Effects of atmospheric compensation techniques on the performance of synchronous receivers

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ABSTRACT

We present recent studies on the impact of atmospheric turbulence on free-space links using either synchronous or nonsynchronous detection. We compare options for atmospheric compensation, including phase compensation and diversity combining techniques. We consider the effects of log-normal amplitude fluctuations and Gaussian phase fluctuations, in addition to local oscillator shot noise, for both passive receivers and those employing active modal compensation of wave-front phase distortion. We compute error probabilities for M -ary phase-shift keying, and evaluate the impact of various parameters, including the ratio of receiver aperture diameter to the wave-front coherence diameter, the number of diversity channels, and the number of modes compensated.

Keywords: Atmospheric turbulence; Statistical optics; Adaptive optics; Optical communications.

1. INTRODUCTION

Evaluating the performance of a heterodyne or homodyne receiver in the presence of atmospheric turbulence is generally difficult because of the complex ways turbulence affects the coherence of the received signal that is to be mixed with the local oscillator. The downconverted heterodyne or homodyne power is maximized when the spatial field of the received signal matches that of the local oscillator. Any mismatch of the amplitudes and phases of the two fields will result in a loss in downconverted power (Fig. 1). The use of adaptive compensation of atmospheric wave-front phase distortions to improve the performance of atmospheric systems has been an important field of study for many years. Phase-compensated receivers offer the potential for overcoming atmospheric limitations by adaptive tracking of the beam wave-front and consequent correction of atmospherically induced aberrations. In particular, the modal compensation method involves correction of several modes of an expansion of the total phase distortion in a set of basis functions.

Here, we study in a unified framework the effects of both phase-front distortion and amplitude scintillation on the performance of synchronous (coherent) and nonsynchronous (noncoherent) receivers utilizing phase-front compensation and diversity-combining techniques. First, we have defined a mathematical model for the received signal after propagation through the atmosphere. By noting that the downconverted signal current can be characterized as the sum of many contributions from different coherent regions within the aperture, we have shown that the probability density function (PDF) of this current can be well-approximated by a modified Rice distribution. In our model, the parameters describing the PDF depend on the turbulence conditions and the degree of modal compensation applied to the receiver. Second, we analyze how diversity-combining of several replicas of the transmitted signal, each corrupted independently by the atmosphere, can provide more reliable communication because of the low probability of deep fades in all the diversity channels. Finally, we have computed the error probability for digital signals in the presence of multiplicative noise from atmospheric turbulence and additive white Gaussian noise (AWGN).

2. STATISTICAL MODEL OF OPTICAL HETERODYNE DETECTION

When a received signal experiences atmospheric turbulence during transmission, both its envelope and its phase fluctuate over time. In the case of coherent modulation, phase fluctuations can severely degrade performance unless measures are taken to compensate for them at the receiver. Here, we assume that after homodyne or heterodyne downconversion is

Fig. 1. A coherent free-space optical communication system is affected by atmospheric turbulence in two ways. First, turbulence induces a decrease of the mean received power level, which translates into a diminished signal-to-noise ratio in the receiver. Second, amplitude scintillation and phase distortion in the receiver plane act as intense sources of noise distorting the quality of the optical signal available for processing. These two effects combine to deteriorate the performance of the system. Phase compensation and diversity combining methods are two techniques that, at least in principle, can mitigate the effects of atmospheric turbulence on system performance.

used to obtain an electrical signal, the receiver is able to track any phase fluctuations caused by turbulence (as well as those caused by laser phase noise), performing ideal coherent (synchronous) demodulation. Under this assumption, analyzing the receiver performance requires knowledge of only the envelope statistics of the downconverted electrical shot-noise signal.

The classical theory of communications was developed originally in the context of linear channels with additive noise. For an additive white Gaussian noise (AWGN) channel, with average power constraint P , noise power spectral density $N_0/2$ and bandwidth B , the SNR is $\gamma_0 = P/N_0B$. The SNR γ_0 for a quantum-limited signal can be interpreted as the detected number of photons (photocounts) per symbol, when the symbol period is $1/B$. Coherently detected signals are modeled as narrowband RF signals with additive white Gaussian noise (AWGN). In free-space optical communication through the turbulent atmosphere, we must consider fading channels, which are a class of channels with multiplicative noise. In the fading AWGN channel with average power constraint P and noise power spectral density $N_0/2$, we let α^2 denote the atmospheric channel power fading and $(P/N_0B)\alpha^2 = \gamma_0\alpha^2$ denote the instantaneous received SNR per symbol. For a shot-noise-limited coherent optical receiver, the SNR can be interpreted as the number of signal photons detected on the receiver aperture γ_0 multiplied by a heterodyne mixing efficiency α^2 . For systems with perfect spatial mode matching the heterodyne mixing efficiency is equal to 1. When the spatial modes are not properly matched, the contribution to the current signal from different parts of the receiver aperture can interfere destructively and result in a reduced instantaneous heterodyne mixing and consequent fading.

