

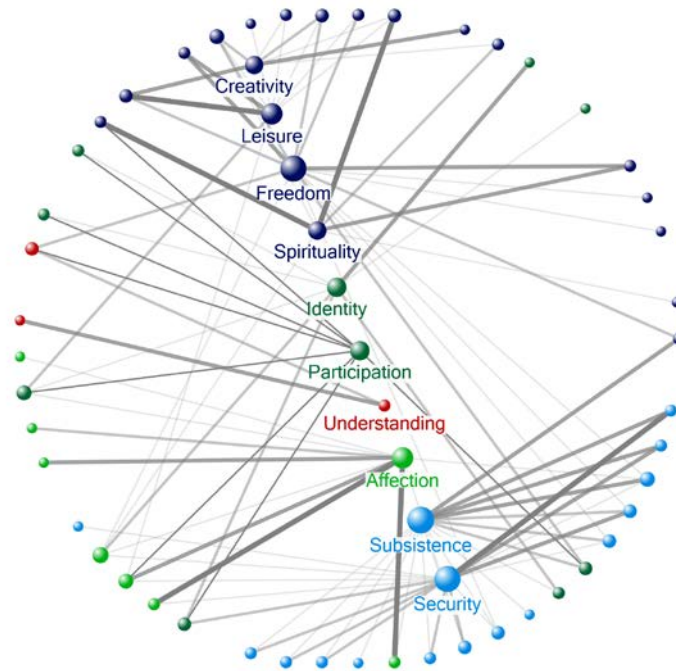
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Unveiling connectivity patterns of categories in complex systems. An application to human needs in urban places.

Abstract

We present a methodology based on weighted networks and dependence coefficients aimed at revealing connectivity patterns between categories. As a case study, it is applied to an urban place and at two spatial levels, neighbourhood and square, where categories correspond to human needs. Our results show that diverse spatial levels present different and non-trivial patterns of need emergence. A numerical model indicates that these patterns depend on the probability distribution of weights. We suggest that this way of analysing the connectivity of categories (human needs in our case study) in social and ecological systems can be used to

define new strategies to cope with complex processes, such as those related to transition management and governance, urban-making and integrated planning.

Keywords: Qualitative and quantitative analysis; Complex networks; Maximal information coefficient; Human-scale development; Urban places

1 Introduction

People use concepts both to provide a taxonomy of things in the world and to express relations between classes in that taxonomy (Smith & Medin, 2014). One component of this taxonomic function is categorisation, the process in which a specific instance is recognised as a member of a concept (i.e., this inanimate thing is an automobile) or that one particular concept is a subset of another (i.e., this automobile is a sport utility vehicle). Concepts and categories thus (a) give our world some coherence and (b) capture the notion that many objects or events are alike in some important respects, and hence can be thought about and responded to in ways we already comprehend. This process has been particularly helpful in assessing the many problems related with socio-ecological systems (SES). SES are usually defined as complex and adaptive 'bio-geo-physical' units, delimited by spatial or functional boundaries, which include social actors and institutions (Gunderson & Holling, 2002). As human impact on nature grows, SES have been increasingly analysed under systemic conceptual frameworks (i.e., systems theory and complexity science) and paradigms (i.e., sustainability), where natural and social sciences confront the need to approaching and complementing each other (Berkes & Folke, 2000; Holling, 2004). In doing so, a process of categorisation must be carried out in order to recognise and differentiate concepts and objects, and, ideally, to illuminate relationships between subjects and objects of knowledge (Cohen & Lefebvre, 2005). In the particular case of sustainability science, scholars have incorporated in recent years many, usually complementary, frames of reference that include different categories, in order to facilitate the vision of sustainable futures (Kajikawa, 2008; Sumi, 2007). Since the Brundtland report (WCED, 1987), which broadened the definition of sustainability to encompass the entire range of human values (Ascher, 2007), this vision has included an evolving definition of the overall human well-being as a function of both the level of human needs met and the extent to which individuals or groups are satisfied with this level

(Costanza et al., 2007; Max-Neef, Elizalde, & Hopenhayn, 1991). This categorisation process (e.g., of well-being through human needs and satisfiers) has allowed researchers to better analyse and understand the social dimension of sustainability. Whereas human needs are constant, finite, few and classifiable, the way in which these needs are satisfied changes over time and between cultures, depending on the prevailing social contract. They must involve various types of capital (i.e., time, built, natural, social or human) and be based on (a) the generation of growing levels of self-reliance, and (b) the construction of organic articulations of people with nature and cultural environments, of global processes with local activity, of the personal with the social, of planning with autonomy, and of civil society with the state (Max-Neef, 1992).

However, any categorisation process implies by definition a loss of some structural information on the overall system. Once concepts, ideas and objects are categorised, the global pattern of possible conceptual connections among them is strongly constrained by the categorising process itself. In this paper, we present a methodology aimed at revealing the connectivity between categories in systems where these classificatory divisions can be defined. We hypothesise that recovering the connectivity pattern of categories in general, and in SES in particular, offers the possibility to add information back to the system in order to facilitate its comprehension and analysis in at least three ways: (1) there is a hidden map of subjacent connexions between categories that cannot be observed unless a process of direct enquiry to the elements (i.e., actors) of a system is used as a proxy of how a community builds its particular social contract based on preferences; (2) the connectivity pattern obtained along this process of information retrieval is necessarily a function of both, spatial and temporal constraints, and the strength of the different connexions between categories; and (3) connectivity patterns can be used to design strategies to act on the function and aspect of the system under study.

