Master in Photonics

MASTER THESIS WORK

INFLUENCE OF THE BEAM DIVERGENCE ON THE SCINTILLATION IN A FREE-SPACE LASER COMMUNICATION LINK: SIMULATIONS AND EXPERIMENTS

Ricardo Barrios Porras

Supervised by Dr. Federico Dios, (UPC)

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Influence of the beam divergence on the scintillation in a free-space laser communication link: simulations and experiments.

Ricardo Barrios
E-mail: ricardo.barrios@tsc.upc.edu

Abstract. In terrestrial free-space laser communication, aside from pointing issues, the major problem that have to be dealt with is the turbulent atmosphere that produces irradiance fluctuations in the received signal, greatly reducing the link performance. Prediction of the scintillation index and the aperture averaging factor for Gaussian beam with currently available theory is compared with data collected experimentally and simulations based in the beam propagation method, where the atmospheric turbulence is represented by linearly spaced random phase screens. Experiments were carried out using a collecting lens with two simultaneous detectors, one of them with a small aperture to emulate an effective point detector, while the other one was mounted with interchangeable diaphragms, hence measurements for different aperture diameters could be made. The testbed for the experiments consists of a nearly horizontal path of 1.2 km with the transmitter and receiver on either side of the optical link. The analysis of the simulated and experimental data is used to characterize the scintillation index and the aperture averaging factor when using convergent and divergent Gaussian beam sources.

Keywords: Free-space laser communications, aperture averaging, laser divergence, scintillation index, beam propagation method.

1. Introduction

In recent years extensive studies have been carried out on free-space laser communication, with great interest on investigating the effects of the turbulent atmosphere on the communication link, which mostly produces irradiance fluctuations in the received signal, phenomena known as scintillation, greatly reducing the link performance. Albeit considerable research has been done characterizing the atmospheric effects rather less attention have been paid to investigate the relevance of the laser parameters, such as the beam divergence, and their role on the scintillation index (SI) and aperture averaging factor, A, at the receiver plane. Nevertheless, most of this works
have been confined to a large extent to the utilization of divergent beams, and virtually none have considered using convergent laser beams. [1–5]

The purpose of this paper is to characterize the relationship of the SI and aperture averaging when using different values of laser divergence with numerical simulations, based on the beam propagation method,[6] and data collected experimentally. While the next section is a brief review of the background theory on optical scintillation, it also make reference of the recently developed SI theory for Gaussian beams and finite receiving apertures, which is essential in defining the aperture averaging factor. Next, all the simulation data for four different values of beam divergence, two of them being positive (divergent beam) while the others are negative (convergent beam), and for various receiving apertures diameters are shown. Here, some of the results, as explained below, would need further analyses that are out of the scope of the present work. Lastly, experimental measurements were conducted in order to validate the conclusions reached from the numerical simulation, which suggest that convergent beams have a better performance regarding to the SI and the aperture averaging factor.

2. Background

2.1. Scintillation

A laser beam propagating through the atmosphere will be altered by refractive turbulence inhomogeneities. At the receiver plane, a random pattern is produced both in time and space.[7] These intensity fluctuations, or scintillation, are produced by changes in the refractive index of the atmosphere, which in turn is mainly due to small temperature variations. The parameter that express these irradiance fluctuations is called scintillation index (SI), and is defined by

\[
\sigma_I^2 = \frac{\langle I^2 \rangle - \langle I \rangle^2}{\langle I \rangle^2} = \frac{\langle I^2 \rangle}{\langle I \rangle^2} - 1, \tag{1}
\]

where \( I \) denotes irradiance of the optical wave and the brackets \( \langle \cdot \rangle \) denote an ensemble average.

The refractive index structure constant \( C_n^2 \) is the quantitative way to measure the atmospheric turbulence strength and it is the most dominant parameter on the SI, followed by the inner scale \( l_0 \) and the outer scale \( L_0 \) of turbulence.[1] Many authors have proposed different models to estimate this parameter, ranging from methods that rely on the atmospheric data in situ [8–10] up to others that extract the \( C_n^2 \) value from experimental scintillation data.[11, 12] The latter approach was used in this work fitting the experimental SI data with numerical simulations of beam propagation to estimate the refractive-index structure constant.

