Belief Increasing in SKL Model Frames

Matías Alvarado
Gustavo Núñez

Report LSI-95-42-R

Facultat d'Informàtica
de Barcelona - Biblioteca

21 AGO. 1995

Belief Increasing in SKL Model Frames *

Matías Alvarado¹ and Gustavo Núñez²

¹ LSI, Technical University of Catalonia
Pau Gargallo 5, 08028 Barcelona, Spain
e-mail: matias@lsi.upc.es

² Integral Tecnology of Information
Diego Rivera 31, 91000 Xalapa, Ver., México
email: GNunez.TSCW@cid.mhs.compuserve.com

Abstract. Three-Valued Strong Kleene Logic [16], provides an adequate framework to deal with belief increasing. In this paper, *undefined* truth-value is used to denote opinionless information. Through an informative refinement, such information could become to be *true* or *false* in a step by step way. Our approach is based upon model frames [9], that are sets of informatively ordered three-valued interpretations. A relation of compatibility among frames, being a partial informative order, is defined. Belief increasing is accomplished by using the compatibility relation that is based on concatenation operation over model frames. A correspondence between model frames and analytic tableaux is outlined. It provides to deal with model frames by using that powerful proof method [11].

1 Introduction

Classical approaches to belief change are the Alchourron-Gärdenfors-Makinson (AGM) [1] and the *Update Theory* of Katsuno and Mendelzohn (KM) [15]. In order to change the logically closed belief base ψ with a sentence μ , AGM proposal provides with *expansion*, *contraction* and *revision* operations. *Expansion* is applied whenever μ is consistent with ψ , obtaining logical closure from $\psi \cup \{\mu\}$; *contraction* to get out from ψ the sentence μ together with some deductively-related formulas; and *revision* being a $\neg\mu$ -contraction followed by a μ expansion. Alchourron *et al.* proposed the so called *rationality postulates* to be satisfied by any *revision operator*. On the other hand, KM approach provides eight postulates to update with μ any arbitrary sets of sentences ψ (not only logically closed ones as AGM does). In case of inconsistency between ψ and μ , semantically, AGM results in the closest model to ψ satisfying μ , while KM release, for each model of ψ , the models of μ closest to it [15].

The main drawback of those fruitful approaches to belief change consists, essentially, in the lack of flexibility to deal with no-considered revision or updating cases, or the contraintuitive results that they provide dealing with ambiguous information. Brewka and Hertzberg [6] as well as Del Val and Shoham [7] propose to use *Action Theory* through axioms of action encoding the *circumstances of especific updating or revision*. We coincide to consider that the point is the capability to handle particular information based on a general framework.

Thus, the paramount of this paper is twofold:

- Provide a intuitive framework for representation and change of belief using model frames defined by Doherty [9]
- Outline an equivalence between model frames and Analytic Tableaux such that this flexible proof method [11] is used to deal with belief in a computational perspective intuitions preserving.

In section 2, three-valued *SKL* is introduced together with Belnap's informative partial order over model frames. From *SKL* and the informative order we outline our epistemic definitions in section 3, developing a (constructive-like) logic of belief. In section 4 correspondence between epistemic model frames and analytic tableaux is mentioned showing that tableaux proof method is well applied in our intend. In the last section a brief comparison between AGM paradigm and *Update Theory* with respect to our approach is developed.

2

2.1 Strong Kleene Logic

For Strong Kleene Logic, propositional language $\mathcal{L}$ is defined from a finite set of sentences Σ and the primitive connectives negation $\neg$ and disjunction $\vee$. From those connectives the conjunction $\wedge$ and implication $\rightarrow$ are defined in the classical way. Let $\mathcal{F}$ be the set of $\mathcal{L}(\Sigma)$ -formulas. Semantical definitions for disjunction and negation in the Strong Kleene Logic are given in Table 1. Let *true*, *false* and *undefined* be truth-values denoted with t , f and u .