Our methodology is based on complex networks and maximal information approaches and it is applied to a case study aimed at revealing the topological structure of human needs (defined as existential categories) in urban places. The term *place* here is used to designate a setting that takes into account various combinations of social, cultural, communal, economic and ecological facets (Marsden, 2013). Urban places are thus complex expressions of the varied interactions between these interconnected and interrelated spheres. The importance of urban contexts as key SES is reflected in the efforts currently devoted to developing a unified theory of urban living (Batty, 2012; Bettencourt & West, 2010). With 52% of people (78% in more developed regions) now living in cities (United Nations, 2014), sustainability science greatly needs to find a predictive framework where dynamics involved in the on-going expansion processes of urban areas can be included. The application of mathematics in social sciences is now essential for the study of society and groups as more and more human systems are complex and interconnected (Bonacich & Lu, 2012). In this regard, our understanding of urban places is being transformed by new approaches where cities are treated as complex adaptive systems—characterised by structures, processes, social and technological networks and interactions—that give rise to morphologies, which illustrate fractal patterns, self-similarity and scaling laws (Batty, 2005; Bettencourt, Lobo, Helbing, Kühnert, & West, 2007). However, results so far have been concentrated essentially in the descriptive-analytical or problem-focused domain, rather than in the transformational or solution-oriented mode (Wiek, Ness, Schweizer-Ries, Brand, & Farioli, 2012). Since most people’s sense of well-being depends on the social and cultural system in which they are living (Sumi, 2007), any approach (coming from complexity science or not) that does not take into account, on an equitable basis, the human needs and the changing aspirations of a community is of very limited use (Kajikawa, 2008).

The body of the paper is structured thus: in the next section, we provide a summary of the methodology, focusing on possible applications for different systems. Then, we present our case study, where we define our study categories, describe our database and provide sample statistics. Once this foundation is laid, we can report the results and main findings. We discuss the results and identify structural connectivity patterns in our context. Finally, we present our conclusions.

2 Materials and methods

The proposed methodology is a mixed method that allows the classification of quantitative data into qualitative categories and the retrieval of the overall connectivity pattern of the variables within the study categories. The particular steps proposed in the methodology are shown in Figure 1.

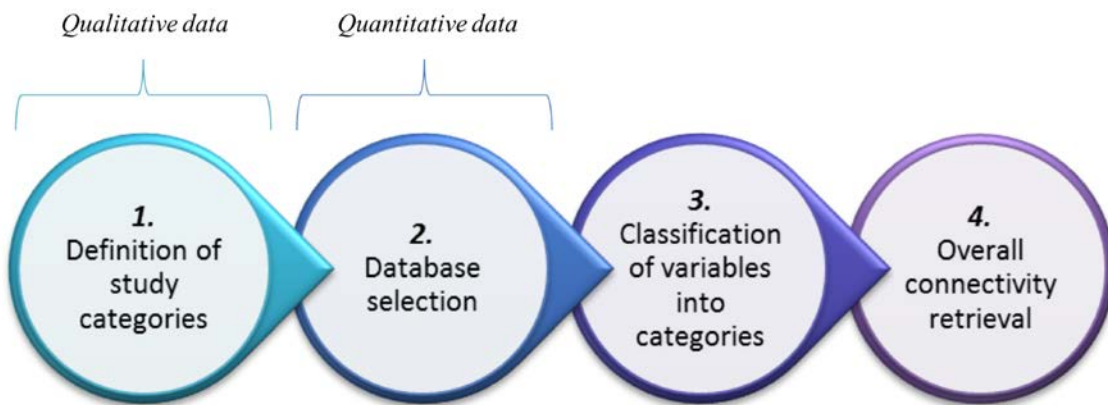


Figure 1: General methodological process.

The *first step* in the process is the definition of the study categories. These categories correspond to the qualitative variables according to which we would like to classify our quantitative ones. Categories could be either pre-established or assumed, depending on the assessment and given that categorisation processes require and consider subjective opinion as

necessary (Cohen & Lefebvre, 2005). Table 1 offers a sample of quantitative and qualitative data systems where this methodology can be applied.

The *second step involves* the database selection. The quantitative variables may correspond to close survey responses, indexes, measures and other numerical data or numeric databases.

Table 1: Examples of qualitative and quantitative data that may be used as study categories and variables correspondingly.

Qualitative Data	Quantitative data
Human needs (Max-Neef et al., 1991)	World Values Survey Database (http://www.worldvaluessurvey.org/)
Constituents of well-being (Millennium Ecosystem Assessment, 2005)	The Survey of Health, Ageing and Retirement in Europe (http://www.share-project.org/)
Gross National Happiness Index Domains (http://www.grossnationalhappiness.com/)	Environmental conflicts (Temper, del Bene, & Martinez-Alier, 2015)
Planetary boundaries (Rockstrom et al., 2009)	

For example (Table 1), we may want to know how the impacts of environmental conflicts (Temper, del Bene, & Martinez-Alier, 2015) may affect the constituents of well-being (Millennium Ecosystem Assessment, 2005). We have a list of variables related to possible impacts of environmental conflicts. In this case, the responses for each conflict are binary (i.e., either yes or no). We also have a list of categories to classify those impacts, corresponding to the constituents of well-being according to the Millennium Ecosystem Assessment. Those are: Existence, Health, Security, Good Social Relations and Freedom of choice and action. Or, we may want to see how environmental impacts related to environmental conflicts (Temper et al., 2015) have an effect on the nine Planetary Boundaries categories corresponding to Climate

change: Ocean acidification, Stratospheric ozone, Biogeochemical nitrogen (N) and phosphorus (P) cycle, Global freshwater use, Land system change, Rate of biological diversity lost, Chemical pollution and Atmospheric aerosol loading (Rockstrom et al., 2009). Another alternative could be to check the fulfilment of human needs of the Human-Scale development paradigm (Max-Neef et al., 1991) for a specific European country (or a comparison between them) according to data coming from the Survey of Health, Ageing and Retirement in Europe (SHARE) questionnaire¹.

