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On the consideration of scaling properties of extreme rainfall in Madrid (Spain) for developing a generalized intensity-duration-frequency equation and assessing probable maximum precipitation estimates

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Abstract

The fractal behavior of extreme rainfall intensities registered between 1940 and 2012 by the Retiro Observatory of Madrid (Spain) has been examined, and a simple scaling regime ranging from 25 min to 3 days of duration has been identified. Thus, an intensity-duration-frequency (IDF) master equation of the location has been constructed in terms of the simple scaling formulation. The scaling behavior of probable maximum precipitation (PMP) for durations between 5 min and 24 h has also been verified. For the statistical estimation of the PMP, an envelope curve of the frequency factor (k_m) based on a total of 10,194 station-years of annual maximum rainfall from 258 stations in Spain has been developed. This curve could be useful to estimate suitable values of PMP at any point of the Iberian Peninsula from basic statistical parameters (mean and standard deviation) of its rainfall series.

Graphical abstract

1. Introduction

The intensity-duration-frequency curves (IDF curves) at a location are an important tool for hydrological planning. At urban scale, IDF values related to short durations are essential for designing rainwater sewer networks (mainly pipes and storm tanks) and surface drainage infrastructures (inlet systems, streams channeling, etc.). Moreover, IDF curves are necessary to evaluate socio-economic risk due to extreme events occurring in urban area. In Spain, some studies (Ferrer and Ardiles 1994; DGC 1999; Salas and Fernández 2007) were devoted to calculate the IDF curves from daily rainfall data applying regional analysis, and using data obtained from the digitization of pluviograph bands, the Meteorological Agency of Spain (AEMET) published IDF values for a selected set of pluviometric stations distributed throughout Spain (INM, AEMET 2003; INM, AEMET 2007), including the observatories of Madrid. In the present work, rainfall data recorded by the Central Meteorological Observatory located in the Parque del Retiro of this city have been analyzed to reproduce and update its current IDF values and to obtain a master IDF equation for this place. With this purpose, the fractal property of self-similarity and scale invariance of its rainfall intensities has been investigated. Many atmospheric processes produced by complex dynamic mechanisms acting in a wide temporal range, as rainfall generation, give rise to phenomena that look the same regardless of the scale where they are contemplated (self-similarity). These processes can

be considered of fractal type, and their properties exhibit power laws of the scale parameter λ , which is the ratio t/t_0 between any two durations t and t_0 within the scaling regime. The fractal self-similarity of natural processes, such as rainfall generation, has a statistical nature, as opposed to mathematical fractals in which the parties are an exact copy of the whole. Therefore, the scaling properties of phenomena like rain are expressed by statistical relationships describing their fractal behavior (Schertzer and Lovejoy 1987; Olsson 1996; Schertzer and Lovejoy 2011). For instance, the probability distribution of the annual maximum rainfall intensities accomplishes scale relationships (Koutsoyiannis and Foufoula-Georgiu 1993; Burlando and Rosso 1996), i.e., the probability distribution for a certain duration t_0 can be related to the distribution at other scale t by means of a factor which is a power function of the scale parameter λ (with $t = \lambda t_0$). In terms of the statistical moments of order q of the rainfall series (Menabde et al. 1999), this scaling relationship can be stated as Eq. (1).

$$\langle I_t^q \rangle = \lambda^{K(q)} \langle I_{t_0}^q \rangle \quad 1$$

AQ2

AQ3

Simple scale (or monofractal) processes are those for which the scale function $K(q)$ is linear, i.e., $K(q) = \beta q$. When this scale function is not linear, processes are multiscalar (or multifractal). Applied on rainfall series recorded in Madrid, the simple scaling hypothesis (Menabde et al. 1999; Yu et al. 2004; Aronica and Freni 2005; Bara et al. 2009) has resulted suitable, and its formulation has been useful to obtain a meaningful and appropriate IDF master equation for the city.