Table 1

φ	ψ	$\varphi \vee \psi$	$\neg \varphi$
t	t	t	f
t	f	t	t
t	u	t	t
f	t	t	f
f	f	f	t
f	u	u	t
u	t	t	t
u	f	u	t
u	u	u	t

Definition 1. A three-valued interpretation I , is a valuation function from $\mathcal{F}$ to $\{t, f, u\}$ in accordance with logical connectives definition. I is equivalent to $I^t \cup I^f \cup I^u$ in such a way that I^t is the set of *true* formulas, I^f the set of *false* formulas and I^u the set of *undefined* formulas. When I^u is the empty set we say that I is complete with respect to *true* and *false* truth-values. Let $\mathcal{I}(\mathcal{F})$ be the set of three-valued interpretations of formulas in $\mathcal{F}$.

2.2 Informative Order

In this section we concern with the approach that use an informative order to deal with knowledge and belief (see e.g [2, 4, 9]), in order not reduce them to

truth-semantic-logical aspect. Herein an informative order over interpretations of sets of beliefs is given. Underline objective is to deal with belief change in a step by step way. The intuitive point is that seems natural to consider the following:

- Having *true*, *false* and *undefined* statements, *true* or *false* ones provide more information than *undefined* statements.

We propose that when undefined information turns up defined, as true or false, beliefs could increase.

We observe that increasing of belief can be considered a belief expansion-like operation of Alchourron, Gärdenfors and Makinson Paradigm [1, 14]. It is well known, however, that in the seminal AGM approach, change of belief includes also elimination and revision of current beliefs by using *contraction* and *revision* operations. On the other hand, *Updating Theory* [15] is the classical approach to belief updating. As mentioned in the introduction, this approach has subtle differences with the AGM one and constitutes the other significant view of belief change. In section 5 we outline the way in which our proposal can deal with contraction, revision and updating operations. Now, the informative partial order defined for Doherty over $\{t, f, u\}$ set [9] is introduced.

Definition 2. The Degree of information among t , f and u truth-values, is defined as follows: $u \preceq_i t$, $u \preceq_i f$ and t, f are not comparable.

Let $\mathcal{T}$ be a subset of $\mathcal{I}(\mathcal{F})$. The informative partial order $\preceq_i$ is introduced over $\mathcal{T}$ in the following way: given an $I, I' \in \mathcal{T}$, $I \preceq_i I'$ is satisfied if and only if $I' \subseteq I''$, $I' \subseteq I'''$ and $I'' \supseteq I'''$. The order $\preceq_i$ is a partial one on $\mathcal{T}$.

Intuition of that order is that given two interpretations of a set $\mathcal{P}$ of formulas, interpretation with more *undefined* formulas is less informed than the one having more *true* or *false* formulas. Then, for $\mathcal{P} \subseteq \mathcal{F}$ such that $I(\mathcal{P}) \preceq_i I'(\mathcal{P})$ for every $P \in \mathcal{P}$, I' is more informed than I with respect to $\mathcal{P}$, denoted $I \preceq_i^{\mathcal{P}} I'$. In this case it is said that I' is an *informative refinement* of I with respect to $\mathcal{P}$.

Example : Let $\mathcal{P} = \{P\}$ be the only sentence of language $\mathcal{L}$; $I_0 = (u)$, $I_1 = (t)$, $I_2 = (f)$ be the interpretations of $\mathcal{F}$. Then I_1, I_2 are informative refinements of I_0 and thus more informed than it.

The following formal structure providing a suitable framework for our epistemic approach is an extension of model frame defined by Doherty [10]. The underling objective is to capture the dynamic manner in which beliefs change. We consider that it depends of *worlds being explored*. Gradual exploration is allowed by using the informative order $\preceq_i$ among model frames.

Definition 3. A model frame is an ordered 3-tuple $\mathcal{M} = (\mathcal{T}, I_0, \preceq_i)$, where $\mathcal{T} \subseteq \mathcal{I}(\mathcal{F})$ is a set of interpretations and $I_0 \in \mathcal{T}$ is called the current (actual or present) interpretation. For any $I' \in \mathcal{T}$, we assume that $I_0 \preceq_i I'$. Thus, any interpretation in $\mathcal{M}$ is at least as informed as I_0 .

Notice that as interpretation I_0 remains fix, it constitutes an invariant and characterizes the model frame. Following definition will be used in the following comments and in belief definition.

Definition 4. An interpretation J is maximal in $\mathcal{M}$ if and only if there is no J' in $\mathcal{M}$ such that $J \prec_i J'$.