Conditional on a realization with gain coefficient α , the atmospheric optical channel is an AWGN channel with instantaneous received SNR $\gamma = \gamma_0\alpha^2$, which is, itself, random. The statistical properties of the atmospheric random channel fade α^2 , with probability density function (PDF) $p_{\alpha^2}(\alpha^2)$, provide a statistical characterization of the SNR $\gamma = \gamma_0\alpha^2$ and, consequently, of the maximal spectral efficiency achievable for the free-space optical link. We have already modeled the impact of atmospheric turbulence-induced phase and amplitude fluctuations on free-space optical links using synchronous detection and found that the SNR γ for a single monolithic-aperture coherent receiver is described by a noncentral chi-square probability distribution function (PDF) with two degrees of freedom [1]:

$$p_{\gamma}(\gamma) = \frac{1+r}{\bar{\gamma}} \exp(-r) \exp\left[-\frac{(1+r)\gamma}{\bar{\gamma}}\right] I_0\left[2\sqrt{\frac{(1+r)r\gamma}{\bar{\gamma}}}\right], \quad \gamma \geq 0, \quad (1)$$

Fig. 2. When a received signal experiences atmospheric turbulence during transmission, both its envelope and its phase fluctuate over time. In the case of coherent modulation, phase fluctuations can severely degrade performance unless measures are taken to compensate for them at the receiver. Here, we assume that after homodyne or heterodyne downconversion is used to obtain an electrical signal, the receiver is able to track any phase fluctuations caused by turbulence (as well as those caused by laser phase noise), performing ideal coherent (synchronous) demodulation. Under this assumption, analyzing the receiver performance requires knowledge of only the envelope statistics of the downconverted electrical signal.

where the average SNR (or average detected photocounts) $\bar{\gamma}$ and the parameter $1/r$ is described below. As the scheme in Fig. 2 shows, the model leading to the PDF in Eq. (1) is based on the observation that the downconverted signal current can be characterized as the sum of many contributions from N different coherent regions within the aperture [1]. In this model, the signal is characterized as the sum of a constant (coherent) term and a random (incoherent) residual halo. The contrast parameter $1/r$ is a measure of the strength of the residual halo relative to the coherent component. The parameter r ranges between 0 and ∞ . It can be shown that when the constant term is very weak ($r \rightarrow 0$), turbulence fading makes the SNR to become negative-exponential-distributed, just as in a speckle pattern. Likewise, when the dominant term is very strong ($r \rightarrow \infty$), the density function becomes highly peaked around the mean value $\bar{\gamma}$, and fading becomes negligible. In Fig. 3, we show the Rician density function as a function of the normalized SNR $\gamma / \bar{\gamma}$ and the Rician factor r .

In order to assess the impact of turbulence on the heterodyne mixing and fading, the field amplitude without the effect of turbulence in the pupil plane must be modified by a multiplicative factor $\exp[\chi(\mathbf{r}) - j\phi(\mathbf{r})]$ where $\chi(\mathbf{r})$ and $\phi(\mathbf{r})$ represent the log-amplitude fluctuations (scintillation) and phase variations (aberrations), respectively, introduced by atmospheric turbulence. Consequently, both $\bar{\gamma}$ and $1/r$ are described in terms of log-normal amplitude fluctuations and Gaussian phase fluctuations as characterized by their respective statistical variances, σ_χ^2 and σ_ϕ^2 ,

$$\begin{aligned}\sigma_\chi^2 &= \log_e(1 + \sigma_\beta^2), \\ \sigma_\phi^2 &= C_j \left(\frac{D}{r_0} \right)^{5/3}.\end{aligned}\quad (2)$$

The intensity variance σ_β^2 is often referred to as the scintillation index [2]. In Eq. (2), the statistics of phase aberrations caused by atmospheric turbulence are characterized, considering a Kolmogorov spectrum of turbulence [3]. In that analysis, classical results for the phase variance σ_ϕ^2 were extended to consider modal compensation of atmospheric phase distortion. In such modal compensation, Zernike polynomials are widely used as basis functions because of their simple analytical expressions and their correspondence to classical aberrations [4]. The coefficient C_j depends on J [3]. A coefficient 1.0299 in the phase variance σ_ϕ^2 assumes that no terms are corrected by a receiver employing active modal compensation. For example, aberrations up to tilt, astigmatism, coma and fifth-order correspond to $J = 3, 6, 10$ and 20 ,

