A model for sand ridges on the shelf: Effect of tidal and steady currents

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Abstract. The dynamics of sand ridges, as observed on the innershelves and midshelves of various coastal seas, are investigated within the framework of an idealized model. These seabed undulations include shoreface-connected sand ridges and tidal sand banks. The new aspects in this study are the explicit incorporation of both steady and tidal currents, a shelf with a sloping bottom, and a statistical approach to describe the sediment transport during storms (linear in the current) and fair weather conditions (cubic in the current). The present model is used to gain understanding of the different ridge characteristics in different geographical regions. This is done by analyzing the initial growth of small bottom perturbations, which evolve on a basic state describing a longshore uniform flow over a reference topography. It is found that basically five different bottom modes can be excited: shoreface-connected sand ridges, Coriolis ridges, friction-induced bars, trapped tidal ridges, and tidal sand banks. Their growth, which is due to different physical mechanisms, strongly depends on the weather climate and the geometrical characteristics of the shelf.

1. Introduction

On the inner shelf and midshelf of many coastal seas (in water depths between 5 and 30 m) bed forms are observed which have a rhythmic structure in the longshore direction. They include shoreface-connected sand ridges, which occur on storm-dominated inner shelves, and tidal sand banks, which appear on midshelves where tidal currents are stronger than 0.5 m s^{-1} . The typical alongshore spacing of these ridges is between 5 and 10 km, and their height is in the range 1–10 m.

Early observations of shoreface-connected ridges were made on the eastern American continental shelf by *Duane et al.* [1972] and *Swift et al.* [1972]. Later on they extended their work to other inner shelves [*Swift et al.*, 1978]. The ridges are also found along the Brazilian

shelf [*Figuieredo et al.*, 1982], the Argentinian shelf [*Parker et al.*, 1982], the Canadian shelf [*Hoogendoorn and Dalrymple*, 1986], the central Dutch coast [*van de Meene*, 1994; *van de Meene et al.*, 1996], and the German coast [*Antia*, 1996]. In these ridge areas, significant net longshore currents are present (typically of the order of 0.5 m s^{-1}) which are driven by winds, tides, and longshore pressure gradients. A striking property is that the crests of the ridges are always rotated against this net current. In other words, the seaward ends of the crests are shifted upstream with respect to their shoreface attachment, thereby forming an angle of 10° – 30° with the coastline. Furthermore, the ridges appear to migrate downstream with typical velocities of 10 m yr^{-1} , with a large scatter depending on local conditions.

The tidal sand banks have dimensions similar to those of the shoreface-connected ridges, but the former only occur when tidal currents are stronger than approximately 0.5 m s^{-1} . They are observed for example on the midshelf of the North Sea [*Stride*, 1982; *Pattiaratchi and Collins*, 1987] but not on the microtidal U.S. shelf. Two essential differences with the shoreface-connected ridges are that the crests of tidal sand banks are always cyclonically rotated (anticlockwise on the North-

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Figure 1. Large-scale bed forms in the North Sea [from *van de Meene*, 1994].

ern Hemisphere) with respect to the dominant tidal current direction and that these ridges do not seem to migrate. As an example, Figure 1 shows the location of both tidal sand banks and shoreface-connected ridges in the North Sea. Note that the tidal sand banks and the shoreface-connected ridges have different orientations with respect to the coastline.

With regard to the origin of sand ridges, different hypotheses have been formulated in the past. It has sometimes been suggested that they are geological relicts; see, e.g., the discussion of *McBride and Moslow* [1991]. Although this may be partly true for some ridges, observations [*Swift et al.*, 1978; *van de Meene*, 1994] show that the U.S. and Dutch ridges are active under the present hydrodynamic conditions. This suggests that their behavior is determined by the interactions between the water motion, the transport of sediment, and the erodible bed.

These considerations motivate the study of idealized models, which only describe a limited number of processes, to identify which physical mechanisms are responsible for the generation of bed forms. Such models are useful tools to investigate the hypothesis that the latter form owing to an inherent instability of a basic state of the coupled water bottom system. In such linear stability models the bottom perturbations are assumed to have an infinitesimally small amplitude which evolves on a basic state describing a longshore flow over a given

reference topography. The success of this method has already been demonstrated in many other cases: sea ripples [*Vittori and Blondeaux*, 1992], bars in rivers [*Schielen et al.*, 1993; *Seminara*, 1995, and references herein], nearshore oblique and transverse bars [*Falqués et al.*, 1996], estuarine bars [*Schuttelaars and de Swart*, 1999; *Seminara and Tubino*, 1998], crescentic bars [*Vittori et al.*, 1999], etc.

For tidal sand banks, such models have been developed and analyzed by *Huthnance* [1982], *Hulscher et al.* [1993], and *de Swart and Hulscher* [1995]. The water motion is modeled by depth-averaged shallow water equations; the basic state describes a pure tidal current over a flat bottom, and a bed load sediment transport formulation is used. The formation of sand banks appears to be due to the combined effect of residual circulations, which develop owing to tide-topography interactions [*Zimmerman*, 1981], and a sediment flux which is a "faster than linear" function of the current velocity (as occurs during fair weather conditions).

With regard to shoreface-connected sand ridges, separate models have been studied. Essential differences with the tidal sand bank model are that the basic state describes a steady current over a shelf with a transverse slope (with respect to the coastline). The first model of this kind was presented by *Trowbridge* [1995]: by analyzing the morphologic instability of the basic state current he was able to model and explain the up-current

rotation of the ridges. Later on, significant physical modifications of the model were added by *Falqués et al.* [1998a, 1998b]. They explicitly modeled the momentum balance of the basic state flow and demonstrated the important role of a longshore pressure gradient and wind for the ridge formation. Incorporation of the effect of local bed slopes on the sediment transport resulted in the prediction of preferred modes having the fastest growth rate. The spacings and migration speeds of these modes appeared to be in agreement with observations along the U.S. and Dutch inner shelf. The main causes for the formation of shoreface-connected ridges are the transverse slope of the inner shelf, in combination with a sediment flux which is linear in the current (as occurs during storm conditions). However, for the North Sea ridges the model is not entirely suitable, since no tidal effects are incorporated.

Therefore in the present paper a morphodynamic model will be discussed which explicitly describes the interaction between a flow, consisting of both steady and tidal components, with the erodible bottom. Another new aspect is the incorporation of a statistical description of the weather conditions. The first objective is to investigate the effect of tidal currents on the dynamics of shoreface-connected sand ridges. Second, it will be demonstrated that the model also allows for different types of bed forms (including tidal sand banks) and that the selection of the most preferred mode strongly depends on the storm fraction, the intensity of the tidal currents, and the geometrical characteristics of the shelf. These results will also be explained in terms of different physical mechanisms.