A model frame can be seen as a set of possible worlds that provides a *description of the state of the things together with its possibles extensions* (see fig 1). Whenever worlds corresponding with the (informatively) maximal interpretation in the model frame contain undefined statements, extension of belief could be performed as it should show below (see 4).

On the other hand, informative order $\preceq_i$ can be considered an accessibility relation among interpretations in $\mathcal{T}$ as follows: I' is accessible (compatible) for I whenever I' is an informative refinement of I . Moreover, if for every $I \in \mathcal{T}$, in a model frame $\mathcal{M}$, $I(\alpha) \in \{t, f\}$, then this last one is a structure of Kripke³.

3 Belief

In this section a belief definition is proposed. Propositional language $\mathcal{L}$ is extended by adding a modal operator of belief, B . The language extended with these modal operators is denoted $\mathcal{BL}$ by belief language. The following satisfaction definition is relative to model frame.

Definition 5. A sentence $\alpha \in \mathcal{F}$ is satisfied by the interpretation $I \in \mathcal{T}$ of the model frame $\mathcal{M}$, denoted $I \models_{\mathcal{M}} \alpha$, if and only if $I(\alpha) = T$. It is said that the frame satisfies α , $\mathcal{M} \models \alpha$, if every $I \in \mathcal{T}$ satisfies α . For a set Γ such that any sentence of Γ is satisfied by $\mathcal{M}$, is said that the frame $\mathcal{M}$ satisfies Γ , $\mathcal{M} \models \Gamma$.

Notice that this satisfaction definition is for no modal formulas only; for belief modal ones is given below. Relativity of model frame satisfaction is welcome in our aim to encourage the point of view considering belief relative to actual information and context. This semantical flexibility can be syntactically well treated using a method based on Analytic Tableaux. Now we give belief definition.

Definition 6. A formula α is believed in the interpretation I of the model frame $\mathcal{M}$, denoted $I \models_{\mathcal{M}} B(\alpha)$, if and only if, for every maximal J of $\mathcal{M}$ with $I \preceq_i J$, $J \models \alpha$ (see fig 2). Whenever for every I of $\mathcal{M}$, $I \models_{\mathcal{M}} B(\alpha)$, it is said that $\models_{\mathcal{M}} B(\alpha)$.

³ A structure of Kripke is a 3-tuple $\langle W, R, \nu \rangle$ such that W is a nonempty set of worlds, R a possibility relation and ν a valuation function [18].

Then, a statement is belief in $\mathcal{M}$ if it is *true* in the interpretation I_0 or if being undefined in I_0 , it becomes *true* in every maximal interpretation of the frame.

Figure 2: Belief

3.1 Consistency over Belief Increasing

Our formalism provides the possibility to reason from initial, possibly not consistent beliefs set, in a not trivial way. Each one of the contradictory sentences, if any, are considered in different *parts* of the total frame. In fact, increasing (expansion) of beliefs happens in different directions, being consistent each one of them. Now we define compatibility relation among model frames; based upon it, in addition to concatenation and unification operations over frames, change of the initial beliefs, as increasing or modification, is considered. (A comparison with Logic of Explicit and Implicit Belief in [19].)

3.2 Compatible frames

We are interested in model frames that *change* in order to model the dynamic character of a cognitive process. A model frame defined by Doherty in [9] is static; is beforehand given and does not change at all. However, solution to our requirements can be done by introducing an operation, namely of *concatenation*, among Doherty's model frames. The intuitive idea of concatenation is the following:

- To add information to a maximally informed world in the frame, if possible, in such a way that it allows to increase beliefs.

Let $\mathcal{M}_1 = \langle \mathcal{T}_1, I_0^1, \preceq_i^1 \rangle$, $\mathcal{M}_2 = \langle \mathcal{T}_2, I_0^2, \preceq_i^2 \rangle$ be two model frames.