Another important tool for hydrological planning is the probable maximum precipitation (PMP), i.e., “*the greatest depth of precipitation for a given duration meteorologically possible for a given size storm area at a particular location at a particular time of year, with no allowance made for long-term climatic trends*” (WMO 1986). Hydrologists and hydraulic engineers use the PMP magnitude to maximize reliability and safety in the design of hydrological and hydraulic infrastructures. Results determining the PMP are always estimations with a risk statement; for instance, a return period between 10^5 and 10^9 years was estimated by the National Research Council (1994) for the PMP in the

USA. To estimate the PMP in a location, several procedures were proposed (e.g., Wiesner 1970; Schreiner and Reidel 1978; WMO 1986), most of them based on physical and meteorological analysis (Collier and Hardaker 1996; Casas et al. 2011; Ishida et al. 2015; Micovic et al. 2015) while some are based on statistical analysis (Hershfield 1961 and 1965; Rakhecha et al. 1992, Řezáčová et al. 2005; Desa et al. 2001; Desa and Rakhecha 2007; Casas et al. 2008). Among the statistical techniques to estimate the PMP, the most extensively used is the method proposed by Hershfield (1961, 1965) which involves the frequency analysis of the annual maximum rainfall series recorded at the location. This technique is based on the general frequency Eqs. (2) and (3) (Chow 1951).

$$PMP = \overline{x}_n + k_m \sigma_n \quad 2$$

$$k_m = \frac{x_M - \overline{x}_{n-1}}{\sigma_{n-1}} \quad 3$$

AQ4

The values of $\overline{x}_n$, σ_n , and x_M are the mean, standard deviation, and highest value of n annual maximum rainfall series for a certain duration, $\overline{x}_{n-1}$ and σ_{n-1} are the mean and standard deviation of these series when the highest value x_M has been excluded, and k_m is a frequency factor whose estimation by using Eq. 3 could lead to an unreliable value of the PMP. Hershfield (1961, 1965) recommended values of the frequency factor depending on the mean rainfall amount of the area of study. The nomograms suggested in the WMO (1986) manual to determine the k_m value for various rainfall durations and mean daily rainfall were derived from the data collected mostly in the USA, and therefore, their application in other places could be sometimes not suitable enough (Řezáčová et al. 2005). In order to adapt the WMO recommendations to their own area of study, some researchers (Rakhecha et al. 1992; Rakhecha and Soman 1994; Koutsoyiannis and Baloutsos 2000; Casas et al. 2008) found an appropriate envelope curve of the observed frequency factor values plotted against the mean rainfall amounts for several stations of their area to obtain reliable estimates of the PMP. In this work, an envelope curve of the frequency factor k_m has been developed based on the actual daily

rainfall data of 258 stations in Spain; a total of 10,194 station-years of annual maximum rainfall. This curve could be useful to estimate suitable values of 1-day PMP at any point of the Iberian Peninsula from basic statistical parameters (mean and standard deviation) of its rainfall series. In case sub-daily PMP values are of interest, there is no need of additional curves as those suggested in the WMO manual for durations lower than 24 h, because such additional curves can be derived from the local IDF relationship (Koutsoyiannis 1999). Assuming that this relationship can be formulated with duration and return period as separate variables, the frequency factor k_m for a certain return period should not depend on duration (Koutsoyiannis 1999); so, the k_m value calculated for 24 h can be used to estimate sub-daily PMP (Casas et al. 2008; Pérez-Zanón et al. 2015). Moreover, the PMP should satisfy the equivalent scaling relationship found for rainfall intensities at the site of interest. In the present work, the scaling behavior of PMP estimates in Madrid for several durations between 5 min and 24 h has been analyzed.

