Titulació:

ENGinyeria Aeronàutica

Alumne:
FRANCESC BETORZ MARTÍNEZ

Títol PFC:
STUDY OF A ZERO-EMISSION AIRSHIP TRANSPORT SYSTEM BASED ON THE GEOSTROPHIC FLIGHT CONCEPT

Director del PFC:
Dr. ROBERTO FLORES LE ROUX

Convocatòria de lliurament del PFC:
GENER 2011

Contingut d’aquest volum:  -TECHNICAL REPORT-
                          -ANNEX-
Acknowledgements

The realization of this study has been possible thanks to many people who supported me throughout the last months:

First of all I would like to thank my supervisor Dr. Roberto Flores for all his support. He showed such high enthusiasm in my field of study and thus provided me with extra motivation. Moreover, I want to point out that his broad knowledge of engineering was a significant key for the successful development of this study. Over all, without his inspiration and encouragement it would not have been possible.

I wish to express my appreciation to the following members of the Servei Meteorològic de Catalunya: Mr. Jordi Cunillera, Mr. Abdel Sairouni and Mr. Manel Bravo. They kindly provided me with weather forecast data for the year 2008 without any costs for my academic research and further assisted me with explanations about technical aspects of weather forecasting. I want to thank them for their positive attitude towards my project and for always being willing to help me out when I had some questions.

Additional data was provided by the appreciated expert balloonist and engineer Mr. Josep M. Lladó. Only thanks to his data of the balloon track I was able to perform the wind model data verification.

Moreover, I would like to express my appreciation to the expert balloonist Uwe Schneider who enriched my work by giving good advice about balloon navigation and weather forecast data.

Of course I would like to thank my family, including my parents and my brother and sisters, for all their love and for supporting me always. I want to mention especially my mother who I would like to thank for her love, for standing beside me at any time and encouraging me constantly with great patience.

My sincere thanks go to my fiancée Linda Moser for her contribution due to her constant support and motivation, her interest in my study, and for proof-reading of my project. I want to particularly emphasize that she was the initial reason to inspire me to do this project because in summer 2009 she enabled me to fly in an airship for the very first time in my life.
I also received a lot of motivation from my department colleagues who worked together with me at ETSEIAT where I was employed as a technician of the aerospace laboratory. Thank you for your help and support.
Table of contents

Acknowledgements ................................................................................................................. 2
Table of contents ....................................................................................................................... 4
List of figures ............................................................................................................................... 6
List of tables ................................................................................................................................. 9
Glossary ....................................................................................................................................... 10
1 Introduction ............................................................................................................................. 12
  1.1 Objective and motivation .................................................................................................... 12
  1.2 Geostrophic flight concept .............................................................................................. 12
  1.3 Airships: a new return? ..................................................................................................... 13
2. Antecedents ........................................................................................................................... 15
  2.1 Airship electric flight .......................................................................................................... 15
  2.2 Geostrophic flight ............................................................................................................ 18
3. Atmospheric characterization ............................................................................................... 19
  3.1 Planetary boundary layer and free atmosphere ............................................................... 19
  3.2 Numeric weather forecast models .................................................................................. 21
  3.3 MM5 model ...................................................................................................................... 22
  3.4 Weather forecasting at Servei Meteorològic de Catalunya ............................................ 23
    3.4.1 Data used in the study ............................................................................................... 24
      3.4.1.1 Data format ........................................................................................................... 26
      3.4.1.2 Wind data validation ........................................................................................... 27
4. Solar power ............................................................................................................................. 31
  4.1 Solar angles ....................................................................................................................... 31
  4.2 Solar irradiance. Available power .................................................................................... 33
  4.3 Components of the solar power system for airship propulsion ....................................... 37
    4.3.1 Solar cells .................................................................................................................... 37
    4.3.2 Energy storage ............................................................................................................. 39
    4.3.3 Electric motor & power conditioner ........................................................................... 40
5. Solar-powered flight ................................................................................................................. 42
5.1 Fundamental equations ................................................................. 42
5.2 Projected area calculation .............................................................. 43
5.3 Conceptual design of “Zero”: a recreational solar airship .................. 45
  5.3.1 Reference airship .................................................................... 45
  5.3.2 Requirements and configuration ............................................... 49
  5.3.3 Size and weights ..................................................................... 50
  5.3.4 Performance ........................................................................... 53
    5.3.4.1 Solar speed ...................................................................... 53
    5.3.4.2 Emergency speed ............................................................. 56
    5.3.4.3 Range and autonomy ......................................................... 57
6. Geostrophic flight ........................................................................... 58
  6.1 General concept .......................................................................... 58
  6.2 Guaranteed covered area ............................................................... 60
  6.3 Trajectory calculation .................................................................. 61
    6.3.1 General considerations .......................................................... 61
    6.3.2 Vertical trajectory ................................................................. 62
    6.3.3 Horizontal trajectory ............................................................. 69
    6.3.4 Determination of the atmospheric parameters along the
        trajectory ................................................................................. 72
      6.3.4.1 Horizontal airship coordinates in the grid ....................... 72
      6.3.4.2 Horizontal coordinates not coinciding with the grid
          points .................................................................................. 75
    6.3.5 Matlab code ......................................................................... 79
7. Case study ...................................................................................... 81
  7.1 GCA for the “Zero” airship .......................................................... 81
8. Environmental implications ............................................................... 95
9. Conclusions and further recommendations ....................................... 96
10. Budget ............................................................................................ 98
11. References ..................................................................................... 99

ANNEX: Matlab Code
List of figures

Figure 1. Airship trajectory during a geostrophic flight in the northern hemisphere

Figure 2. Tissandier brothers electric flight in 1883 [2]

Figure 3. Airship “La France” in 1884 [2]

Figure 4. Khoury’s Sunship. 1978 [3]

Figure 5. Planetary boundary layer structure [6]

Figure 6. Skill of the 36 hour and 72 hour 500 hPa forecasts produced at NCEP [8]

Figure 7. Geographical coverage of the three domains [10]

Figure 8. Covered area, 12-km SMC’s domain, mesh of 69x69 points

Figure 9. 12-hour short term prediction

Figure 10. Balloon trajectory [Google Earth]

Figure 11. Flight altitude profile

Figure 12. Real and predicted balloon trajectories

Figure 13. Distance between real and computed final points

Figure 14. Solar angles (altitude A and azimuth Z) [12]

Figure 15. Latitude, hour angle and declination [12]

Figure 16. Sun’s declination during solstices [12]

Figure 17. Direct solar irradiance in Barcelona at 2000 meters altitude

Figure 18. Direct solar irradiance in Barcelona at sea level

Figure 19. Projected area normal to the solar flux

Figure 20. Main components of the solar power system for airship propulsion [4]

Figure 21. Solar cell efficiency evolution (NREL laboratory) [16]

Figure 22. Amorphous silicon cells on a flexible polymer substrate [17]
Figure 23. Permanent magnet brushed DC motor D135RAG [22]

Figure 24. Determination of solar cells' projected area

Figure 25. AU-12 during takeoff [24]

Figure 26. Flight altitude as function of effective payload (ISA conditions)

Figure 27. Performance comparison between airship AU-12 (blue) and first iteration for “Zero” (red).

Figure 28. Performance comparison between airship AU-12 (blue) and second iteration for “Zero” (red).

Figure 29. Solar cells’ projected area for airship “Zero”

Figure 30. Maximum speed of airship “Zero”

Figure 31. Airship geostrophic flight

Figure 32. Mean wind speed at 850, 795 and 700 hPa (year 2008)

Figure 33. GCA example: ZERO_SP_41.38_2.18_2000_02_1100_13102008

Figure 34. “Adapted” helium volume

Figure 35. Constant helium volume

Figure 36. Second order modified Euler method

Figure 37. Flight altitude between geopotential altitudes

Figure 38. Wind velocity for grid points at 2000 m. 12:00 h, 19th November 2008

Figure 39. Horizontal trajectory coordinates among grid points

Figure 40. Element shape in local axes

Figure 41. Isoparametric transformation

Figure 42. Wind velocity interpolation

Figure 43a. ZERO_SP_41.38_2.18_2000_03_1130_15012008

Figure 43b. Range as function of the airship’s azimuth

Figure 44a. ZERO_SP_41.38_2.18_2000_03_1130_15022008
Figure 44b. Range as function of the airship’s azimuth

Figure 45a. ZERO_SP_41.38_2.18_2000_03_1130_15032008
Figure 45b. Range as function of the airship’s azimuth

Figure 46a. ZERO_SP_41.38_2.18_2000_03_1130_15042008
Figure 46b. Range as function of the airship’s azimuth

Figure 47a. ZERO_SP_41.38_2.18_2000_03_1130_15052008
Figure 47b. Range as function of the airship’s azimuth

Figure 48a. ZERO_SP_41.38_2.18_2000_03_1130_15062008
Figure 48b. Range as function of the airship’s azimuth

Figure 49a. ZERO_SP_41.38_2.18_2000_03_1130_15072008
Figure 49b. Range as function of the airship’s azimuth

Figure 50a. ZERO_SP_41.38_2.18_2000_03_1130_15082008
Figure 50b. Range as function of the airship’s azimuth

Figure 51a. ZERO_SP_41.38_2.18_2000_03_1130_15092008
Figure 51b. Range as function of the airship’s azimuth

Figure 52a. ZERO_SP_41.38_2.18_2000_03_1130_15102008
Figure 52b. Range as function of the airship’s azimuth

Figure 53a. ZERO_SP_41.38_2.18_2000_03_1130_15112008
Figure 53b. Range as function of the airship’s azimuth

Figure 54a. ZERO_SP_41.38_2.18_2000_03_1130_15122008
Figure 54b. Range as function of the airship’s azimuth
List of tables

Table 1. Comparison of planetary boundary layer and free atmosphere characteristics [6]

Table 2. Comparative values of the real path and the predicted balloon trajectory

Table 3. AU-12 airship technical data [24]

Table 4. Au-12 net weight breakdown

Table 5. “Zero” propulsion system weight. First iteration

Table 6. OEW breakdown for airship “Zero”. Second iteration

Table 7. “Zero” airship main dimensions and motorization

Table 8. Maximum daily differences in T and P for each month (2008)

Table 9. Values of $a_{up}$ and $a_{lo}$

Table 10. GCAs' mean range

Table 11. PFC cost
Glossary

DC: Direct Current

DELAG: Deutsche Luftschiffahrts-Aktiengesellschaft ("German Airship Travel Corporation")

ETSEIAT: Escola Tècnica Superior Enginyeries Industrial i Aeronàutica de Terrassa

FORTRAN: Formula Translating System

FW: Fuel Weight

GCA: Guaranteed Covered Area

GPS: Global Positioning System

HYSPLIT: Hybrid Single Particle Lagrangian Integrated Trajectory Model

ISA: International Standard Atmosphere

LMC: Lynch Motor Company

LTA: Lighter Than Air

MATLAB: Matrix Laboratory

MM5: Fifth-Generation PSU/NCAR Mesoscale Model

NCAR: National Center for Atmospheric Research

NCEP: National Centers for Environmental Prediction

NOAA: National Oceanic and Atmospheric Administration

NREL: National Renewable Energy Laboratory

NW: Net Weight

OEW: Operational Empty Weight

PBL: Planetary Boundary Layer

PFC: Projecte de Fi de Carrera

PSU: Pennsylvania State University
SMC: Servei Meteorològic de Catalunya

TOW: Take Off Weight

UTC: Coordinated Universal Time

WWII: World War II
1 Introduction

1.1 Objective and motivation

The objective of this project is to study the feasibility of a sustainable and non-contaminant recreational airship flight by using accurate weather forecasting data and an electric propulsion system powered by solar cells.

1.2 Geostrophic flight concept

Above the planetary boundary layer, in the free atmosphere, wind is barely affected by the Earth's surface friction and it can be very well approximated by the geostrophic wind. In the free atmosphere there is much less turbulence and wind speed is much more uniform. The performance of an airship greatly depends on atmospheric weather conditions. Therefore flying above the planetary boundary layer should represent a more stable, comfortable and predictable flight. Consequently, by geostrophic flight we understand a flight whose trajectory is determined in advance using accurate weather forecast data and that is performed mainly over the planetary boundary layer.

Figure 1. Airship trajectory during a geostrophic flight in the northern hemisphere
As an example imagine that we want to travel from point 1 to point 5 (Figure 1). It's clear that in absence of wind the only manner for an airship to fly within two points is using some kind of propulsion. Let’s consider now a more realistic atmospheric scenario. In the northern hemisphere, cyclones (B) rotate counter clockwise and anticyclones (A) rotate the opposite. Then, a possible wind field distribution over the planetary boundary layer could be for example the one shown in Figure 1.

In this hypothetical steady wind scenario a possible flying strategy to travel from point 1 to point 5 could be the following:

i) Free ballooning flight from the starting point 1 to point 2. Even though the flight is not propelled, it is indeed “controlled” since with numeric wind models we are able to predict the trajectory that the airship will follow.

ii) Propelled flight from point 2 to point 3. A propulsive maneuver is needed; otherwise the airship cannot escape the cyclone wind field. Numeric wind models are also used to take into account the drift due to the wind field and to determine the proper course to follow.

iii) Once the airship reaches point 3, again free ballooning controlled flight from point 3 to point 4.

iv) Finally, at point 4 we perform another propulsive maneuver to escape the anticyclone wind field and be able to reach point 5, the final destination.

Depending on the starting and ending points and the wind conditions, geostrophic flight can be performed either entirely without engine, powered throughout all the flight, or by a combination of both as shown in the previous example. In the case of the last scenario, the emissions should decrease dramatically if the airship uses a conventional fuel engine. Moreover, if electric engines and solar cells are used for propulsion, emissions should be zero in any case.

1.3 Airships: a new return?

Airships belong to the category of the so-called "lighter than air" aircrafts (LTA). While conventional "heavier than air" aircrafts get the lift thanks to aerodynamic forces, airships obtain the lifting force mainly thanks to aerostatic buoyancy. The main advantage of airships with respect to fixed-wing and rotary-wing aircrafts is that they require much less power or speed to fly. Moreover, the fact that the lifting force is independent from the thrust makes airship’s specific consumption
much lower than those of conventional fixed-wing aircrafts and theoretically allows flights for large periods of time. Further advantages of the airship include their ability to hover over one place for an extended period of time, to land with very little supporting infrastructure and their ability to fly over the city centre due to their quietness, maneuverability and inherent safety. On the other hand the main disadvantage of airships is their inherent high volume and therefore their large wetted surface and frontal area. This causes a very high drag that limits their maximum speed to not more than 150 km/h. This relatively low speed and high volume makes them more susceptible to weather conditions than the "heavier than air" aircrafts.

Airships were widely used before the 1940s, but their use decreased over time as their capabilities were surpassed by those of airplanes. Little after the Hindenburg accident in 1937, airships were abandoned as air transportation vehicles and conventional wing fixed airplanes took their place. After WWII airships were definitely abandoned as passenger and cargo transport.

Today the use of airships is marginal and represents only a tiny percentage of the aerospace industry. Airships are still used today, but only in certain niche applications like aerial advertising and observation, tourism flights and camera platforms for outdoor events where the capability to hover over one place for an extended period of time is the main flight requirement. Modern airships are purpose-built with a range of advanced technologies that ensure a safe and low-CO₂ emission flight. They are fitted with efficient fuel engines with the latest technology for propulsion, and regarding buoyancy they rely on a combination of helium and propellers.

There are several factors that may allow a gradual return of airships as a means of transport:

- Growing environmental awareness demanding less contaminant vehicles.
- Dramatic improvement of weather forecast models.
- Availability of advanced technologies such as composite materials and modern electronic systems.

Even though a modern airship like Zeppelin NT pollutes much less than a conventional modern airplane, the aim of this project is to find out if an airship zero emission flight could be possible by studying the performances of a recreational non-emissive airship. In order to achieve this goal already-operational technologies and services such as numeric weather forecasting and solar electric propulsion are combined.
2. Antecedents

2.1 Airship electric flight

Historically airships have been the pioneers of the conquest of the air. Some of the great achievements made by these lighter than air aircrafts include: the first controlled flight (Giffard 1852), first return Atlantic crossing (R34 1919), the first commercial airline (DELAG 1910), the first flight over the North Pole (Nobile, Amundsen 1926) and the first nonstop aerial circumnavigation around the world (Graf Zeppelin 1929).

Airships were also the first aircrafts to be powered with electric engines and, therefore, the first to perform zero emission flights. More specifically, the first electric-powered flight in history was conducted by Albert and Gaston Tissandier in 1883 (figure 2). The Tissandier brothers constructed a 1.062 cubic meter airship propelled by a battery-powered electric motor. The power produced by the motor was 1.1 kilowatts at 180 revolutions per minute and drove a large two-bladed pusher propeller through reduction gearing. The maximum speed achieved in calm air was only 4.8 kilometers per hour since the power to weight ratio of the engine was still very low and no better than the one achieved by Giffard in his first controlled flight [1].

Figure 2. Tissandier brothers electric flight in 1883 [2]
This was the first recorded electric-powered flight, though it was not to be the last. One year later in 1884 Charles Renard and Arthur Krebs (inventors and military officers in the French Army Corps of Engineers) duplicated this feat with their electric-powered airship La France, which notably was also the world’s first fully-controllable airship (figure 3). La France was the first airship that could return to its starting point in light wind conditions and represented a vast improvement over earlier models. It was 50.3 meters long, its maximum diameter was 8.2 meters, and it had an envelope volume of 1.869 cubic meters. Like the Tissandiers’ airship, an electric battery-powered motor propelled La France, but this one produced 5.6 kilowatts, five times the available power in Tissandier’s airship. The first flight of La France took place on August 9, 1884. After a fully controlled flight of 8 kilometers and 23 minutes, Renard and Krebs landed successfully at the same point where they had taken off. Although the batteries limited its flying range, La France demonstrated that controlled flight was possible if the airship had a sufficiently powerful lightweight motor [1].

Figure 3. Airship “La France” in 1884 [2]
Despite the success achieved by airship La France, the rapid improvement of internal combustion engines in the late nineteenth century made them to become the most suitable for airship propulsion. These new engines offered much higher power to weight ratios than their equivalent heavy electric motors. Moreover, the range and autonomy using internal combustion engines were not limited as they were when using batteries. Thus, La France was the last manned electric-powered airship. Since then all manned airships incorporate internal combustion engines.

Concerning solar powered manned airships the first theoretical proposal was made by Khoury and Mowforth in 1978 [3]. In their work the concept of the “Sunship” was introduced. The idea basically consisted in covering the envelope surface of a conventional helium airship with an array of flexible thin film solar cells (figure 4). In theory the solar cells generate enough power to feed the two main electric motors. Khoury justifies his idea with these words: “Solar energy is attractive in an environmental conscious age. Sunlight is a renewable, free, non-polluting and non-inflammable fuel. A solar-powered airship would not require refuelling when operating in remote sunny areas of the world or at high altitudes. A solar-powered airship is defined as an airship that attains its power for propulsion primarily from solar energy, albeit in conjunction with on-board energy storage for operations at night and in windy conditions” [4].