The *third step* corresponds to the classification of the variables into the selected categories. For this classification, a focus group is needed. The focus group may consist of an expert group on the subject in particular, of a sample of the local and directly affected population or even of some random population sample. The matching of the variables to one or more categories is seen as a subjective choice related to individual understanding and interpretation. During this process a variable can be related to more than one category (Table 2). Variables are classified into categories by following and averaging the binary decision process from individuals belonging to the focus group. In Table 2, category *A* is assumed to be defined by variables $1, 2, \dots, m$ and by individuals a, b, \dots, n . The importance of variable m in defining category *A* is the average of individuals who acknowledge its importance in this particular category.

Table 2: Variables are classified into categories by following and averaging a simple binary decision from the focus group. Here, category *A* is assumed to be defined by variables $1, 2, \dots, m$ and by individuals a, b, \dots, n . The importance of variable m in defining category *A* is the average of individuals who acknowledge its importance in this particular category.

¹ <http://www.share-project.org/>

Need A					
Question	Expert a	Expert b		Expert n	Average
1	1	1	...	1	3 / n
2	1	1		0	2 / n
...					
m	1	0		i	$\frac{\sum_{i=1}^n}{n}$

For the sake of clarity, we may

consider again examples in Table 1. World Values Survey (WVS) Database² can be used to measure the Gross National Happiness Index (GNH)³ levels per country, continent or globally. To do so, variables in WVS database should be classified by a focus group to the GNH's domains (or categories) corresponding to Psychological well-being, Standard of living, Good governance, Health, Education, Community vitality, Cultural diversity and resilience, Time use and Ecological diversity and resilience. Each variable may belong to more than one domain according to each individual perception coming from the members of the focus group. For instance, a question coming from the WVS like "How democratically is this country being governed today" may be directly classified under GNH categories like "Good Governance", but for some it may also affect "Psychological well-being". Another question like "How secure somebody feels in the neighbourhood" may be classified under "Psychological well-being", "Standard of Living" and/or "Community vitality", among others.

In order to (a) detect the relevance of each variable in defining each category, and (b) balance the amount of variables per category, so every category is defined by a similar number of variables, variables per category are selected in decreasing average order (see last column of Table 2). When the average punctuation for a variable is equivalent for more than one need, a sequential criterion is used, based firstly on equal number of variables per need, and secondly on a random selection of pertinence. So as to detect any possible bias in the results when

² <http://www.worldvaluessurvey.org/>

³ <http://www.grossnationalhappiness.com/>

introducing this randomising step, this process must be reproduced several times with different sets of randomised values.

In the *fourth* and final *step*, the overall connectivity between categories is retrieved. This process makes use of a correlation value between variables in order to establish how strongly they are connected. Depending on the researcher's personal choice and the type of variables, different methods can be used. Linear (i.e., Pearson, χ^2 , Spearman, etc.) and/or non-linear (i.e., maximal information) correlation coefficients are more suitable for analogic variables, while methods like entailment analysis (White, Burton, & Brudner, 1977; White, 1996) or partial order models (Wiley & Martin, 1999) can be more suitable in the particular case of binary only data. Although a process of dichotomisation of variables is always possible, it usually comes as a trade-off for complexity, and it must be implemented with care to avoid an oversimplification of our connectivity map. The relation between two particular variables (or the relation between the categories those variables are associated with) is not of interest in this methodology, rather the overall connectivity map between variables and how this map emerges is relevant. To provide an intuitive and efficient interpretation of how the different variables in a dataset are related to each other, a network-type visualisation of the datasets is constructed. Variables in the dataset are represented by nodes, while relationships between variables are represented by edges, weighted according to the results of the selected coefficient for each pair of variables.

Finally, the weighted degree of each node is obtained as a proxy of its significance and as a function of its relations with the rest of variables. In network theory, the degree of a node is simply the number of connections or edges that the node has to other nodes. Weighted degree has generally been extended to the sum of weights when analysing weighted networks (Barrat, Barthélemy, Pastor-Satorras, & Vespignani, 2004; M. E. J. Newman, 2001; Opsahl,

Agneessens, & Skvoretz, 2010), and it is labelled *node strength*. This measure is formalised as follows:

$$s_i = \sum_j^N w_{ij} \quad (1)$$

where w is the weighted adjacency matrix, in which w_{ij} is greater than 0 if the node i is connected to node j and the value represents the weight of the tie (i.e., correlation coefficient). Once a network is thus created, graph theory can be applied to reveal other structural properties based on network centrality measures (M. E. J. Newman, 2010). Here, strength (i.e., weighted degree) cumulated probability distribution $P_{>}(s_i)$ is primarily used, which measures the percentage of nodes with strength s_i or higher. A remarkable attribute of cumulated distributions, compared to histograms and other binning procedures, is that no statistical information is lost. In many ways, it is a much more useful and convenient method for plotting the data (M. Newman, 2005).

2.1 Case study

The proposed methodology (see Figure2) was applied to two different, although related, urban SES within the same area: the neighbourhood of Vila de Gràcia (Gn) and Plaça de la Virreina square (Vs), respectively, both of them located in Barcelona (Spain). Here dwellers' perceptions on these urban places act as variables and fundamental human needs (Max-Neef et al., 1991) act as categories. In doing so we aim at unveiling the complex expressions of human needs in urban places in order to better understand the social dimension of sustainability.