Definition 7. Model frame $\mathcal{M} = \langle \mathcal{T}, \preceq_i, I_0^1 \rangle$ is the *concatenation* of $\mathcal{M}_2$ with $\mathcal{M}_1$, denoted $\mathcal{M} = \mathcal{M}_1 \odot \mathcal{M}_2$, if and only if

- 1) I_0^2 is a maximal interpretation of $\mathcal{M}_1$.
- 2) $\mathcal{T} = \mathcal{T}_1 \cup \mathcal{T}_2$,

By definition, $I_0^1 \preceq I'$ for every $I' \in \mathcal{T}_1 \cup \mathcal{T}_2$. Notice that concatenation is defined over maximal interpretations only. Thus, further information is added to maximally informed worlds in the frame. It could be of interest to define

concatenation for every interpretation in the model frame but this is behind the scope of this work.

Notice that frame concatenation is based upon the informative orders of model frames. Then, the possibility to concatenate $\mathcal{M}_2$ to $\mathcal{M}_1$ depends on the fact that interpretations in $\mathcal{M}_2$ could be informative refinements of a maximal interpretation in $\mathcal{M}_1$ (see fig 3 and the next example). Concatenation is applied to increase belief from *undefined* statements by changing them to *true* or *false*, providing a constructive way to obtain beliefs from opinionless information.

Fig. 3

Now we define an accessibility relation among model frames. Jaako Hintikka's intuitions about use accessibility relation of Kripke' semantics of possible worlds [18] to model knowledge and belief [14], are very suited, being the formalisation weakness that does not capture adequately such intuitions⁴ Using concatenation, a relation of possibility, called of compatibility, is *defined*. That relation captures the constructive and dynamic manner in which we think epistemic and doxastic alternatives (worlds) are conceived by a limited rational agent. The intuitive idea of compatibility is the following:

- Given a current frame describing a situation, any frame being coherently more informed than it, is compatible with it. (It is worth to think in the particular case when frames are defined by a single world.)

Definition 8. A model frame $\mathcal{M}_2$ is *compatible with* model frame $\mathcal{M}_1$ if and only if, there is a succession of model frames $\mathcal{N}_1, \dots, \mathcal{N}_n$ such that $\mathcal{N}_1 = \mathcal{M}_1$, $\mathcal{N}_n = \mathcal{M}_2$, and $\mathcal{N}_{i+1}$ is concatenated to $\mathcal{N}_i$ for $i = 1, \dots, n - 1$.

Then, compatible model frames with $\mathcal{M}_1$ correspond to informative refinements that can be obtained from a maximal world in $\mathcal{M}_1$ (see example 2). Thus, $\mathcal{M}_2$ is compatible for $\mathcal{M}_1$ if it is more informed than $\mathcal{M}_1$ in some of the directions with undefined information that $\mathcal{M}_1$ has. By definition, any concatenated frame to $\mathcal{M}_1$ is compatible with it.

⁴ In [19] a wide analysis about this point is developed.

Let $\mathcal{N}_1, \dots, \mathcal{N}_l$ model frames with interpretation set $\mathcal{T}_1, \dots, \mathcal{T}_l$, respectively. The following is a remarkable fact.

Proposition 9. *If each set of interpretations $\mathcal{T}_i$ for $i = 1, \dots, l$ is a singleton, then each model frame is a world. In that case, accessibility relation is over possible worlds.*

Thus, compatibility relation is a step by step possibility relation among sets of possible worlds in Kleene's three-valued logic.

Proposition 10. *Partial order over $\mathcal{T}$ set of a model frame $\mathcal{M}$ corresponds to an accessibility relation inside $\mathcal{M}$.*

Example : Let the following set of statements, with initial interpretation I_0 such that $I_0(P) = u$ for every $P \in \mathcal{P}$.

$\mathcal{P} = \{ \text{Misery} \rightarrow \text{Rebellion},$	(1)
$\text{Rebellion} \rightarrow \text{Negotiation} \vee \text{Repression},$	(2)
$\text{Rebellion} \wedge \text{Negotiation} \rightarrow \text{Solution},$	(3)
$\text{Rebellion} \wedge \text{Repression} \rightarrow \text{War},$	(4)
$\text{Solution} \rightarrow \neg \text{Misery},$	(5)
$\text{War} \rightarrow \text{MoreMisery}$	(6)}

Informative order is indicated in the following model frame (fig 4). In the figure we use the initial letter of each predicate to deal with it, except to denote *Rebellion* with *Rb* and *Repression* with *Rp*. Formulas without indicated truth-value are considered undefined. It is assumed that *true* and *false* formulas in upper interpretations are present in the following ones.