Fig. 3. By noting that the downconverted signal current can be characterized as the sum of many contributions from different coherent regions within the aperture, we show that the probability density function (PDF) of this current can be well-approximated by a modified Rice distribution. In (a), we show the Rician density function as a function of the normalized SNR and the Rician factor r . In (b), we plot the PDF of the received SNR γ_{MRC} at the output of a perfect L -branch MRC coherent combiner in the atmosphere as a function of the normalized SNR and the number of combining branches L .

respectively. Ideally, it is desirable to choose J large enough that the residual variance σ_ϕ^2 becomes negligible. In Eq.(2), the receiver aperture diameter D is normalized to the wavefront coherence diameter r_0 , which describes the spatial correlation of phase fluctuations in the receiver plane [5].

We consider MRC diversity combining of the received signal. MRC schemes assume perfect knowledge of the branch amplitudes and phases, require independent processing of each branch, and require that the individual signals from each branch be scaled by the complex conjugates of their respective fading gains before being summed together (assuming equal additive noises in each branch). The instantaneous SNR γ_{MRC} for a coherent MRC combiner is the power ratio of the phase-coherent addition of the signal amplitudes from each element of the combiner to the incoherent addition of the noises. Under these assumptions, the resultant composite SNR for an L -element MRC combiner is [6]

$$\gamma_{MRC} = \sum_{l=1}^L \gamma_l. \quad (3)$$

Therefore, the instantaneous MRC combiner SNR γ_{MRC} is the sum of the component array element SNR's. For independent branch signals and equal average branch SNR $\bar{\gamma}$,

$$\bar{\gamma}_l = \bar{\gamma} \quad \text{for all } l \in \{1, 2, \dots, L\}, \quad (4)$$

the PDF of the received SNR γ_{MRC} at the output of a perfect L -branch MRC coherent combiner in the atmosphere would be described a sum of L independent and identically distributed non-central chi-squared random variables with two degrees of freedom. This random variable has a noncentral chi-square distribution with $2L$ degrees of freedom [6]:

$$p_\gamma(\gamma_{MRC}) = \left(\frac{1+r}{\bar{\gamma}}\right)^{\frac{L+1}{2}} \left(\frac{1}{Lr}\right)^{\frac{L-1}{2}} \exp(-Lr) \exp\left[-\frac{(1+r)\gamma_{MRC}}{\bar{\gamma}}\right] I_{L-1}\left[2\sqrt{\frac{L(1+r)r\gamma_{MRC}}{\bar{\gamma}}}\right]. \quad (5)$$

Here, the mean SNR $\bar{\gamma}_{MRC}$ is simply L times the single-element mean SNR $\bar{\gamma}$ or $L\bar{\gamma}$. In order to get a unified approach to performance evaluation, this result can equivalently be expressed in terms of moment generating function (MGF)

associated with SNR γ_{MRC} , which it is defined by means of the expected value of $\exp(s \gamma_{MRC})$ and depends only on the fading channel model assumed. After some algebra, we obtain the MGF

$$M_{MRC}(s) = \int_0^\infty d\gamma_{MRC} \exp(s \gamma_{MRC}) p_\gamma(\gamma_{MRC})$$

$$= \left[\frac{1+r}{1+r-s\gamma} \exp\left(\frac{rs\bar{\gamma}}{1+r-s\gamma}\right) \right]^L.$$
(6)

Replacing s by $-s$ in Eq. (6), we can refer to this expectation value as the Laplace transform of the random variable γ_{MRC} .

3. PERFORMANCE OF COHERENT RECEIVERS

In the presence of turbulence, the received power is scaled by the fading intensity α^2 , a random variable with PDF $p_{\alpha^2}(\alpha^2)$, which depends on atmospheric propagation. At the receiver, the signal is perturbed by AWGN, which is statistically independent of the fading intensity α^2 . Hence, the instantaneous SNR γ is proportional to α^2 . The symbol-error probability (SEP) $p_s(E)$ of an ideal coherent receiver is obtained by averaging the SEP conditioned on the SNR γ over the PDF of the instantaneous SNR, $p_\gamma(\gamma)$:

$$p_s(E) = \int_0^\infty d\gamma p_s(E|\gamma) p_\gamma(\gamma)$$
(7)