Gn and Vs were essentially chosen for their cohesive urban and social fabric, providing high levels of participation and public engagement. Gn is characterised by an irregular urban grid

with narrow streets and 16 public squares, many of which are considered to be emblematic⁴. The neighbourhood occupies the third position in terms of population in the city of Barcelona, with 50,448 inhabitants (Ajuntament de Barcelona, 2014) out of 120,273 living in the Gràcia district, distributed within 1.3 km² and with a population density of 38,806/km². The neighbourhood is characterised for preserving its ‘village’ identity with (still) strong social cohesion. The use of public spaces in this neighbourhood is very intense and subject to high demand, often creating the need for balance between the well-being of the residents and the activities in the public space. One of its most emblematic public spaces is Plaça de la Virreina. Vs was built in 1878 (when Gràcia was still a village on the outskirts of Barcelona) and continues to be one of the places within the area that gives the neighbourhood its “sense of village”⁵. This impression is created by the parish church of Sant Joan and a set of low-rise houses located to the right of the square, originally inhabited by workers from Vila de Gràcia’s once very important textile industry.

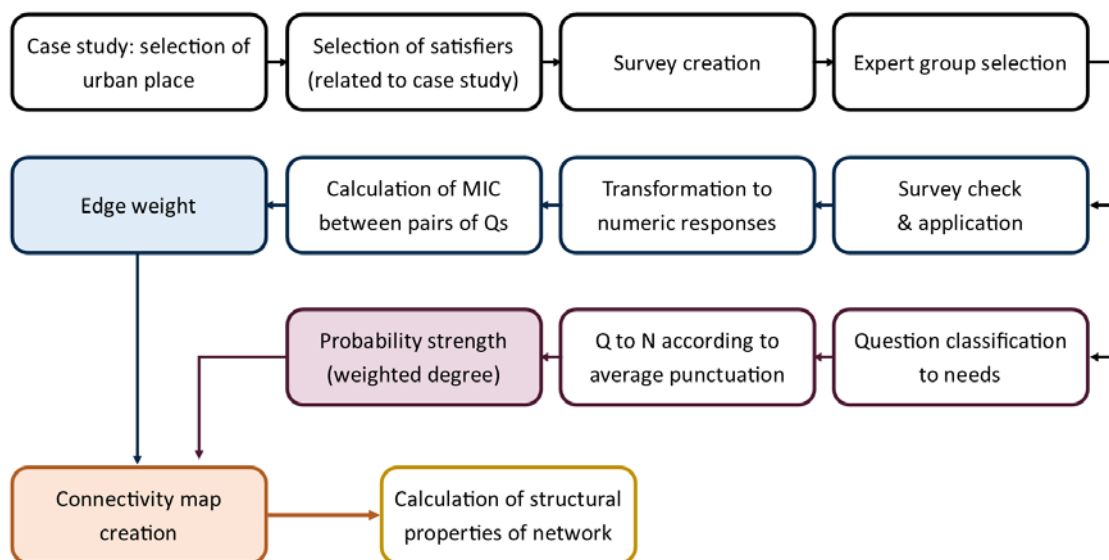


Figure 1: Methodological steps for the case study.

⁴ <http://lameva.barcelona.cat/gracia/ca/home/el-barri-de-la-vila-de-gracia>

⁵ <http://graciapedia.gracianet.cat>

2.1.1 Definition of study categories

The study categories applied in this analysis were adopted from the Human-Scale Development (H-SD) paradigm developed by Manfred Max-Neef et al. (1991), partially modified by Costanza et al. (2007). Human needs indicate deprivations and, at the same time, individual and collective human potential. Needs are seen as finite, few and classifiable, changing only at a very slow pace along with the evolution of our kind. They can be satisfied according to many criteria. In this case, the axiological needs were used, with categories corresponding to Subsistence, Protection, Affection, Understanding, Participation, Leisure, Creation, Identity and Freedom. Protection was changed by Security, as per Costanza et al. (2007), and Subsistence was considered within Reproduction, the latter being understood as a part of the former. Spirituality was also included as a study category, because of its importance in the assessment as a need (O'Brien, 2005; Van Dierendonck, 2011).

The fulfilment of all needs (or categories) is considered equally important. Any unsatisfied or inadequately satisfied human need reveals a form of human poverty, hindering happiness, and thus, having the capacity to develop potential pathologies (Cruz, Stahel, & Max-Neef, 2009). The satisfiers of these needs change over time and between cultures. There is no one-to-one correspondence between needs and satisfiers. A satisfier may contribute simultaneously to the satisfaction of different needs or, conversely, a need may require various satisfiers in order to be met. These relations are not fixed; they may vary according to time, place and circumstance (Max-Neef et al., 1991). Each economic, social and political system adopts different methods for the satisfaction of the same fundamental human needs. In every system, they are satisfied (or not satisfied) through the generation (or non-generation) of different types of satisfiers.

2.1.2 Database selection

For the database creation, and in order to assess possible differences in spatial scales in terms of perceptions related to urban places, two simple and accessible surveys were created, one for Vila de Gràcia neighbourhood (Gn) and the other for Virreina square (Vs), which could be completed both online and in person (Papachristou & Rosas-Casals, 2015)⁶. Surveys were made available between May–June 2012 (Gn) and September–October 2014 (Vs). Our aim was to question people **using** the space and not only people **living** there. For the Gn case, the surveys could be completed both online and in person. Here we undertook a process of engagement (of the people) by contacting local associations and social groups. The online format of the survey was proposed by most of those groups as a means of reaching their members easily. To complement this, in-situ surveys were used in order to obtain a complete view of the perceptions of the people using the space. For the Vs case, surveys could only be completed in person due to its reduced spatial scale. That is to say there were no active associations or social groups related to it to be addressed by the researchers and as a result the online format of the survey did not apply. The total number of completed surveys was 174 and 51 for Gn and Vs, respectively. The main statistics of the samples are shown in Table 3.

Table 3: Main statistics of the two samples Gn (neighbourhood) and Vs (square).

	Groups	Gn%	Vs%
Gender	Female	50.6	58.8
	Male	49.4	41.2
Age	14-17	0.6	0.0
	18-24	10.9	5.9
	25-30	21.3	33.3
	31-44	35.6	41.2
	45-64	28.2	15.7
	65+	3.5	3.9

⁶ Survey questions can be found in the Supplementary material.