2. Methodology

Rainfall data used in this study were recorded by two instruments located at the Parque del Retiro of Madrid, at 667 m of altitude, and were provided by the AEMET: daily data from a totalizer rain gauge from 1951 to 2012, and the digitalized data from a tipping bucket rain gauge recording between 1940 and 2012. The data from the tipping bucket gauge were compiled as series of 5-min rainfall amounts between 1940 and 1999 and maximum rainfall amounts for durations from 10 min to 72 h for everyday between 1940 and 2012. All these series were considered stationary since no significant trends (p value <0.05) were detected by a Mann–Kendall test (Yue et al. 2002). From the tipping bucket data set, the annual maximum rainfall series for several durations between 5 min and 72 h were obtained. These amounts are true maxima because they were calculated by the use of unrestricted temporal intervals, without the established beginning and ending times of the common fixed intervals. Among the diverse theoretical functions tested to analyze their statistical distribution providing satisfactory fit for the data, a function previously applied (Ferrer and Ardiles 1994; INM 2003; INM 2007; Ballesteros-Cánovas et al. 2013) for the statistical analysis of annual maximum daily rainfall series in Spain, the square-root exponential type extreme distribution SQRT-ET-max proposed by Etoh et al. (1987), was chosen. The distribution function of the SQRT-ET-max is expressed by Eq. (4),

$$F(x) = \exp [-k (1 + \sqrt{\alpha x}) \exp (-\sqrt{\alpha x})] \quad 4$$

with x being the maximum rainfall amount and k and α the frequency and scale parameters which were estimated for each series using a method of moments developed by Zorraquino (2004). All the fittings made using the SQRT-ET-max passed the Kolmogorov–Smirnov test at a significance level of 0.05.

2.1. Determination of the IDF master equation of Madrid

From the fitted SQRT-ET-max distributions, the maximum rainfall amounts for every duration t and return periods T of 2, 5, 10, 25, 50, 100, 200, and 500 years have been estimated (Table 1), as well as the IDF values $I(t, T)$. Among the different empirical functions for IDF curves, one of the most commonly used to describe rainfall intensity-duration relationship $I(t)$ for a given return period T is the Sherman equation (5) (Chen 1983; Chow et al. 1988; Casas et al. 2004; Pérez-Zanón et al. 2015),

$$I(t) = \frac{a}{(b + t)^c} \quad 5$$

where a , b , and c are the constant parameters. A generalized IDF formula involving various return periods and durations can be obtained (Chen 1982; Casas et al. 2004) fitting the Sherman equation (2) to values of the ratio $\frac{I(t,T)}{I(60,T)}$ between the maximum intensity values $I(t,T)$ for each return period T and every duration t (in minutes), and the 1-h maximum intensity $I(60,T)$ for the same return period. Since parameter c can be related with the temporal scaling properties of rainfall (Pérez-Zanón et al. 2015), its value has been determined first. In terms of IDF values, the scaling relationship (1) between rainfall intensity for any duration t and its corresponding hourly intensity for the same return period leads to Eq. (6):

$$I(t, T) = I(60, T) \left(\frac{t}{60} \right)^\beta \quad 6$$

Table 1

Estimated maximum rainfall intensity (in mm/h) for every duration t and return period T

	T (years)							
t (min)	2	5	10	25	50	100	200	500
5	53.3	78.0	96.6	123	144	166	190	224
10	39.3	55.3	67.2	83.8	97.1	111	126	147
15	31.5	43.7	52.8	65.3	75.5	86.1	97.4	113
20	27.2	38.0	46.1	57.2	66.2	75.7	85.8	99.9
30	20.7	29.6	36.3	45.6	53.1	61.0	69.5	81.3
45	15.2	21.7	26.5	33.3	38.8	44.6	50.7	59.4
60	12.5	17.3	20.8	25.6	29.6	33.7	38.0	44.2
90	9.2	12.8	15.4	19.0	21.9	24.9	28.2	32.7
120	7.7	10.4	12.4	15.1	17.3	19.6	22.0	25.3
240	4.8	6.4	7.5	9.1	10.4	11.7	13.2	15.1
360	3.7	4.7	5.5	6.5	7.3	8.2	9.1	10.4
480	3.0	3.9	4.6	5.5	6.2	7.0	7.8	8.9
600	2.6	3.4	4.0	4.8	5.5	6.2	6.9	7.9
720	2.2	2.9	3.4	4.1	4.6	5.2	5.8	6.6
1080	1.7	2.2	2.6	3.1	3.6	4.0	4.5	5.1
1440	1.4	1.8	2.2	2.6	2.9	3.3	3.7	4.2