Figure 4. Khoury’s Sunship. 1978 [3]
The proposed Sunship would be 80 meters long with a diameter of 23 meters and a total hull volume of about 22,000 cubic meters. Considering the maximum direct solar energy available at an altitude of 1000 meters at solar noon and using reasonable values for the different efficiencies involved, the theoretical maximum attainable speed would be 83 km/h. The maximum speed would increase to 101 km/h if diffuse and reflected solar radiation would also be considered for an airship totally covered with solar cells [4]. Besides Khoury’s theoretical work, no manned solar powered airships have been built up to date.

2.2 Geostrophic flight

The use of the winds for air navigation goes back to the early era of balloons and airships. In 1910 the famous German meteorologist Hugh Hergesell already anticipated in his book "With Zeppelin to Spitzbergen" that for a hypothetical trip to the North Pole by airship "a special method of navigating should be developed where the track to follow was chosen by taking into account a weather forecast chart, in order to use the favorable winds instead of going against them" [5]. Subsequently and until the disappearance of the great airships in the 1940s the use of weather forecast data was common practice in airship route planning. However, the available weather forecast in the first half of the twentieth century was of poor quality. Until the advent of computers and numerical models it was not possible to make accurate short and medium term weather forecasts. Thus, a type of navigation like the one proposed in this study would have been impossible in the golden era of the airships due to the lack of reliable weather forecast data.

Nowadays the situation is paradoxically the opposite. Meteorological numerical models are able to produce accurate weather predictions but, as mentioned in chapter 1, airships play a secondary role in the aviation industry and are no longer used for passenger or cargo transport.

The aim of this project is to study the feasibility of a recreational two-seat zero-emission airship by combining the idea of the solar airship with a navigation based on accurate weather forecasting data.
3. Atmospheric characterization

3.1 Planetary boundary layer and free atmosphere

The troposphere is the first layer of the atmosphere and extends from the ground up to an average altitude of 11 kilometers. The troposphere itself can be divided in two zones: a planetary boundary layer (PBL) near the Earth’s surface and the free atmosphere above it. According to Stull’s definition, “the planetary boundary layer can be defined as the part of the troposphere that is directly influenced by the presence of the Earth’s surface, and responds to surface forcings with a timescale of about an hour or less” [6]. The planetary boundary has a complex structure (figure 5) and its thickness is variable in time and space, ranging from hundreds of meters to a few kilometers. One of the characteristics that make this boundary layer different from the rest of the atmosphere is its turbulent nature. The wind speed suffers from rapid fluctuations and the vertical mixing is strong. Table 1 summarizes the most important differences between the PBL and the free atmosphere.

<table>
<thead>
<tr>
<th>Property</th>
<th>Planetary boundary layer</th>
<th>Free atmosphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbulence</td>
<td>Almost continuously turbulent over its whole depth</td>
<td>Sporadic clear air turbulence in thin layers of large horizontal extent</td>
</tr>
<tr>
<td>Friction</td>
<td>Strong drag against the Earth’s surface. Large energy dissipation</td>
<td>Small viscous dissipation</td>
</tr>
<tr>
<td>Dispersion</td>
<td>Rapid turbulent mixing in the vertical and horizontal</td>
<td>Small molecular diffusion. Often rapid horizontal transport by mean wind</td>
</tr>
<tr>
<td>Winds</td>
<td>Near logarithmic wind speed profile in the surface layer. Subgeostrophic, cross-isobaric flow common</td>
<td>Winds nearly geostrophic</td>
</tr>
<tr>
<td>Thickness</td>
<td>Varies between 100m to 3km in time and space. Diurnal oscillations over land</td>
<td>Less variable, thickness ranging from 8 to 18km. Slow time variations</td>
</tr>
</tbody>
</table>

Table 1. Comparison of planetary boundary layer and free atmosphere characteristics [6]

Above the PBL, in the free atmosphere, winds are almost geostrophic. Geostrophic wind is a theoretical wind which results from the equilibrium between
the pressure gradient force and the Coriolis force. Geostrophic wind is parallel to
the isobars and assumes that there is no friction and that the isobars are perfectly
straight. A better approximation of the real wind in the free atmosphere is the so-
called gradient wind where, besides the geostrophic balance, also the centrifugal
force due to the curvature of the isobars is taken into account.

Weather conditions play a very important role in airships’ performance given the
relative low speed and high volume of these aircrafts. Taking into account the
characteristics of the atmospheric layers mentioned above, an airship flying in the
free atmosphere should experience the following advantages with respect to
flying in the PBL:

- Less turbulence and wind gusts which guarantee a more comfortable and
  stable flight.
- Better wind prediction since numerical weather forecast models better
  predict the winds in the free atmosphere due to the turbulent nature and
  friction forces present in the PBL.
- Flying above the planetary boundary layer avoids fog and low clouds and
  allows better visibility.
- Winds are more constant in intensity and direction in the free atmosphere
  and therefore, allow the implementation of effective flying strategies.

In this study, an airship flight whose trajectory is determined in advance using
accurate weather forecast data and that is performed mainly over the planetary
boundary layer is defined as a geostrophic flight.
3.2 Numeric weather forecast models

Weather forecasting is the core element for the proposed geostrophic flight, where the idea is to take advantage of the predicted wind fields and use them favorably for navigation. More specifically (see chapter 6), the atmospheric variables that must be accurately predicted in order to forecast the trajectory of an airship are temperature, pressure distribution and the zonal and meridional winds. Zonal wind is the wind component along the local parallel whereas meridional wind is the wind component along the local meridian.

Nowadays numerical weather models are the fundamental tool for accurate weather forecasting. Although the first efforts to accomplish regular weather forecasts were done almost a century ago, it wasn’t until the advent of computer simulation that regular weather predictions became feasible [7].

Numerical weather prediction simulates the evolution of the main meteorological parameters that determine the state of the atmosphere by using mathematical models. These models are based on the physical laws that describe the evolution of the atmosphere and constitute a system of equations. Given the complexity of these equations and the huge volume of data involved in the calculations, they can only be solved approximately using computers. The maximum time range for which these equations are solved is known as the prediction horizon. Despite of the atmosphere being a continuous medium, these equations are solved only at certain points to reduce computation time. All these points where the equations are solved constitute a three-dimensional mesh. If the distance between these points (called grid) decreases, the model is capable of simulating meteorological phenomena of smaller scale and has therefore a higher resolution. However, it also means an increase of the computing time needed to run the simulation. On the other hand, the effects of physical processes that the model is unable to solve explicitly are estimated by using parameterizations. Parameterizations are a set of mathematical formulas that approximately describe atmospheric smaller scale phenomena (clouds, radiative fluxes, turbulence, convection, etc.).

Numerical weather predictions have improved dramatically since their origin. The remarkable progress in forecasting over the past 50 years is illustrated by the record of skill of the 500 hPa forecasts produced at the National Centers for Environmental Prediction (NCEP) [8]. Forecast skill scores, expressed as percentages of perfect forecasts, have improved steadily over the past 50 years and each introduction of a new prediction model has resulted in further improvement (figure 6). For a 36 hour forecast the skill score was almost 80% in
2004. This percentage increases for very-short term predictions which are to ones to be used in geostrophic flight.

Although all numerical weather forecast models are based on the same physical laws, there are differences in their mathematical formulation and also in the numerical techniques which are used to solve the equations. Moreover, a distinction is made between the models whose geographic area or domain covers the whole planet (global models) and the models that only cover a specific area (limited-area models). The MM5 model is a limited-area model.

![Figure 6. Skill of the 36 hour and 72 hour 500 hPa forecasts produced at NCEP](image)

### 3.3 MM5 model

The PSU/NCAR (Pennsylvania State University/National Center for Atmospheric Research) model (known as MM5) is a limited-area, hydrostatic or non-hydrostatic, terrain-following sigma-coordinate model designed to simulate or predict mesoscale and regional-scale atmospheric circulation [9]. These include phenomena that occur at spatial scales ranging from a few to several hundred kilometers, such as storms, breezes, or frontal systems amongst others. The model is supported by several pre- and post-processing programs, which collectively form the MM5 modeling system. The MM5 modeling system software is provided free of charge and mostly written in FORTRAN. It is continuously
being improved by contributions from users at several universities, laboratories and weather forecast agencies like “Servei Meteorològic de Catalunya”.

The MM5 model was initially developed at Pennsylvania State University in the early 70’s. Since then, it has undergone many changes designed to broaden its usage. These include:

(i) A multiple-nest capability.
(ii) Non-hydrostatic dynamics, which allow the model to be used at a few-kilometer scale.
(iii) Multitasking capability on shared- and distributed-memory machines.
(iv) Four-dimensional data-assimilation capability
(v) More physics options.

Since the MM5 is a regional model, it requires an initial condition as well as lateral boundary conditions to run. The initial atmospheric state is determined by performing a process of data assimilation before the numerical models begin their calculations. This process consists in feeding the model with data from several meteorological observations (rawinsondes, surface stations, satellite, radar, etc.) that describe the initial state of the atmosphere.

3.4 Weather forecasting at Servei Meteorològic de Catalunya

Servei Meteorològic de Catalunya (SMC) is the Catalan agency for weather forecasting [10]. Currently, SMC carries out several numerical simulations using the MM5 model every day in order to perform the weather forecast for Catalonia. The three different domains (limited areas) in which the model is run are shown in figure 7. The biggest domain has a grid of 36 km, the medium one of 12 km and the smallest one of 4 km. The model is run twice a day, at 00 and 12 UT, for the three different domains but only produces graphical outputs of the first two domains, the 36 and 12 km grid, which are distributed through the SMC website. First of all an initial simulation is carried out in the larger domain (36 km grid) and a 72-hour prognosis is completed. Due to the fact that this is a limited-area model, additional data from a global model is needed in order to determine the initial meteorological variables in the domain boundaries. The initial conditions are improved with the assimilation of meteorological observational data from rawinsondes and surface stations, in order to match the initial state as close as possible to reality. Another simulation is performed using the 12-km grid domain and a 48-hour prognosis. The initial and boundary conditions for this simulation are derived out of the output data obtained from the previous simulation on the large domain. As the grid is smaller in this case it is able to simulate atmospheric
phenomena that occur at smaller scales. In addition, short-term predictions (12-hour prognosis) are computed eight times a day (00, 03, 06, 09, 12, 15, 18 and 21 UTC) using the same 12-km grid domain.

3.4.1 Data used in the study

The weather forecast data used in this project to study the feasibility of the geostrophic flights was supplied by SMC. The supplied data corresponds to the 12-hour short-term prediction data in the 12-km grid domain and for the entire year 2008. Due to the dimensions of the covered area and the grid distance, the mesh is composed by 69x69 points (figure 8). These short-term forecast predictions are computed every 3 hours and predict the following 12 hours. It is assumed that the first 3 hours of each run represent the most accurate prediction, and therefore only data from the first 3 hours of each run is used (figure 9). The computation was carried out for the entire year 2008 and for 5 different pressure levels (on the ground, 950 hPa, 850 hPa, 700 hPa and 500 hPa).
Thus, every 3 hours the temperature (T), zonal and meridional wind (u,v) and geopotential altitude (h) were computed at each of those pressure levels mentioned above.

Figure 8. Covered area, 12-km SMC’s domain, mesh of 69x69 points

Figure 9. 12-hour short term prediction
3.4.1.1 Data format

The SMC weather forecasting data used in this project was provided in grid files. Each file contains the values of the predicted variable at the mesh points at a given pressure level and at a given time.

The files are named as "ddmhhvn.grd". Where:

- **dd** is the day of the month
  - From 1 to 29, 30 or 31 depending on the month
- **m** is the month of the year
  - e=January
  - f=February
  - m=March
  - a=April
  - y=May
  - j=June
  - l=July
  - g=August
  - s=September
  - o=October
  - n=November
  - d=December
- **hh** is the hour of the day (UTC)
  - 00, 03, 06, 09, 12, 15, 18, 21
- **v** is the predicted variable
  - 1=u=zonal wind
  - 2=v=meridional wind
  - 3=T=temperature
  - 4=h=geopotential altitude
- **n** is the pressure level
  - 1=ground level
  - 2=950 hPa
  - 3=850 hPa
  - 4=700 hPa
  - 5=500 hPa

For instance the file “19d1534.grd” contains the prediction for the temperature at the pressure level 700 hPa, at 15:00h on December 19th, 2008.
3.4.1.2 Wind data validation

In order to estimate the reliability of the wind forecast data supplied by SMC a wind data validation was made. This validation was done by comparing a reference trajectory followed by a manned hot air balloon with the equivalent one predicted by a code implemented in Matlab. As an input this code takes the starting point of the flight and the altitude of the balloon along the track. It also uses the predicted weather variables supplied by the MM5 model (wind speed and direction, temperature and geopotential height at 700 and 850 hPa) for the day of the flight, with the same characteristics as the data that will be used for studying the geostrophic flight.

The reference trajectory was the real path followed by a hot air balloon piloted by Mr. Josep M. Lladó over the Pyrenees on November 14th, 2009. The actual flight time was 106 minutes (from 6:48 UTC to 8:34 UTC) and the trajectory of the balloon was monitored by an onboard GPS receiver. Figure 10 shows the path of the balloon across the Pyrenees. The starting point was the village of Gòsol in Catalonia (42.238ºN latitude and 1.661ºE longitude) and the arrival point was a field in France near the border town of Le Perthus. The average flight altitude was 2304 meters. The balloon traveled 94.5 kilometers at an average speed of 53.5 km/h.

Figure 10. Balloon trajectory [Google Earth]
The flight altitude profile provided by the GPS receiver along the balloon’s path is shown in figure 11. The figure also shows the geopotential height corresponding to 700 hPa and 850 hPa provided by the MM5 model.

The actual path of the balloon was also compared to that obtained by the trajectory simulation program HYSPLIT (Hybrid Single Particle Lagrangian Integrated Trajectory Model) of the National Oceanic and Atmospheric Administration (NOAA) [11].

Figure 12 shows the results obtained. In red the real path of the balloon is shown, in blue the computed track using data from the MM5 weather forecast model and in green the track obtained using HYSPLIT. As shown in figure 12, the computed trajectory using data from the MM5 model fits reasonably well to the actual path of the balloon. Table 2 shows some comparative values of the real path and the predicted trajectory using the MM5 wind data.
Figure 12. Real and predicted balloon trajectories

<table>
<thead>
<tr>
<th></th>
<th>Latitude last point (°)</th>
<th>Longitude last point (°)</th>
<th>Total distance (Km)</th>
<th>Average speed (Km/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real trajectory</td>
<td>42,425</td>
<td>2,773</td>
<td>94.49</td>
<td>53.5</td>
</tr>
<tr>
<td>Computed trajectory using MM5 data</td>
<td>42,435</td>
<td>2,806</td>
<td>97.01</td>
<td>55.1</td>
</tr>
</tbody>
</table>

Table 2. Comparative values of the real path and the predicted balloon trajectory

The distance between the real and the computed final point (absolute error) is 2.9 km (figure 13).
However the relative error must be determined in order to obtain an indication of the accuracy of the prediction. The relative error can be calculated (given the absolute error and the total distance of the real trajectory) as:

\[ E = \frac{\text{absolute error}}{\text{total distance}} = \frac{2.9}{94.49} = 0.031 = 3.1\% \]

A relative error of 3.1% for the trajectory prediction is considered to be acceptable for this study. Therefore the MM5 weather prediction data of the year 2008 provided by Servei Meteorològic de Catalunya will be used in chapter 6 and 7 in order to study the performance and feasibility of the solar geostrophic flight.
4. Solar power

4.1 Solar angles

In order to determine the available solar irradiance at a given point over the Earth’s surface we first have to determine the position of the Sun relative to an appropriate horizontal coordinate system. This local coordinate system uses the observer's local horizon as the fundamental reference plane. The solar altitude angle $A$ is defined as the angle between the sunrays and the horizontal reference plane containing the observer $P$ (figure 14). The other angle for defining the position of the Sun is the solar azimuth angle $Z$. It is defined as the angle measured from the South-pointing coordinate axis to the projection of the Sun’s central ray (figure 14).

![Figure 14. Solar angles (altitude A and azimuth Z) [12]](image)

Both the solar altitude and the solar azimuth depend on the latitude of the observer, the hour of the day and the Earth’s declination angle.
The solar altitude \( A \) can be determined using the following formula [4]:

\[
\sin A = \cos N \cdot \cos H \cdot \cos L + \sin N \cdot \sin L
\]

where \( L \) is the latitude of the observer \( P \) in degrees, \( H \) is the hour angle time after solar noon and \( N \) is the Sun’s declination which represents the seasonal variation in the Sun’s apparent motion (figure 15).

![Diagram of solar altitude and declination](image)

Figure 15. Latitude, hour angle and declination [12]

The hour angle \( H \) can be calculated in degrees as:

\[
H = 15 \cdot (12 - \text{hour of the day})
\]

\( H \) is 0 at noon, where the solar altitude is at its maximum. The Sun’s declination \( N \) varies between +23.45° at summer solstice and -23.45° at winter solstice (figure 16). Measuring the time of the year in days from the spring equinox the declination can be calculated in degrees by:

\[
N = 23.45 \cdot \sin \left( \frac{2\pi d}{365} \right)
\]

Finally, the solar azimuth \( Z \) can be calculated as [4]:

\[
Z = \arcsin \left( \frac{\cos N \cdot \sin H}{\cos A} \right)
\]
Figure 16. Sun’s declination during solstices [12]

4.2 Solar irradiance. Available power

The solar energy reaching the upper limits of the Earth’s atmosphere has a yearly average value of 1.366 W/m². This value is called the solar constant $D_0$. Due to the fact that the Earth’s orbit is slightly elliptical the solar constant varies by ±3.4 percent throughout the year with the maximum irradiance occurring at the perihelion (Earth closest to the Sun) and the minimum at the aphelion. When travelling through the Earth’s atmosphere, direct solar radiation is progressively attenuated until it reaches the ground level. The degree of attenuation depends on the amount of air mass encountered by the solar rays in its path through the atmosphere, which mainly depends on the solar altitude. The longer the path length of the solar rays through the atmosphere the greater the attenuation. Likewise the lower the solar altitude the greater is the solar irradiance attenuation. Direct solar radiation at sea level and for clear sky conditions can be expressed as a function of the solar altitude as [13]:

$$D = D_0 \cdot e^{-c \left( \frac{1}{\sin A} \right)^s}$$

where $D_0$ is the solar constant, $A$ is the solar altitude and $c=0.357$ and $s=0.678$ are two empirical constants. This equation is valid only for solar altitudes greater than 20º. Direct solar flux increases with altitude above sea level and it reaches the solar constant outside the atmosphere. In order to take into account this effect, the previous equation has to be modified [14]:
where $h$ is the altitude above sea level in kilometers and $a=0.14$ is another empirical constant. This formula is valid only for the first few kilometers above sea level.

The total solar radiation that reaches a point on the Earth's surface is the sum of the direct, the diffused and the reflected solar radiation. Diffuse solar radiation is the portion of solar radiation that is scattered downwards by the molecules of the atmosphere. During clear days, the magnitude of diffuse radiation is no greater than 10% of the total solar radiation received at the Earth's surface. However, clouds have a significant influence on diffuse radiation and only this kind of radiation may reach the Earth's surface during extremely cloudy days. When the solar radiation irradiates upon a surface which is opaque like clouds and ground surface, a portion of the radiation is absorbed and the remaining portion is reflected. The amount of reflected radiation depends on the albedo or surface reflectance of the object.