The organization of the paper is as follows. Equations of motion and boundary conditions are presented in section 2. This is followed by a discussion in section 3 of a reference state, which describes a longshore uniform flow with both steady and tidal components. The stability properties of this morphodynamic equilibrium with respect to small bottom perturbations are analyzed in section 4. Results are presented in section 5, and the final section contains the discussion and conclusions.

2. Governing Equations and Boundary Conditions

Although the vertical structure of the currents can have an important role in the ridge area, it is hypothesized that a depth-averaged model already can describe the ridge formation. So we will consider the 2DH shallow water momentum equation

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} + f \vec{e}_z \times \vec{v} = -g \vec{\nabla} z_s - \mu \frac{\vec{v}}{z_s - z_b} + \frac{\vec{\tau}^*}{\rho(z_s - z_b)} \quad (1)$$

and the mass conservation equation (1)

$$\frac{\partial(z_s - z_b)}{\partial t} + \vec{\nabla} \cdot [(z_s - z_b) \vec{v}] = 0. \quad (2)$$

Here $\vec{v}$ is the depth-averaged velocity, f is the Coriolis parameter, $\vec{e}_z$ is a unity vector in the vertical direction, z_s and z_b are the free surface and the bottom elevation, ρ is the water density, $\vec{\tau}^*$ is the wind stress, g is the acceleration due to gravity, and μ is a bottom friction coefficient.

Note that a linear, rather than a quadratic, bed shear stress formulation is used. The motivation for this choice is that it captures most of the essential properties of the bottom friction and at the same time keeps the subsequent analysis as simple as possible. For purely tidal currents the concept of this Lorentz linearization procedure of the bed shear stress is discussed by *Zimmerman* [1992] and has been applied in the tidal sand bank model by *Hulscher et al.* [1993]. In the case of steady currents it has been demonstrated by *Falqués et al.* [1998b] that, with regard to the formulation for shoreface-connected ridges, results are not sensitive to replacing a quadratic bottom stress by a linear stress formulation.

The bottom evolution is governed by a sediment conservation equation:

$$\frac{\partial z_b}{\partial t} + \vec{\nabla} \cdot \vec{q} = 0, \quad (3)$$

where $\vec{q}$ stands for the volumetric sediment flux per unit width. A parameterization for this transport will be given later on.

The equations are considered on a half-open domain, as sketched in Figure 2. An orthogonal coordinate system is taken, such that $x = 0$ marks the transition from inner shelf to shoreface, which is assumed to be parallel to the straight coastline. The x , y , and z axes thus point in the cross-shore, longshore, and vertical directions, respectively. Here $z = 0$ represents the still water level and $\vec{\nabla} = (\partial/\partial x, \partial/\partial y)$. The boundary conditions imposed are that the cross-shore flow component u vanishes at $x = 0$ and far offshore (i.e., $x \rightarrow \infty$) and that the bed level z_b has a fixed value at these locations. This means that there is no net water exchange between the inner shelf and the shoreface.

Figure 2. Sketch of the geometry and coordinate system. For explanation of the symbols, see the text.

The equations of motion are made dimensionless by introducing the transformations

$$\begin{aligned} t &= \sigma^{-1} t', & (x, y) &= L_S(x', y'), & z_b &= L_V z'_b, \\ \vec{v} &= U \vec{v}', & z_s &= \frac{U \sigma L_S}{g} z'_s, & \vec{q} &= Q \vec{q}', \end{aligned}$$

with σ ($\sim 10^{-4} \text{ s}^{-1}$) the tidal frequency, L_S ($\sim 10 \text{ km}$) the characteristic width of the inner shelf, L_V ($\sim 15 \text{ m}$) the depth at the transition $x = 0$, U ($\sim 0.5 \text{ m s}^{-1}$) a characteristic magnitude for the current, and Q ($\sim 10^{-4} \text{ m}^2 \text{ s}^{-1}$) a scale for the volumetric sediment flux per unit width.

If we drop the primes in the variables for convenience, the nondimensional momentum and mass conservation equations read

$$\begin{aligned} \frac{\partial \vec{v}}{\partial t} + \lambda(\vec{v} \cdot \vec{\nabla})\vec{v} + \hat{f}\vec{e}_z \times \vec{v} = \\ - \vec{\nabla} z_s - \frac{r \vec{v}}{F^2 z_s - z_b} + \frac{\vec{\tau}_s}{F^2 z_s - z_b} \end{aligned} \quad (4)$$

$$\frac{\partial(F^2 z_s - z_b)}{\partial t} + \lambda \vec{\nabla} \cdot ((F^2 z_s - z_b)\vec{v}) = 0, \quad (5)$$

where $\lambda = U/(\sigma L_S)$, $\hat{f} = f/\sigma$, $r = \mu/(\sigma L_V)$, $F^2 = \sigma L_S/\sqrt{g L_V}$, and $\vec{\tau}_s = \vec{\tau}_s^*/(\rho \sigma U L_V)$. Typical values of the parameters are $\lambda \sim 0.4$, $\hat{f} \sim 0.8$, $r \sim 0.5$, and $|\vec{\tau}_s| \sim 0.1$. Note that the Froude number is very small ($F^2 \sim 0.01$); therefore the free surface terms which are measured by F^2 can be neglected in equations (4) and (5).

Now we turn to the sediment mass conservation equation (3). By using the previous scaling its dimensionless form reads (primes are again dropped)

$$\frac{\partial z_b}{\partial t} + \frac{Q}{\sigma L_V L_S} \vec{\nabla} \cdot \vec{q} = 0.$$

Since Q is in the range ($10^{-4} - 10^{-5}$) $\text{m}^2 \text{ s}^{-1}$, it follows that the parameter which measures the magnitude of the sediment flux divergence is very small. Thus the bottom does not evolve on the tidal timescale σ^{-1} , but rather on the morphological timescale $T_m = L_V L_S/Q$, which is much longer (decades to centuries). Consequently, the time derivative of the bed level z_b in equation (5) can be neglected. Furthermore, the bottom evolution is not determined by the oscillatory sediment fluxes during the tidal cycle, but rather by the tidally averaged fluxes. Hence the final dimensionless bottom evolution equation becomes

$$\frac{\partial z_b}{\partial \tau} + \vec{\nabla} \cdot \langle \vec{q} \rangle = 0, \quad (6)$$

where angle brackets denote an average over the tidal period and $\tau = t/T_m$ is a slow nondimensional time coordinate.