AQ5

The exponent c in Eq. (6) can be identified as $-\beta$ if $b = 0$ in the Sherman equation (5). Casas et al. (2004) and Rodríguez et al. (2014) obtained a value of b around 10 min for the city of Barcelona, which for durations longer than 1 h can be considered small. The scaling exponent β has been calculated from the Retiro Observatory records using the statistical moments of their annual maximum rainfall series, as proposed by Menabde et al. (1999). A simple scaling regime between 25 min and 72 h has been found for these series (Fig. 1), with a value of the scaling exponent of $\beta = -0.73 \pm 0.01$. Then, ratio $\frac{I(t,T)}{I(60,T)}$ has been fitted to the Sherman equation (5) with the parameter c fixed to 0.73. The fitting parameters a and b resulted as $a = 21.3 \pm 0.2$, close to the value $(1/60)^{-0.73}$ of Eq. (6), and $b = 3.0 \pm 0.2$ min. Thus, the fitting by the Sherman equation can be expressed as Eq. (7),

$$\frac{I(t,T)}{I(60,T)} = \frac{21.3}{(3.0 + t)^{0.73}} \quad 7$$

Fig. 1

Statistical moments for different values of q of the annual maximum intensity of the Retiro series, as a function of the duration (between 5 min and 72 h). The *fitted straight lines* indicate scale invariance over a range from 25 min to 72 h. Their slopes determine the scale function $K(q)$. The *inserted figure* shows a linear regression between $K(q)$ and the order q of the moments

where $I(t, T)$ is the intensity (in mm/min) for a duration t (min) with a return period of T (years). Equation 8 expresses the logarithmic dependence of the intensity of reference $I(60, T)$ (in mm/min) with the return period T (in years).

$$I(60, T) = 0.218 \log T + 0.13 \quad 8$$

The combination of Eqs. (7) and (8) leads to the master IDF equation for the Retiro Observatory (Eq. (9)).

$$I(t, T) = \frac{4.6 \log T + 2.8}{(3.0 + t)^{0.73}} \quad 9$$

Figure 2 shows curves for 2-, 5-, 10-, 25-, and 100-year return periods. The master equation (9) fits satisfactorily the IDF points listed in Table 1 with an average relative error of 6% for all durations and return periods.

Fig. 2

Intensity-duration-frequency values $I(t,T)$ (in mm/h) and fitted curves for durations t between 5 min and 24 h and return periods T from 2 to 100 years

2.2. Estimation of the probable maximum precipitation

As commented in Sect. 1, an envelope curve to describe the dependence between the frequency factor k_m and the mean daily rainfall amount in different locations of the Iberian Peninsula has been determined in order to obtain reliable estimates of PMP. This curve should be an adaptation to the area of study of the nomograms proposed by the WMO (1986). With an analogous proposal, Casas et al. (2008) found an envelope curve to estimate the 1-day PMP values at 145 stations of Catalonia (Spain) from their annual maximum daily rainfall series. In this work, an updated and extended version of the envelope curve obtained by Casas et al. (2008) has been calculated adding the $(\bar{x}_n, k_m)$ points corresponding to 113 more AEMET stations in Spain (Fig. 3). Figure 4 shows the envelope curve drawn with the help of the seven upper points (thick dots in the figure) related to the stations which have maximum k_m versus $\bar{x}_n$ values.