2.2. Aperture averaging

To counterbalance the scintillation effects on the optical link performance, it is desirable to have a large area detector in order to integrate as much light as possible. As the
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Intensity fluctuations are produced by inhomogeneities of different spatial scale sizes within the atmospheric inertial range, comprised from $l_0$ to $L_0$, all the effects of the scale sizes smaller than the receiver area will be averaged. From the ray optics point of view more rays, which all travel through distinct optical paths, can be collected by means of a lens and integrated on the photodetector and the measured scintillation index will be less compared to that of a point receiver. This phenomenon, called aperture averaging, have been extensively addressed and expressions have been developed for plane and circular waves, [7, 11, 13] and more recently for Gaussian beams.[14, 15]

The mathematical expression for the aperture averaging factor $A$ is defined by

$$A(D) = \frac{\sigma_I^2(D)}{\sigma_I^2(0)},$$

(2)

where $\sigma_I^2(D)$ and $\sigma_I^2(0)$ are the SI of a receiving aperture with diameter $D$ and a point receiver, respectively. In practical terms an aperture can be considered as a effective point receiver if it is much smaller than $\sqrt{\lambda L}$ and the inner scale $l_0$.[16]

Theory developed to calculate the aperture averaging for a Gaussian beam is based on the modified Rytov theory for the large-scale $\sigma_{inx}^2(D)$ and small-scale $\sigma_{iny}^2(D)$ log-irradiance flux variances.[14] Thereby, the SI can be expressed in the form

$$\sigma_I^2(D) = \exp \left[ \sigma_{inx}^2(D) + \sigma_{iny}^2(D) \right] - 1.$$  

(3)

This expression can applied in the moderate-to-strong turbulence regime, thanks to the use of the asymptotic theory and spatial filter functions for its derivation. Furthermore, Eq. 3 in combination with Eq. 2 can be used to estimate the aperture averaging factor by using the definition of an effective point receiver mentioned above.

3. Simulations

Since its introduction by Fleck et al. the beam split-step method has been widely used to simulate the propagation of electromagnetic waves, where the effects produced by the turbulent atmosphere are simulated by a series of linearly spaced random phase screens. The most widespread technique use to generate such screens is based on the spectral method, in which phase screen are generated in the spectral domain by means of filtering Gaussian white noise with the second-order statistics of the selected turbulence power spectrum.[17–19] The fractal method is an alternative approach to reproduce the phase screens directly in the spatial domain by successive interpolations from a set of random numbers that obey the desired structure function.[20] In this work the wave optics code used to perform the numerical simulations is based on a improvement of the fractal method, using the Kolmogorov spectrum of turbulence and randomized interpolation techniques.[21]

3.1. Simulation scenario

In order to carry out the numerical simulations the generation of fractal phase screens was performed in two steps. First, an exact low-resolution screen, of $16 \times 16$ points,
was obtained by means of the covariance method. Next, successive interpolation steps were executed to produce the desired grid size, generally $512 \times 512$ (occasionally $1024 \times 1024$) points with grid spacing of 0.5–2 mm. All the simulations were conducted for a link range of 1300 m using 26 random phase screens, and for different values of the refractive index structure constant $C_n^2$. The wavelength was set to match the laser wavelength used for the experiments ($\lambda = 780\text{nm}$). A minimum of 3000 realizations were run to reduce the statistical uncertainties of the numerical simulations for the scintillation data. In addition, the scintillation index was estimated for a set a receiving apertures with different diameters $D$.

![Figure 1](image)

**Figure 1.** Simulation data and theoretical curves for a beam divergence of $\theta = +66\mu\text{rad}$, link range $L = 1.2\text{km}$ and three different aperture sizes. The theoretical curves were generated using an inner scale size $l_0 = 8\text{mm}$.

The values of the diameter for the receiving apertures were chosen considering the actual aperture sizes used during the experiments. The consequence of this criterion, and the adoption of the effective point receiver notion proposed by Ochs *et. al.* combined with the fact that the Kolmogorov spectrum neglects the effects of the inner scale, is to adopt a 3 mm diameter receiving aperture as point receiver; which is used for the derivation of the simulated aperture averaging factor. Furthermore, simulations were run for divergent and convergent Gaussian beams denoted, as a convention, with positive and negative beam divergence respectively.