In the example we can draw the possible scenarios that could be allowed from a (unhappy) situation. It depends on the way in which opinionless statements describing the initial situation become *true*. At the final steps of current frame, there are two compatible (possible) evolving frames —not resolved yet in the example— of current frame. One with a good (desired) evolution corresponds with that eventually statement (3) is *true*; the other, with unhappy evolve, corresponds with having as *true* statement (4). (It depends on the beliefs of the active (implicit) agents.) In each case a compatible model frame can be concatenated to the current one.

4 Analytic Tableaux as Frames of Beliefs

We will use as proof system the analytic tableaux method [11] because its flexibility is adequated to introduce doxastic conditions over sets of formulas [2, 9, 17]. There are already characterization of Gentzen type provers for three-valued logics [3]. In fact, Avron distinguishes between the ones that are based upon a logic in which the *undefined* truth-value denotes incomplet or unknown information, from those that use *undefined* denoting inconsistent information. The

most known of the first ones corresponds with Kleene's and Lukasiewicz' logics⁵. Moreover, there are multivalued-logic theorem provers [5, 20].

Fig. 4 Analytic Tableaux as Frame of Belief

Our proposed approach uses *undefined* as incomplete or unknown. We can add doxastic conditions through tableaux like rules, or as belief statements in open branches of tableaux containing current interesting information. It is easy to show from the last example the natural manner in which a frame of beliefs can be represented through an analytic tableaux (see fig 4). The correspondence can be established under the following considerations:

- Current model frame is contained in the set of paths of the associated tableaux.
- The informative order over interpretations in the model frame allows the order in which classical tableaux rules are applied over the set of formulas.
- Whenever a formula becomes *true* in the frame, the closure that such change of value conveys is performed by application of classical tableaux rules.
- Classical α, β, π, ν -rules of tableaux [11] when applied over true formulas evolve in tableaux paths such that its model frames counterparts are the virtual concatenations of current model frame.
- Virtual extensions (by concatenation) of the model frame correspond with tableaux extensions through open branches.

Incorporation of above considerations can be done appending to classical tableaux rules, the following one denoted UD:

- Whenever an *undefined* formula becomes *true* or *false*, then a tableaux is opened and all its logical consequences are deduced.

⁵ An example of the inconsistent view is the Paraconsistent Logics of Newton d'Acosta (see [3]).

According to UD rule, whenever an *undefined* formula becomes *true* or *false*, a tableaux is opened and classical rules are applied. In the associated tableaux of the example, β -rule was applied over implication and disjunctive formulas as soon as they turned up *true*, and its logical consequences obtained. It is assumed that every tableaux satisfies all the formulas in upper tableaux. Closed tableaux are marked with X and corresponds with incompatible (impossible) situations for model frame.

5 Related Works and Discussion

In the introduction was mentioned that classical approaches to belief change are those of AGM paradigm [1, 14] and Update Theory (UT) [15]. The first one defines operations of *expansion*, *contraction* and *revision* over logically closed sets (called belief sets,) while the second deals with *belief updating* for arbitrary sets. In the proposed approach, using concatenation operation, it follows increasing (expansion) of belief. Now we can outline the way in which it is possible to deal with contraction, revision and update of beliefs in a nonless natural way.

Intuitive idea of revision of a beliefs set ψ with new information μ is to modify as little as possible ψ in such a way that it entails μ , preserving consistency. In our approach it conveys the modification of the frame $\mathcal{M}$ such that μ becomes *true* in all maximal interpretation of $\mathcal{M}$.

If μ is *true* or *undefined* in every interpretation of $\mathcal{M}$ then is an μ -belief expansion on $\mathcal{M}$, while if μ is false in some interpretation, it arises a contradiction in $\mathcal{M}$. In this case is proceeded in the following manner:

- It should be *erased* interpretations of the frame, by turning *undefined* formulas that being *true* conveys contradictions with formula μ .