Taking into account the previous formulas, direct solar radiation for clear sky conditions can be calculated for a given latitude, day of the year, local solar time and altitude above sea level. Figure 17 and 18 show the daily direct solar radiation for a point located over Barcelona (latitude 41ºN) at 2000 meters of altitude and at sea level, respectively. The blue, green and red lines correspond respectively to the summer solstice, the winter solstice and equinoxes. Any other day of the year will be represented by a line which shall be between the lines of the summer and winter solstice.

The maximum direct solar irradiance through the year is obtained at noon during the summer solstice. At 2000 meters altitude over Barcelona the maximum direct solar irradiance is 1.062 W/m² whereas at sea level is 944 W/m². It is also remarkable that at 2000 meters altitude the direct solar radiation at noon is always greater than 900 W/m² in clear sky conditions. As seen in figures 17 and 18 and as mentioned above the altitude has a significant influence on the value of direct radiation.
Figure 17. Direct solar irradiance in Barcelona at 2000 meters altitude

Figure 18. Direct solar irradiance in Barcelona at sea level
Finally, the available solar energy reaching a surface $S$ can be obtained by multiplying the incident direct solar radiation $D_r$ by the projected area $S_p$ normal to the solar flux (figure 19).

Figure 19. Projected area normal to the solar flux
4.3 Components of the solar power system for airship propulsion

The main components of a solar power system for airship propulsion are the solar cells, the energy storage elements, the electric motor and the power conditioner [4].

4.3.1 Solar cells

Solar cells are the main element of the system since they are responsible for transforming the incident solar radiation into electric power. The photovoltaic effect is the basis for the conversion of solar radiation into electricity in solar cells, which are made of a semiconductor material. Conversion efficiency is the most important parameter of a solar cell \( \eta_{cell} \) and is defined as the ratio of the electrical power output of the cell to the solar energy input onto the cell:

\[
\eta_{cell} = \frac{W_{out}}{W_{in}}
\]
As shown in figure 21 solar cell efficiencies have increased significantly since the 1970s. Although laboratory specimens have reached efficiencies around 40%, typical silicon commercial solar cells have an efficiency of around 12% and those using the much more expensive gallium arsenide have an efficiency not greater than 20% [15].

Flexible thin film solar cells are the ones suitable for electrical airship propulsion. Their mechanical flexibility and light weight make them ideal for covering the top surface of the airship’s envelope. Commercial amorphous silicon cells have a proven efficiency of 12% (figure 2) and a specific power of 1000 W/Kg [17].
4.3.2 Energy storage

Although electric power for solar flight is obtained mainly through the solar cells, it is essential to have an electric storage system on board not only to feed the instrumentation and navigation systems on board but also to overcome the following situations:

- Failure of the solar cell system: Should a malfunction occur in the solar cell system the energy storage devices on board must be able to supply enough energy and during a sufficient period of time in order to guarantee a safe landing.
- Temporary little incident solar radiation due to clouds or other causes: In this case it will be necessary to partially feed the engine with electrical energy from the batteries during the time period of low solar irradiation.
- Occasional needs for maximum power: In general the highest power supplied by the solar cells will be less than the maximum power that can be delivered by the electric motors. Occasionally there may be situations where the maximum engine power is needed (e.g. strong wind gusts, collision avoidance, etc.). The additional energy to the solar power in order to reach maximum power comes from the energy storage system.
The most suitable system for energy storage on an airship are rechargeable batteries. The fundamental parameter that determines the performance of a battery is its specific energy. Typical values of specific energy range from 40 Wh/kg for lead acid batteries to 350 Wh/kg for advanced lithium sulfur batteries [18] [19]. Batteries have been used successfully in recent manned and unmanned airplane electric flights. In 2007 the electric airplane “Electra” performed a 48-minutes manned flight using lithium polymer batteries with a specific energy of 200 Wh/Kg [20]. On the other hand, in 2008 the unmanned electric airplane “Zephyr-6” performed a 3-day flight using advanced lithium sulfur batteries with a specific energy of 350 Wh/kg [21]. Batteries are the heaviest element of the solar propulsion system and therefore they must be accurately adopted according to the operational requirements of the aircraft.

4.3.3 Electric motor & power conditioner

The power conditioner is responsible for regulating the electrical power supplied to the engine under the pilot’s demand and for feeding the auxiliary navigation systems. It is also responsible for the global management of the variable direct current produced by the solar cells and the energy stored in the batteries. Finally, the electric motor drives the propeller producing the desired thrust. The characteristics required in an airship electric motor are: low weight, high electrical performance, minimal maintenance and low cost. DC electric motors are the most suitable given their easy regulation and due to the nature of the available power on board (direct current). The high weight of conventional DC motors has been dramatically reduced since steel of the frames has been replaced by aluminum. Additionally, permanent magnet motors have also significantly improved the performance of DC motors and are potential candidates for electric airship propulsion.
Figure 23. Permanent magnet brushed DC motor D135RAG [22]

Figure 23 shows the permanent magnet brushed DC motor D135RAG of the Lynch Motor Company. This engine, which was used successfully by the electric airplane “Electra” in 2007 [20] develops a rated power of 16.8kW, weighs approximately 11 kg and has a peak efficiency of 91% [22].
5. Solar-powered flight

5.1 Fundamental equations

The advantages of airships with respect to conventional fixed wing airplanes concerning solar-powered flight are:

i) As the lifting force is produced mainly by buoyancy, there is no need for minimum speed for the airship. That means that less power is needed to remain airborne.

ii) The inherent big volume of the airship means a big envelope surface and therefore a greater available area for solar cells.

In order to deduce the fundamental equations driving the solar electric flight a horizontal steady symmetric flight is considered. Under these circumstances the horizontal equilibrium of forces yields to:

\[ F = D \]

Where \( F \) is the thrust provided by the engine and \( D \) is the drag.

\[ D = \frac{1}{2} \rho v^2 C_{Dw} S_{wet} \]

Where:

\( \rho \) = Air density

\( v \) = True airspeed

\( C_{Dw} \) = Drag coefficient referred to the wetted surface area of the airship

\( S_{wet} \) = Airship’s wetted surface area

The equation can be written in terms of power as:

\[ F \cdot v = D \cdot v \]

\( F \cdot v \) is the useful work and can be written as:

\[ F \cdot v = W_{av} \cdot \eta_p \]

Where \( \eta_p \) is the propulsive efficiency and \( W_{av} \) is the available power for the propeller in order to produce thrust. The available power is function of the available solar power, the projected area, and some efficiencies:
\[ W_{av} = N \cdot S_p \cdot \eta_{cell} \cdot \eta_{el} \cdot \eta_{mec} \]

Where:

\( N \) = Incident (direct) normal solar flux (W/m\(^2\))

\( S_p \) = Solar cells' projected area (m\(^2\))

\( \eta_{cell} \) = Solar cells conversion efficiency

\( \eta_{el} \) = Electrical components' (e.g. motor) efficiency

\( \eta_{mec} \) = Mechanical efficiency

Therefore the equation in terms of power can be written as:

\[ N \cdot S_p \cdot \eta_{cell} \cdot \eta_{el} \cdot \eta_{mec} \cdot \eta_p = \frac{1}{2} \rho v^3 C_{Dw} S_{wet} \]

Finally we can obtain the speed:

\[ v = \sqrt[3]{\frac{2 \cdot N \cdot \eta_{cell} \cdot \eta_{el} \cdot \eta_{mec} \cdot \eta_p \cdot (S_p)}{\rho \cdot C_{Dw} \cdot (S_{wet})}} \]

A first result from the equation above is that at equal surface ratio and efficiencies, the speed increases with altitude. This is because as seen in chapter 4 solar radiation increases with altitude and the air density decreases with height. Therefore, flying above the planetary boundary layer not only provides the advantage of more stable wind conditions but also allows higher flight speeds.

The solar cells’ projected area plays an important role in the airship’s speed. For a horizontal flight it depends on the solar altitude and on the relative azimuth between the Sun and the airship. Consequently the projected area changes continuously in time and so does the maximum airship speed. For a solar airship in horizontal flight with the whole upper surface envelope covered by flexible solar cells, the value of the projected area increases with the solar altitude, reaching its maximum when the solar altitude is 90\(^\circ\).

**5.2 Projected area calculation**

All values of the previous equation are relatively easy to calculate except for the solar cells’ projected area \( S_p \). The projected area depends on the orientation of the airship, the solar altitude and the solar azimuth. Consequently, the value of the projected area changes continuously in time and, given an arbitrary shape of...
the solar cells surface, its calculation is not trivial. In general the envelope’s surface of a conventional airship can be very well approximated by a revolution ellipsoid [23] and, as a result, the solar cells surface is convex. For our case of horizontal airship flight, with convex surface of solar cells covering the top of the airship and solar altitudes between 0° and 90°, the solar cells’ projected area can be calculated using the following procedure (figure 24):

1) Projection of the solar cells’ surface on the projection plane \((xp, yp)\): The projection plane \((xp, yp)\) is generated from the coordinate system \((xp, yp, zp)\). The zp axis is the axis that passes through the fixed origin of coordinates and has the direction of the Sun’s rays. The xp axis is located below the horizontal reference plane forming an angle of 90° with the zp axis. The xp axis is contained in the plane formed by the zp axis and its vertical projection on the horizontal reference plane. Finally, the yp axis forms a right-handed triad with the other two. The projection of the solar cells’ surface on the plane \((xp, yp)\) is performed by making two changes of coordinates. In the first change, the new coordinate axes \((x_1, y_1, z_b)\) are obtained by rotating the initial airship body axes \((x_b, y_b, z_b)\) around the \(z_b\) axis in an angle of \(\theta_3\). The matrix that performs the first change is:

\[
R(z_b, \theta_3) = \begin{pmatrix}
\cos \theta_3 & -\sin \theta_3 & 0 \\
\sin \theta_3 & \cos \theta_3 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

Where \(\theta_3 = Zd-Z\)

\(Zd\) is the airship’s azimuth from the South

\(Z\) is the solar azimuth

In the second change, the new coordinate axes \((xp, yp=y_1, zp)\) are obtained by rotating the previous axes \((x_1, y_1, z_b)\) around the \(y_1\) axis in an angle of \(\theta_1\). The matrix that performs the second change is:

\[
R(y_1, \theta_1) = \begin{pmatrix}
\cos \theta_1 & 0 & \sin \theta_1 \\
0 & 1 & 0 \\
-\sin \theta_1 & 0 & \cos \theta_1
\end{pmatrix}
\]

Where \(\theta_1 = -(90-A)\)

\(A\) is the solar altitude

2) Once the solar cells’ surface has been projected in the plane \((xp, yp)\) the value of the solar cells’ projected area \(S_p\) can be easily obtained by calculating the area of the projected surface.
5.3 Conceptual design of “Zero”: a recreational solar airship

5.3.1 Reference airship

In order to make a conceptual preliminary design of a recreational solar airship a conventional modern airship has been taken as a reference. The reference airship used for this purpose is the AU-12 from the Russian manufacturer RosAeroSystems [24]. This is a low-volume two-seat certificated airship with the following main characteristics:
The AU-12 was initially designed for visual and instrumental monitoring of gas and oil pipelines, surveillance of roads and urban territories, advertising, high quality aerial photography, and rescue operations.

A breakdown of AU-12's net weight has been made in order to determine the weight of its main subcomponents: hull group, tail group, gondola and propulsion...
system. In order to do that the technical data provided by the manufacturers [24] [25] has been used as well as the weight estimation techniques and formulas proposed by Khoury in “Airship Technology” for each of the main components [4].

<table>
<thead>
<tr>
<th>Component</th>
<th>Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hull group</td>
<td>404.4</td>
</tr>
<tr>
<td>Tail group</td>
<td>131.8</td>
</tr>
<tr>
<td>Gondola</td>
<td>123.5</td>
</tr>
<tr>
<td>Propulsion system</td>
<td>120.6</td>
</tr>
<tr>
<td><strong>TOTAL NET WEIGHT</strong></td>
<td><strong>780.3</strong></td>
</tr>
</tbody>
</table>

Table 4. Au-12 net weight breakdown

The total net weight calculated above corresponds to the one specified by the manufacturer. As a result, it can be assumed that the breakdown shown in table 4 represents the distribution of the mass among the airship’s main components. Fuel weight (FW) has to be added to the net weight (NW) in order to obtain the operational empty weight (OEW). The fuel weight has been estimated taking into account the engine performance and fuel consumption graphs provided by the manufacturer [25] and knowing that the maximum autonomy of the airship at full speed is 2 hours. According to the previous specifications the fuel tank capacity is 50 liters and therefore the total mass of fuel is 36 kg (density of AVGAS 100 LL is 0.721 kg/l). Consequently the OEW is:

\[ OEW = 780 + 36 = 816 \text{ kg} \]

Likewise, the cruising power has been estimated given that the autonomy at cruising speed is 6 hours and the fuel tank capacity is 50 liters. The value obtained for the cruising power is 43 kW.

Modern airships are designed to have a slightly positive static heaviness. Static heaviness is defined as the takeoff weight minus the buoyancy force due to the lifting gas (see chapter 6). Maximum payload, and therefore the take off weight (TOW), is variable in airships and depends on the desired maximum operational flight altitude: the higher the maximum flight altitude the lower the maximum payload. Under ISA conditions, the maximum payload for the minimum flight altitude (payload 1) is obtained by filling all the available envelope volume with lifting gas at sea level by completely deflating the ballonets. However, this is only a theoretical reference value since the maximum operational altitude of this payload is 0. On the other hand the maximum payload for the maximum flight
altitude (payload 2) is obtained by completely filling the balloonets with air at sea level. In this case, as the airship ascends, the helium gas expands and the balloonets diminish their volume. The maximum flight level of payload 2 will be achieved when the balloonets are completely deflated and the helium gasbags are completely expanded occupying the entire envelope (this altitude is the so-called “pressure height”). Considering ISA conditions the values of payload 1 and 2 for the AU-12 airship are:

\[
\text{payload 1} = V_{\text{max helium}} \cdot (\rho_{\text{air}} - \rho_{\text{helium}})_{sl} - \text{OEW}
\]

\[
\text{payload 1} = 1250 \cdot (1.225 - 0.19) - 816 = 477 \text{ kg}
\]

\[
\text{payload 2} = V_{\text{min helium}} \cdot (\rho_{\text{air}} - \rho_{\text{helium}})_{sl} - \text{OEW}
\]

\[
\text{payload 2} = 938 \cdot (1.225 - 0.19) - 816 = 154 \text{ kg}
\]

The maximum flight level for payload 2 under ISA conditions can be obtained knowing that at the pressure height the helium has expanded completely:

\[
\sigma = \frac{\rho_{\text{air max altitude}}}{\rho_{\text{air sl}}} = \frac{938}{1250} = 0.75 \quad \rightarrow \quad h_{\text{max payload 2}} = 2896 \text{ m}
\]

For any other payload having a mass between payload 1 and 2 the maximum flight altitude can be calculated by interpolating the altitudes obtained above for payload 1 and 2 (figure 26).

Figure 26. Flight altitude as function of effective payload (ISA conditions)
Additionally, it has to be mentioned that the “effective” payload of the airship can be a value lower than payload 2 (e.g. only the pilot as payload). However, as stated above, airships operate in almost neutral buoyancy conditions (with slightly positive static heaviness). It means that if the “effective” payload is lower than payload 2, ballast has to be added as additional payload in order to guarantee slightly positive static heaviness conditions.

5.3.2 Requirements and configuration

The requirements for the “Zero” airship are the following:

- Two-seat airship with at least the same payload capability as airship AU-12.
- Same or better flight altitude capability as airship AU-12.
- Same length/diameter ratio as airship AU-12.
- Electrical propulsion system with zero emissions of pollutants.
- Back-up power system in case of malfunction of the primary feeding electric system. The back-up power system must allow the normal operation of the electric engines at its nominal rated power for at least 1 hour.

Under these requirements the proposed configuration for the “Zero” airship is the following:

- Solar powered airship based on the “Sunship” concept [3] with a grid of thin film flexible solar cells (acting as a primary feeding electric system) covering the whole upper surface on the airship’s envelope. The selected flexible cells are amorphous silicon cells mounted on a flexible polymer substrate. These cells have a proven efficiency of 12% and a specific power of 1000 W/kg [17].
- 2 permanent magnet brushed DC motors D135RAG of the Lynch Motor Company [22]. Each motor has a nominal rated power of 16.8 kW and a peak power of 34.3 kW. The weight of each motor is 11 kg.
- Back-up power system based on secondary batteries. The selected batteries are lithium polymer batteries with a specific energy of 200 Wh/kg [26].
- Size and dimensions of the envelope and control surfaces are proportional to those of the airship AU-12.
5.3.3 Size and weights

The determination of the size and weight of the “Zero” airship given its requirements and general configuration is done by an iterative process.

The starting point for the sizing of “Zero” is to consider an airship with the original size and hull dimensions of the AU-12 airship. In this scenario the weights of the hull group, tail group and gondola do not change. The weight of the solar propulsion system can be estimated taking into account the following considerations:

- The weight of the solar cells is proportional to the upper surface of the airship’s envelope. The considered surface density for the solar cells is 0.13 kg/m² [17].
- The weight of the electric motors and the power conditioner are provided by the manufacturers [22].
- The weight of the batteries is chosen to fit the design requirements given its specific energy of 200 Wh/kg.
- The rest of the components are estimated taking into account the same criteria used for airship AU-12.

Table 5 shows the weight breakdown for the propulsion system in the first iteration.

<table>
<thead>
<tr>
<th>Sub-component</th>
<th>Mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Installed engine</td>
<td>24.0</td>
</tr>
<tr>
<td>Flexible solar cells</td>
<td>47.0</td>
</tr>
<tr>
<td>Collector grid network</td>
<td>14.1</td>
</tr>
<tr>
<td>Power conditioner</td>
<td>35.0</td>
</tr>
<tr>
<td>Ducted propeller</td>
<td>10.0</td>
</tr>
<tr>
<td>Duct</td>
<td>30.0</td>
</tr>
<tr>
<td>Transmission system</td>
<td>10.0</td>
</tr>
<tr>
<td>Vector system</td>
<td>4.8</td>
</tr>
<tr>
<td>Secondary batteries</td>
<td>100</td>
</tr>
<tr>
<td><strong>TOTAL PROPULSION SYSTEM WEIGHT</strong></td>
<td><strong>274.9</strong></td>
</tr>
</tbody>
</table>

Table 5. “Zero” propulsion system weight. First iteration

In this case no fuel weight has to be added and therefore the operational empty weight is:

\[ OEW = 404.4 + 131.8 + 123.5 + 274.9 = 934.6 \text{ kg} \]
And payload 1 and 2 for the first iteration are:

\[
payload_1 = 1250 \cdot (1.225 - 0.19) - 934.6 = 359 \text{ kg}
\]

\[
payload_2 = 938 \cdot (1.225 - 0.19) - 934.6 = 36 \text{ kg}
\]

Both values are lower than the original ones obtained for airship AU-12. The performance in terms of payload and flight altitude is shown in figure 27.

![Figure 27](image)

Figure 27. Performance comparison between airship AU-12 (blue) and first iteration for “Zero” (red).

The requirement in terms of payload is not satisfied so another iteration has to be done. A bigger volume envelope for “Zero” has to be selected in order to generate more lift.