Four of these seven stations are placed in coastal locations of the Mediterranean Sea: 8019 (Alicante, $k_m = 10.4$, $\bar{x}_n = 46.3$ mm), 7031 (San Javier, Murcia, $k_m = 8.7$, $\bar{x}_n = 63.8$ mm), 6155A (Málaga, $k_m = 7.7$, $\bar{x}_n = 75.4$ mm), and g0433 (Cadaquès, $k_m = 5.1$, $\bar{x}_n = 104.4$ mm), whereas the other three are in mountainous places near the Pyrenees at heights higher than 850 m: g9584 (Puigcerdà, $k_m = 8.7$, $\bar{x}_n = 64.0$ mm), b0079 (La Pobla de Lillet, $k_m = 7.3$, $\bar{x}_n = 80.6$ mm), and g0311 (Camprodon, $k_m = 5.0$, $\bar{x}_n = 103.6$ mm). The equation of the envelope curve is as follows:

$$k_m = 19.6 \exp(-0.0128 \bar{x}_n) \quad 10$$

Fig. 3

Location of the AEMET stations over Spain used in this study. *White*, the seven stations with maximum frequency factor k_m versus the mean value. *Black*, the Retiro Observatory of Madrid

Fig. 4

Frequency factor k_m versus the mean daily precipitation $\bar{x}_n$. Envelope frequency factor curve for Spain, fitted to the seven upper points (*thick dots*) corresponding to stations 8019, 7031, 6155A, g0433, g9584, b0079, and g0311. *Dark dots*, corresponding to the Retiro Observatory of Madrid

As the envelope curve proposed by the WMO (1986) and Rakhecha and Soman (1994), Eq. (10) provides values of the frequency factor depending decreasingly with the mean daily rainfall from a maximum value close to 20 (19.6 in this case). Applied to all the considered stations of the Iberian Peninsula, it assigns values of this factor between 4.5 and 14.4.

Using the proposed envelope curve, the PMP in Madrid has been estimated from records of the two different instruments located in the Retiro Observatory mentioned before: daily registers from a totalizer rain gauge between 1951 and 2012 and the series of true maxima recorded by a tipping bucket pluviograph between 1940 and 2012. A 62-year annual maximum daily rainfall series were obtained from the daily records of the totalizer gauge, with a mean daily rainfall of 33.5 mm and a standard deviation of 10.7 mm. Taking into consideration Eq. (10), the frequency factor assigned to this mean value is $k_m = 12.8$, which results in a first estimate of the 1-day PMP for Madrid using Eq. (2) of 192 mm. From the pluviograph records, a second statistical estimation of the PMP in Madrid has been done. In this case, since annual series of true maximum rainfall for durations from 5 min to 72 h had been calculated previously, the PMP has been estimated not only for 24 h but for sub-daily durations also. The statistical parameters and the observed frequency factors of these series for 20 durations between 5 min and 24 h have been shown in Table 2. The mean value of the true annual maximum rainfall in 24 h registered in the period 1940–2012 in Madrid-Retiro was 35.6 mm (see Table 2). As

expected, this mean value resulted slightly higher than that of 33.5 mm obtained from the totalizer gauge. An adjusting multiplicative factor of 1.13 (WMO 1986) has been considered for the annual maximum rainfall amounts, which were registered in a single fixed time interval, to approximately result in true maximum values which would be obtained with unrestricted temporal intervals. Thus, the mean true rainfall of 35.6 mm in 24 h would correspond, dividing by the adjusting factor 1.13, to approximately 31.5 mm for a 24-h fixed interval. From Eq. (10), a frequency factor of 13.1 is assigned to this mean value of 31.5 mm, very similar to that obtained using the totalizer records (12.8), and higher than any of the observed frequency factors of Table 2, varying from 3 to 6.8. The highest values of these factors have been found for durations from 8 to 16 h, with a maximum of 6.8 for approximately 12 h. The frequency factor k_m for a given return period should not depend on duration (Koutsoyiannis 1999); so, the highest value 13.1 has been taken as a maximized frequency factor common for all durations. Using Eq. (2), the PMP for every duration t can be calculated (Table 3, Fig. 5). The estimated value of the 1-day PMP for Madrid has resulted 187 ± 19 mm, an amount with a return period of 100,000 years according to the master IDF Eq. (9) obtained in Sect. 2.1, at the low end of the PMP range considered by the NRC (1994). The return periods of the sub-daily PMP estimates range between 10^5 and 10^6 years. The uncertainty of the PMP values, approximately 10% for every of the considered durations, has been determined using a technique proposed by Salas et al. (2014) for PMP statistical estimates.