Activity	Public sector	38.9	17.7
	Private sector	27.9	37.3
	Student	25.0	19.6
	Unemployed	3.5	3.9
	Pensioner	3.5	3.9
	Self-employed	3.4	11.8

2.1.3 Classification of variables into categories

In order to classify survey questions into the ten fundamental human needs (or study categories), a focus group of experts was selected. In this case it was formed by researchers of the Sustainability Measurement and Modelling Lab⁷ (SUMMLab) and the University Research Institute for Sustainability Science⁸ (IS.UPC), both at the Universitat Politècnica de Catalunya – Barcelona Tech. They were selected considering their knowledge on subjects related to sustainability issues. Questions (and groups of questions) associated to satisfiers before the group of experts weighted them into needs can be found in the Supplementary Material. The selected study group was asked (i) to review the questionnaires to detect any missing aspect, and (ii) to match the given questions to the needs (Table 2). The first task was undertaken in group, while the second was performed individually, bearing in mind that a question could be related to more than one need (e.g., a question such as “How satisfied are you with your health” can be categorised under Subsistence, Security, Freedom and/or Spirituality).

2.1.4 Overall connectivity

The correlation coefficient used in our case study was the maximal information coefficient (MIC) which allows many-dimensional datasets to be explored, assuming generality (i.e., it captures a wide range of associations, not limited to specific functions such as linear, exponential, etc.) and equitability (i.e., it gives similar scores to equally noisy relationships of

⁷ <http://summlab.upc.edu/en>

⁸ https://is.upc.edu/?set_language=en

different types) (Reshef et al., 2011). It is calculated from a matrix of scores generated from a given set of two-variable data⁹. This matrix is created by searching for grids that maximise the penalised mutual information of the distribution induced on each grid's cells by the data. Different relationship types give rise to characteristic matrices with different properties. For instance, strong relationships yield characteristic matrices with high peaks, monotonic relationships yield symmetric characteristic matrices, and complex relationships yield characteristic matrices whose peaks are far from the origin. In our case, the input data is a matrix where columns correspond to variables (i.e., survey questions), rows correspond to individual subjects (i.e., survey responses) and a MIC value is obtained for each pair of variables.

MIC values for independently taken pairs of variables cannot give a true account of the aggregate outcome of the answers. To show how the different questions in the dataset are related to each other, a network-type visualisation of the dataset was obtained. Questions in the dataset were represented by nodes, while relationships between questions were represented by edges, weighted according to the MIC strength of each pair of variables (equation 1). Once a network was thus created for each case (Gn and Vs), graph theory was applied to obtain weighted degree cumulated probability distributions $P_{>}(s_i)$. In order to be able to compare the two networks, s_i was normalised using the highest degree s_i^{max} for each network as the normalising constant:

$$\hat{s}_i = \frac{s_i}{s_i^{max}} \quad (2)$$

⁹ MIC code can be downloaded from <http://www.exploredata.net>. The page also offers real data examples, the necessary steps to compute MIC value and an explanation regarding its parameters.

3 Results

Results presented here are related to the two test cases of the previous section. The outcomes from Table 2 are shown in Figure 3. This representation shows our first classification of the survey questions into fundamental human needs by the experts. Nodes in green represent questions and groups of questions (used when one question in the survey was split into others and to prevent the figure from being overloaded). Nodes in blue colour represent needs. Node size is proportional to the number of connections a need or question has. Edge width is proportional to the importance of that question in defining that need (i.e., the average of experts who acknowledge its importance, a value taken from column *Average* in Table 2). Despite the fact that a majority of questions are connected to more than one need, we observe that each question is notably important in defining one particular need alone (i.e., its connecting edge having a wider width).