Regarding the sediment transport, we use formulations derived from the basic concept already introduced by *Bailard* [1981]. We will distinguish the situations for storms, fair weather, and a combined state.

2.1. Storms

Suspended load sediment transport dominates over bed load transport, and wave-induced velocities are large in comparison with the current velocity. Under these weather conditions the suspended load contribution of *Bailard* [1981], in nondimensional form, becomes

$$\vec{q}_1 = \vec{v} - \gamma_1 \vec{\nabla} h', \quad (7)$$

with a scale of the volumetric sediment flux Q_1 . Here h' is the difference between the actual bed level and that of a reference bottom, as will be discussed in the next section. A sediment transport linear in the depth-averaged velocity was already used by *Trowbridge* [1995], while *Falqués et al.* [1998b] also take into account the bed slope effect, measured by parameter γ_1 . The physics underlying this formula is that sediment is stirred by waves and transported by currents.

2.2. Fair weather

Sediment transport takes place mainly as bed load and wave stirring is negligible. The nondimensional bed load contribution of *Bailard* [1981] reads

$$\vec{q}_3 = |\vec{v}|^3 \left(\frac{\vec{v}}{|\vec{v}|} - \gamma_3 \vec{\nabla} h' \right), \quad (8)$$

with a scale of the volumetric sediment flux Q_3 . This parameterization was used by *Hulscher et al.* [1993]. In this case the influence of local bed slopes is measured by parameter γ_3 .

2.3. Combined state

To deal with a long-term situation in which both periods of fair and storm weather conditions are present, the previous transports are combined in a probabilistic way:

$$\vec{q} = \beta \vec{q}_1 + \epsilon(1 - \beta) \vec{q}_3. \quad (9)$$

Here β is the fraction of time during which storms occur. The scale of the volumetric sediment flux is Q_1 , and ϵ is the ratio between scale of the sediment flux during fair weather and storm conditions, i.e., $\epsilon = Q_3/Q_1$.

Note that the sediment transport is expressed in terms of the state variables of the system. Typical values for the parameters Q_1 , γ_1 , Q_3 , γ_3 , and β will be given in section 5 (results).

3. Basic State

The model appears to have a basic state which describes a longshore current $\vec{v} = (0, V(x, t))$ over a sloping bottom $z_b = -H(x)$. In this state the current is allowed to vary with time, but the divergence of the net sediment transport must vanish so that the bottom does not vary on the morphological timescale. In this case the sediment and the mass conservation equations are automatically obeyed so that we only have to consider the momentum equations (4) which become

$$-\hat{f}V = -\frac{\partial \xi}{\partial x} + \frac{\tau_{sx}}{H}, \quad \frac{\partial V}{\partial t} = -\frac{\partial \xi}{\partial y} - r\frac{V}{H} + \frac{\tau_{sy}}{H}, \quad (10)$$

where $z_s = \xi(x, y, t)$ is the free surface elevation.

Note that in this model the longshore pressure gradient $\partial \xi / \partial y$ must be prescribed. The condition that the velocity profile must be longshore uniform and the solvability condition for the free surface imply that this forcing term can only be a function of time. Here we choose

$$\frac{\partial \xi}{\partial y} = s_0 - |s_1| \cos(t),$$

which represents a steady and one tidal component. From this the longshore flow $V(x, t)$ can be computed from the second equation in (10), whereas the first equation in (10) determines the cross-shore setup or setdown of the water level due to cross-shore winds and Ekman effects. It is assumed that the wind stress is constant and that the longshore wind stress and longshore pressure gradient drive a current in the same direction. This implies that the parameters τ_{sy} and s_0 have opposite signs.

Upon writing the basic state velocity as a superposition of a steady and a time-dependent current, i.e., $V(x, t) = V_0(x) + V_1(x, t)$, the solution reads

$$V(x, t) = \alpha U_0(x) + (1 - |\alpha|) U_1(x) \sin[t + \varphi(x)], \quad (11)$$

with

$$U_0(x) = 1 + a[H(x) - 1], \quad U_1(x) = \frac{H\sqrt{1+r^2}}{\sqrt{H^2+r^2}},$$

$$\alpha = \frac{-s_0 + \tau_{sy}}{r}, \quad \varphi(x) = \arctan\left(\frac{r}{H}\right),$$

and

$$a = \frac{|s_0|}{|-s_0 + \tau_{sy}|}.$$

Here we have assumed that

$$|\alpha| + \frac{1}{\sqrt{1+r^2}}|s_1| = 1, \quad (12)$$

which states that the velocity scale U is defined as the maximum flow amplitude at the shoreface boundary $x = 0$. This quantity can be computed for a given longshore pressure gradient, longshore wind stress, tidal frequency, and water depth L_V at $x = 0$, because the applied scaling yields that

$$s_n = \frac{gs_n^*}{\sigma U} \quad (n = 0, 1), \quad \tau_{sy} = \frac{\tau_{sy}^* / \rho}{\sigma U L_V},$$

where asterisks refer to dimensional variables.

The coefficient α is the dimensionless amplitude of the steady current at the transition $x = 0$ from the inner shelf to the shoreface. It also determines the relative contribution of the steady current to the total flow, i.e., $-1 \leq \alpha \leq 1$. In general, the value of this parameter during storms will differ from that during fair weather.

The steady flow is driven both by wind and a longshore pressure gradient; the relative effect of the latter to the total steady flow is measured by parameter a ($0 \leq a \leq 1$).

4. Linear Stability Analysis

The next step is to investigate the possible formation of bed forms, with a rhythmic structure in the alongshore direction, which evolve on the basic state discussed in the previous section. Thus we consider solutions of the form

$$\vec{v} = (u', V + v'), \quad z_s = \xi + \eta', \quad z_b = -H + h'.$$

Here the perturbations u' , v' , and η' in the water motion are functions of x , y , t , and τ , whereas the bottom perturbation h' only depends on x , y , and τ . Owing to their structure and the imposed boundary conditions, they describe solutions with a periodic structure in the longshore direction, e.g., $u'(x, y, t) = \text{Re} [\tilde{u}(x, t, \tau) \exp(iky)]$, where k is the wave number and Re denotes the real part of a solution.