Table 2

Mean $\bar{x}_n$, standard deviation σ_n , maximum value x_M , and observed frequency factor k_m of the annual maximum rainfall series of the Retiro Observatory for durations t from 5 min to 24 h

t (min)	$\bar{x}_n$ (mm)	σ_n (mm)	x_M (mm)	k_m
5	4.4	2.2	10.4	3.0
10	6.8	3.0	16.0	3.7
15	8.3	3.5	18.3	3.1
20	9.7	4.2	22.5	3.4

t (min)	$\overline{x_n}$ (mm)	σ_n (mm)	x_M (mm)	k_m
30	11.4	5.2	32.8	4.8
45	12.5	5.8	35.3	4.7
60	13.6	5.6	36.0	4.5
90	15.3	6.3	36.2	3.7
120	16.8	6.3	37.1	3.4
240	20.7	7.5	50.5	4.8
360	23.5	7.4	57.1	5.4
480	25.9	8.5	65.4	6.0
600	28.0	9.5	74.5	6.6
720	28.9	9.5	78.4	6.8
840	30.9	10.0	80.3	6.7
960	32.1	10.3	80.6	6.2
1080	33.1	10.8	80.6	5.5
1200	34.3	11.2	81.6	5.2
1320	35.3	11.4	82.0	5.0
1440	35.6	11.6	83.8	5.0

Table 3

PMP values in the Retiro Observatory for every considered duration t in minutes

t (min)	PMP (mm)	Error (mm)
5	33	4
10	46	5
15	54	6
20	65	7
30	80	9
45	88	11
60	87	9
90	98	12
120	100	11
240	119	14
360	120	12
540	138	16
600	152	18
720	153	16
840	162	18
960	167	19
1080	174	20

t (min)	PMP (mm)	Error (mm)
1200	181	21
1320	185	21
1440	187	19

Fig. 5

PMP values for durations between 5 min and 24 h

3. Discussion

The IDF values for Madrid-Retiro found in this work have been obtained from a rainfall data series 13-year longer than that considered in the last previous studies estimating IDF for this station (INM 2003 ; INM 2007 ; Salas and Fernández

2007), which means a 23% increase in the number of data. The present IDF values have resulted almost coincident to those published by the DGC (1999) for 24 h, and very similar to those obtained by the AEMET (INM 2003): a slight decrease of 9% on average for durations between 20 min and 12 h for return periods longer than 50 years, and 5% for shorter periods, whereas differences resulted lower than 4% for 24 h and all the return periods considered. Higher differences can be found, 18% on average, comparing to IDF values obtained from the method of regionalization of Salas and Fernández (2007): for durations shorter than 30 min the present IDF values resulted 20% higher on average for all return periods, while for longer durations 6 and 12 h resulted 18% lower and for 24 h 11% lower on average for all return periods. In this case, some of the discrepancies can be attributed to the different methodologies; the regionalization method of IDF laws used by Salas and Fernández (2007) and the present at-site method.