With the intention of comparing the simulated data results with the theory for Gaussian beams,[14] a simulation of 3000 realizations was run with a screen size of $512 \times 512$ point and 1.63 mm of resolution. One effect of using a finite grid is that, although a Kolmogorov spectrum is used, the minimum effective scale size that produces scintillation is not zero, but rather it is determined by twice the grid spacing $\Delta$ owing to Fourier spectral considerations.[22] Thus, as shown in figure 1, the simulated data had to be compared with a theoretical curve including the effects of a non-zero inner scale size $l_0$. Nevertheless, when observing the simulated data it was detected that the theory tends to underestimate the effects of the inner scale, hence, a higher value ($l_0 = 8\text{mm}$
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instead of the corresponding value \( l_0 = 2\Delta = 3.26\text{mm} \) was used in the theoretical expression.

3.2. Simulation data

The vast majority of the recently works conducted on scintillation and aperture averaging have assumed or made use of divergent beams. Although Recolons et al. [23] have presented simulation data for a case of study with a focused beam, in the weak turbulence regime with \( C_n^2 = 1.39 \cdot 10^{-16} \text{m}^{-2/3} \) and Rytov variance of \( \sigma_R^2 = 0.1817 \). In this work simulations have been carried out using values of the refractive index structure constant up to \( 4 \cdot 10^{-13} \text{m}^{-2/3} \), thereby leading to values of the Rytov variance exceeding unity, which corresponds to the moderate-to-strong turbulence regime. Additionally, all turbulence conditions were simulated with semiangle beam divergences of \( \theta = \pm 37\text{µrad} \) and \( \theta = \pm 66\text{µrad} \), where the negative and positive signs stand for convergent and divergent beams, respectively.

The simulation results obtained for four different values of beam divergence and three receiving aperture diameters are shown in figure 2, where the plots for the SI as well as for the aperture averaging factor are presented as a function of the Rytov variance. It is clearly noticed that the curves have a strong dependence on the beam divergence in the weak turbulence zone \( (\sigma_R^2 < 1) \). Moreover, the data suggest that by the utilization of convergent beams there is an improvement on the scintillation index and the aperture averaging factor, respect the use of divergent ones. It is also observable that the convergent beam with the lowest absolute value of beam divergence is the one that provides the best results for the two plotted parameters, as compare with the others beam divergence values, either convergent or divergent. This trend is preserved in the moderate turbulence regime up to \( \sigma_R^2 \approx 5 \) where the curves start to overlap, indicating that the SI as well as the aperture averaging become insensitive to beam divergence when a stronger turbulence regime is achieved.

An interesting behavior of the SI is observed in figure 2(c) where the curves suggest that for a given receiving aperture and link distance there is an trend inversion for \( \sigma_R^2 > 10 \). If more data points were available it would be possible to verify if this inversion, where the convergent beams suffer higher scintillation than the divergent ones, is continued or on the contrary a saturation regime is reached with all the beam divergences curves converging to the same scintillation index value.

4. Experiments

4.1. Experimental setup

The experiments were conducted on October the 14th and November the 13th of 2009 at Barcelona, Spain, in a 1.2 km optical path, shown in figure 3(a), between the rooftops of two buildings along a medium density residential terrain. A 780 \( \mu \text{m} \) continuous-wave diode laser at 15 mW (12 dBm) from LISA Laser (HL25/MIII), with built-in collimator,
was used in combination with a beam expander of diameter 3.2 mm. The testbed selected for the experiments consisted in a nearly horizontal path with the transmitter and receiver on either side of the optical path. On the receiver side the light was detected using a 15 cm focal length Fresnel lens with two simultaneous photodetectors, both with bandpass interference filter with a 3 dB bandwidth of 10 nm to remove the out-of-band background radiation, as depicted in figure 3(b). One of the detectors was mounted with a pinhole aperture to emulate a point detector, while the other one had interchangeable diaphragms, hence, measurements for different aperture diameters were possible.