In this second iteration we consider a hull length of 36 meters and a maximum envelope diameter of 9 meters. The new envelope volume is 1526 m$^3$ and the volume of the ballonets is up to 378 m$^3$. The weight of the main components changes except for the gondola. The operational empty weight (OEW) is obtained proceeding in a similar way as done previously. Table 6 shows the main component’s breakdown for the second iteration.
Table 6. OEW breakdown for airship “Zero”. Second iteration

Payload 1 and 2 for the second iteration are:

\[ \text{payload 1} = 1526 \cdot (1.225 - 0.19) - 1033 = 546 \text{ kg} \]

\[ \text{payload 2} = 1148 \cdot (1.225 - 0.19) - 1033 = 155 \text{ kg} \]

Both values are greater than the original ones obtained for airship AU-12. The maximum flight level for payload 2 under ISA conditions for the second iteration is:

\[ \sigma = \frac{\rho_{\text{air max altitude}}}{\rho_{\text{air st}}} = \frac{1148}{1526} = 0.75 \rightarrow h_{\text{max payload 2}} = 2896 \text{ m} \]

The performance in terms of payload and flight altitude for the second iteration is shown in figure 28.

Figure 28. Performance comparison between airship AU-12 (blue) and second iteration for “Zero” (red).
Iteration 2 fulfills the payload and flight altitude requirements and is therefore selected as the final configuration for airship “Zero”. Table 7 summarizes its main dimensions and its motorization.

<table>
<thead>
<tr>
<th>Envelope volume</th>
<th>1526 m$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>including air balloonets, up to</td>
<td>378 m$^3$</td>
</tr>
<tr>
<td>Length/diameter ratio</td>
<td>4</td>
</tr>
<tr>
<td>Max. envelope diameter</td>
<td>9 m</td>
</tr>
<tr>
<td>Envelope length</td>
<td>36 m</td>
</tr>
<tr>
<td>Solar cells surface</td>
<td>411 m$^2$</td>
</tr>
<tr>
<td>Batteries weight</td>
<td>100 kg</td>
</tr>
<tr>
<td>Operational empty weight</td>
<td>1033 kg</td>
</tr>
<tr>
<td>Engine type</td>
<td>2 x D135RAG (LMC)</td>
</tr>
<tr>
<td>Max. engine power</td>
<td>68.6 kW</td>
</tr>
</tbody>
</table>

Table 7. “Zero” airship main dimensions and motorization

### 5.3.4 Performance

#### 5.3.4.1 Solar speed

The speed performance of the solar airship “Zero” is estimated using the equation for the speed obtained in 5.1 and assuming clear sky conditions. Unlike conventional powered airships, the maximum reachable speed of a solar powered airship in a given time depends on the latitude and altitude of the aircraft and also on the hour and day of the year.

Without feeding the electric engines with the secondary batteries the available power for propulsion is entirely obtained from the solar cells. Therefore, it is function of the solar radiation reaching the top of the airship, and it is also function of the projected area of the solar cells. Solar radiation depends on the hour, day of the year, latitude and altitude as has been explained in chapter 4. The projected area of the solar cells depends on the azimuth of the airship, the solar altitude and the solar azimuth as discussed in this chapter.

In order to save weight, the “Zero” airship has been designed with only the upper surface of the envelope covered by flexible solar cells. Due to this configuration only the direct solar radiation is considered when calculating the available solar radiation at any time. For clear sky conditions the reflected radiation corresponds
mainly to the radiation reflected by the ground and consequently this kind of radiation which is reaching the upper surface of the envelope is negligible. On the other hand during clear days, the contribution of diffuse radiation to the total solar radiation is no greater than 10% [4]. The exact contribution of diffuse radiation is difficult to predict in advance since it depends on air humidity conditions, suspension particles content, contamination, etc. Consequently only direct solar radiation is taken into account for calculating the speed performance of the solar airship. By neglecting the reflected and diffuse solar radiation the available power obtained at any moment is slightly underestimated. However, due to the cubic root relationship between available power and speed it can be easily found that for the “Zero” airship, on a clear day a maximum error of 3% is made by not considering the reflected and diffuse solar radiation, which is considered acceptable for the present study.

The speed performance of the “Zero” airship will be analyzed at solar noon for a latitude of 41ºN (Barcelona) and 2000 m of altitude. The four most significant solar days of the year will be considered: summer solstice, autumn equinox, winter solstice and spring equinox. The following efficiencies and parameters are considered:

- \( \rho = 1 \text{ kg/m}^3 \) (ISA conditions)
- \( C_{Dw} = 0.0045 \) \([27][28]\)
- \( \eta_{cell} = 0.12 \) \([16]\)
- \( \eta_{el} = 0.95 \) \([22]\)
- \( \eta_{mec} = 0.9 \) \([28]\)
- \( \eta_p = 0.8 \) \([28]\)
- \( S_{wet} = 822 \text{ m}^2 \)

Daily direct solar radiation for a point located over Barcelona (latitude 41ºN) at 2000 meters of altitude has been calculated in chapter 4 for the selected days. The values at noon of the direct solar radiation to be considered are:

- \( N_{\text{summer solstice}} = 1062 \text{ W/m}^2 \)
- \( N_{\text{spring & autumn equinoxes}} = 1020 \text{ W/m}^2 \)
- \( N_{\text{winter solstice}} = 905 \text{ W/m}^2 \)

Finally, the solar cells’ projected area \( S_p \) is calculated applying the methodology explained in this chapter and for values of the airship azimuth (from southern direction) ranging from 0º to 180º. In order to do that a Matlab code has been implemented and the results are shown in figure 29.
Figure 29. Solar cells’ projected area for airship “Zero”

The blue, green and red lines correspond respectively to the summer solstice, the winter solstice and the equinoxes. Like for the direct solar radiation, any other day of the year will be represented by a line which shall be between the lines of the summer and winter solstice. The following considerations can be deduced from figure 29:

- The higher the solar altitude the higher the projected area for a given airship azimuth.
- As the solar altitude increases the variation of the projected area with the airship azimuth is less pronounced.
- The maximum solar cells’ projected area for a given solar altitude is always obtained when the airship’s azimuth is perpendicular to the solar azimuth.

Finally, the theoretical speed of the solar airship is shown in figure 30.
The maximum attainable solar speed is 22.7 m/s (81.7 km/h) and corresponds to “Zero” flying with an azimuth of 90° at the noon of the summer solstice (blue line). On the other hand the lowest solar speed is 16.72 m/s (60.2 km/h) and corresponds to “Zero” flying with an azimuth of 0° at the noon of the winter solstice (green line). The following considerations can be deduced from figure 30:

- The higher the solar altitude, the higher the maximum speed for a given airship azimuth.
- As the solar altitude increases the variation of the maximum speed with the airship azimuth is less pronounced. Moreover, during half of the year, in summer and spring (between the blue and red line), the maximum speed is practically independent from the airship’s azimuth.
- The maximum speed for a given solar altitude is always obtained when the airship’s azimuth is perpendicular to the solar azimuth.

5.3.4.2 Emergency speed

In case of malfunction of the solar cells the secondary batteries must feed the electric motors in order to guarantee a normal airship operation for at least one hour. Assuming a maximum battery discharge of 70%, the available power is:

\[
100 \text{ kg} \cdot 200 \frac{Wh}{kg} \cdot 0.7 = 14000 \text{ Wh}
\]

This means an available power of 14000 W during 1 hour or 28000 W during half an hour. Considering the same efficiency values as the ones considered above.
the emergency speed (or speed of the airship powered only by the batteries) is $v_{1\,\text{hour}} = 62.2 \, \text{km/h}$ during 1 hour or $v_{30\,\text{minutes}} = 78.4 \, \text{km/h}$ during half an hour.

\[
v_{1\,\text{hour}} = \sqrt{\frac{2 \cdot 14000 \cdot \eta_l \cdot \eta_{\text{mec}} \cdot \eta_p}{\rho \cdot C_D \cdot S_{\text{wet}}}} = 62.2 \, \text{km/h}
\]

\[
v_{30\,\text{minutes}} = \sqrt{\frac{2 \cdot 28000 \cdot \eta_l \cdot \eta_{\text{mec}} \cdot \eta_p}{\rho \cdot C_D \cdot S_{\text{wet}}}} = 78.4 \, \text{km/h}
\]

### 5.3.4.3 Range and autonomy

Like the maximum available speed, range and autonomy for a solar airship depend on the day of the year, latitude and altitude of flight. Considering a minimum value for the incoming solar irradiation of 750 W/m² to be acceptable the autonomy of the “Zero” airship is 12 hours in the summer solstice and 6 hours in the winter solstice according to figure 17. For any other day of the year the autonomy of the “Zero” will be a value between 6 and 12 hours.

For a solar airship the range not only depends on the parameters mentioned above but also on the airship’s azimuth and the wind conditions along the track. Consequently the traditional concept of range is not accurate enough to characterize the performance of a solar powered airship. As a result a new equivalent concept for range called the guaranteed covered area (GCA) will be introduced in chapter 6. A three-hour flight range characterization of the “Zero” airship using the GCA concept is presented in chapter 7.
6. Geostrophic flight

6.1 General concept

As introduced in chapter 1, by airship geostrophic flight we understand a flight whose trajectory is determined in advance using accurate weather forecast data and that is performed mainly over the planetary boundary layer. Figure 31 shows a first classification of geostrophic flight based on the airship’s propulsion system and the energy source.

![Figure 31. Airship geostrophic flight](image_url)

The key element in geostrophic flight is the availability of accurate short-term weather forecast data in the area where the flight shall be performed. As seen in chapter 3 this type of data is currently available and can be used for trajectory prediction. Moreover, the condition to fly over the planetary boundary layer represents many advantages as discussed in chapter 3. These advantages are:

- Less turbulence and wind gusts which theoretically guarantees a more comfortable and stable flight.
- Better wind forecast since numerical weather models predict better the winds in the free atmosphere due to the turbulent nature and friction forces present in the PBL.
- Theoretically flying above the planetary boundary layer avoids fog and low clouds and allows better visibility.
- Winds are more constant in intensity and direction in the free atmosphere and, therefore, allow the implementation of effective flying strategies.
The disadvantage of flying above the PBL is that the higher the altitude the lower the effective payload.

Since the wind plays a very important role in geostrophic flight, it is necessary to have an estimation of the wind speed above the planetary boundary layer. In order to do that the MM5 wind data provided by SMC corresponding to the year 2008 has been analyzed. The mean wind speed for each month has been calculated for the pressure altitudes of 850, 795 and 700 hPa which approximately correspond to 1500, 2000 and 3000 m of altitude, respectively. The value of the wind speed at the grid points has been averaged along each month and only data corresponding to diurnal time has been considered. The result is shown in figure 32.

![Figure 32. Mean wind speed at 850, 795 and 700 hPa (year 2008)](image)

As expected the wind speed grows with altitude. The yearly mean values for 850, 795 and 700 hPa are 7.2, 8.0 and 9.6 m/s, respectively. For a flight at 2000 m, over the planetary boundary layer, the mean wind speed of 8.0 m/s = 28.8 km/h is lower than the cruising speed of a conventional airship like AU-12 (90 km/h) and is also lower than the maximum attainable cruising speed calculated for the solar-powered airship “Zero” in chapter 5 (81.7 km/h). Therefore an airship flight at 2000 m can be considered acceptable in terms of ground speed.
6.2 Guaranteed covered area

When forecasting the trajectory of an airship in a geostrophic flight the weather conditions along the track are taken into account. If the airship is solar-powered also the solar azimuth, the solar altitude and the airship's azimuth have to be considered at all times. Consequently, to better characterize the range of a geostrophic flight the concept of guaranteed covered area (GCA) is introduced. The guaranteed covered area of a geostrophic flight is defined as the effective area that can be reached by an airship given its initial position, initial time, flight altitude and flight duration. The GCA is represented on a map by the area enclosed by a closed curve that connects the final points of regularly spaced predicted trajectories as described below (figure 33). In this case, the GCA is obtained from 12 tracks (starting from the initial point) that are also included in the GCA map. These 12 “radii” are the trajectories of the forecast geostrophic flights corresponding to flying conditions of constant airship azimuth (from 0 to 330° with an interval of 30°). The following notation is used to describe a GCA:

\[
\text{NAME}_\text{PP}_\text{LAT}_\text{LONG}_\text{FFFF}_\text{TT}_\text{HHHH}_\text{DDMMYYYY}
\]

Where:

- \text{NAME} is the name of the airship.
- \text{PP} is the available cruising power. For a solar-powered flight it is SP. For a conventional powered airship it is the available cruising power in kW.
- \text{LAT} is the initial latitude in degrees.
- \text{LONG} is the initial longitude in degrees.
- \text{FFFF} is the flight altitude in meters.
- \text{TT} is the flight duration in hours
- \text{HHHH} is the flight starting time in hours and minutes.
- \text{DDMMYYYY} is the day, month and year of the flight.
Figure 33 shows, as an example, a guaranteed covered area for the solar-powered airship “Zero”: ZERO_SP_41.38_2.18_2000_02_1100_13102008. It represents the GCA for the airship “Zero” with starting point in Barcelona, flight altitude of 2000 m, flight duration of 2 hours and starting time at 11:00, October 13\textsuperscript{th} 2008.

It is important to clarify that, in general, the GCA is not the area of maximum range since maximum range can be achieved through a variable airship’s azimuth flight.

6.3 Trajectory calculation

6.3.1 General considerations

In order to calculate the geostrophic flight trajectory of an airship the general following assumptions are considered:

- The Earth is an inertial reference system.
- Atmospheric air and helium are ideal gases.
- The airship is treated as a point mass when considering the forces acting on it.
- Horizontal airship flight.
- The aerodynamic lift due to the envelope and fins is negligible. The only existing lifting force is due to static lift (buoyancy).
- At any time the temperature of the lifting gas is the same as the surrounding air.

6.3.2 Vertical trajectory

Airships obtain the lift force to stay aloft mainly thanks to buoyancy. The gross static lift \( L_g \) of an airship is equal to the weight of the air displaced by the envelope of the airship:

\[
L_g = (V_n \cdot \rho_a) \cdot g
\]

Where \( \rho_a \) is the air density, \( V_n \) is the envelope volume occupied by the lifting gas (helium in airships) and \( g \) is the standard gravity.

At the starting point of the trajectory calculation the airship is assumed to be already at the cruising altitude. The groundspeed is assumed to be zero. This cruising altitude is the design altitude, which is the altitude for which the airship is meant to be operational. At this initial point, the weight of the airship \( W \) is compensated by the net static lift \( L_n \). The net static lift is the gross static lift \( L_g \) less the weight of the lifting gas (helium) contained in the envelope volume:

\[
L_n = L_g - M = (V_n \cdot \rho_a - V_n \cdot \rho_g) \cdot g = V_n \cdot (\rho_a - \rho_g) \cdot g = V_n \cdot \rho_n \cdot g
\]

Where \( \rho_g \) is the helium density, \( M \) is the weight of the lifting gas and \( V_n \) is the volume occupied by the lifting gas (helium). The net density \( \rho_n \) is also called lift density.

When computing the trajectory of the airship we can consider two possibilities depending on how we take into account the variation of temperature and pressure of air and helium over space and time. The two possibilities are:

A. In the airship, the volume of the lifting gas \( V_n \) is variable (by inflating or deflating the ballonets) so the pressure of the lifting gas is the same as the surrounding air at any time. In this case there is no pressure differential across the envelope. We assume that the ballonets are big enough to accommodate the required changes in volume. In this case the net static lift remains constant over the trajectory.
B. The volume of the lifting gas remains constant along the flight. As a result, the density of the lifting gas remains constant. There is a pressure differential across the envelope of the airship. We assume that the envelope is able to resist the pressure differential. In this case the net static lift changes over the trajectory.

Option A. No pressure differential across the envelope. “Adapted” helium volume

In this case the airship is able to change the volume of the helium envelope in order to let the helium reach the same pressure as the atmospheric air (figure 34).

![Figure 34. “Adapted” helium volume](image)

It is considered that at the initial altitude $h_1$ the airship stands still (without vertical speed) and that the net static lift $L_{n1}$ compensates the weight $W$ of the airship. As a result there is no net vertical force acting on the vehicle and therefore the airship has neutral buoyancy:

$$L_{n1} = V_{n1} \cdot (\rho_{a1} - \rho_{g1}) \cdot g = W$$

where $\rho_{a1} = \frac{P_{a1}}{R_a T_{a1}}$ is the density of the air surrounding the airship.
\[ \rho_{g1} = \frac{p_{g1}}{R_g T_{g1}} = \frac{m_g}{V_{n1}} \]
is the density of the helium contained in the gas envelope.

\[ T_{a1} \text{ and } p_{a1} \]
are the temperature and pressure of the air at altitude \( h_1 \).

\[ T_{g1} \text{ and } p_{g1} \]
are the temperature and pressure of the helium.

\[ R_a \text{ and } R_g \]
are the gas constants for air and helium.

\[ m_g \]
is the mass of helium gas, which is constant.

\[ V_{n1} \]
is the volume of the envelope containing the helium.

The temperatures \( T_{a1} \) and \( T_{g1} \) are the same as assumed by hypothesis and in order to avoid differential of pressure across the envelope we impose that \( p_{g1} \) is equal to \( p_{a1} \). It means that the helium is “adapted” and has the same static pressure as the atmospheric air at altitude \( h_1 \).

The gas envelope has an initial volume \( V_{n1} \). It is obtained given the total weight of the airship and the initial altitude \( h_1 \). When \( V_{n1} \) and \( p_{g1} \) are fixed we can easily obtain the mass of helium \( m_g \) which remains constant along the flight (we assume no losses of gas).

The needed volume of helium \( V_{n1} \) (painted yellow in figure 34) is obtained by adequately regulating the volume of the air ballonet. The initial volume of the ballonet is \( V_{b1} \). The total volume \( V_t \) of the envelope is the sum of the lifting gas volume and the ballonet volume. \( V_t \) is fixed and constant.

\[ V_t = V_{n1} + V_{b1} \]

Now let’s consider an increment of time \( dt \). The temperature and pressure values of the atmospheric air have changed to \( T_{a2} \) and \( p_{a2} \). The density of the air has changed to \( \rho_{a2} \). The density ratio is:

\[ \frac{\rho_{a2}}{\rho_{a1}} = \frac{p_{a2}}{p_{a1}} \cdot \frac{T_{a1}}{T_{a2}} = \sigma \]

The new helium temperature \( T_{g2} \) is equal to \( T_{a2} \) as assumed by hypothesis. The air ballonet is inflated or deflated as required in order to achieve \( p_{g2} = p_{a2} \). The new volume of helium is \( V_{n2} \) and the new volume of the ballonet \( V_{b2} \). The density ratio for helium is:
The net static lift \( L_{n2} \) is:

\[
L_{n2} = V_{n2} \cdot (\rho_{a2} - \rho_{g2}) \cdot g
\]

\[
L_{n2} = V_{n2} \cdot (\sigma \rho_{a1} - \sigma \rho_{g1}) \cdot g
\]

\[
V_{n2} = \frac{m_g}{\rho_{g2}} \quad ; \quad m_g = \rho_{g1} \cdot V_{n1}
\]

\[
L_{n2} = \frac{\rho_{g1} \cdot V_{n1}}{\rho_{g2}} \cdot (\sigma \rho_{a1} - \sigma \rho_{g1}) \cdot g
\]

\[
L_{n2} = \frac{V_{n1}}{\sigma} \cdot (\sigma \rho_{a1} - \sigma \rho_{g1}) \cdot g
\]

\[
L_{n2} = V_{n1} \cdot (\rho_{a1} - \rho_{g1}) \cdot g = L_{n1} = W
\]

The net static lift does not change and neither does the airship weight. There is no net force acting upwards or downwards and the airship maintains its neutral buoyancy. As a result the altitude of the airship can be easily maintained constant. If the airship ascends or descends due to turbulence or wind gusts the pilot can incline the vectored thrust accordingly in order to compensate the perturbation and recover the desired flight altitude. Another option for the pilot is to slightly incline the airship’s hull upwards or downwards in order to produce an additional aerodynamic lift to compensate the perturbation.