The application of the envelope curve of the frequency factor k_m obtained in Sect. 2.2 clearly improves the estimation of PMP that can be made using the nomograms of the WMO (1986) in locations of the Iberian Peninsula, which is usually too high. It provides reasonable PMP estimates for all the considered stations of the Iberian Peninsula, excluding some of that used to draw the curve, for which the estimated PMP, as expected, results a bit low. In terms of the ratio of the estimated PMP using the envelope curve to the maximum observed value of the sample, in front of the values around 2 found for most of the stations, 2.3 on average, for three of the seven enveloping stations (g0433, g9584, and b0079) values between 1.15 and 1.18 have resulted. These ratio values can be interpreted as the maximum observed values in these locations achieved more than the 80% of the PMP estimated using the envelope curve. Anyway, in some arid places of the Iberian Peninsula, the PMP provided by this envelope is probably still a bit high and should be considered as a first estimate. The effort to find a suitable division of the Iberian Peninsula with different frequency factor curves is an issue to be dealt with in future work. In the case of Madrid-Retiro, the value around 190 mm estimated for the 1-day PMP can be considered a reliable estimation since its return period resulted to be 100,000 years and its ratio to the maximum observed value (83.8 mm) resulted to be 2.3.

Regarding the sub-daily PMP values estimated for Madrid-Retiro, it is worth to mention that, as expected, these values satisfy the same scale behavior as found in Sect. 2.1 for intensities. Equation (6) can be expressed in terms of the scaling relationship between the 1-day PMP and any sub-daily PMP (t) with duration t in minutes as in Eq. (11):

$$PMP(t) = PMP(1440) \left(\frac{t}{1440} \right)^{\beta+1} \quad 11$$

Thus, considering the first estimation of 192 mm obtained from the daily records of the totalizer gauge as the 1-day PMP in Madrid and the scaling exponent of $\beta = -0.73$ found in Sect. 2.1, Eq. (11) results in Eq. (12), a relationship accomplished by PMP values of Table 3 with a coefficient of linear correlation of 0.98 between the logarithms of both variables PMP (in mm) and duration t (in min).

$$PMP(t) = 27t^{0.27} \quad 12$$

This was an expected result since the magnitude $PMP(t)/t$ can be considered as an intensity with a very high return period; a return period which should be common to all durations because the same frequency factor was considered for all them (Koutsoyiannis 1999). Applying the generalized IDF equation (9), the return period of the PMP values can be estimated to be around 100,000 years between 2 and 24 h.

4. Conclusions

A simple scaling regime between 25 min and 72 h, with a scaling exponent of $\beta = -0.73 \pm 0.01$, has been observed for the maximum rainfall intensities series obtained from records of a tipped bucket rain gauge located in the Retiro Observatory of Madrid between 1940 and 2012. A master equation of the IDF relationship in Madrid which takes into consideration the simple scaling formulation has been obtained. This master equation ($I(t, T) = \frac{4.6 \log T + 2.8}{(3.0 + t)^{0.73}}$, with t in minutes, I in mm/min, and T in years) fits satisfactorily the IDF values calculated from the frequency analysis of the Retiro rainfall series, with an average relative error of 6% for all the durations and return periods considered.

The same simple scaling behavior has been observed also for the estimated values of PMP in Madrid for durations

between 5 min and 24 h. For this estimation, an envelope curve for the relationship between the frequency factor k_m and the mean annual maximum daily rainfall $\bar{x}_n$ ($k_m = 19.6 \exp(-0.0128 \bar{x}_n)$) has been developed for the Iberian Peninsula using a total of 10,194 station-years of annual maximum rainfall from daily rainfall data recorded in 258 Spanish locations. This curve, an adaptation for our region of the nomograms suggested by the WMO (1986) manual, could provide reliable 1-day PMP estimates at any point of the Iberian Peninsula from basic statistical parameters (mean and standard deviation) of its daily rainfall series. In particular, the 1-day PMP in Madrid has been estimated as 187 ± 19 mm, an amount with a return period of 100,000 years according to the master IDF equation, whereas the return periods of the sub-daily PMP estimates range between 10^5 and 10^6 years. The uncertainty of the PMP values has resulted approximately 10% for every of the considered durations.

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Compliance with ethical standards

Conflicts of interest The authors declare that they have no conflicts of interest.

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