The photodetectors were equipped with an offset knob, thus resulting in no need of subtracting the background radiation in the data processing as this could be corrected before each run. The detected signal was captured with a National Instrument data-acquisition card (PCI-6221), using preamplification, at 10 kHz of sampling rate and a custom-written LabVIEW program. Data were taken in individuals runs for the

Figure 2. Scintillation index $\sigma_I^2$ and aperture averaging factor $A$ simulated data for various divergence and receiving aperture diameter values, with propagation distance (a)(b) $L = 600m$ and (c)(d) $L = 1.2km$. 

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receiving apertures of 5 min each. The diaphragms used for the experiments had aperture diameters of 25 mm, 40 mm, 60 mm and 80 mm.

4.2. Data analysis

To obtain the experimental SI and aperture averaging collected data were processed with a 5 seconds width unweighted moving average window, also known as simple moving average SMA, thus resulting in a method to measure their behavior in time. With this strategy the mean value of the SI and aperture averaging for the data runs were obtained, as well as the their standard deviations. The experiments were conducted using a beam divergence of $\theta = -66 \mu \text{rad}$ for October the 14th and $\theta = +37 \mu \text{rad}$ November the 13th, in order to coincide with the simulations presented in Section 3.

After the SMA was applied to the data a linear interpolation technique was used in order to fit its mean value to the SI simulation curves, next, the estimated refractive index structure constant $C_n^2$ for each aperture was used as the abscissa coordinate for the experimental aperture averaging factor as presented in figure 4, where the experimental data point are plotted with circles and their respective standard deviation are shown as error bars. This fitting procedure give rise to a $C_n^2 = 2.10 \cdot 10^{-14} m^{-2/3}$ with standard deviation $\sigma_{C_n^2} = 1.28 \cdot 10^{-15} m^{-2/3}$ and $C_n^2 = 4.59 \cdot 10^{-14} m^{-2/3}$ with $\sigma_{C_n^2} = 1.94 \cdot 10^{-15} m^{-2/3}$ for figure 4(a) and figure 4(c), respectively. On the other hand, for the aperture averaging plots in figure 4 most of the experimental data points fall, within one standard deviation, on the simulation curves with exception of the receiving aperture of 40 mm on both cases. This discrepancy is mainly due to the fact that for the experimental aperture averaging a finite effective point receiver was used, and also the fact that the effects of the inner
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Figure 4. Fitting of the experimental data points (circles) with error bars, to the simulation curves. The date and the beam divergence used for the experiments are shown in each plot.

scale on the numerical simulations were neglected.

It is evident that the simulation curves and the experiments are in good agreement as all the experimental data points fall around a mean value of $C_n^2$ with an error one order of magnitude lower, which provides a procedure that makes possible a first estimation of the refractive index structure constant. Nevertheless and additional source of error is added when taking the projection of the uncertainty imposed by the experimental data standard deviation along the SI curve over the abscissa axis.

5. Conclusions

From Section 3 it can be concluded that the simulations performed with the improvement proposed in Ref. 19 are in good agreement with the theory, developed by Andrews et. al. for Gaussian beams, although from the comparison it seems that the theory tends to underestimate the influence of the inner scale $l_0$. Issue that should be analyzed with
more detail in further studies. In addition, simulations suggest that convergent beams perform better than divergent ones for the same given link conditions up to $\sigma_R^2 \approx 5$ where the behavior of both type of beams overlap to each other, indicating that the SI as well as the aperture averaging become insensitive to beam divergence as a stronger turbulence regime is achieved. It is also observable from figure 2(c) that for $\sigma_R^2 > 10$ there is an inversion of the trends for convergent and divergent beams. An extension to the simulations conducted have been foreseen with the purpose of investigating if this inversion tendency either is maintained or reversed in the turbulence saturation regime, or perhaps all the curves for the different beam divergences converge to a same value.

Finally, the experimental measurements helped to validate the simulation predictions for SI and aperture averaging factor, specially on the moderate turbulence regime as the Rytov variance for the experiments conducted on October the 14th and November the 13th of 2009 with $\sigma_R^2 = 1.25$ and $\sigma_R^2 = 2.74$, respectively. A first estimation of the refractive index structure constant $C_n^2$ was possible by fitting the experimental data points to the simulated curves, although, this method should be assessed in order to verify its validity with other $C_n^2$ measuring techniques.

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References

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