The linearized equations for the amplitudes of the perturbations read

$$\frac{\partial \tilde{u}}{\partial t} + ik\lambda V \tilde{u} - \hat{f} \tilde{v} = -\frac{\partial \tilde{\eta}}{\partial x} - \frac{r}{H} \tilde{u} + \frac{\tau_{sx}}{H^2} \tilde{h} \quad (13a)$$

$$\frac{\partial \tilde{v}}{\partial t} + \lambda \left(\frac{\partial V}{\partial x} \tilde{u} + ikV \tilde{v} \right) + \hat{f} \tilde{u} =$$

$$-ik\tilde{\eta} - \frac{r}{H} \tilde{v} + \frac{s_0}{H} \tilde{h} - rV_1 \frac{\tilde{h}}{H^2} \quad (13b)$$

$$H \frac{\partial \tilde{u}}{\partial x} + \frac{dH}{dx} \tilde{u} + ikH \tilde{v} - ikV \tilde{h} = 0 \quad (13c)$$

$$\frac{\partial \tilde{h}}{\partial \tau} + \beta \langle \tilde{S}_1 \rangle + \epsilon(1 - \beta) \langle \tilde{S}_3 \rangle = 0. \quad (13d)$$

Here

$$\tilde{S}_1 = \frac{\partial \tilde{u}}{\partial x} + ik\tilde{v} - \gamma_1 \left(\frac{\partial^2 \tilde{h}}{\partial x^2} - k^2 \tilde{h} \right)$$

and

$$\tilde{S}_3 = |V|^2 \left[\frac{2}{V} \frac{\partial V}{\partial x} \tilde{u} + \frac{\partial \tilde{u}}{\partial x} + 3ik\tilde{v} \right.$$

$$\left. - \gamma_3 |V| \left(\frac{3}{V} \frac{\partial V}{\partial x} \frac{\partial \tilde{h}}{\partial x} + \frac{\partial^2 \tilde{h}}{\partial x^2} - k^2 \tilde{h} \right) \right]$$

denote the amplitude of the perturbed sediment flux divergence during storms and fair weather, respectively. We remark that the hydrodynamic problem for a specific reference bottom $H(x)$ and bottom perturbation $\tilde{h}$ (the so-called “flow over topography” (FOT) problem), as defined by equations (13a)-(13c), is solved separately for the two different flow regimes occurring in the model, i.e., storms and fair weather conditions with

frequencies β and $1 - \beta$, respectively. Ultimately, these results are combined in a statistical sense in the bottom evolution equation (13d) to solve for the bottom perturbations.

The amplitude equations have solutions of the type

$$[\tilde{u}(x, t, \tau), \tilde{v}(x, t, \tau), \tilde{\eta}(x, t, \tau), \tilde{h}(x, \tau)] = e^{\omega\tau} [u(x, t), v(x, t), \eta(x, t), h(x)], \quad (14)$$

where ω is a complex frequency which describes the evolution of the amplitudes on the morphological timescale and is to be determined from the dynamics. Its real part, $\Omega_r = \text{Re}[\omega]$, is the growth rate of the perturbations, and $c = -\text{Im}[\omega]/k$ is the corresponding phase speed.

First we analyze the dynamics on the short tidal timescale. From cross-differentiation of the momentum equations (13a) and (13b) and substituting (14), we obtain the Fourier-transformed vorticity equation. An expression for the alongshore velocity perturbation as a function of h and u follows from the continuity equation (13c):

$$v = \frac{V}{H}h + \frac{i}{k} \left(\frac{\partial u}{\partial x} + \frac{1}{H} \frac{dH}{dx} u \right). \quad (15)$$

Substitution of this expression in the Fourier-transformed vorticity equation yields a single equation for the cross-shore velocity u as a function of the bottom perturbation h :

$$\begin{aligned} \mathcal{U}_{12} \frac{\partial^3 u}{\partial t \partial x^2} + \mathcal{U}_{11} \frac{\partial^2 u}{\partial t \partial x} + \mathcal{U}_{10} \frac{\partial u}{\partial t} \\ + \mathcal{U}_{02} \frac{\partial^2 u}{\partial x^2} + \mathcal{U}_{01} \frac{\partial u}{\partial x} + \mathcal{U}_{00} u = \mathcal{H}_1 \frac{\partial h}{\partial x} + \mathcal{H}_0 h, \end{aligned} \quad (16)$$

where the coefficients are specified in Appendix A.

The linearized bottom evolution equation (13d) becomes, after substitution of (14) and (15),

$$\omega h = -\beta \langle S_1 \rangle - \epsilon(1 - \beta) \langle S_3 \rangle, \quad (17)$$

where

$$S_1 = -\frac{1}{H} \frac{dH}{dx} u + ik \frac{V}{H} h - \gamma_1 \left(\frac{\partial^2 h}{\partial x^2} - k^2 h \right) \quad (18a)$$

$$\begin{aligned} S_3 = |V|^2 \left[\left(-\frac{3}{H} \frac{dH}{dx} + \frac{2}{V} \frac{dV}{dx} \right) u - 2 \frac{\partial u}{\partial x} + 3ik \frac{V}{H} h \right. \\ \left. - \gamma_3 |V| \left(\frac{\partial^2 h}{\partial x^2} + \frac{3}{V} \frac{dV}{dx} \frac{\partial h}{\partial x} - k^2 h \right) \right]. \end{aligned} \quad (18b)$$

Together with the boundary conditions $u(0) = u(\infty) = 0$, $h(0) = h(\infty) = 0$, equations (16)–(18) define an eigenvalue problem which determines the complex frequency ω of the perturbations as a function of the wave number k and the model parameters. This problem is solved by a numerical spectral method (collocation in x , Fourier-Galerkin in t ; see Calvete [1999] for further details).

The physical interpretation of the different terms in (18) is as follows. The terms multiplied by the coefficients γ_1 and γ_3 describe a diffusive decay of the bed forms due to bed slope effects. The term in equations (18) preceding the diffusive contributions are proportional to ih and therefore describe a migration of bed forms. The remaining contributions are potential sources of instabilities, and they appear to be of three different kinds. The first term in equations (18) only contribute to the bottom evolution if the reference topography contains a cross-shore gradient. In case of linear transport (S_1), which is considered by Trowbridge [1995], this is the only source of morphologic instabilities. In case of a faster than linear transport (S_3) there are additional contributions. The second term in S_3 is active if the basic flow contains a cross-shore shear ($\partial V / \partial x \neq 0$). The third term in S_3 is due to spatial gradients in the cross-shore flow perturbations. This mechanism also appears in the tidal morphological models of Huthnance [1982] and Hulscher *et al.* [1993].