As a result, if the ballonet has enough capacity to adapt its volume to the changing values of air temperature and pressure the altitude of the airship along the trajectory can be easily remained constant.

**Option B. Pressure differential across the envelope. Constant helium volume**

In this case the volume of the helium envelope remains constant along the flight (figure 35). The envelope has to be able to resist the stress due to the pressure differential between inside and outside the volume. The altitude of the airship changes as the air temperature and pressure change with time.

As in the previous case, at the initial position the net static lift is \( L_{n1} \) and the altitude is \( h_1 \).

\[
L_{n1} = V_{n1} \cdot (\rho_{a1} - \rho_{g1}) \cdot g = W
\]
After an increment of time $\mathrm{dt}$, the density of the atmospheric air will change due to the variation of air temperature and pressure values. As already seen before the density ratio is:

$$\frac{\rho_{a2}}{\rho_{a1}} = \frac{p_{a2}}{p_{a1}} \cdot \frac{T_{a1}}{T_{a2}} = \sigma$$

However, now the volume of the ballonet does not change and nor does the volume of helium. As a result the density of helium remains constant.

$$V_{b2} = V_{b1} \quad V_{n2} = V_{n1}$$

$$\rho_{g2} = \frac{m_g}{V_{n2}} = \frac{m_g}{V_{n1}} = \rho_{g1}$$

The new net static lift is:

$$L_{n2} = V_{n2} \cdot (\rho_{a2} - \rho_{g2}) \cdot g$$

$$L_{n2} = V_{n1} \cdot (\sigma \cdot \rho_{a1} - \rho_{g1}) \cdot g$$
As a result there is a net vertical force acting on the vehicle. The airship will ascend or descend depending on the value of $\sigma$:

$$\sigma > 1 \quad \Rightarrow \quad L_{n2} > W \quad \Rightarrow \quad \text{airship ascends}$$

$$\sigma < 1 \quad \Rightarrow \quad L_{n2} < W \quad \Rightarrow \quad \text{airship descends}$$

To summarize, the airship is in neutral buoyancy and therefore it can easily maintain a constant cruising altitude if the ballonet is capable to adapt its volume to the changing atmospheric conditions of pressure and temperature during the flight.

Taking into account a cruising altitude of 2000 m, flight duration less than one day and the MM5 forecast weather data provided by SMC for the year 2008, we will estimate the required volume needed by the ballonets in order to perform a constant altitude flight. Then we will compare the required ballonet volume with the one available in the airship “Zero” in order to decide if we can assume option A in our vertical trajectory calculation. A Matlab code has been implemented to calculate the maximum daily differences (for each month) in temperature and pressure at 2000 m throughout the considered region as well as the monthly maximum and minimum pressure and temperature at the same altitude (Table 8). The calculation of $\sigma$ has been done by using those maximum daily differences and the minimum monthly values for temperature and pressure (most unfavorable situation).

The upper and lower values of sigma ($\sigma_{up}$ and $\sigma_{lo}$) have been computed as follows (Table 9):

$$\sigma_{up} = \left( \frac{P_{a2} - P_{a1}}{P_{a1}} \right) \cdot \left( \frac{T_{a1}}{T_{a2}} \right)$$

$$\sigma_{lo} = \left( \frac{P_{a2} - P_{a1}}{P_{a1}} \right) \cdot \left( \frac{T_{a1}}{T_{a2}} \right)$$

The required maximum and minimum helium volume will be:
\[(V_{n2})_{max} = \frac{V_{n1}}{\sigma_{lo}} \quad ; \quad (V_{n2})_{min} = \frac{V_{n1}}{\sigma_{up}}\]

And the required ballonet volume:

\[(V_{b2})_{max} = V_t - (V_{n2})_{min}\]

\[(V_{b2})_{min} = V_t - (V_{n2})_{max}\]

<table>
<thead>
<tr>
<th>Month</th>
<th>Min (T) month (K)</th>
<th>Max diff. (T) day (K)</th>
<th>Min (P) month (Pa)</th>
<th>Max diff. (P) day (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>265.3</td>
<td>14.0</td>
<td>77189.0</td>
<td>2208.2</td>
</tr>
<tr>
<td>February</td>
<td>266.1</td>
<td>11.4</td>
<td>78184.0</td>
<td>2083.8</td>
</tr>
<tr>
<td>March</td>
<td>259.4</td>
<td>18.5</td>
<td>77150.0</td>
<td>2551.1</td>
</tr>
<tr>
<td>April</td>
<td>267.0</td>
<td>16.5</td>
<td>76807.0</td>
<td>2414.0</td>
</tr>
<tr>
<td>May</td>
<td>270.0</td>
<td>17.1</td>
<td>78612.0</td>
<td>1281.7</td>
</tr>
<tr>
<td>June</td>
<td>272.0</td>
<td>14.9</td>
<td>78960.0</td>
<td>1204.4</td>
</tr>
<tr>
<td>July</td>
<td>274.8</td>
<td>20.2</td>
<td>79471.0</td>
<td>1233.8</td>
</tr>
<tr>
<td>August</td>
<td>274.0</td>
<td>22.3</td>
<td>79022.0</td>
<td>1457.2</td>
</tr>
<tr>
<td>September</td>
<td>271.9</td>
<td>19.1</td>
<td>78872.0</td>
<td>1715.7</td>
</tr>
<tr>
<td>October</td>
<td>265.4</td>
<td>15.9</td>
<td>77622.0</td>
<td>1861.7</td>
</tr>
<tr>
<td>November</td>
<td>261.8</td>
<td>18.0</td>
<td>76907.0</td>
<td>2218.8</td>
</tr>
<tr>
<td>December</td>
<td>260.4</td>
<td>14.1</td>
<td>77263.0</td>
<td>1993.0</td>
</tr>
</tbody>
</table>

Table 8. Maximum daily differences in \(T\) and \(P\) for each month (2008)

<table>
<thead>
<tr>
<th>Month</th>
<th>(\frac{P_{a2}}{P_{a1}})</th>
<th>(\frac{T_{a1}}{T_{a2}})</th>
<th>(\sigma_{up})</th>
<th>(\frac{P_{a2}}{P_{a1}})</th>
<th>(\frac{T_{a1}}{T_{a2}})</th>
<th>(\sigma_{lo})</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>1.029</td>
<td>1.053</td>
<td>1.083</td>
<td>0.972</td>
<td>0.950</td>
<td>0.923</td>
</tr>
<tr>
<td>February</td>
<td>1.027</td>
<td>1.043</td>
<td>1.071</td>
<td>0.974</td>
<td>0.959</td>
<td>0.934</td>
</tr>
<tr>
<td>March</td>
<td>1.033</td>
<td>1.071</td>
<td><strong>1.107</strong></td>
<td>0.968</td>
<td>0.933</td>
<td><strong>0.907</strong></td>
</tr>
<tr>
<td>April</td>
<td>1.031</td>
<td>1.062</td>
<td>1.095</td>
<td>0.970</td>
<td>0.942</td>
<td>0.913</td>
</tr>
<tr>
<td>May</td>
<td>1.016</td>
<td>1.063</td>
<td>1.081</td>
<td>0.984</td>
<td>0.940</td>
<td>0.925</td>
</tr>
<tr>
<td>June</td>
<td>1.015</td>
<td>1.055</td>
<td>1.071</td>
<td>0.985</td>
<td>0.948</td>
<td>0.934</td>
</tr>
<tr>
<td>July</td>
<td>1.016</td>
<td>1.074</td>
<td>1.090</td>
<td>0.985</td>
<td>0.931</td>
<td>0.917</td>
</tr>
<tr>
<td>August</td>
<td>1.018</td>
<td>1.081</td>
<td>1.101</td>
<td>0.982</td>
<td>0.925</td>
<td>0.908</td>
</tr>
<tr>
<td>September</td>
<td>1.022</td>
<td>1.070</td>
<td>1.094</td>
<td>0.979</td>
<td>0.934</td>
<td>0.914</td>
</tr>
<tr>
<td>October</td>
<td>1.024</td>
<td>1.060</td>
<td>1.085</td>
<td>0.977</td>
<td>0.943</td>
<td>0.921</td>
</tr>
<tr>
<td>November</td>
<td>1.029</td>
<td>1.069</td>
<td>1.100</td>
<td>0.972</td>
<td>0.935</td>
<td>0.909</td>
</tr>
<tr>
<td>December</td>
<td>1.026</td>
<td>1.054</td>
<td>1.081</td>
<td>0.975</td>
<td>0.949</td>
<td>0.925</td>
</tr>
</tbody>
</table>

Table 9. Values of \(\sigma_{up}\) and \(\sigma_{lo}\)
According to Table 9 the maximum and minimum values of sigma (both in March) are \( \sigma_{up} = 1.107 \) and \( \sigma_{dp} = 0.907 \). The required helium volume for the airship “Zero” flying at 2000 m with 150 kg of effective payload and standard conditions is \( V_{n1} = 1380 \text{ m}^3 \) and therefore the volume of the ballonets is \( V_{b1} = 146 \text{ m}^3 \). The total envelope volume of airship “Zero” is 1526 m³.

The required ballonet volumes to perform a constant 2000 m altitude flight are:

\[
(V_{b2})_{\text{max}} = V_t - (V_{n2})_{\text{min}} = 1526 - \frac{1380}{1.107} = 279 \text{ m}^3
\]

\[
(V_{b2})_{\text{min}} = V_t - (V_{n2})_{\text{max}} = 1526 - \frac{1380}{0.907} = 4 \text{ m}^3
\]

The required maximum ballonet volume \((V_{b2})_{\text{max}}\) is smaller than the maximum available ballonet volume of airship “Zero” which is 378 m³. In his turn the required minimum ballonet volume \((V_{b2})_{\text{min}}\) is greater than 0 which is the lowest possible value. As a result we can consider option A for vertical trajectory calculation. There is no pressure differential across the envelope and the airship maintains the neutral condition buoyancy during all the flight. Besides other considerations this result is important because the pilot has an easy control over the vertical coordinate and is able to maintain the selected cruising altitude by means of the vectored thrust or by generating an appropriate aerodynamic lift by tilting the hull.

\[
h_1 = h_2 = \text{cte} \quad \rightarrow \quad \frac{dh}{dt} = 0
\]

When computing the predicted geostrophic flight trajectories of the airship, the “real” required changing volume of the ballonets along the track will also be calculated. The “real” ballonet volume is expected to be smaller than the maximum obtained before since the temperature and pressure conditions that the airship will encounter will be less extreme than the ones considered for estimating the required ballonet volume \((V_b)_{\text{req}}\).

6.3.3 Horizontal trajectory

The net force \( \vec{F}_{\text{net}} \) acting on the airship at any time is the resultant of the thrust \( \vec{T} \) provided by the airship engines and the drag force \( \vec{F}_D \) due to the relative velocity between the airship velocity vector and the wind velocity vector.

\[
\vec{F}_{\text{net}} = \vec{T} + \vec{F}_D
\]

The drag force is, more specifically:
Where: \( \rho_a \) is the air density at cruising altitude

\[ \dot{v}_{rel} = \dot{v} - \dot{v}_{wind} \]  
Relative velocity vector

\[ \ddot{v} = \text{Airship velocity vector} \]

\[ \dot{v}_{wind} = \text{Wind velocity vector} \]

\[ C_{Dw} = \text{Resistance coefficient} \]

\[ S_w = \text{Wet surface of the airship} \]

\[ \frac{\ddot{v}_{rel}}{|\ddot{v}_{rel}|} = \text{Relative velocity unitary vector} \]

The equation of motion is therefore:

\[ \ddot{F}_{net} = \ddot{T} + \ddot{F}_D = M \cdot \ddot{a} \]

Or:

\[ \ddot{T} - \frac{1}{2} \cdot \rho_a \cdot |\ddot{v}_{rel}|^2 \cdot C_{Dw} \cdot S_w \cdot \frac{\ddot{v}_{rel}}{|\ddot{v}_{rel}|} = M \cdot \ddot{a} = M \cdot \frac{d\ddot{v}}{dt} \]

Where \( M \) is the total mass of the airship and \( \ddot{a} \) its acceleration vector.

The equation of motion is an ordinary differential equation and is solved by using a second order modified Euler method. This method uses an auxiliary intermediate point to better estimate velocities and accelerations at each time increment \( \Delta t \).
The steps are:

1) At the initial point \(i\) the horizontal coordinates \(n_i\), altitude \(h_i\), ground velocity \(\vec{v}_i\) and thrust \(\vec{T}_i\) of the airship are known. As shown later in this chapter by using accurate weather forecast data we can also obtain the vector wind velocity \(\vec{v}_{\text{wind}_i}\), air temperature \(T_i\), air pressure \(P_i\) and thus air density \(\rho_i\) for this initial point. Finally we can also determine the vector acceleration at point \(i\) using the equation:

\[
\vec{a}_i = \left( T_i - \frac{1}{2} \cdot \rho_i \cdot |\vec{v}_{\text{rel}_i}|^2 \cdot C_{DW} \cdot S_w \cdot \frac{\vec{v}_{\text{rel}_i}}{|\vec{v}_{\text{rel}_i}|} \right) / M
\]

Where \(\vec{v}_{\text{rel}_i} = \vec{v}_i - \vec{v}_{\text{wind}_i}\).

2) Next step is to consider a time increment \(\Delta t / 2\) and calculate the horizontal coordinates, velocity and acceleration of an intermediate auxiliary point \(i + \frac{1}{2}\). The two first are direct:

\[
\vec{n}_{i+\frac{1}{2}} = n_i + \frac{\Delta t}{2} \vec{v}_i
\]

\[
\vec{v}_{i+\frac{1}{2}} = v_i + \frac{\Delta t}{2} \vec{a}_i
\]

In order to calculate \(\vec{a}_{i+\frac{1}{2}}\) we must proceed in a similar way as we did to calculate \(\vec{a}_i\).
Finally, after a time increment $\Delta t$ we calculate the new position and velocity of the airship at the next point $i + 1$:

$$\vec{r}_{i+1} = \vec{r}_i + \Delta t \cdot \vec{v}_{i+1/2}$$

$$\vec{v}_{i+1} = \vec{v}_i + \Delta t \cdot \vec{a}_{i+1/2}$$

Repeating the procedure we can advance the airship trajectory. The elapsed flight time is therefore $D = (i - 1) \cdot \Delta t$.

### 6.3.4 Determination of the atmospheric parameters along the trajectory

#### 6.3.4.1 Horizontal airship coordinates in the grid

First, the airship is considered to have the same horizontal coordinates (latitude, longitude) as one of the MM5 grid points. Usually the cruising altitude of the airship at this location will not coincide with any of the geopotential altitudes corresponding to the available pressure levels. In general the airship altitude will be between two geopotential altitudes (figure 37). Furthermore, some kind of interpolation is needed in order to calculate the temperature $T$, pressure $P$ and the wind speed $(u,v)$ at the cruising altitude, which is assumed in this case to be 2000 m.
In order to calculate the temperature $T_{2000}$ at the flight altitude, a linear variation of the temperature with altitude is considered [29] [30]:

$$a = \frac{T_4 - T_3}{h_4 - h_3}$$

$$T_{2000} = T_3 + a \cdot (2000 - h_3)$$

Where:
- $T_3$ = Temperature of the grid point at pressure level 3
- $T_4$ = Temperature of the grid point at pressure level 4
- $h_3$ = Geopotential altitude of the grid point at pressure level 3
- $h_4$ = Geopotential altitude of the grid point at pressure level 4

Considering the atmospheric air as a perfect gas and assuming hydrostatic equilibrium, the pressure $P_{2000}$ and density $\rho_{2000}$ at the flight altitude are [29] [30]:

$$P_{2000} = P_3 \cdot \left(\frac{T_{2000}}{T_3}\right)^{-\frac{\gamma}{R}}$$

$$\rho_{2000} = \frac{P_{2000}}{R \cdot T_{2000}}$$
Where:

- $P_3$ = Pressure level 3 (850 hPa)
- $R$ = Gas constant for air
- $g$ = Standard gravity.

Finally the wind velocity vector $(u_{2000}, v_{2000})$ can be interpolated as:

$$b_u = \frac{u_4 - u_3}{P_4 - P_3}$$

$$b_v = \frac{v_4 - v_3}{P_4 - P_3}$$

$$u_{2000} = u_3 + b_u \cdot (P_{2000} - P_3)$$

$$v_{2000} = v_3 + b_v \cdot (P_{2000} - P_3)$$

Where:

- $P_4$ = Pressure level 4 (700 hPa)
- $u_3$ = Zonal wind speed of the grid point at pressure level 3
- $u_4$ = Zonal wind speed of the grid point at pressure level 4
- $v_3$ = Meridional wind speed of the grid point at pressure level 3
- $v_4$ = Meridional wind speed of the grid point at pressure level 4

Figure 38 shows an example of wind velocity determination for the grid points at 2000 m. The values correspond to 19th November 2008 at 12:00 h.
6.3.4.2 Horizontal coordinates not coinciding with the grid points

By repeating the previous procedure we can obtain the values of temperature, pressure and wind velocity at the grid points and at a given altitude. In general, however, the horizontal coordinates of the airship’s trajectory will not coincide with any of those of the grid points (figure 39).
The values of the temperature, pressure and wind velocity of the trajectory points can be obtained by doing a bilinear interpolation after an isoparametric transformation.

The property $\varphi$ that we want to calculate for the trajectory points is considered to be known for the grid points. In our case $\varphi$ is temperature, pressure and wind velocity and, as seen before in this chapter, those parameters can be easily calculated for the grid points for a given flight altitude.

Given the coordinates of the trajectory point in a local axes coordinate system (figure 40), the property for the trajectory point ($P(\xi, \eta)$) can be calculated by doing a bilinear interpolation:
\[ \varphi_p = \sum_{i=1}^{4} N_i(\xi_p, \eta_p) \cdot \varphi_i \]

Where \( \varphi_i \) is the value of the property in the grid points and \( N_i \) are the interpolation weights, which are defined as follows:

\[ N_1 = \frac{(1 - \xi) \cdot (1 - \eta)}{4} \]
\[ N_2 = \frac{(\xi + 1) \cdot (1 - \eta)}{4} \]
\[ N_3 = \frac{(\xi + 1) \cdot (\eta + 1)}{4} \]
\[ N_4 = \frac{(1 - \xi) \cdot (1 + \eta)}{4} \]

\[ N_i(\xi_i, \eta_i) = 1 \quad i = 1 \div 4 \]
\[ N_i(\xi_j, \eta_j) = 0 \quad i \neq j \]

However, before performing the bilinear interpolation an isoparametric transformation has to be done (figure 41) due to the fact that the MM5 grid geometry is not uniform because the grid points are obtained from a polar stereographic projection. As a result the horizontal and vertical distances among the grid points change with latitude and longitude.