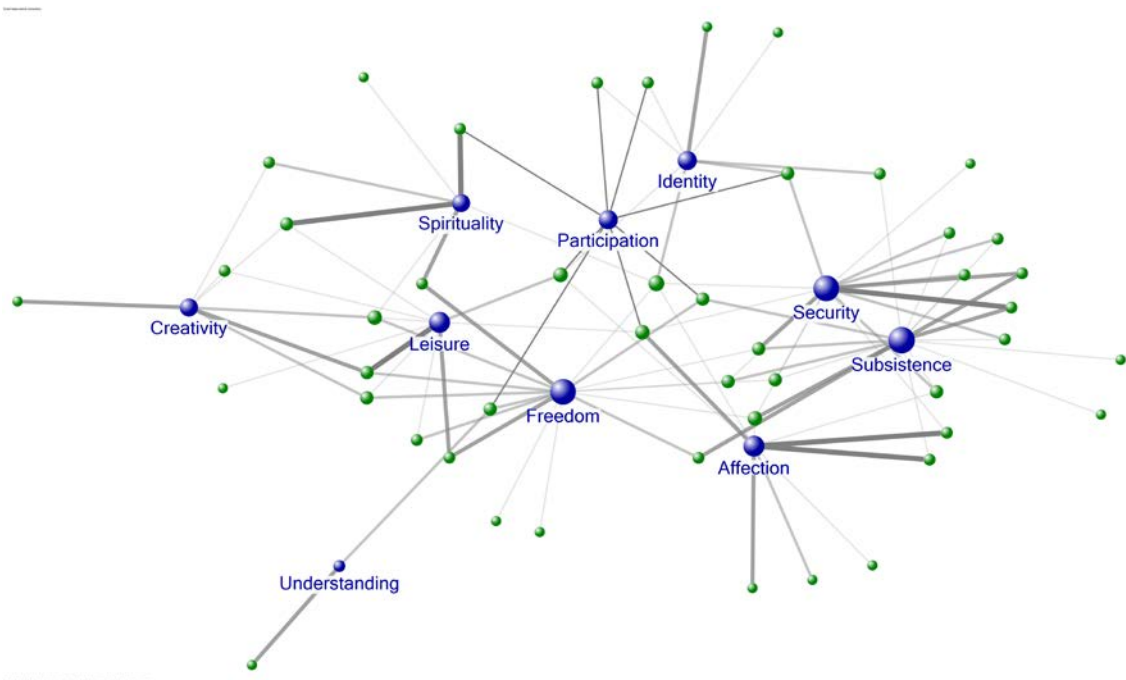


Figure 2: Connectivity of questions (nodes in green) and needs (nodes in blue). Node size is proportional to the number of connections (i.e., degree) that a need or question has. Edge width is proportional to the importance of that question in defining that need (i.e., the average of experts who acknowledge its importance). Graph figure created with NodeXL (<http://nodexl.codeplex.com>).

Although many questions (or group of questions) were originally related to more than one need, a subsequent processing of sequential selection was performed, where questions were selected in decreasing average order per need (see previous section). This balanced the number of questions per need, so every need was defined by a similar number q of questions on average; in this case $\bar{q} = 12$ and standard deviation $SD_{\bar{q}} = 2.5$.

As previously commented, here strength (i.e., weighted degree) cumulated probability distribution $P_{>}(s_i)$ was used as the fundamental measure to reveal the connective pattern of categories (i.e., human needs). Since $P_{>}(s_i)$ measures the percentage of nodes (i.e., questions) in the network with strength equal to or greater than s_i and, at the same time, these nodes have been divided into needs, $P_{>}(s_i)$ offers the possibility to observe the evolution in the emergence of each need in accordance with the appearance of each question in the network, ordered from highest to lowest strength. This process can be qualitatively (Figure 4) and quantitatively (Figure 5) presented. Figure 4 shows two particular screenshots corresponding to the emergence of the graph for G_n at two stages, with questions spatially grouped and coloured according to H-SD categories. The first stage is when 15% of the nodes with the highest strength are shown (a), where Affection and Security appear as the two most prominent needs in terms of correlations between questions. When strength is decreased in order to make 70% of the nodes present (b), nodes related to Affection and Security have fully appeared, while other needs' nodes have not appeared yet (i.e., Participation). At this second stage, the graph is still far from being fully connected and all non-present nodes have strengths lower than 0.35.

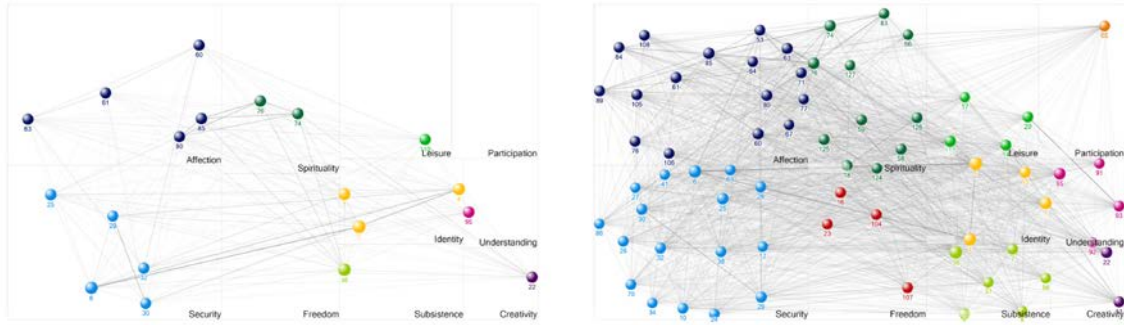


Figure 3: Appearance of needs (different colors) using strength as a filter for the Vila de Gràcia network. Graph snapshots when 15% (left) and 70% (right) of the nodes with the highest strength are shown, respectively. Graph figures created with NodeXL (<http://nodexl.codeplex.com>).

Results for the normalised strength cumulated probability distribution for both networks, Gn and Vs, are shown in Figure 5. $P_{>}(s_i)$ exhibits a bimodal distribution for both networks, with two similar sigmoid behaviours after and before strength value $\hat{s}_l = 0.35$ for Gn, and $\hat{s}_l = 0.25$ for Vs. Figure 5(a) shows its deviation from the corresponding cumulated probability distribution for a random graph (i.e., a graph with Gaussian degree distribution) with the same average node strength and number of nodes (shown with dashed lines). A random graph is a graph in which properties such as the number of graph vertices, graph edges, and connections between them are determined in some random way (Bollobás, 2001) and thus, uses the least amount of assumptions in its generation process. This deviation is more substantial in the case of Vila de Gràcia. Figure 5(b) shows $P_{>}(s_i)$ for the same nodes, but this time, it is grouped by needs. Here, the particular characteristics of every need can be observed and how its importance (in terms of more connected nodes or questions) emerges gradually from the entire connected network and at some particular transitional zones, numbered from (I) to (III). The features that can be observed are the following:

- One striking difference between both distributions is shown in Figure 5(a). Gn presents a more spread normalised strength distribution than Vs. This can be clearly observed

by the number of nodes existing in the range $0.5 < \hat{s}_i < 1.0$ for both networks: $P_{>}^{Gn}(s_i \geq 0.5) = 0.4$ whereas $P_{>}^{Vs}(s_i \geq 0.5) = 0.02$. Although qualitatively similar, this characteristic keeps both distributions segregated and quantitatively distinct, as it will be commented later.