5. Results

5.1. Parameter Values: Dutch Shelf

The required input of the numerical model (called MORFO30) consists of the shelf and flow characteristics of the reference state. It is important to realize that many of the model parameters generally have different values during storms than during fair weather. In the next subsection the values are chosen such that they are representative for the situation on the shelf along the central Dutch coast.

First, we will specify default values for the part of the input which is independent of the flow regime, i.e., the tidal frequency σ , the Coriolis parameter f , and the shelf characteristics. Near the Dutch coast the frequency of the M_2 tide is $\sigma = 1.4 \times 10^{-4} \text{ s}^{-1}$, and the Coriolis parameter at 52° N is $f = 1.12 \times 10^{-4} \text{ s}^{-1}$; consequently, the nondimensional parameter $\hat{f} = 0.8$. Furthermore, the inner shelf has a width $L_S = 12 \times 10^3 \text{ m}$ and a rather constant slope, the water depth at the transition inner shelf-shoreface is $L_V = 15 \text{ m}$, and the outer shelf has a constant depth of 20 m. This motivates the choice of the following dimensionless reference topography:

$$H(x) = \begin{cases} 1 + sx & \text{if } 0 \leq x < 1 \\ 1 + s & \text{if } x \geq 1, \end{cases}$$

where $s = s_* L_S / L_V$ and s_* is the shelf slope; here $s = 0.33$ is considered.

Regarding the hydrodynamic parameters, the tidal velocity amplitude has been taken as $U_T = 0.45 \text{ m s}^{-1}$ from van de Meene [1994], and the bottom friction coefficient was taken as $\mu = 8 \times 10^{-4} \text{ m s}^{-1}$ during both weather conditions, which is consistent with Lentz *et al.* [1999]. The nondimensional friction parameter then reads $r = 0.38$. During storm the steady velocity amplitude has been taken as $U_S = 0.35 \text{ m s}^{-1}$

Figure 3. Contour plot of the largest growth rate of perturbations as a function of wave number k and parameter α , which defines the relative contribution of the steady flow to the total flow. Storm-dominated case; for parameter values, see the text.

which yields a value of the total velocity amplitude of $U = 0.80 \text{ m s}^{-1}$ and a default $\alpha = -0.44$ and $\lambda = 0.48$. During fair weather conditions the steady velocity amplitude has been reduced to $U_S = 0.05 \text{ m s}^{-1}$; then $U = 0.50 \text{ m s}^{-1}$, $\lambda = 0.30$, and $\alpha = -0.10$.

As far as sediment transport is concerned, parameters have been computed from the formulation of *Bailard* [1981] for medium sand ($d_{50} = 0.4 \text{ mm}$) with porosity $p = 0.4$. In case of storms the suspended load contribution has been averaged over a wave period for waves which propagate almost parallel to the current and which have an orbital velocity $u_w = 1.0 \text{ m s}^{-1}$. The scale of the volumetric sediment flux (including the pores) becomes $Q_1 = 8.8 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$, the corresponding morphological timescale $T_M = 65 \text{ years}$, and the bed slope parameter $\gamma_1 = 9.4 \times 10^{-5}$. During fair weather, waves do not contribute to the bed load transport, and the resulting parameters are $Q_3 = 7.7 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$, $T_M = 750 \text{ years}$, and $\gamma_3 = 1.8 \times 10^{-3}$.

From the values given above it follows that parameter $\epsilon = 8.7 \times 10^{-2}$.

5.2. Storms

Here we investigate the model behavior during storm conditions. This means that in the bottom evolution equation (6) the flux $\bar{q}$ is taken as $\bar{q}_1$, given in (7). Test runs showed that in this case, accurate results are obtained with 60 collocation points and 3 Fourier modes in time (representing the principal tidal component and two overtones).

First, experiments have been carried out with all parameters having their default values, except that the tidal parameter α is varied. Since the total velocity scale U is kept fixed, this implies that the intensity of both the steady and the tidal current is varied. In Figure 3 a contour plot is shown of the maximum growth rates of the perturbations in the k - α parameter space.

In Figure 4 the growth rates and migration speeds are shown versus the wave number for different but fixed values of α : -0.8 , -0.4 , 0 , 0.4 , and 0.8 . Each growth rate curve has a clear maximum which corresponds to the preferred wave number for this set of parameter values. Other experiments (not shown) have demonstrated that the results shown are quite generic for the model: they are only weakly sensitive to variations in the parameters r , f , and λ .

These results show that maximum growth rates and migration speeds decrease with decreasing values of $|\alpha|$. This means that by increasing the relative contribution of tides to the total velocity field (keeping the velocity scale U fixed), the instability process becomes less effective. However, a different conclusion is reached in case the steady velocity amplitude $U_S (\equiv |\alpha|U)$ is kept fixed and then the tidal velocity amplitude $U_T (\equiv (1 - |\alpha|)U)$ is increased. It then turns out that growth rates hardly depend on the tidal current amplitude, while migra-

Figure 4. (left) Growth rate Ω_r versus wave number k for different values of parameter α ; storm-dominated case. Other parameters as in the previous plot. (right) Like the left panel, but for migration speed c .

Figure 5. Spatial patterns of the most preferred bed forms for different values of parameter α ; other parameter values as in Figure 3: (a) $\alpha = -0.4$, $k = 4.8$; (b) $\alpha = 0.0$, $k = 2.6$; (c) $\alpha = 0.4$, $k = 1.6$. Bars and pools are indicated by solid and dashed lines, respectively.

tion speeds decrease with increasing tidal velocity amplitudes.

In case of the Dutch shelf ($\alpha = -0.44$) the maximum growth rate is attained for a dimensionless wave number $k = 8.5$, which corresponds to a dimensional longshore wavelength of about 9 km. The spatial pattern of this preferred bed form is shown in Figure 5a, together with the perturbed tidally averaged velocity. Note that the pattern resembles that of shoreface-connected ridges which are trapped to the inner shelf. In particular, the seaward end of the crestlines are shifted upstream with respect to their shoreface attachments. This preferred bottom mode has a nondimensional growth rate of 0.05, which corresponds to an e -folding timescale of approximately 1400 years. Furthermore, its dimensionless migration speed $c = -0.5$, which implies a dimensional migration speed of about 80 m s^{-1} . Both the spatial pattern and e -folding timescales are realistic and agree with those found by Trowbridge [1995]. In the latter study it was also shown that the occurrence of these bottom modes is due to a mechanism related to the transverse slope of the reference bottom. The latter causes the flow to have an offshore deflection over the bars, which can also be observed in Figure 5.