Knowing the real coordinates of the trajectory point \( P(x, y) \), the coordinates in the local axes coordinate system \((\xi_p, \eta_p)\) can be determined by solving the following equation system:
This is a non linear system and the solution is obtained by linearization and iteration. Considering \((x_0, y_0)\) as initial guess:

\[
x_p = x_0 + \delta x_p
\]

\[
y_p = y_0 + \delta y_p
\]

\[
\delta x_p \approx \frac{\partial x_p}{\partial \xi} \delta \xi + \frac{\partial x_p}{\partial \eta} \delta \eta
\]

\[
\delta y_p \approx \frac{\partial y_p}{\partial \xi} \delta \xi + \frac{\partial y_p}{\partial \eta} \delta \eta
\]

Using Newton’s method:

\[
x_{p} - x_{0} = \left. \frac{\partial x_p}{\partial \xi} \right|_{0} \delta \xi + \left. \frac{\partial x_p}{\partial \eta} \right|_{0} \delta \eta
\]

\[
y_{p} - y_{0} = \left. \frac{\partial y_p}{\partial \xi} \right|_{0} \delta \xi + \left. \frac{\partial y_p}{\partial \eta} \right|_{0} \delta \eta
\]

Solving the previous linear system we obtain \((\delta \xi, \delta \eta)\) and a new guess for \((\xi_p, \eta_p)\):

\[
\xi \equiv \xi_0 + \delta \xi
\]

\[
\eta \equiv \eta_0 + \delta \eta
\]

In general, given an iteration point \((\xi_i, \eta_i)\) we can obtain the next iteration point \((\xi_{i+1}, \eta_{i+1})\) as follows:

\[
A_i = \begin{bmatrix}
\frac{\partial x_p}{\partial \xi} & \frac{\partial x_p}{\partial \eta} \\
\frac{\partial y_p}{\partial \xi} & \frac{\partial y_p}{\partial \eta}
\end{bmatrix}_i
\]

\[
\begin{bmatrix}
\xi_{i+1} \\
\eta_{i+1}
\end{bmatrix} = \begin{bmatrix}
\xi_i \\
\eta_i
\end{bmatrix} + A_i^{-1} \begin{bmatrix}
x_p - x_i \\
y_p - y_i
\end{bmatrix}
\]
The iteration process has to be repeated until \((\xi_{i+1}, \eta_{i+1}) \equiv (\xi_i, \eta_i)\), given a convergence criterion.

Once the values for \((\xi_p, \eta_p)\) are obtained the temperature, pressure and wind velocity values for the trajectory point can be derived applying the bilinear interpolation presented above.

Figure 42 shows an example for wind velocity interpolation. The velocity in the grid points is shown in red and the interpolated wind velocity for the trajectory points is shown in blue.

**6.3.5 Matlab code**

In order to compute the geostrophic flight trajectories of a solar airship (and therefore the guaranteed covered area GCA) a Matlab code (annex 1) has been developed taking into account the hypothesis and the calculation methodology explained in 6.3.3.

The code consists of a main program called *dirig16.m*, three functions called *projectarea2.m*, *solar3.m* and *wind6.m* and two sub-functions called *isop.m* and *jaco.m*.

The main program *dirig16.m* computes the geostrophic flight trajectory (or trajectories if the GCA is calculated) of a solar airship given the following inputs:

- Airship characteristics: mass, drag coefficient, envelope dimensions, wetted surface area, propeller area.
- Starting flight time, day and month.
- Flight duration.
- Coordinates of the starting point.
- Flight altitude.
- Constant airship azimuth.
- Solar cells, electric components and mechanic efficiencies.
- Propulsive efficiency.

The main program calls the three functions mentioned above in order to perform specific repetitive computation tasks. The input/output values of these functions are:

- projectarea2.m: computes the solar cells' projected area taking as input the variables solar altitude, solar azimuth, airship's azimuth and the solar cells surface.

- solar3.m: computes the direct solar irradiation, the solar altitude and the solar azimuth taking as input variables the time, day, month, latitude and flight altitude.

- wind6.m: computes the temperature, pressure and wind velocity values for a trajectory point taking as input variables the latitude, longitude and altitude of the corresponding trajectory point. This function calls the sub-functions isop.m and jaco.m which perform auxiliary tasks for calculating the isoparametric transformation and the bilinear interpolation.
7. Case study

7.1 GCA for the “Zero” airship

In order to study the range performance of the airship “Zero”, twelve guaranteed covered areas (GCA) are studied in this chapter. Each GCA has been calculated for the 15th day of each month of the year 2008, considering the following flight characteristics:

- Airship: “Zero”
- Energy source: Solar power
- Starting point: Barcelona
- Flight altitude: 2000 m
- Flight duration: 3 hours
- Starting time: 11:30 (UT)
- Day: 15th of each month
- MM: ranging from 01 to 12

And consequently, as explained in chapter 6.2 the notation is as follows:

ZERO_SP_41.38_2.18_2000_03_1130_15MM2008

The payload is defined to be 200 kg and the following efficiencies and parameters have been taken into account:

- \( C_{Dw} = 0.0045 \) [27] [28]
- \( \eta_{cell} = 0.12 \) [16]
- \( \eta_{el} = 0.95 \) [22]
- \( \eta_{mec} = 0.9 \) [28]
- \( \eta_{p} = 0.8 \) [28]
- \( S_{wet} = 822 \text{ m}^2 \)

The GCAs have been obtained using the Matlab code developed in the framework of this study which is described in chapter 6 (annex 1). The trajectory corresponding to a flight with constant airship azimuth of 0° is painted in red in the GCA charts. A further graph shows the range (in km) as function of the azimuth of the airship (in degrees).
Figure 43a. ZERO_SP_41.38_2.18_2000_03_1130_15012008

Figure 43b. Range as function of the airship’s azimuth
Figure 44a. ZERO_SP_41.38_2.18_2000_03_1130_15022008

Figure 44b. Range as function of the airship’s azimuth

Figure 44b. Range as function of the airship’s azimuth
Figure 45a. ZERO_SP_41.38_2.18_2000_03_1130_15032008

Figure 45b. Range as function of the airship’s azimuth
Figure 46a. ZERO_SP_41.38_2.18_2000_03_1130_15042008

Figure 46b. Range as function of the airship's azimuth
Figure 47a. ZERO_SP_41.38_2.18_2000_03_1130_15052008

Figure 47b. Range as function of the airship's azimuth
Figure 48a. ZERO_SP_41.38_2.18_2000_03_1130_15062008

Figure 48b. Range as function of the airship's azimuth

87
Figure 49a. ZERO_SP_41.38_2.18_2000_03_1130_15072008

Figure 49b. Range as function of the airship's azimuth

Figure 49b. Range as function of the airship's azimuth
Figure 50a. ZERO_SP_41.38_2.18_2000_03_1130_15082008

Figure 50b. Range as function of the airship’s azimuth
Figure 51a. ZERO_SP_41.38_2.18_2000_03_1130_15092008

Figure 51b. Range as function of the airship's azimuth
Figure 52a. ZERO_SP_41.38_2.18_2000_03_1130_15102008

Figure 52b. Range as function of the airship's azimuth
Figure 53a. ZERO_SP_41.38_2.18_2000_03_1130_15112008

Figure 53b. Range as function of the airship's azimuth
Figure 54a. ZERO_SP_41.38_2.18_2000_03_1130_15122008

Figure 54b. Range as function of the airship's azimuth
Table 10 shows the mean range for each of the GCAs determined above. As expected the mean range is higher during the summer and spring months due to the greater available solar power. Also, as seen in the GCA graphs the wind conditions encountered along the trajectories have a great impact on the shape of the GCA.

<table>
<thead>
<tr>
<th>Flight</th>
<th>GCA mean range (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>189.2</td>
</tr>
<tr>
<td>February</td>
<td>185.9</td>
</tr>
<tr>
<td>March</td>
<td>218.5</td>
</tr>
<tr>
<td>April</td>
<td>228.6</td>
</tr>
<tr>
<td>May</td>
<td>231</td>
</tr>
<tr>
<td>June</td>
<td>235.8</td>
</tr>
<tr>
<td>July</td>
<td>227.4</td>
</tr>
<tr>
<td>August</td>
<td>244.6</td>
</tr>
<tr>
<td>September</td>
<td>216.3</td>
</tr>
<tr>
<td>October</td>
<td>195</td>
</tr>
<tr>
<td>November</td>
<td>186.5</td>
</tr>
<tr>
<td>December</td>
<td>180</td>
</tr>
</tbody>
</table>

Table 10. GCAs’ mean range
8. Environmental implications

The objective of this project is to study the feasibility of sustainable and non-contaminant recreational airship flight by using accurate weather forecasting data and an electric propulsion system powered by solar cells. The solar airship uses solar power as primary energy source in order to feed an electric engine and therefore no emissions of contaminant gases occur during flight.

A comparison with the airship AU-12, which is used as a reference to design “Zero”, is performed in terms of pollution production per year. We can compare them since they have similar payloads (the one of “Zero” even being slightly higher) and are also comparable in terms of autonomy (AU-12 has a maximum autonomy of 6 hours whereas “Zero” has a minimum autonomy of 6 hours, depending on the day of the year). The type of fuel used by the airship AU-12 is Avgas 100 LL which has an emission coefficient of 2.2 kg of CO$_2$ per liter [31]. Considering one 6-hour flight at cruising speed the total consumed fuel of the AU-12 airship is 50 liters. The CO$_2$ production will therefore be:

$$50l \times \frac{2.2 \text{ kg CO}_2}{1l} = 110 \text{ kg CO}_2$$

As a consequence the utilization of the “Zero” airship saves 110 Kg of CO$_2$ with respect to AU-12, for a same duration of flight. If a flight like this would be performed every day of the year, 40150 kg of CO$_2$ (over 40 tons) less would be ejected to the atmosphere. To summarize, a solar-powered airship with an electric engine will produce zero emissions during flight.

However, not only zero emissions are considered but also other implications on the environment. Thus a very important aspect is that electric flight is practically noiseless compared to a conventional internal combustion engine.

Nowadays the amount of airship activities is low. However, the very low environmental impact of solar geostrophic flight may foster its development into a widespread sustainable aviation activity. In addition, the demonstration that a means of transport can be designed which dramatically reduces the emissions of contaminants may have a great impact on the public which should push conventional means of transport for a reduction of their emissions.
9. Conclusions and further recommendations

This study has demonstrated that recreational airship solar geostrophic flight is feasible with current technology. The key element of geostrophic flight is the availability of accurate and reliable short term weather forecast data. In this sense the weather forecast data provided for this study by Servei Meteorològic de Catalunya has been able to accurately predict the flight trajectory of a hot-air balloon over the Pyrenees and therefore has been considered acceptable to be used for this study.

In general, the dramatic improvement of numeric weather forecast models over the past 50 years may overcome one of the disadvantages of airship flight: its relatively low speed which makes airships more dependent on wind conditions. By knowing the weather conditions for the flight in advance, on the one hand potential dangerous wind situations can be avoided and on the other hand faster flight trajectories can be implemented in case of favorable wind conditions. The second main disadvantage of airships is its inherent big volume and envelope surface. Paradoxically this apparent disadvantage makes airships the ideal candidates for solar flight. Indeed, the still low efficiencies of thin film solar cells require the solar cells to be spread over a large surface, which is available on the upper side of the airship’s envelope.

In this study a conceptual design of “Zero”, a recreational solar airship, has been made using already existing technology. Its performance in terms of payload and speed is similar to the one of the reference airship (AU-12) which is used for the design of “Zero”, with the additional advantage of producing zero contaminant gases during its flight. The range of the “Zero” airship has been studied for the year 2008 using the introduced technique of the Guaranteed Covered Area (GCA). As expected, during summer and spring the flight range of the airship is greater due to the greater available solar power. Because of the dependency of the GCA with the day of the year and location of the starting point, in the opinion of the author, nowadays the possible implementation of the solar geostrophic airship flight is restricted to the field of sportive recreational aviation.

As a conclusion, given the large surface area inherent in airships and the potential to use this large area for the production of photovoltaic solar energy, the possible use of this lighter than air aircrafts as a complement to the traditional aviation industry should be considered positively, since the availability of accurate short- and midterm weather forecast data can make the 21st century airship flight safe and reliable.
Finally, further recommendations for future studies include:

- Consider not only the direct but also the reflected and diffuse radiation in the incoming solar radiation. Consider also other situations rather than clear sky conditions.
- Study optimal airship envelope shapes in order to maximize the incoming solar power and static lift but minimize the drag.
- Compare the results obtained for the geostrophic flight using the weather forecast data from the MM5 model with other regional and global weather prediction models.
10. Budget

This chapter contains the estimated budget for the realization of the "Study of a zero-emission airship transport system based on the geostrophic flight concept". The costs of this study are only related to the author's working hours and to the cost of the educational license for the software Matlab. The weather forecast data of the year 2008 from SMC has been provided for free due to the academic background of this study.

In order to determine the cost, the following considerations have been taken into account:

- The duration of the project has been eight months and the author has invested a total of 600 hours.
- An 8 €/hour tariff is assumed for the author.
- The chosen currency is Euro.
- Taxes are not included.

The cost breakdown is shown in table 11.

<table>
<thead>
<tr>
<th>Concept</th>
<th>Quantity</th>
<th>Units</th>
<th>Unitary cost</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Author working hours</td>
<td>600</td>
<td>hours</td>
<td>8 €/hour</td>
<td>4800 €</td>
</tr>
<tr>
<td>Matlab license</td>
<td>1</td>
<td>-</td>
<td>87 €</td>
<td>87 €</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td></td>
<td></td>
<td></td>
<td><strong>4887 €</strong></td>
</tr>
</tbody>
</table>

Table 11. PFC cost

The total cost for the realization of this study is therefore 4887 €.
11. References


[27] Hoerner, S.F. Fluid-dynamic drag. Hoerner fluid dynamics, USA. 1965


Annex: Matlab Code
STUDY OF A ZERO-EMISSION AIRSHIP TRANSPORT SYSTEM BASED ON THE GEOSTROPHIC FLIGHT CONCEPT

% Programa dirig16.m
%
% Programa que determina les Guaranteed Covered Areas (GCA) del dirigible solar "Zero"
% fixada un llei de l'alçada de vol
% Predicció del camp de vent del 2008 amb model MM5 del Servei Meteorològic de Catalunya
%
% Novembre 2010
% Autor: Francesc Betorz Martínez
%
clear %inicialitza les variables
%
% Lectura punts malla
[lat,long] = textread('MM5latlonalt.txt','%f %f %*f'); %Lectura coordenades punts de la malla
MM5 (latitud, longitud). 69x69 punts
%
% Directori amb les dades de vent
directorTemp='D:\Aeronautica\PFC\Meteocat_Temp2008\'; %ruta a la carpeta de temperatures

directorGeop='D:\Aeronautica\PFC\Meteocat_Geopot2008\'; %ruta a la carpeta d'alçades
geopotencials

directorVent='D:\Aeronautica\PFC\bones_smc_Vent_2008\'; %ruta a la carpeta de dades de
vent
%
% Dibuix de la zona de treball
latlim = [37.2 44.2]; lonlim = [-3.3 6];
figure
worldmap(latlim,lonlim)
cont=0; %contador
%
% Eficiències
rend_cell=0.12; % Rendiment cel.lules solars
rend_elec=0.95; % Rendiment elèctric motor
rend_mec=0.9; % Rendiment mecànic
rend_prop=0.8; % Rendiment propulsiu
Area_disc=12.56; % Area discal en m2
%
% Definim elipsoide en posició horitzontal. Semieixos 18 i 4.5
i=1;
for phi=0:0.1:3.14
    for teta=0:0.01:3.14
        x(i)=18*sin(phi)*cos(teta);
        z(i)=4.5*sin(phi)*sin(teta);
        y(i)=4.5*cos(phi);
        i=i+1;
    end
end
for angw=0:30:330  % bucle amb diferents rumbs per trovar la GCA
    angw
    cont=cont+1
    % Dades i paràmetres inicials
    dia='15'; % Dia del mes amb 2 dígits
    day=15;
    hora='10'; % Hora del dia amb 2 dígits (0 a 24)
    hour=10;
    minut='30'; % Minuts horaris, de 00 a 59
    minute=30;
    mes='d'; % Mes de l’any segons conveni . P.e. d es desembre
    levels=[101300 95000 85000 70000 50000]; %Nivells de pressió del model MM5 (Pa)
    vector_uns=ones(4761,1); % Vector auxiliar
    m=1309; % massa dirigible en Kg 1000
    cd=0.04; % coef de resistència volumètric (relatiu a V^2/3)
    swet=116; % V^2/3
    angw=deg2rad(angw); %canvi a radians per poder entrar en els sinus i cosinus
    alvol=2000; % Alçada de vol en m, es considera constant.
    delta=10; %increment de temps en segons
    rterra=6371; % Radi de la Terra en km
    % Prediccions són cada tres hores. Hem d’adaptar les hores.
if hour>=0 & hour<03
    minute=minute+((hour-0)*60);
    hour=0;
end
if hour>=03 & hour<06
    minute=minute+((hour-3)*60);
    hour=3;
end
if hour>=06 & hour<09
    minute=minute+((hour-6)*60);
    hour=6;
end
if hour>=09 & hour<12
    minute=minute+((hour-9)*60);
    hour=9;
end
if hour>=12 & hour<15
    minute=minute+((hour-12)*60);
    hour=12;
end
if hour>=15 & hour<18
    minute=minute+((hour-15)*60);
    hour=15;
end
if hour>=18 & hour<21
    minute=minute+((hour-18)*60);
    hour=18;
end
if hour>=21 & hour<24
    minute=minute+((hour-21)*60);
hour=21;

end

%Càlculs inicials

%coordenades del punt inicial on calcularem la trajectòria

latini=41.38; %Latitud inicial Barcelona 41.38
longini=2.18; %Longitud inicial Barcelona 2.18

%Llei d'alçada de vol
hvol=alvol*ones(7201,1); %Llei alçada de vol en metres.

