown in Figure 8.5:
Figure 8.5: Theoretical and simulated shifts for different standard deviations, where the circle represents a radius \( r=2m \).
Figure 8.6: Theoretical and simulated mean shift for different standard deviation of the random phases

If the random phases are not zero mean, the shift does not have zero mean. To prove this, it has been added a constant phase for receivers of the same arm: +5 degrees for receivers of arm A, -5° for receivers of arm B and +10° for receivers of arm C.

The results of this simulation are shown in Figure 8.7:
8.6 Conclusions

The Redundant Space Calibration method is a powerful tool to assess the phase calibration performance only using the visibility samples measured by the instrument.

This method yields linear errors in each arm in the phase retrieval which are translated into pointing errors in the images. These pointing errors can be corrected by means of a point source at a known location, if required (in the following section it will be seen that the pointing error is negligible in the SMOS case).

It also corrects the image degradation by decreasing the secondary lobes, returning to the ideal values.
Chapter 9

9 Implementation of the RSC method in SMOS

Now, the theory developed in this project can be applied to the SMOS data, which will be used to retrieve the relative phases.

9.1 Relative phase retrieval

The relative phase of the receivers is going to be retrieved using the RSC method. To do this, real data from several days between February 2010 and September 2010 has been used. The chosen orbits and the selected areas used to retrieve the phases can be seen in Appendix 2: Selected orbits and filtering areas while the selected days and the time filtering intervals are shown in Appendix 3: Selected days and time filtering intervals.

The samples of the visibility have been obtained from orbits above oceans, where the measures are known to be RFI free and more stable than in land. As explained before, it has been applied a time filtering to the visibility only where the phases of the baselines are known to be quite flat. After that, these filtered values have been averaged.

The results of the relative phase retrieval for both polarizations can be seen in Figure 9.1. Each point of the same column represents a retrieval of a different day and orbit:
Figure 9.1: Retrieved relative phases from several days: a) Horizontal and b) Vertical

The final value of the relative phases will be the mean value of all the retrievals. The reason for calculating the mean value is because averaging, the pointing errors are reduced considerably due to the fact that the mean value of the pointing error is much lower when an average is performed than for any single retrieval.

Figure 9.2 shows the mean value of the retrievals for each receiver plus/minus the standard deviation for each polarization:
Figure 9.2: Mean retrieved relative phases: a) Horizontal and b) Vertical

The mean value of each relative phase has uncertainty, which means that the real value of the mean is inside the confidence interval that can be calculated using equation 9.1:

$$< m_N >_u = < m_N > \pm \frac{\sigma_N}{\sqrt{\text{number of retrievals}}}$$  \hspace{1cm} 9.1

These intervals have been calculated for all receivers and both polarizations in Appendix 4: Mean phase uncertainty and are also shown in Figure 9.3:
Figure 9.3: Mean uncertainty of the relative phases: a) Horizontal and b) Vertical

From the results in Figure 9.2, the standard deviation of the mean value of the relative phases can be calculated:

\[
\sigma_{SMOS}^H = 5.97^\circ \\
\sigma_{SMOS}^V = 3.32^\circ
\]

In order to estimate the shift in an SMOS image if a correction is not applied, the information of Figure 8.6 and the standard deviation of the retrieved relative phases in 9.2 will be used. Figure 9.4 shows the estimated mean shift calculated from the values of the shift simulation in 8.5 and the results in 9.2:

Figure 9.4: Estimated mean shift if no correction is applied: a) Horizontal and b) Vertical

From the estimated mean shift values previously retrieved, the pointing errors can be calculated from the equation of the spatial resolution.
\[ \Delta r_H = 0.00065957 \quad \rightarrow \quad \Delta L_H = \frac{r \cdot \sin^{-1} (\Delta r_H)}{\cos (\theta)} = 0.76 \text{ km} \]

\[ \Delta r_V = 0.00037492 \quad \rightarrow \quad \Delta L_V = \frac{r \cdot \sin^{-1} (\Delta r_V)}{\cos (\theta)} = 0.43 \text{ km} \]

As it can be seen, this pointing error is below 2% of SMOS boresight spatial resolution (42 km). It means that, from a practical point of view, the pointing error is negligible.

### 9.2 Comparison of different sets of solutions

Although all sets of solutions should be the same except for small errors, in practice that does not happen. This is caused because different sets of measured data have different random errors, yielding slightly different pointing errors.

However, every solution should have an error that is linear in each arm as proved in the previous study of the RSC method. According to this, if the difference between two solutions is calculated, this difference must be also linear in each arm, as proved before:

\[ \phi_{\text{retrieved}}^1 = \phi_{IVT, \text{error}} + \phi_{\text{pointing error}} \]

\[ \phi_{\text{retrieved}}^2 = \phi_{IVT, \text{error}} + \phi_{\text{pointing error}} \]

\[ \phi_{\text{retrieved}}^2 - \phi_{\text{retrieved}}^1 = \phi_{\text{pointing error}}^2 - \phi_{\text{pointing error}}^1 \]

Calculating the difference between pairs of sets of solutions is a good strategy to determine if a given solution is a good or a bad estimation. If the difference is not linear, at least one of the solutions is a bad estimation.

Figure 9.5 shows two examples of bad estimations due to non-linear phase differences:
Figure 9.5: Two examples of bad estimations due to non-linear difference

Figure 9.6 shows two examples of good estimations with linear differences.

Figure 9.6: Two examples of good estimations with linear difference
9.3 Conclusions

In order to accept a phase retrieval as a good one, the difference with all other possible solutions must be linear. Retrievals that do not verify this must be discarded.

The different retrievals for the same receiver have different values due to the presence of pointing errors. However, if the mean of all the retrievals is calculated, this pointing error is reduced considerably. Moreover, with the results obtained, it has been proved that the shift due to the pointing error can be neglected.
Chapter 10

10 Conclusions and further work

After the realization of this project, the main conclusions that have been obtained are:

- RSC method cannot be used to track fast phase drifts such as LO phase drift because it will be need long averaging. However, it can be used to recalculate the fixed phase term between CIP and HAP/VAP (inter-element relative phases), which remain out of the internal calibration loop by correlated noise (note that noise injection takes place at the switch plane and it does not take into account the phase term between the switch and the antenna phase centre)

- This method yields linear errors in each arm which are translated into pointing errors in the images

- The pointing error can be reduced by averaging the different retrievals. Nevertheless, it also has been proved that the shift due to the pointing error can be neglected in the SMOS case (errors below 2% of SMOS spatial resolution)

- The quality of different sets of RSC phase retrievals can be easily assessed as follows: the difference between different retrievals must be linear. If this does not happen, that means the solutions are contaminated (e.g., small RFI contributions) and must be discarded before computing the definitive values

The main outcomes of this project have been recently presented in the following workshop:


Figure 10.1: Presentation in ‘One year of SMOS data’ workshop
In addition, the main results have been submitted to the following international symposium:


Further work that must be done out of the results of this project are:

- Validate the residual phases obtained by applying the RSC method using a point source (an interference)
- Correct the values of the inter-element phases of the MDBFactory file of MTS (if needed)

These two activities are currently undertaken in the frame of the PhD activities of Wu Lin (UPC Remote Sensing Laboratory): ‘SMOS pixel bias assessment and spatial error correction’.
## Appendix 1: Receivers nomenclature

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Appendix 2: Selected orbits and filtering areas
Appendix 2 – Selected orbits and filtering areas
Appendix 3: Selected days and time filtering intervals

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## Appendix 4: Mean phase uncertainty

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