- For both networks, nodes related with Identity are detected first – reading the graphs from the right to the left –, indicating their highest normalised strength (ie., $\hat{s}_i = 1.0$). In the case of the Gn network, nodes representing Security and Understanding appear as early as for $\hat{s}_i > 0.89$ and $\hat{s}_i > 0.67$, respectively.
- Zone (I). For $\hat{s}_i = 0.55$, a first sudden transition for Gn is observed when nodes related with Affection, Creativity, Freedom, Leisure, Participation and Spirituality categories mostly appear. This implies the presence of 30% of the nodes in Gn, while barely 2% of the nodes in Vs are present for this same value of \hat{s}_i . Nodes related with Subsistence have already completely appeared in the distribution from normalised strength $\hat{s}_i = 0.45$.
- Zone (II). There is not a second transition until $\hat{s}_i = 0.34$, this time, for the Vs network. At this point, nodes related to all categories have already appeared, although some of them only with a reduced presence (i.e., for Freedom, Leisure and Participation). Here, 70% of the nodes are already present in the Gn connected component of the graph, while only 35% of their counterparts in Vs have appeared.
- Zone (III). Finally, for $\hat{s}_i < 0.3$, the remaining nodes appear in a process of slow convergence between the two networks shown in Figure 5(a). For the Vs case, however, some sudden transitions occur for the Creativity, Freedom, Leisure and Participation categories. Curiously enough, the two only needs in Vn that reach a final presence in the network with percentages similar to those of Gn are Affection and Security.

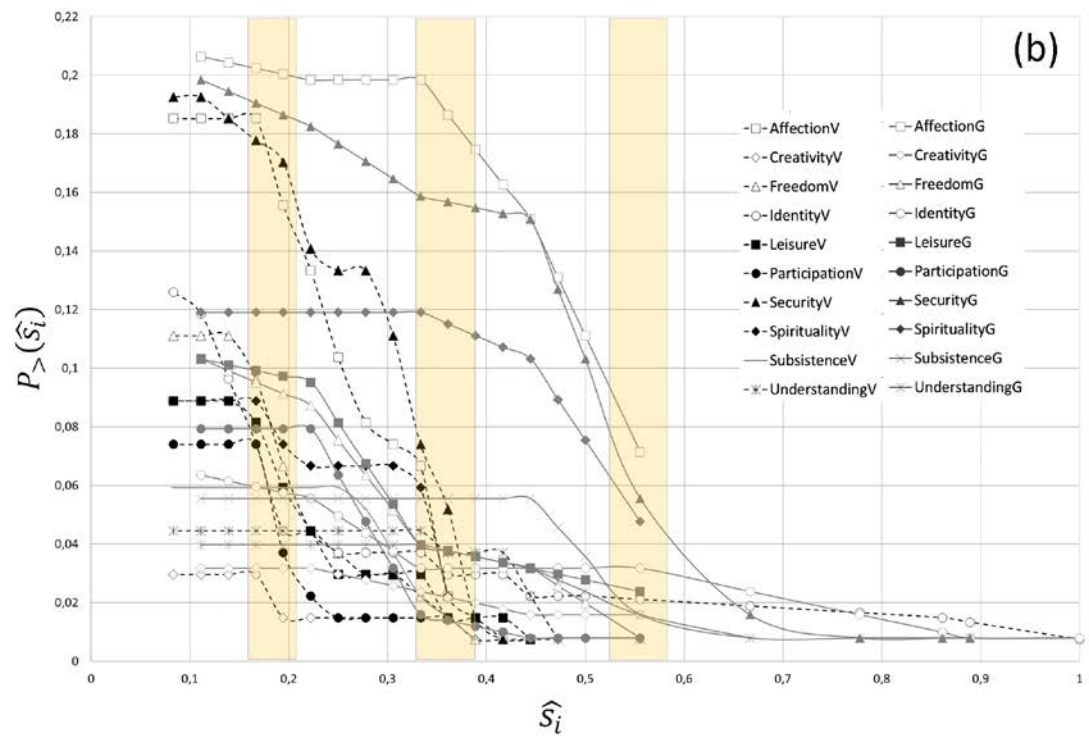
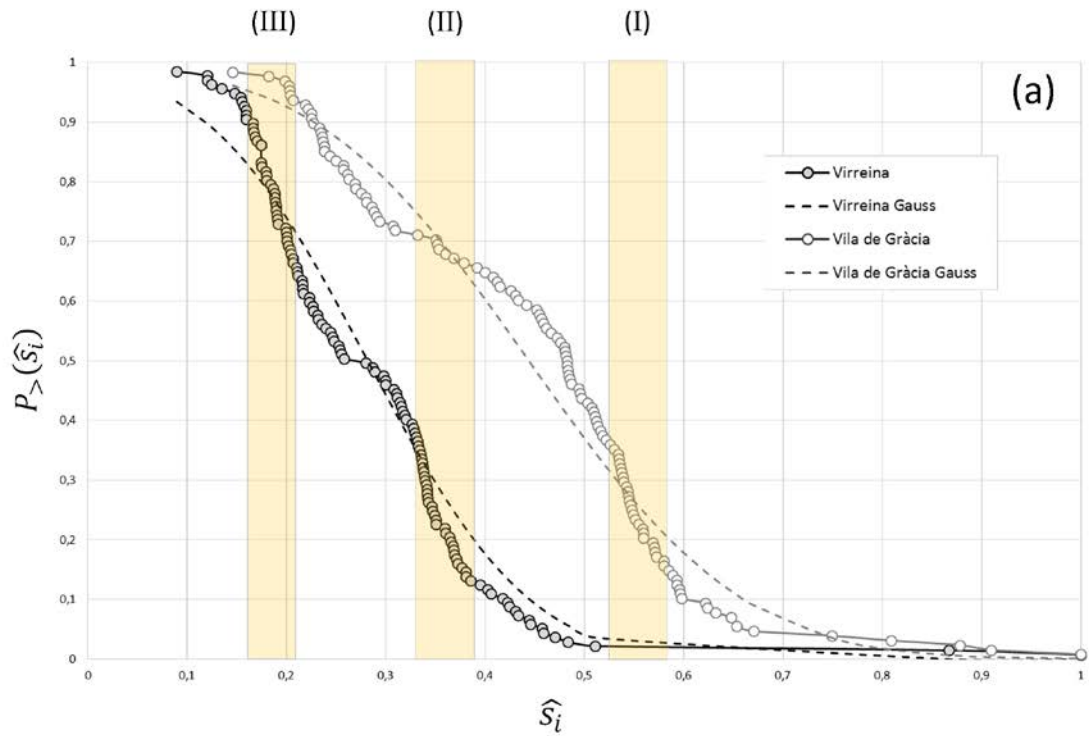