%determinació del punt de la malla mes pròxim al punt inicial
distmin=100; %distància al punt de malla més pròxim
posini=1; %inici contador de la posició. Posini es el punt de malla més pròxim al punt

for k=1:4761

dist= distance(lat(k),long(k),latini,longini); %calcula distancia mínima (ortodròmica) en graus entre els 2 punts

distkm=deg2km(dist);

if distkm<distmin

distmin=distkm; %determina distància mínima

posini=k; %determina punt de la malla més pròxim
end

end

% Interpolació temporal inicial (suposem dades subministrades cada 3 hores)

hores='00010203040506070809101112131415161718192021'; %variable alfanumèrica amb les hores de les prediccions

nivell='12345'; %variable alfanumèrica amb els nivells de pressió

hour1=hores((hour*2)+1:(hour*2)+2); %limit inferior horari

hour2=hores((hour*2)+7:(hour*2)+8); %limit superior horari

pes1=1-(minute/180); %pes sobre hour1

pes2=(minute/180); %pes sobre hour2

% Determinació dels nivells de pressió. Els nivells de pressió són lower i upper

lower=3; %nivell inferior de pressió. Perque alçada constant de vol a 2000m

upper=4; %nivell superior de pressió. Perque alçada constant de vol a 2000m
% Càlcul trajectòria

hour1=hores((hour*2)+1:(hour*2)+2); % límit inferior horari

hour2=hores((hour*2)+7:(hour*2)+8); % límit superior horari

lowniv=nivell(lower:lower); % nivell inferior de pressió

upniv=nivell(upper:upper); % nivell superior de pressió

% Lectura dades de temperatura, alçades geopotencials i camp de vent en els nivells de pressió inferior i superior a hores 1 i 2

% nom fitxers de dades a hora 1

nomfitxer_t_low_1=[directoriTemp dia mes hour1 '3' lowniv '.grd'];
nomfitxer_t_up_1=[directoriTemp dia mes hour1 '3' upniv '.grd'];
nomfitxer_h_low_1=[directoriGeop dia mes hour1 '4' lowniv '.grd'];
nomfitxer_h_up_1=[directoriGeop dia mes hour1 '4' upniv '.grd'];
nomfitxer_u_low_1=[directoriVent dia mes hour1 '1' lowniv '.grd'];
nomfitxer_u_up_1=[directoriVent dia mes hour1 '1' upniv '.grd'];
nomfitxer_v_low_1=[directoriVent dia mes hour1 '2' lowniv '.grd'];
nomfitxer_v_up_1=[directoriVent dia mes hour1 '2' upniv '.grd'];

% nom fitxers de dades a hora 2

nomfitxer_t_low_2=[directoriTemp dia mes hour2 '3' lowniv '.grd'];
nomfitxer_t_up_2=[directoriTemp dia mes hour2 '3' upniv '.grd'];
nomfitxer_h_low_2=[directoriGeop dia mes hour2 '4' lowniv '.grd'];
nomfitxer_h_up_2=[directoriGeop dia mes hour2 '4' upniv '.grd'];
nomfitxer_u_low_2=[directoriVent dia mes hour2 '1' lowniv '.grd'];
nomfitxer_u_up_2=[directoriVent dia mes hour2 '1' upniv '.grd'];
nomfitxer_v_low_2=[directoriVent dia mes hour2 '2' lowniv '.grd'];
nomfitxer_v_up_2=[directoriVent dia mes hour2 '2' upniv '.grd'];

[t_low_1] = textread(nomfitxer_t_low_1,'%f','headerlines',5); %Temperatura en K al nivell inferior de pressió, hora 1

[t_up_1] = textread(nomfitxer_t_up_1,'%f','headerlines',5); %Temperatura en K al nivell superior de pressió, hora 1

[h_low_1] = textread(nomfitxer_h_low_1,'%f','headerlines',5); %Alçada geopotencial en m del nivell inferior, hora 1
STUDY OF A ZERO-EMISSION AIRSHIP TRANSPORT SYSTEM BASED ON THE GEOSTROPHIC FLIGHT CONCEPT

\[
[h_{up,1}] = \text{textread}(\text{nomfitxer}_h_{up,1},'%f','headerlines',5); \quad \% \text{Alçada geopotencial en m del nivell superior, hora 1}
\]

\[
[u_{low,1}] = \text{textread}(\text{nomfitxer}_u_{low,1},'%f','headerlines',5); \quad \% \text{Velocitat oest-est en m/s al nivell inferior, hora 1}
\]

\[
[u_{up,1}] = \text{textread}(\text{nomfitxer}_u_{up,1},'%f','headerlines',5); \quad \% \text{Velocitat oest-est en m/s al nivell superior, hora 1}
\]

\[
[v_{low,1}] = \text{textread}(\text{nomfitxer}_v_{low,1},'%f','headerlines',5); \quad \% \text{Velocitat nord-sud en m/s al nivell inferior, hora 1}
\]

\[
[v_{up,1}] = \text{textread}(\text{nomfitxer}_v_{up,1},'%f','headerlines',5); \quad \% \text{Velocitat nord-sud en m/s al nivell superior, hora 1}
\]

\[
[t_{low,2}] = \text{textread}(\text{nomfitxer}_t_{low,2},'%f','headerlines',5); \quad \% \text{Temperatura en K al nivell inferior de pressió, hora 2}
\]

\[
[t_{up,2}] = \text{textread}(\text{nomfitxer}_t_{up,2},'%f','headerlines',5); \quad \% \text{Temperatura en K al nivell superior de pressió, hora 2}
\]

\[
[h_{low,2}] = \text{textread}(\text{nomfitxer}_h_{low,2},'%f','headerlines',5); \quad \% \text{Alçada geopotencial en m del nivell inferior, hora 2}
\]

\[
[h_{up,2}] = \text{textread}(\text{nomfitxer}_h_{up,2},'%f','headerlines',5); \quad \% \text{Alçada geopotencial en m del nivell superior, hora 2}
\]

\[
[u_{low,2}] = \text{textread}(\text{nomfitxer}_u_{low,2},'%f','headerlines',5); \quad \% \text{Velocitat oest-est en m/s al nivell inferior, hora 2}
\]

\[
[u_{up,2}] = \text{textread}(\text{nomfitxer}_u_{up,2},'%f','headerlines',5); \quad \% \text{Velocitat oest-est en m/s al nivell superior, hora 2}
\]

\[
[v_{low,2}] = \text{textread}(\text{nomfitxer}_v_{low,2},'%f','headerlines',5); \quad \% \text{Velocitat nord-sud en m/s al nivell inferior, hora 2}
\]

\[
[v_{up,2}] = \text{textread}(\text{nomfitxer}_v_{up,2},'%f','headerlines',5); \quad \% \text{Velocitat nord-sud en m/s al nivell superior, hora 2}
\]

% Càlcul de la temperatura, pressió i camp de vent a l'alçada de vol en els punts de malla

% Interpolació temporal

\[
t_{low}=(pes1*t_{low,1})+(pes2*t_{low,2}); \quad \% \text{Temperatura en K al nivell inferior de pressió a l'instant de càlcul}
\]

\[
t_{up}=(pes1*t_{up,1})+(pes2*t_{up,2}); \quad \% \text{Temperatura en K al nivell superior de pressió a l'instant de càlcul}
\]

\[
h_{low}=(pes1*h_{low,1})+(pes2*h_{low,2}); \quad \% \text{Alçada geopotencial en m al nivell inferior de pressió a l'instant de càlcul}
\]

\[
h_{up}=(pes1*h_{up,1})+(pes2*h_{up,2}); \quad \% \text{Alçada geopotencial en m al nivell superior de pressió a l'instant de càlcul}
\]

\[
u_{low}=(pes1*u_{low,1})+(pes2*u_{low,2}); \quad \% \text{Velocitat del vent zonal al nivell inferior de pressió en m/s a l'instant de càlcul}
\]
STUDY OF A ZERO-EMISSION AIRSHIP TRANSPORT SYSTEM BASED ON THE GEOSTROPHIC FLIGHT CONCEPT

\[ u_{up} = (p_{e1} \cdot u_{up_1}) + (p_{e2} \cdot u_{up_2}); \] % Velocitat del vent zonal al nivell superior de pressió en m/s a l'instant de càlcul

\[ v_{low} = (p_{e1} \cdot v_{low_1}) + (p_{e2} \cdot v_{low_2}); \] % Velocitat del vent meridional al nivell inferior de pressió en m/s a l'instant de càlcul

\[ v_{up} = (p_{e1} \cdot v_{up_1}) + (p_{e2} \cdot v_{up_2}); \] % Velocitat del vent meridional al nivell superior de pressió en m/s a l'instant de càlcul

% Càlcul de la temperatura de l'aire a l'alçada de vol. Es suposa variació lineal de la temperatura entre els dos nivells

\[ \text{dis} = h_{up} - h_{low}; \] % Distància en metres entre nivells inferior i superior

\[ a = (t_{up} - t_{low}) / \text{dis}; \] % Gradient de temperatura en K/m. El punt es perquè la divisió es faci element a element

\[ t_{vol} = t_{low} + a \cdot (h_{vol} - h_{low}); \] % Temperatura en K a l'alçada vol suposant variació lineal amb l'alçada.

% Càlcul de la pressió de l'aire a l'alçada de vol. Es suposa relació hidrostàtica

\[ g = 9.8; \] % Acceleració de la gravetat (m/s^2)

\[ R = 287; \] % Constant dels gasos per l'aire (J/KKg)

\[ \text{pres}_{level\_lower} = \text{levels}(\text{lower}) \cdot \text{vector\_uns}; \] % Pressió al nivell inferior en tots els punts de malla (Pa)

\[ \text{pres}_{level\_upper} = \text{levels}(\text{upper}) \cdot \text{vector\_uns}; \] % Pressió al nivell superior en tots els punts de malla (Pa)

\[ \text{for } i = 1:4761 \] % per tots els punts de la malla, 69x69 = 4761
\[ \text{pvol1}(i) = \text{levels}(\text{lower}) \cdot ((t_{vol}(i) / t_{low}(i))^{(-g / (R \cdot a(i)))}); \] % Pressió en Pa a l'alçada de vol
\[ \text{end} \]

\[ \text{pvol1}' = \text{pvol1}^t; \] % el trasposem per poder operar amb vectors u3 i v3

% Càlcul velocitat del vent a l'alçada de vol. Suposem variació lineal amb la pressió

\[ \text{kwindu} = (u_{up} - u_{low}) / (\text{pres}_{level\_upper} - \text{pres}_{level\_lower}); \]

\[ u = (u_{low} + \text{kwindu} \cdot (\text{pvol1} - \text{pres}_{level\_lower}); \]

\[ \text{kwindv} = (v_{up} - v_{low}) / (\text{pres}_{level\_upper} - \text{pres}_{level\_lower}); \]

\[ v = (v_{low} + \text{kwindv} \cdot (\text{pvol1} - \text{pres}_{level\_lower}); \]

\[ \text{pvol1}' = \text{pvol1}^t; \] % tornem a trasposar per consistència amb el bucle

% % Càlcul de la trajectòria. latp, longp són els vectors amb les coordenades de la trajectòria

%
% Càlculs en el punt inicial
% Coordenades i velocitat absoluta del punt inicial
latp(1)=latini; % Latitud punt inicial
longp(1)=longini; % Longitud punt inicial
vabslat(1)=0; % Velocitat meridional punt inicial
vabslong(1)=0; % Velocitat zonal punt inicial
[upunt,vpunt,tpunt,ppunt,pos]=wind6(latp(1),longp(1),lat,long,u,v,tvol,pvol1,posini);
vwindlat(1)=vpunt;
vwindlong(1)=upunt;
temptrack(1)=tpunt; %temperatura en punt inicial
prestrack(1)=ppunt; %pressió en punt inicial
rotrack(1)=prestrack(1)/(R*temptrack(1)); % densitat en punt inicial
posini=pos;
vrrellat(1)=vabslat(1)-vwindlat(1);
vdraglat(1)=-vrrellat(1);
vrrellong(1)=vabslong(1)-vwindlong(1);
vdraglong(1)=-vrrellong(1);
[Irrad,solaraltd,solarazimd]=solar3(day,mes,hour,minute,latp(1),alvol); %Calcul irradiació directa i altura i azimut solars
[Sproj]=projectarea2(solaraltd,angw,solarazimd,x,y,z); %Càcul superficie cel.lules solars projectada
power_av=Irrad*Sproj*rend_cell*rend_elec*rend_mec; % Potència disponible a l'eix hèlix
thrust_zero=(power_av^2*(2*rotrack(1)*Area_disc))^(1/3); % Empenta a punt fixe
vrel=(vrrellat(1)^2+vrrellong(1)^2)^(1/2); % Mòdul velocitat relativa
if vrel<5
  vrel=5; %Per evitar divisió per 0 i valors grans
end
thrust=(power_av*rend_prop)/vrel; % Mòdul de l'empenta
if thrust>thrust_zero
  thrust=thrust_zero; % Limit del mòdul d'empenta és empenta a punt fixe
end
thrustlat=cos(angw)*thrust; % Empenta meridional
thrustlong=sin(angw)*thrust; % Empenta zonal
vdrag=vdraglong(1)^2+vdraglat(1)^2+(1/2);
dragvalor=0.5*rotrack(1)*(vdrag^2)*cd*swet;
alfadrag=atan2(vdraglong(1),vdraglat(1));
draglat(1)=dragvalor*cos(alfadrag);
alat(1)=(thrustlat+draglat(1))/m;
draglong(1)=dragvalor*sin(alfadrag);
along(1)=(thrustlong+draglong(1))/m;

% Resta de punts
for i=2:1081 %3 hores
    minute=minute+(delta/60);
    hour;
    if minute<180 % perque els intervals de prediccions són 3 hores
        pes1=1-(minute/180); %pes sobre hour1
        pes2=(minute/180); %pes sobre hour2
    end
    if minute>=180
        hour=hour+3;
        minute=0;
        hour1=hores((hour*2)+1:(hour*2)+2); %límit inferior horari
        hour2=hores((hour*2)+7:(hour*2)+8); %límit superior horari
        pes1=1-(minute/180); %pes sobre hour1
        pes2=(minute/180); %pes sobre hour2
        lowniv=nivell(lower:lower); % nivell inferior de pressió
        upniv=nivell(upper:upper); % nivell superior de pressió
    % Lectura dades de temperatura, alçades geopotencials i camp de vent en els nivells de
    % pressió inferior i superior a hores 1 i 2
    % nom fitxers de dades a hora 1
    nomfitxer_t_low_1=[directoriTemp dia mes hour1 '3' lowniv '.grd'];
    nomfitxer_t_up_1=[directoriTemp dia mes hour1 '3' upniv '.grd'];
nomfitxer_h_low_1=[directoriGeop dia mes hour1 '4' lowniv '.grd'];
nomfitxer_h_up_1=[directoriGeop dia mes hour1 '4' upniv '.grd'];
nomfitxer_u_low_1=[directoriVent dia mes hour1 '1' lowniv '.grd'];
nomfitxer_u_up_1=[directoriVent dia mes hour1 '1' upniv '.grd'];
nomfitxer_v_low_1=[directoriVent dia mes hour1 '2' lowniv '.grd'];
nomfitxer_v_up_1=[directoriVent dia mes hour1 '2' upniv '.grd'];
% nomfitxers de dades a hora 2
nomfitxer_t_low_2=[directoriTemp dia mes hour2 '3' lowniv '.grd'];
nomfitxer_t_up_2=[directoriTemp dia mes hour2 '3' upniv '.grd'];
nomfitxer_h_low_2=[directoriGeop dia mes hour2 '4' lowniv '.grd'];
nomfitxer_h_up_2=[directoriGeop dia mes hour2 '4' upniv '.grd'];
nomfitxer_u_low_2=[directoriVent dia mes hour2 '1' lowniv '.grd'];
nomfitxer_u_up_2=[directoriVent dia mes hour2 '1' upniv '.grd'];
nomfitxer_v_low_2=[directoriVent dia mes hour2 '2' lowniv '.grd'];
nomfitxer_v_up_2=[directoriVent dia mes hour2 '2' upniv '.grd'];
[t_low_1] = textread(nomfitxer_t_low_1,'%f','headerlines',5); %Temperatura en K al nivell inferior de pressió, hora 1
[t_up_1] = textread(nomfitxer_t_up_1,'%f','headerlines',5); %Temperatura en K al nivell superior de pressió, hora 1
[h_low_1] = textread(nomfitxer_h_low_1,'%f','headerlines',5); %Alçada geopotencial en m del nivell inferior, hora 1
[h_up_1] = textread(nomfitxer_h_up_1,'%f','headerlines',5); %Alçada geopotencial en m del nivell superior, hora 1
[u_low_1] = textread(nomfitxer_u_low_1,'%f','headerlines',5); %Velocitat oest-est en m/s al nivell inferior, hora 1
[u_up_1] = textread(nomfitxer_u_up_1,'%f','headerlines',5); %Velocitat oest-est en m/s al nivell superior, hora 1
[v_low_1] = textread(nomfitxer_v_low_1,'%f','headerlines',5); %Velocitat nord-sud en m/s al nivell inferior, hora 1
[v_up_1] = textread(nomfitxer_v_up_1,'%f','headerlines',5); %Velocitat nord-sud en m/s al nivell superior, hora 1
[t_low_2] = textread(nomfitxer_t_low_2,'%f','headerlines',5); %Temperatura en K al nivell inferior de pressió, hora 2
[t_up_2] = textread(nomfitxer_t_up_2,'%f','headerlines',5); %Temperatura en K al nivell superior de pressió, hora 2
\[ [h_{\text{low}}_2] = \text{textread}(\text{nomfitxer}_\text{h}_\text{low}_2,'%f','\text{headerlines}',5); \] % Alçada geopotencial en m del nivell inferior, hora 2

\[ [h_{\text{up}}_2] = \text{textread}(\text{nomfitxer}_\text{h}_\text{up}_2,'%f','\text{headerlines}',5); \] % Alçada geopotencial en m del nivell superior, hora 2

\[ [u_{\text{low}}_2] = \text{textread}(\text{nomfitxer}_\text{u}_\text{low}_2,'%f','\text{headerlines}',5); \] % Velocitat oest–est en m/s al nivell inferior, hora 2

\[ [u_{\text{up}}_2] = \text{textread}(\text{nomfitxer}_\text{u}_\text{up}_2,'%f','\text{headerlines}',5); \] % Velocitat oest–est en m/s al nivell superior, hora 2

\[ [v_{\text{low}}_2] = \text{textread}(\text{nomfitxer}_\text{v}_\text{low}_2,'%f','\text{headerlines}',5); \] % Velocitat nord-sud en m/s al nivell inferior, hora 2

\[ [v_{\text{up}}_2] = \text{textread}(\text{nomfitxer}_\text{v}_\text{up}_2,'%f','\text{headerlines}',5); \] % Velocitat nord-sud en m/s al nivell superior, hora 2

end

% Interpolació temporal
\[ t_{\text{low}}=(\text{pes1} \times t_{\text{low}}_1)+(\text{pes2} \times t_{\text{low}}_2); \] % Temperatura en K al nivell inferior de pressió a l'instant de càlcul

\[ t_{\text{up}}=(\text{pes1} \times t_{\text{up}}_1)+(\text{pes2} \times t_{\text{up}}_2); \] % Temperatura en K al nivell superior de pressió a l'instant de càlcul

\[ h_{\text{low}}=(\text{pes1} \times h_{\text{low}}_1)+(\text{pes2} \times h_{\text{low}}_2); \] % Alçada geopotencial en m al nivell inferior de pressió a l'instant de càlcul

\[ h_{\text{up}}=(\text{pes1} \times h_{\text{up}}_1)+(\text{pes2} \times h_{\text{up}}_2); \] % Alçada geopotencial en m al nivell superior de pressió a l'instant de càlcul

\[ u_{\text{low}}=(\text{pes1} \times u_{\text{low}}_1)+(\text{pes2} \times u_{\text{low}}_2); \] % Velocitat del vent zonal al nivell inferior de pressió en m/s a l'instant de càlcul

\[ u_{\text{up}}=(\text{pes1} \times u_{\text{up}}_1)+(\text{pes2} \times u_{\text{up}}_2); \] % Velocitat del vent zonal al nivell superior de pressió en m/s a l'instant de càlcul

\[ v_{\text{low}}=(\text{pes1} \times v_{\text{low}}_1)+(\text{pes2} \times v_{\text{low}}_2); \] % Velocitat del vent meridional al nivell inferior de pressió en m/s a l'instant de càlcul

\[ v_{\text{up}}=(\text{pes1} \times v_{\text{up}}_1)+(\text{pes2} \times v_{\text{up}}_2); \] % Velocitat del vent meridional al nivell superior de pressió en m/s a l'instant de càlcul

% Càlcul de la temperatura de l’aire a l’alçada de vol. Es suposa variació lineal de la temperatura entre els dos nivells
\[ \text{dis}=h_{\text{up}}-h_{\text{low}}; \] % Distància en metres entre nivells inferior i superior

\[ \text{a}=\frac{t_{\text{up}}-t_{\text{low}}}{\text{dis}}; \] % Gradient de temperatura en K/m. El punt és perquè la divisió es faci element a element

\[ t_{\text{vol}}=t_{\text{low}}+\text{a} \times (h_{\text{vol}}(i)-h_{\text{low}}); \] % Temperatura en K a alçada vol suposant variació lineal amb l’alçada.