As much as needed to preserve consistency in the frame are the erased interpretations. Syntactical operativity is performed using the analytic tableaux associated with the model frame. Contradiction in analytic tableaux appears whenever an open branch of the tableaux becomes close. The erasure of contradictory interpretation in the frame corresponds with the erasure of the closed branches in the tableaux that were generated by making μ *true*. Unconsistency over tableaux branch is backward translated until it disappears. Thereafter contradiction is eliminated, is possible that μ becomes to be *true* in all maximal interpretation of the *revised* frame $\mathcal{M}'$.

We observe that erasure can be done in several ways. It is necessary to consider additional criteria in order to obtain certain erasure(s) among the possible ones. Adopted criteria can be translated over Analytic Tableaux as rules, constraints or strategies over deduction process [19].

Now we make some observations:

- 1.) In the case of expansion, there are open branches in the associated tableaux to the frame, over which the operation is done.
- 2.) The erasure of frame interpretations could be considered the contraction operation over the frame.

3.) In AGM approach, revision operation of ψ by μ is equivalent to contraction for $\neg A$ and expansion by μ . This is the manner in which revision of a model frame is performed.

Thus, expansion, contraction and revision operation can be performed over model frames.

With respect with belief updating a similar thing should be done. Intuitive idea of UT is that the world has changed and is necessary to update beliefs. Whenever the updating formula μ do not conveys contradiction with the old beliefs in ψ , revision and updating are equivalent. The case in which contradictions arise, seems that UT generalizes the AGM approach [7]. There are situations in which UT allows different alternatives to belief updating that AGM does not.

However, UT axiomatization, given certain information, results in unintuitive updatings. This is the case when μ is a disjunction that is not in ψ but is entailed by it (remember that UT deals with arbitrary sets.) This situation corresponds with update ψ with so-called *ambiguous* information [6]. Minimal change criterion of UT expressed by axiom U2 (see [15]), conveys unintuitive results in spite of being information describing so simple situations; *e.g.* flipping a coin, the result provided by UT due axiom U2, is the last visible-side-coin, that is a biased one. The same is applicable to deal with the undefinition of a person that having a job, receives a new (might be better) job proposal. *UT* chooses always the old job.

Model frames approach provides to deal with different alternatives for updating with ambiguous information. We have reasons to be optimistic to deal with *problematic* questions of belief change using the proposed approach. Alternative scenarios can be coherently included in a model frame without any kind of implicit bias. While an alternative is not justified enough, it cannot be a belief in the frame, but nor is excluded. In this step it is a local belief satisfied under more specific circumstances. Further information is needed in order to become a global belief. In this case, the other alternative statements, if any, is neither globally rejected. It could remain as local information as well. Further details constitute the material of our current research.

Concluding Remarks

This paper proposes to deal with belief increasing (expansion) using model frames that are sets of informatively ordered possible worlds in three-valued Strong Kleene Logic. The *undefined* truth-value is used to indicate current but opinionless information. *True* and *false* information is considered more informed than the *undefined* one, and it constitutes an informative order. From that order, a compatibility relation over model frames is defined, such that in addition to concatenation frames operation, belief change is allowed. The proposed formalism provides to deal, locally, with contradictory or incoherent beliefs. Using the informative order in a step by step manner ideal reasoning and logical omniscience are avoided. The epistemic framework has a suited syntactical proof method using analytic tableaux.

Part of our forthcoming work is the introduction as far as possible, of strategies for tableaux calculi in modal logic that has been recently developed in [8].

Aknowledgments. This work was benefitted with worth comments from Ll. Godó and R. Rodríguez (IIIA-CSIC, Spain), J. Larrosa (LSI-UPC, Spain), S. Demri (LIFIA-IMAG, France) and anonymous referees. First author is supported by a PhD fellowship of Universidad Nacional Autónoma de México. Participation of second author was during a *Visiting Professor Stage* in Technical University of Catalonia.