The differences between the model of Trowbridge [1995] and the present model during storm conditions is that also bottom friction, Coriolis forces, tides, and the effect of the bottom slopes on the sediment transport are accounted for. Apparently, tides and vorticity-generating forces do not have a strong influence on the behavior of the bed forms during storms. Including the slope term in the sediment transport causes the model to have a preferred longshore wavelength of the bottom perturbations, which agrees well with the observed spacings of shoreface-connected ridges on the Dutch shelf [van de Meene et al., 1996]. In contrast, the migration speeds are a factor of 10 too large. This discrepancy can be lifted by taking account of the fact that storms only occur for a certain fraction of the time. However, this correction will also cause the e -folding timescale to become unrealistically large. Apparently, the present model is not able to give correct values of both the e -folding timescale and the migration speed.

From Figures 3-5 it can be seen that, for a fixed value of $|\alpha|$, the results strongly depend on the direction of the steady flow. For positive α values, upcurrent-rotated bed forms are obtained, but their growth rates and wavelengths are much larger than those found for neg-

ative α values. It is important to realize at this point that all shoreface-connected ridges in the field occur for negative α/f . This corresponds to downwelling conditions in which the Ekman transport is directed onshore and a setup of the sea level is established.

Another interesting result is that in case $\alpha = 0$ (tides only) there is positive feedback between the tidal current and the erodible bottom. According to Figure 3 these growing modes seem to be connected to those which appear for positive α . This result differs from the conclusions stated by *Huthnance* [1982] and *Hulscher et al.* [1993] that a faster than linear sediment transport is required in order to generate bed forms due to tide-topography interaction. However, they considered a reference flow over a flat bottom, whereas in the present model the inner shelf has a transverse slope. In Appendix B it is demonstrated, by using physical arguments, that indeed the presence of the transverse slope is responsible for the instability process in this case. The preferred bottom mode in case of pure tides has a longshore wavelength of about 20 km, which is close to the tidal excursion length $U_T T$ (U_T is the tidal current amplitude and T is the tidal period). Furthermore, the bar crests are rotated cyclonically with respect to the tidal current direction, and anticyclonic residual circulations occur around the bars. The centers of these circulation cells are slightly shifted in the negative y direction, with respect to the bar crests, such that there is an offshore deflection of the net current over the bars.

Finally, we remark that the results are rather sensitive to parameter a in the steady flow, which indicates the relative contribution of longshore pressure gradients in driving the reference flow. If also the forcing due to a longshore wind stress is taken into account, growth rates become substantially smaller. We will not pursue this aspect here but simply state that quite realistic ridges are obtained in the case that forcing by a longshore pressure gradient is used.

5.3. Fair Weather

In this case the sediment transport is cubic in the instantaneous flow velocity; thus $\bar{q}$ in equation (6) is given by $\bar{q}_3$ of equation (8). Experiments revealed that in this case more collocation points were required to obtain converging solutions. The results described below have been computed for 120 collocation points and 3 tidal harmonics. The default situation is discussed in section 5.1 and is representative of the Dutch shelf during fair weather conditions.

In Figure 6 a contour plot is shown of the largest growth rate of the perturbations as a function of wave number k and parameter α . Note that a dimensionless growth rate of 1 now corresponds to a dimensional e -folding timescale of 750 years. A remarkable difference with the storm-dominated case (compare with Figure 3) is that this plot is nearly symmetric with respect to the line $\alpha = 0$. Hence the direction of the steady current is

Figure 6. Contour plot of the maximum growth rate of perturbations as a function of wave number k and parameter α , which defines the relative contribution of the steady flow to the total flow. Fair weather conditions; for parameter values, see the text.

no longer important for the characteristics of the resulting bed forms. Furthermore, in this plot clearly three different instability regimes can be identified. The first occurs for large values of $|\alpha|$ (nearly steady flow) and is characterized by wave numbers $k \sim 3$, i.e., longshore spacings of approximately 25 km. A second regime occurs for moderate values of $|\alpha|$ (of order 0.5) and has wave numbers $k \sim 8$. The final instability regime is observed for small values of $|\alpha|$ (dominant tidal flow) and has wave numbers $k \sim 10$. The characteristic nondimensional migration speeds (not shown) are of order 1 if $|\alpha| \sim 1$ (these bed forms migrate about 16 m yr^{-1}) and they linearly decay to zero with decreasing values of $|\alpha|$.

In Figure 7, contour plots of the most preferred bed forms, together with the perturbed tidally averaged velocity field, are shown for several characteristic preferred modes. For $|\alpha|$ close to 1 the most preferred mode is trapped to the coast, and its crests are rotated cyclonically with respect to the dominant current direction. Its formation is related to vorticity production due to Coriolis torques which are exerted if a steady flow is moving over this bar [see *van de Meene*, 1994]; hence it has been called a Coriolis bar. The mode shown in Figure 7d occurs on the outer shelf and is characterized by a series of alternating shoals and pools. This type of bed form is related to vorticity production induced by frictional torques and is also observed in the case of pure steady flow [*Falqués et al.*, 1996]; it is called a friction-induced bar.

The third instability regime is linked to the presence of tidal currents, and the corresponding bed forms resemble tidal sand banks, as can be seen in Figures 7b and 7c. Remarkably enough there are two different modes with almost the same growth rates. One corresponds to the classical offshore tidal sand bank which was already described by *Huthnance* [1982] and *Hulscher et al.* [1993]; it occurs on the outer shelf, the

Figure 7. Spatial patterns of the most preferred bed forms for different values of parameter α ; other parameter values as in Figure 6: (a) $\alpha = -0.8$, $k = 2.8$; (b) $\alpha = 0.0$, $k = 10.0$, a trapped tidal sand ridge; (c) $\alpha = 0.0$, $k = 10.0$, an offshore tidal sand bank; (d) $\alpha = -0.4$, $k = 7.6$. Bars and pools are indicated by solid and dashed lines, respectively.

crests are rotated cyclonically with respect to the tidal current direction, and anticyclonic residual circulations exist around the bars. The other mode appears to be a tidal sand ridge which is trapped to the inner shelf and has cyclonically rotated crests.

Another remarkable difference with the storm-dominated case is that the results are not strongly dependent on the value of the parameter a in the steady current profile. If the value of a is reduced (i.e., increasing influence of longshore wind stress with respect to longshore

Figure 8. Contour plot of the maximum growth rate of perturbations as a function of wave number k and the storm fraction parameter β . Here parameters are indicated for the shelf along the central Dutch coast; see the text.

pressure gradient), growth rates weakly decrease, but they are still substantially large, even in the case $a = 0$. Thus the results discussed here are rather representative for the model; also the sensitivity with respect to the parameter λ^{-1} is rather weak. Furthermore, the sizes of the different instability regions in plots like Figure 6 depend on the choices for the friction and Coriolis parameter. However, no new phenomena are detected if the values of these coefficients are varied.