% Càlcul de la pressió de l’aire a l’alçada de vol. Es suposa relació hidrostàtica
preslevel_lower=levels(lower)*vector_uns; % Pressió al nivell inferior en tots els punts de malla (Pa)

preslevel_upper=levels(upper)*vector_uns; % Pressió al nivell superior en tots els punts de malla (Pa)

for k=1:4761 % per tots els punts de la malla, 69x69=4761
    pvol(k)=levels(lower)*((tvol(k)/t_low(k))^(-g/(R*a(k)))); %Pressió en Pa a l'alçada de vol
end

pvol=pvol'; % el trasposem per poder operar amb vectors u3 i v3

% Càlcul velocitat del vent a l'alçada de vol. Suposem variació lineal amb la pressió
kwindu=(u_up-u_low)/(preslevel_upper-preslevel_lower);
u=(u_low+kwindu.*(pvol-preslevel_lower));

kwindv=(v_up-v_low)/(preslevel_upper-preslevel_lower);
v=(v_low+kwindv.*(pvol-preslevel_lower));

pvol=pvol'; % el trasposem de nou per consistència amb el bucle

% Posició i velocitat del punt intermig auxiliar
lataux=latp(i-1)+km2deg((delta/2)*vabslat(i-1))/1000);
longaux=longp(i-1)+km2deg((delta/2)*vabslong(i-1))/1000,rterra*cos(deg2rad(lataux)));

longvaux=vabslong(i-1)+((delta/2)*along(i-1));
latvaux=vabslat(i-1)+((delta/2)*alat(i-1));

% Posició nou punt
latp(i)=latp(i-1)+km2deg((delta*latvaux)/1000);
longp(i)=longp(i-1)+km2deg((delta*longvaux)/1000,rterra*cos(deg2rad(latp(i))));

% Acceleració punt intermig auxiliar
[upunt,vpunt,tpunt,ppunt,pos]=wind6(lataux,longaux,lat,long,u,v,tvol,pvol,posini);

longvrelaux=longvrelaux-upunt;
longvdragaux=-longvrelaux;
latvrelaux=latvrelaux-vpunt;
latvdragaux=-latvrelaux;
tempaux=tpunt;

presaux=ppunt;
oraux=presaux/(R*tempaux);
posini = pos;

vdrag = (longvdragaux^2 + latvdragaux^2)^(1/2);

[Irrad, solaraltd, solarazimd] = solar3(day, mes, hour, minute, lataux, alvol); % Calcul irradiació directa i altura i azimut solars

[Sproj] = projectarea2(solaraltd, angw, solarazimd, x, y, z); % Càlcul superfície cel.lules solars projectada

power_av = Irrad * Sproj * rend_cell * rend_elec * rend_mec; % Potència disponible a l'eix hèlix

thrust_zero = (power_av^2 * (2 * roaux * Area_disc))^(1/3); % Empenta a punt fixe

vrel = (longvrelaux^2 + latvrelaux^2)^(1/2); % Mòdul velocitat relativa

if vrel < 5
    vrel = 5; % Per evitar divisió per 0 i valors grans
end

thrust = (power_av * rend_prop) / vrel; % Mòdul de l'empenta

if thrust > thrust_zero
    thrust = thrust_zero; % Limit del mòdul d'empenta és empenta a punt fixe
end

thrustlat = (cos(angw)) * thrust;

thrustlong = (sin(angw)) * thrust;

dragvalor = (0.5 * roaux * (vdrag^2) * cd * swet);

alfadrag = atan2(longvdragaux, latvdragaux);

draglongaux = dragvalor * sin(alfadrag);

longaux = (thrustlong + draglongaux) / m; % Acceleració zonal punt auxiliar

draglataux = dragvalor * cos(alfadrag);

lataux = (thrustlat + draglataux) / m; % Acceleració meridional punt auxiliar

% Velocitat nou punt

vabslong(i) = vabslong(i-1) + (delta * longaux); % Velocitat zonal nou punt

vabslat(i) = vabslat(i-1) + (delta * lataux); % Velocitat meridional nou punt

% Acceleració nou punt

[upunt, vpunt, tpunt, ppunt, pos] = wind6(latp(i), longp(i), lat, long, u, v, tvol, pvol, posini);

vrellong(i) = vabslong(i) - upunt;

vdraglong(i) = vrellong(i);
STUDY OF A ZERO-EMISSION AIRSHIP TRANSPORT SYSTEM BASED ON THE GEOSTROPHIC FLIGHT CONCEPT

vrellat(i)=vabslat(i)-vpunt;
vdraglat(i)=-vrellat(i);
temptrack(i)=tpunt;
prestrack(i)=ppunt;
rotrack(i)=prestrack(i)/(R*temptrack(i));
posini=pos;
vdrag=(vdraglong(i)^2+vdraglat(i)^2)^(1/2);
[Irrad,solaraltd,solarazimd]=solar3(day,mes,hour,minute,latp(i),alvol); %Calcul irració directa i altura i azimut solars
[Sproj]=projectarea2(solaraltd,angw,solarazimd,x,y,z); %Càcul superficie cel.lules solars projectada
power_av=Irrad*Sproj*rend_cell*rend_elec*rend_mec; % Potència disponible a l'eix hèlix
thrust_zero=(power_av^2*(2*rotrack(i)*Area_disc))^(1/3); % Empenta a punt fixe
vrel=(vrellong(i)^2+vrellat(i)^2)^(1/2); % Mòdul velocitat relativa
if vrel<5
vrel=5; %Per evitar divisió per 0 i valors grans
end
thrust=(power_av*rend_prop)/vrel; % Mòdul de l'empenta
if thrust>thrust_zero
thrust=thrust_zero; % Limit del mòdul d'empenta és empenta a punt fixe
end
thrustlat=(cos(angw))*thrust;
thrustlong=(sin(angw))*thrust;
dragvalor=(0.5*rotrack(i)*vdrag^2)*cd*swet);
alfadrag=atan2(vdraglong(i),vdraglat(i));
draglong(i)=dragvalor*sin(alfadrag);
along(i)=(thrustlong+draglong(i))/m; % Acceleració zonal nou punt
draglat(i)=dragvalor*cos(alfadrag);
alat(i)=(thrustlat+draglat(i))/m; % Acceleració meridional nou punt
i
latpf(cont)=latp(1081);
longpf(cont)=longp(1081);
% Representació gràfica
quiverm(lat,long,v,u,'r');
plotm(lat,long);
plotm(latp(1081),longp(1081),'*');
end
plotm(latp(1),longp(1),'*');
% Representació corva GCA
[kp,vp]=convhull(latp,longp);
plotm(latp(kp),longp(kp),'-');
Function\(\text{wind6.m}\)

Function\(\text{[upunt, vpunt, tpunt, ppunt, pos]} = \text{wind6(latpunt, longpunt, lat, long, u, v, t2000, p2000, posini]}\)

% Càlcul de la nova posició de pos, el punt de malla més proper al punt

daux(1)=distance(lat(posini-70),long(posini-70),latpunt,longpunt);
dauxkm(1)=deg2km(daux(1));
daux(2)=distance(lat(posini-69),long(posini-69),latpunt,longpunt);
dauxkm(2)=deg2km(daux(2));
daux(3)=distance(lat(posini-68),long(posini-68),latpunt,longpunt);
dauxkm(3)=deg2km(daux(3));
daux(4)=distance(lat(posini-1),long(posini-1),latpunt,longpunt);
dauxkm(4)=deg2km(daux(4));
daux(5)=distance(lat(posini),long(posini),latpunt,longpunt);
dauxkm(5)=deg2km(daux(5));
daux(6)=distance(lat(posini+1),long(posini+1),latpunt,longpunt);
dauxkm(6)=deg2km(daux(6));
daux(7)=distance(lat(posini+68),long(posini+68),latpunt,longpunt);
dauxkm(7)=deg2km(daux(7));
daux(8)=distance(lat(posini+69),long(posini+69),latpunt,longpunt);
dauxkm(8)=deg2km(daux(8));
daux(9)=distance(lat(posini+70),long(posini+70),latpunt,longpunt);
dauxkm(9)=deg2km(daux(9));

\([dmin, ind] = \text{min(dauxkm)}; \) %ind indica en quina posició del vector dauxkm es troba el mínim. dmin es aquest mínim.

if ind==1
    pos=posini-70;
end
if ind==2
    pos=posini-69;
end
if ind==3
    pos=posini-68;
end
if ind==4
    pos=posini-1;
end
if ind==5
    pos=posini;
end
if ind==6
    pos=posini+1;
end
if ind==7
    pos=posini+68;
end
if ind==8
    pos=posini+69;
end
if ind==9
    pos=posini+70
end

% determinació dels punts del quadrilàter

dist1=distance(lat(pos+69),long(pos+69),latpunt,longpunt);
dist1km=deg2km(dist1);
dist2=distance(lat(pos-69),long(pos-69),latpunt,longpunt);
dist2km=deg2km(dist2);
dist3=distance(lat(pos+1),long(pos+1),latpunt,longpunt);
dist3km=deg2km(dist3);
dist4=distance(lat(pos-1),long(pos-1),latpunt,longpunt);
dist4km=deg2km(dist4);

if (dist1km>=dist2km) & (dist4km>=dist3km)
    xcon(1)=long(pos-69);
    ycon(1)=lat(pos-69);
    xcon(2)=long(pos-68);
    ycon(2)=lat(pos-68);
    xcon(3)=long(pos+1);
    ycon(3)=lat(pos+1);
    xcon(4)=long(pos);
    ycon(4)=lat(pos);
end

if (dist1km>=dist2km) & (dist3km>dist4km)
    xcon(1)=long(pos-70);
    ycon(1)=lat(pos-70);
    xcon(2)=long(pos-69);
    ycon(2)=lat(pos-69);
    xcon(3)=long(pos);
    ycon(3)=lat(pos);
    xcon(4)=long(pos-1);
    ycon(4)=lat(pos-1);
end

if (dist2km>dist1km) & (dist4km>=dist3km)
    xcon(1)=long(pos);
    ycon(1)=lat(pos);
    xcon(2)=long(pos+1);
    ycon(2)=lat(pos+1);
    xcon(3)=long(pos+70);
    ycon(3)=lat(pos+70);
    xcon(4)=long(pos+69);
    ycon(4)=lat(pos+69);
end

if (dist2km>dist1km) & (dist3km>dist4km)
    xcon(1)=long(pos-1);
    ycon(1)=lat(pos-1);
    xcon(2)=long(pos);
    ycon(2)=lat(pos);
    xcon(3)=long(pos+69);
    ycon(3)=lat(pos+69);
    xcon(4)=long(pos+68);
    ycon(4)=lat(pos+68);
end

%punt a posicionar
xp=longpunt;
yp=latpunt;

%valors de chi i nu inicials
chi=0;
nu=0;

for i=1:5
    [xpos,ypos] = isop(chi,nu,xcon,ycon);
    [jac11, jac12, jac21, jac22]=jaco(chi,nu,xcon,ycon);
    A=[jac11 jac12; jac21 jac22];
    delta=A\[xp-xpos;yp-ypos];
    chi=chi+delta(1);
    nu=nu+delta(2);
end

N1=(1-chi)*(1-nu)/4;
N2=(chi+1)*(1-nu)/4;
N3=(chi+1)*(nu+1)/4;
N4=(1-chi)*(1+nu)/4;

if (dist1km>=dist2km) & (dist4km>=dist3km)
STUDY OF A ZERO-EMISSION AIRSHIP TRANSPORT SYSTEM BASED ON THE GEOSTROPHIC FLIGHT CONCEPT

upunt=(N1*u(pos-69))+(N2*u(pos-68))+(N3*u(pos+1))+(N4*u(pos));
v punt=(N1*v(pos-69))+(N2*v(pos-68))+(N3*v(pos+1))+(N4*v(pos));
tpunt=(N1*t2000(pos-69))+(N2*t2000(pos-68))+(N3*t2000(pos+1))+(N4*t2000(pos));
ppunt=(N1*p2000(pos-69))+(N2*p2000(pos-68))+(N3*p2000(pos+1))+(N4*p2000(pos));

end

if (dist1km>=dist2km) & (dist3km>dist4km)
  upunt=(N1*u(pos-70))+(N2*u(pos-69))+(N3*u(pos))+(N4*u(pos-1));
v punt=(N1*v(pos-70))+(N2*v(pos-69))+(N3*v(pos))+(N4*v(pos-1));
tpunt=(N1*t2000(pos-70))+(N2*t2000(pos-69))+(N3*t2000(pos))+(N4*t2000(pos-1));
ppunt=(N1*p2000(pos-70))+(N2*p2000(pos-69))+(N3*p2000(pos))+(N4*p2000(pos-1));
end

if (dist2km>dist1km) & (dist4km>=dist3km)
  upunt=(N1*u(pos))+(N2*u(pos+1))+(N3*u(pos+69))+(N4*u(pos+68));
v punt=(N1*v(pos))+(N2*v(pos+1))+(N3*v(pos+69))+(N4*v(pos+68));
tpunt=(N1*t2000(pos))+(N2*t2000(pos+1))+(N3*t2000(pos+69))+(N4*t2000(pos+68));
ppunt=(N1*p2000(pos))+(N2*p2000(pos+1))+(N3*p2000(pos+69))+(N4*p2000(pos+68));
end

if (dist2km>dist1km) & (dist3km>dist4km)
  upunt=(N1*u(pos-1))+(N2*u(pos))+(N3*u(pos+69))+(N4*u(pos+68));
v punt=(N1*v(pos-1))+(N2*v(pos))+(N3*v(pos+69))+(N4*v(pos+68));
tpunt=(N1*t2000(pos-1))+(N2*t2000(pos))+(N3*t2000(pos+69))+(N4*t2000(pos+68));
ppunt=(N1*p2000(pos-1))+(N2*p2000(pos))+(N3*p2000(pos+69))+(N4*p2000(pos+68));
end
% **Function isop.m**

```
function [xpos, ypos] = isop(chi, nu, xcon, ycon)

xpos = (xcon(1)*(1-chi)*(1-nu)/4) + (xcon(2)*(chi+1)*(1-nu)/4) + (xcon(3)*(chi+1)*(nu+1)/4) + (xcon(4)*(1-chi)*(nu+1)/4);

ypos = (ycon(1)*(1-chi)*(1-nu)/4) + (ycon(2)*(chi+1)*(1-nu)/4) + (ycon(3)*(chi+1)*(nu+1)/4) + (ycon(4)*(1-chi)*(nu+1)/4);
```

%

% **Function jaco.m**

```
function [jac11, jac12, jac21, jac22] = jaco(chi, nu, xcon, ycon)

jac11 = (-xcon(1)*(1-nu)/4) + (xcon(2)*(1-nu)/4) + (xcon(3)*(nu+1)/4) + (-xcon(4)*(nu+1)/4);

jac12 = (-xcon(1)*(1-chi)/4) + (-xcon(2)*(chi+1)/4) + (xcon(3)*(chi+1)/4) + (xcon(4)*(1-chi)/4);

jac21 = (-ycon(1)*(1-nu)/4) + (ycon(2)*(1-nu)/4) + (ycon(3)*(nu+1)/4) + (-ycon(4)*(nu+1)/4);

jac22 = (-ycon(1)*(1-chi)/4) + (-ycon(2)*(chi+1)/4) + (ycon(3)*(chi+1)/4) + (ycon(4)*(1-chi)/4);
```
% Function solar3.m

function [Irrad,solaraltd,solarazimd] = solar3(day,mes,hour,minute,latinut,hvol)
% càlculs de la funció
hour=hour+(minute/60);
if mes=='e'
    day=day;
end
if mes=='f'
    day=day+31;
end
if mes=='m'
    day=day+60;
end
if mes=='a'
    day=day+91;
end
if mes=='y'
    day=day+121;
end
if mes=='j'
    day=day+152;
end
if mes=='l'
    day=day+182;
end
if mes=='g'
    day=day+213;
end
if mes=='s'
    day=day+244;
end
if mes=='o'
    day=day+274;
end
if mes=='n'
    day=day+305;
end
if mes=='d'
    day=day+335;
end

day=day-81; % portem origen dies a 21 de març
if day<0
    day=366+day;
end

Irr_const=1366; % constant solar W/m2
a=0.14;
c=0.357;
s=0.678;
N=23.5*sin(2*3.14159*day/366); % declinació de la Terra en graus
L=latitud; % latitud en graus
H=15*(12-hour); % angle horari en graus
N=deg2rad(N);
L=deg2rad(L);
H=deg2rad(H);
sinaltitude=cos(N)*cos(H)*cos(L)+sin(N)*sin(L);
solaralt=asin(sinaltitude); % altitud solar en radian
solarazim=asin((cos(N)*sin(H))/cos(solaralt)); % azimut solar en radian
\[ \text{Irrad} = (\text{Irr}_\text{const} \cdot (1 - a\cdot(h\text{vol}/1000))) \cdot \exp(-c \cdot (1/sin\text{altitude})^s) + (a\cdot(h\text{vol}/1000)\cdot\text{Irr}_\text{const}); \]

% irradiacio solar directe en W/m²

solaraltd = rad2deg(solaralt); % altitud solar en graus

solarazimd = rad2deg(solarazim); % azimut solar en graus
% Function projectarea2.m

function [Sproj] = projectarea2(solaraltd,angw,solarazimd,x,y,z)

% gir al voltant de l'eix y
alfa=solaraltd; %Alçada solar en graus
giry=(90-alfa); %Angle de gir al voltant eix y en graus
giry=deg2rad(giry); % passem a radiants
RY=[cos(giry) 0 sin(giry); 0 1 0; -sin(giry) 0 cos(giry)];

% gir al voltant de l'eix z
azdir=(angw-180); %passem azimut dirigible respecte el nord a azimut respecte el sud
girz=azdir-solarazimd; %Azimut del dirigible respecte l'azimut solar. Angle de gir al voltant eix z en graus

girz=deg2rad(girz); % passem a radiants
RZ=[cos(girz) -sin(girz) 0; sin(girz) cos(girz) 0; 0 0 1];

RGIR=RY*RZ;

for i=1:10080
    [temp]=RGIR*[x(i);y(i);z(i)];
    x2(i)=temp(1);
    y2(i)=temp(2);
    z2(i)=temp(3);
end

[ka,v]=convhull(x2,y2); %calcula area de la projeccio de la superficie en el pla
Sproj=v;