References

1. C. Alchourron, P. Gärdenfors and D. Makinson: *On the Logic of Theory Change: Partial Meet Contraction and Revision Function*. Journal of Symbolic Logic. Vol. 50 (1985) 510-530.
2. M. Alvarado: *A Posteriori Knowledge: from Ambiguous Knowledge and Undefined Information to Knowledge*. In Procc. of ECSQARU '95 *Lecture Notes in Artificial Intelligence* 946. Fribourg, Switzerland (1995).
3. A. Avron: *Natural 3-Valued Logics Characterization and Proof Systems*. Journal of Symbolic Logic Vol. 56 No. 1, March 1991.
4. N. Belnap: *A useful four-valued logic* In Dunn and Epstein (Eds.) *Modern Uses of Logic*. Reidel Publishing Co, 1976.
5. Beckert: *STAP Tableau-Based Theorem Prover*. Karlsruhe University, Germany (1994)
6. G. Brewka and J. Hertzberg: *How to do thing with worlds*. Journal of Logic and Computation Vol No. (1993).
7. A. Del Val and Y. Shoham: *A unified view of Belief Revision and Update*. Journal of Logic and Computation Vol. 4 No. 5 pp 797-810 (1994)
8. S. Demri: *Uniform and Non uniform Strategies for Tableaux Calculi for Modal Logic*. In Procc. of Workshop on Logics in Artificial Intelligence JELIA'94, LNAI 738, Springer Verlag (1994)
9. P. Doherty, W. Lukasciewicz: *NML3. A Non-Monotonic Logic with Explicit Defaults*. Journal of Applied Non-Classical Logic Vol. 2 No. 1 (1992)
10. C. Geisler and K. Konolige: *A resolution method for quantified modal logics of knowledge and belief*. In Procc of Reasoning About Knowledge Conference, (1986).
11. M. Fitting: *First-Order Modal Tableaux*. Journal of Automated Reasoning 4 (1988) pp. 191-213.
12. P. Gärdenfors: *Knowledge in Flux*. The MIT Press (1988).
13. M. Ginsberg: *Bilattices and Modal Operators*.
14. J. Hintikka: *Knowledge and Belief* (Cornell University Press 1962).
15. H. Katsuno and A. Mendelzon *Propositional knowledge base revision and minimal change*. Artificial Intelligence 52 (1991) 263 - 294.
16. S. Kleene: *Introduction to Metamathematical*. North Holland Ed. (1952)
17. K. Konolige: *A Deduction Model of Belief and its Logics* (Morgan Kaufmann, 1986).
18. S. Kripke: *A completeness theorem in modal logic*. The Journal of Symbolic Logic. 24 - 1 (1959) pp. 1-14.
19. G. Núñez, M. Alvarado: *An Approache to Belief in Strong Kleene Logic*. Report de Recerca LSI-95-27, Technical University of Catalonia (1995)
20. N. Zabel: *Nouvelles Techniques de Deduction Automatique en Logiques Polyvalentes Finies et Infinies du Premier Ordre*, PhD Thesis. Institut National Polytechnique de Grenoble (1993)

Departament de Llenguatges i Sistemes Informàtics
Universitat Politècnica de Catalunya

Research Reports – 1995

- LSI-95-1-R “Octree simplification of polyhedral solids”, Dolors Ayala and Pere Brunet.
- LSI-95-2-R “A note on learning decision lists”, Jorge Castro.
- LSI-95-3-R “The complexity of searching implicit graphs”, José L. Balcázar.
- LSI-95-4-R “Design quality metrics for object-oriented software development”, Alonso Peralta, Joan Serras, and Olga Slavkova.
- LSI-95-5-R “Extension orderings”, Albert Rubio.
- LSI-95-6-R “Triangles, ruler, and compass”, R. Juan-Arinyo.
- LSI-95-7-R “The modifiability factor in the LESD project: definition and practical results”, Nuria Castell and Olga Slavkova.
- LSI-95-8-R “Learnability of Kolmogorov-easy circuit expressions via queries”, José L. Balcázar, Harry Buhrman, and Montserrat Hermo.
- LSI-95-9-R “A case study on prototyping with specifications and multiple implementations”, Xavier Franch.
- LSI-95-10-R “Evidence of a noise induced transition in fluid neural networks”, Jordi Delgado and Ricard V. Solé.
- LSI-95-11-R “Supporting transaction design in conceptual modelling of information systems”, Joan A. Pastor-Collado and Antoni Olivé.
- LSI-95-12-R “Computer (PC) assisted drawing of diagrams for forecasting soaring weather”, Lluís Pérez Vidal.
- LSI-95-13-R “Animats adaptation to complex environments as learning guided by evolution”, Mario Martin, Màrius Garcia, and Ulises Cortés.
- LSI-95-14-R “Learning to solve complex tasks by reinforcement: A new algorithm”, Mario Martin and Ulises Cortés.
- LSI-95-15-R “Analysis of Hoare’s FIND algorithm with median-of-three partition”, Peter Kirschenhofer, Conrado Martínez, and Helmut Prodingier.
- LSI-95-16-R “MDCO to B-Rep conversion algorithm”, Dolors Ayala, Carlos Andújar, and Pere Brunet.