5.4. Probabilistic Mode

In this subsection the model is used as a tool to obtain information about the dynamics of bed forms in a probabilistic sense. Thus a shelf is considered on which stormy conditions occur during a time fraction β , whereas in the remaining periods (time fraction $1 - \beta$) the weather is calm. Thus $\bar{q}$ in equation (6) is given by expression (9). Once again we remark that the physical conditions (like wave intensity, steady currents, type of sediment transport) during storms and fair weather strongly differ.

Results are again presented for the default (Dutch) shelf discussed in section 5.1. Figure 8 shows the contour plot of the largest growth rates as a function of k and storm fraction β . In the regime $0 \leq \beta < 0.6$ the spatial pattern of the preferred modes is similar to that shown in Figures 7b and 7c (i.e., tidal sand ridges and tidal sand banks). In the regime $0.8 < \beta \leq 1$ the preferred mode resembles a pattern of shoreface-connected sand ridges, similar to that shown in Figure 5a.

The bed forms in the transitional regime ($0.6 \leq \beta \leq 0.8$) (not shown) have either the mixed characteristics of a trapped tidal ridge and shoreface-connected ridge or that of pure offshore tidal sand banks (without morphologic activity on the inner shelf). Also for other parameter settings, representing a storm-dominated regime and a fair weather tidal regime, identical results were obtained. Apparently, according to this model, the observed presence of both offshore tidal sand banks

and shoreface-connected ridges cannot be attributed to one preferred mode, which is forming during different weather conditions, but is rather due to the presence of different bottom modes which simultaneously develop and have almost equal growth rates.

6. Discussion and Conclusions

In this paper an idealized morphodynamic model has been analyzed to study the joint interaction of a steady and tidal flow with the erodible bed on the inner and outer shelf. The first objective was to find out which types of large-scale alongshore rhythmic bottom patterns can form as inherent free instabilities of the coupled water-bottom system. Further aims were to understand the underlying physics of these bed forms and to relate their characteristics to those of morphological features observed in the field.

The water motion in the model is described by the depth-averaged shallow water equations, and for the sediment transport a local parameterization is used. The reference flow consists of a steady part and a time oscillatory part (representing an M_2 tide). Furthermore, a probabilistic concept is used in the sense that the model distinguishes between storms and fair weather, during which the sediment transport is linear and cubic in the instantaneous velocity, respectively. During storms the sediment is stirred by the waves and transported by the current. Here spatially uniform wave conditions have been assumed; thus parameters like the wave stirring coefficient, bed slope coefficient γ_1 , and bottom friction parameter during storms are assumed to be constant. In the storm mode the model is a generalization of that studied by *Trowbridge* [1995] in the sense that the effect of bottom slopes in the sediment transport is accounted for. Furthermore, vorticity-generating forces (due to bottom friction and Earth rotation) and tides are incorporated. In the fair weather mode the model resembles the *Huthnance* [1982] model, but the proximity of the coast and the sloping bottom of the inner shelf are accounted for.

It appears that the model allows for basically five different types of bed forms: shoreface-connected ridges, Coriolis-induced ridges, friction-induced bars, offshore tidal sand banks, and trapped tidal ridges. They occur for different values of the tidal parameter α and the storm fraction parameter β , as is summarized in Table 1.

The formation of shoreface-connected ridges, which are trapped to the inner shelf and have upcurrent rotated crestlines, occurs most effectively during storms and with a strong steady flow component. Their formation is related to the presence of the transverse slope of the inner shelf, as was already explained by *Trowbridge* [1995]. If for a fixed steady current the amplitude of the tidal current is increased, the growth rates are hardly affected, migration speeds decrease, long-shore spacings become longer, and the orientation of the crestlines changes from upcurrent to cyclonic. For

Table 1. Classification of the Five Different Types of Preferred Bottom Modes Predicted by the Model^a

Weak Tides		Strong Tides
Storm dominance	shoreface-connected ridges	trapped tidal ridges
Fair weather	Coriolis ridges	trapped tidal ridges
dominance	friction-induced bars	tidal sand banks

^aStrong and weak tides refer to values of the $|\alpha|$ parameter close to 0 and 1, respectively. Fair weather dominance and storm dominance refer to values of the β parameter close to 0 and 1, respectively.

strong tidal currents the preferred bed form becomes a trapped tidal sand ridge, a feature which has not been described in earlier studies. It owes its existence to the shift of tidal residual circulations due to the transverse slope of the inner shelf, as is explained in Appendix B of this paper.

According to the model, Coriolis-induced ridges mainly form during calm weather and dominant steady current; with increasing tidal influence (or increasing the value of the bottom friction coefficient), friction-induced alternate bars are predicted. These two modes are modifications of bed forms which were already identified and explained by *Falqués et al.* [1998a]. For strong tidal currents and a faster than linear sediment transport, offshore tidal sand banks occur, which were modeled by *Huthnance* [1982] and *Hulscher et al.* [1993].

The ultimate structure of the bed forms strongly depends on the value of the storm fraction parameter β . For β smaller than a critical value β_c the preferred bed forms primarily form during calm weather conditions. For our simulations (Dutch shelf) the value of β_c is about 0.7; this depends on the choice of the morphological timescales during calm weather and storms. For β somewhat larger than β_c the preferred bed forms have more of the characteristics of storm-dominated features. In the transition regime $\beta \sim \beta_c$ the spatial pattern of the bottom modes has characteristics of both types of bed forms. This suggests that the simultaneous presence of offshore tidal sand banks and shoreface-connected ridges on the shelf along the central Dutch coast could be understood from the characteristics of one preferred mode, which is forming during different weather conditions. However, a series of model experiments has shown that, in case of the Dutch shelf, the simultaneous presence of different bottom modes can only be explained as the result of the growth of different bottom modes which have almost identical growth rates.

It is remarkable that the model predicts five different types of preferred bottom modes, whereas only two of them (shoreface-connected ridges and offshore tidal sand banks) are well known from field observations. This is probably due to the fact that the parameter values for which the other bed forms are most favorably excited correspond to situations which usually do not

occur in nature. On the other hand, the accurate data are relatively scarce, since they have been mainly obtained on either storm-dominated shelves, characterized by strong steady currents (like the U.S. and Argentinian shelves), or on shelves where calm weather prevails together with strong tidal currents (like the North Sea).