Although our model can accommodate various modulation/detection schemes, in this work, we consider M -ary phase-shift keying (M -PSK) with ideal coherent detection based on maximum-likelihood principles. In this case, the SEP conditioned on the instantaneous SNR is given by [8]

$$p_s(E|\gamma) = \int_{-\pi/2}^{\pi/2-\pi/M} d\theta \exp\left(-\gamma \frac{\sin^2 \frac{\pi}{M}}{\sin^2 \theta}\right)$$
(8)

Using equations (1) and (8) in Eq. (7), after some algebra, we obtain

$$p_s(E) = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2-\pi/M} d\theta \frac{(1+r) \sin^2 \theta}{(1+r) \sin^2 \theta - \bar{\gamma} \sin^2 \frac{\pi}{M}} \exp\left[-\frac{r\bar{\gamma} \sin^2 \frac{\pi}{M}}{(1+r) \sin^2 \theta - \bar{\gamma} \sin^2 \frac{\pi}{M}}\right]$$
(9)

Although this integral cannot be put in a closed form, we are able to carry out the integration in Eq. (9) using a simple 30-point Gaussian-Legendre quadrature formula, which yields high accuracy. To obtain the unconditional SEP $p_s(E)$ when multichannel reception is used, we must average the multichannel conditional SEP $p_s(E|\gamma_{MRC})$ over the joint PDF of the instantaneous SNR sequence $p_\gamma(\gamma_{MRC})$. Altogether, the average SEP with multichannel reception reduces to a single finite-range integral whose integrand contains the moment generating function MGF (6) of the SNR γ_{MRC} .

Figures 4-6 show the effect of atmospheric turbulence on QPSK with coherent detection using modal-compensated heterodyne or homodyne receivers. We study the SEP Eq. (7) as a function of several parameters: the average turbulence-free SNR γ_0 per symbol, the receiver aperture diameter D , the number of spatial modes J removed by the compensation system, the number L of combiner branches, and the strength of atmospheric turbulence. Turbulence is quantified by two parameters: the phase coherence length r_0 and the scintillation index σ_β^2 . We consider a nonzero value of the scintillation index. The value $\sigma_\beta^2 = 1$ corresponds to strong scintillation, but still below the saturation regime.

Fig. 4. Symbol-error probability (SEP) vs. turbulence-free SNR per symbol γ_0 for QPSK with coherent detection and additive white Gaussian noise (AWGN). Performance is shown for different values of: (a) the normalized receiver aperture diameter D/r_0 , and (b) the number of modes J corrected by adaptive optics. Amplitude fluctuations are neglected by assuming $\sigma_p^2=0$. Turbulence is characterized by the phase coherence length r_0 . In (a), D/r_0 ranges from 0.1 (weak turbulence) to 10 (strong turbulence). In (b), the compensating phases are expansions up to tilt ($J=3$), astigmatism ($J=6$), and 5th-order aberrations ($J=20$). The no-correction case ($J=0$) is also considered. The no-turbulence case is indicated by black lines.

When the turbulence reaches the saturation regime, wavefront distortion becomes so severe that it would be unrealistic to consider phase compensation. In most practical free-space links, amplitude fluctuations are not saturated [8].

Figures 4 and 5 show the effect of atmospheric turbulence on QPSK with coherent detection using modal-compensated heterodyne or homodyne receivers. We study the symbol-error rate as a function of several parameters: the average signal-to-noise ratio per symbol, the strength of atmospheric turbulence, the receiver aperture diameter D , and the number of spatial modes J removed by the compensation system. Turbulence effects are described by two parameters. Fried's coherence length r_0 describes the coherent diameter of the distorted wavefront phase. The scintillation index σ_p^2 describes the intensity of amplitude fluctuations. In our modeling, we have chosen typical values of these two parameters. While our model can be applied to various modulation and detection methods, here, we have shown the performance of QPSK using coherent detection in the presence of AWGN. Also, we limit our analysis to receivers that implement the maximum-likelihood decision rule in order to minimize the error probability.

Figure 4 details the dependency of the error rate with turbulence for different degrees of phase compensation. As expected, in the event that atmospheric turbulence is present, the effect of turbulence is to reduce the coherence area of the signal. This in turn reduces the heterodyne or homodyne downconversion detection efficiency and, consequently, the receiver electrical SNR. When phase correction is used and a set of spatial modes is compensated, the downconversion efficiency grows quickly and the improvement caused by the compensation technique is substantial. In most situations, the effect of phase correction is significant even when just a few modes are corrected. Figure 5 considers the effect of receiver aperture on performance. The optimal aperture diameter, which minimizes the error rate, exhibits two different regimes in our studies. When the aperture is relatively small, amplitude scintillation is dominant, and performance is virtually unaffected by phase-front corrections. When the aperture is larger, phase distortion becomes dominant, and high-order phase corrections may be needed to improve performance to acceptable levels.