- LSI-95-17-R "Augmented regular expressions: A formalism to describe, recognize, and learn a class of context-sensitive languages", René Alquézar and Alberto Sanfeliu.
- LSI-95-18-R "On parallel versus sequential approximation", Maria Serna and Fatos Xhafa.
- LSI-95-19-R "A set of rules for a constructive geometric constraint solver", Robert Juan-Arinyo and Antoni Soto.
- LSI-95-20-R "From degenerate patches to triangular and trimmed patches", Marc Vigo, Núria Pla, and Pere Brunet.
- LSI-95-21-R "Learning Ordered Binary Decision Diagrams", Ricard Gavaldà and David Guijarro.
- LSI-95-22-R "The complexity of learning minor closed graph classes", Carlos Domingo and John Shawe-Taylor.
- LSI-95-23-R "Approximating the permanent is in RNC", J. Díaz, M. Serna, and P. Spirakis.
- LSI-95-24-R "Constraint satisfaction as global optimization", Pedro Meseguer and Javier Larrosa.
- LSI-95-25-R "A rule-constructive geometric constraint solver", R. Juan-Arinyo and Antoni Soto.
- LSI-95-26-R "External schemas in object oriented databases" (written in Spanish), José Samos.
- LSI-95-27-R "An approach to belief in strong Kleene logic", Gustavo Núñez and Matías Alvarado.
- LSI-95-28-R "Dynamic belief modeling", Antonio Moreno and Ton Sales.
- LSI-95-29-R "Proceedings ICLP Workshop on Deductive Databases and Abduction", to appear. H. Decker, U. Geske, A. Kakas, C. Sakama, D. Seipel, and T. Urpí (editors).
- LSI-95-30-R "Difference lists and difference bags for logic programming of categorial deduction", F. Xavier Lloré and Glyn Morrill.
- LSI-95-31-R "An experiment on automatic semantic tagging of dictionary senses", German Rigau.
- LSI-95-32-R "Slices of meaning: a study on nouns of portions", Salvador Climent and Maria Antònia Martí.
- LSI-95-33-R "An HPSG grammar of Spanish implemented in ALE: A solution for dealing with subcategorisation alternances", Salvador Climent and Xavier Farreres.
- LSI-95-34-R "Optimal and nearly optimal static weighted skip lists", Conrado Martínez and Salvador Roura.
- LSI-95-35-R "Evaluation of expressions in a multiparadigm framework", Xavier Burgués and Xavier Franch.
- LSI-95-36-R "Equivalence between executable OOZE and algebraic specification", Vicent-Ramon Palasí Lallana.
- LSI-95-37-R "Automatic deduction of the behavioral equivalence between two algebraic specifications", Vicent-Ramon Palasí Lallana.

- LSI-95-38-R "A method for explaining the behaviour of conceptual models", Antoni Olivé and Maria-Ribera Sancho.
- LSI-95-39-R "Simple PAC learning of simple decision lists", Jorge Castro and José L. Balcázar.
- LSI-95-40-R "Parallel approximation of MAX SNP and MAX NP problems", Maria Serna and Fatos Xhafa.
- LSI-95-41-R "Automatically extracting translation links using a wide coverage semantic taxonomy", German Rigau, Horacio Rodriguez, and Jordi Turmo.
- LSI-95-42-R "Belief increasing in SKL model frames", Matías Alvarado and Gustavo Núñez.

Hardcopies of reports can be ordered from:

Nuria Sánchez
Departament de Llenguatges i Sistemes Informàtics
Universitat Politècnica de Catalunya
Pau Gargallo, 5
08028 Barcelona, Spain
secrelsi@lsi.upc.es

See also the Department WWW pages, <http://www-lsi.upc.es/www/>