The characteristics of the preferred modes in the fair weather mode and for strong tidal current are in satisfactory agreement with other model results [*Huthnance*, 1982; *Hulscher et al.*, 1993] and with field data on the Dutch inner and outer shelves. The spatial pattern of the modeled preferred bed forms during storms, and weak tides also compare well with those found with other models [*Trowbridge*, 1995] and with observed shoreface-connected sand ridges. Also realistic e -folding timescales (about 1500 years) are obtained in case of continuous storms, albeit that the migration speed of the ridges on the Dutch inner shelf are a factor of 10 too large. A good agreement is found by taking a realistic storm fraction, but this causes the e -folding timescale to become rather large. Apparently, the model is not yet able to give in all circumstances both a satisfactory e -folding timescale and correct migration speed. Another result is that the model only yields realistic shoreface-connected ridges in case the reference flow is forced by a longshore pressure gradient. Although there are indications that longshore pressure gradients on the inner shelf are important [cf. *Lentz et al.*, 1999] field data also indicate that the influence of the longshore wind stress cannot be neglected.

Another aspect that needs further attention is the investigation of the sensitivity of model results to incorporation of a second tidal harmonic. The reference flow in the present model consists of a steady component and one tidal constituent; overtides are only generated internally owing to tide-topography interaction. However, both observations and models of tidal currents in the North Sea [cf. *Shinha and Pingree*, 1997] demonstrate that the water motion along the central part of the Dutch coast also has a strong external M_4 component.

Finally, we remark that the present study is limited by the linear analysis which only governs the initial formation of the bed forms. Recently, *Calvete et al.* [1999] developed a model describing the long-term behavior of

shoreface-connected ridges in the steady current regime. They have shown that it is possible to simulate the amplitude behavior and that the results compare rather well with measured ridge amplitudes in the field. It will be worthwhile to extend this analysis such that the dynamics during calm weather periods and the effects of tides are also accounted for. Hence it seems that the present model setup and results contribute to a further understanding of the dynamics of sand ridges on the inner and outer shelves.

Appendix A: Coefficients of the FOT Problem

The coefficients which appear in equation (16) are

$$u_{12} = 1$$

$$u_{11} = \frac{H_x}{H}$$

$$u_{10} = -k^2 + \frac{H_{xx}}{H} - \frac{H_x^2}{H^2}$$

$$u_{02} = \frac{r}{H} + ik\lambda V$$

$$u_{01} = ik\lambda V \frac{H_x}{H}$$

$$u_{00} = r \left(\frac{-k^2}{H} + \frac{H_{xx}}{H^2} - \frac{2H_x^2}{H^3} \right) + ik\lambda \left[-k^2 V - V_{xx} \right.$$

$$\left. + \frac{V_x H_x + V H_{xx} + (\hat{f}/\lambda) H_x}{H} - \frac{V H_x^2}{H^2} \right] \\ \mathcal{H}_1 = -\lambda \frac{k^2 V^2}{H} + ik \left[\frac{V_t}{H} + \frac{r(2V - \alpha U_0)}{H^2} - \frac{s_0}{H} \right] \\ \mathcal{H}_0 = \lambda k^2 \left[\frac{-2V V_x - (\hat{f}/\lambda) V}{H} + \frac{V^2 H_x}{H^2} \right] - k^2 \frac{\tau_{sx}}{H^2} \\ + ik \left\{ \frac{V_{tx}}{H} - \frac{V_t H_x}{H^2} + \frac{s_0}{H^2} H_x \right. \\ \left. + r \left[\frac{2V_x - \alpha U_{0x}}{H^2} - \frac{2(2V - \alpha U_0) H_x}{H^3} \right] \right\}.$$

Here a subscript denotes differentiation with respect to that variable, except for the parameter τ_{sx} , which denotes the cross-shore component of the wind stress.

Appendix B: Physical Mechanism of Trapped Tidal Sand Ridges

The formation of trapped tidal sand ridges resembles that of tidal sand banks in the sense that tidal residual circulations play an important role. The important difference is that in this case the reference bottom has a transverse slope.

For a ridge on a flat bottom, *Zimmerman* [1981] has shown that, owing to the tidal flow moving over a cy-

Figure B1. (a) Situation sketch (top view) of a water parcel moving over a cyclonically rotated bar on a shelf with a transverse slope. The flow bends toward the crest owing to continuity effects. At the crest the upper part of the parcel is in deeper water than its lower part. Thus the Coriolis force experienced by the parcel is slightly smaller on the upper part than at the lower part and directed to the right; hence a positive torque occurs (indicated by solid arrows). Likewise, since the left part of the parcel is in shallower water than the right part, the frictional force on the left side is slightly larger than that on the right (indicated by dashed arrows). Consequently, also the friction force induces a positive torque. Since velocity component v is positive in this case, the vorticity flux is positive at the crest. (b) Net vorticity flux induced by the transverse slope, which causes a shift in the negative y direction of the anticyclonic residual circulation (indicated by the dashed contours). Consequently, $u > 0$ above the crest, and instability occurs.

clonically rotated ridge, vorticity is generated by both Coriolis and frictional torques. Both torques exactly vanish at the crest. This results in an anticyclonic residual circulation (clockwise in the Northern Hemisphere) around the ridges which follows the topographic contours. Subsequently, it was shown by Huthnance [1982] that, if the sediment transport is a faster than linear function of the instantaneous flow velocity, a positive feedback between the water motion and the bottom occurs such that the amplitude of the ridge will grow.

However, if the undisturbed water depth is increasing in the offshore direction, both the Coriolis torque and frictional torque are such that the net vorticity flux above the crest is positive; see Figure B1a. Together with the vorticity flux generated owing to the sole effect of the ridge (i.e., as if it were located on a flat bottom), it follows that the net vorticity flux due to the transverse slope causes the center of the anticyclonic residual circulation to shift in the negative y direction, as is indicated in Figure B1b. Consequently, a positive correlation between the offshore flow component u and the perturbed bottom h occurs, and thus the transverse slope mechanism causes a net convergence of sediment above the crests, similar to the case of shoreface-connected ridges. This shift in the residual circulation and the corresponding positive correlation between u and h are clearly visible in Figure 5b.

The mechanism described above is less effective for anticyclonically rotated ridges, because in that case the Coriolis and frictional torques have opposite signs, such that the tidal residual circulations are much weaker.

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