Figure 6 shows the dependency of the error rate with turbulence on QPSK with coherent detection for different values of the number L of combiner branches. MRC combining is employed. The case $L=1$ corresponds to no receive diversity. The area πD^2 describes the combined, multi-aperture system equivalent aperture. When no receive diversity is considered, D equals the receiver aperture diameter. If a L -aperture system is analyzed, each one of the aperture

Fig. 5. SEP vs. normalized receiver aperture diameter D/r_0 for QPSK with coherent detection and AWGN. In (a), no phase compensation is employed, and performance is shown for different values of the scintillation index σ_β^2 . In (b), the scintillation index is fixed at $\sigma_\beta^2 = 0.3$, and performance is shown for different values of J , the number of modes corrected by adaptive optics. In all cases, the turbulence-free SNR per symbol γ_0 is proportional to the square of the aperture diameter D . For the smallest aperture considered, we assume $\gamma_0 = 10$ dB. In (a), σ_β^2 ranges from 0.3 (weaker turbulence) to 1 (stronger turbulence). In (b), the compensating phases are expansions up to tilt ($J=3$), astigmatism ($J=6$), and 5th-order aberrations ($J=20$). The no-turbulence case is indicated by black lines.

diameters equals $D/\sqrt{L}$. For the smallest aperture considered, we assume γ_0 is equal to 10 photons-per-symbol. For any other aperture diameter, the value of γ_0 is proportional to D^2 . Fig. 6 (a) presents the SEP vs. turbulence-free SNR γ_0 . In (b), Fig. 6 considers the effect of aperture diameter on the error rate. It presents the error rate as a function of the normalized aperture D/r_0 for a constant phase coherence length r_0 .

4. CONCLUSIONS

We have studied the impact of atmospheric turbulence-induced scintillation and phase aberrations on the performance of free-space optical links in which the receiver uses modal wavefront compensation and synchronous homodyne or heterodyne detection. We have defined a mathematical model for the signal received after propagation through the atmosphere and after modal compensation. By noting that the downconverted electrical signal current can be characterized as the sum of many contributions from different coherent regions within the aperture, we showed that the PDF of this signal can be described by a modified Rice distribution. The result is easily generalized to account for diversity combining techniques. The parameters describing the PDF depend on the turbulence conditions and the number of modes compensated at the receiver.

We have provided analytical expressions for the symbol-error probability (SEP) for synchronous detection of M -PSK with additive white Gaussian noise, and have used them to study the effect of various parameters on performance, including signal level, aperture diameter, turbulence strength, diversity branches, and the number of modes compensated. We have separately quantified the effects of amplitude fluctuations and wavefront phase distortion on system performance, and have identified two different regimes of turbulence, depending on the receiver aperture diameter normalized to the coherence diameter of the wavefront phase. When the normalized aperture diameter is relatively small, amplitude scintillation dominates and, as phase fluctuations have little impact, performance is virtually independent of the number of modes compensated.

When the normalized aperture is larger, amplitude fluctuations become negligible, and phase fluctuations become dominant, so that high-order phase compensation may be needed to improve performance to acceptable levels. We have found that for most typical link designs, wavefront phase fluctuations are the dominant impairment, and compensation of a modest number of modes can reduce performance penalties by several decibels. We have also separately identified the

Fig. 6. SEP vs. number of diversity branches for QPSK with coherent detection and AWGN. The case $L=1$ corresponds to no receive diversity (green lines). The area πD^2 describes the combined, multi-aperture system equivalent aperture. No phase compensation is employed and amplitude fluctuations are neglected by assuming $\sigma_\beta^2=0$. In (a), SEP is presented as a function of turbulence-free photons per symbol γ_0 . In (b), SEP is shown as a function of the normalized receiver aperture diameter D/r_0 . In (a), turbulence is characterized by a phase coherence length r_0 such as $D/r_0=10$. In (b), the turbulence-free SNR per symbol γ_0 is proportional to the square of the aperture diameter D . For the smallest aperture considered, we assume $\gamma_0 = 10$ dB. The no-turbulence case is indicated by black lines.

impact of the combiner number of branches on the symbol error rates. In most situations considered, for typical turbulence conditions, sizeable gains in symbol error rates are realizable by allowing for a rather small number of apertures. More detailed results and comments on our analysis will be presented at the meeting.

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