Figure 4: Strength (i.e., weighted degree) cumulated probability distribution and the emergence of the dataset network. (a) Degree cumulated probability distributions for Plaça de la Virreina square and Vila de Gràcia neighbourhood, compared to a normal distribution with the same average weighted degree and standard error (dashed line). (b) Segregated by needs. (V stands for Plaça de la Virreina; G stands for Vila de Gràcia). Grey colour was used to reduce overlapping. Coloured strips (I), (II) and (III) are related with transitional zones (see text).

4 Discussion

SES are both complex and adaptive, meaning that they require continuous testing, learning about, and developing knowledge and understanding in order to cope with change and uncertainty (Carpenter & Gunderson, 2001). Any process of categorisation thereupon must be regarded as provisional and subject to modification as new evidences arise and call for adaptation. While choosing the categories, the researcher should keep in mind that those should be understandable, specific, operational, critical and propositional in order to allow finding a relation with the variables (Max-Neef et al., 1991). The subjective element introduced during the third step of the methodology may lead to even more variability during the classification and each application will give different results as an outcome of subjective choice.

Following this rationale, the structure of the connectivity of human needs presented in our case study must be understood as a global emergent trait from a particular spatial and temporal sample, and hence, cannot be taken out of this particular context. It would also be inappropriate to examine the importance of the different questions by means of their individual connectivity, i.e., which particular question is connected to which other particular question. Although we acknowledge the necessity for an axiomatic approach to the concept of complexity and its many applications in the form of network in general, and social networks in particular (Butts, 2000), the analysis introduced in this paper aims at presenting a proxy of how a community builds its particular social contract based on preferential needs. Other kind of conclusions that may be derived from it must be taken with due care for at least three reasons.

Firstly, null models for comparative purposes cannot be formulated in the case of such specific graph construction processes. Secondly, the strength probability used as a question classifier is a statistical index derived from maximal information coefficients between pairs of questions. Thus, in a group of n questions, the importance of a question depends on its (co-)relation with the remaining $(n - 1)$ questions. Lastly, the classification of questions into human needs comes, in this case, from a pool of people who, despite being experts in their fields, share a particular and time- and space-limited vision and definition of H-SD.

That being said, what is observed here is the appearance of the complex expression of human needs in one particular urban place and time, and on two scales: neighbourhood (Gn) and square (Vs). From an overall point of view, some important features can be highlighted and commented in terms of \hat{s}_i and $P_{>}(s_i)$. First of all, and on both scales, Identity appears as the most prominent need. Under H-SD's existential categories (i.e., being, having, doing and interacting), Identity implies belonging (to a place), language, habits, traditions and values. It is thus a fundamental need in the definition of any social group and a reference for the recognition of one's self in the social sphere. This is the only common trait shared by both networks. At the local scale (Gn), Security appears in second place, a need related with solidarity, family, rights and job. Again, it is a fundamental need in the definition of vital and social domains. On the other hand, the striking difference between Gn and Vs networks is the \hat{s}_i value at which the rest of the needs appear: above and below $\hat{s}_i = 0.5$, respectively. This difference comes essentially from a fundamental change in the strength homogeneity. Gn presents a much more evenly spread distribution in terms of \hat{s}_i than Vs: in Vs questions for all needs appear suddenly, in a much more concentrated manner. In Gn, normalised strength spans in a wider range than in Vs, with nodes occupying the whole spectrum of \hat{s}_i values. From a structural point of view, this means that some particular questions in the Gn network are connected with stronger links than in the Vs one. These questions make Affection (related with

self-esteem, friendship and family), Creativity, Leisure and Participation (related with imagination, humour and curiosity), and Spirituality (related with beliefs and personal growth) characteristic needs at this level of scale.

Although we assume the impossibility of formulating a generative null model for comparative purposes in this case, a very simple numerical model can help us comprehend the influence of the distribution of weights in the observed probability distributions of Figure 4. Modelling in this case is not used to predict experimental outcomes, neither to generate testable hypotheses. We use it as a vehicle of understanding for our particular case study (Lander, 2010). Let assume a fully connected graph $G(m, n)$ with n nodes and $m = n(n - 1)/2$ edges, as it is the case in our original network, where every node i is connected to every other node j . Our objective is to detect qualitative differences in cumulated probability distributions of strengths coming from different probability density functions of weights $f(W_{i,j})$. To do so, the following algorithm is used:

1. A number $m_{W_{i,j}} = n(n - 1)/2$ of weights is generated from two different probability density functions: exponential ($W_{i,j} \leftarrow q(x) \sim e^{-x}$) and power law ($W_{i,j} \leftarrow p(x) \sim x^{-\beta}$). We have considered these two functions as they imply the statistical signature of two extreme phenomena commonly considered in the literature (Gros, 2015; Mitchell, 2009): that of randomness and that of some sort of complexity, respectively. In the particular case of networks, a fat-tailed probability distribution signature (i.e., power law) in terms of degree indicates a hub-dominated topology, where the probability of finding a node with high degree is significantly higher than in a homogenous graph case (M. E. J. Newman, 2010). In other words, these networks do not arise by chance alone and non-trivial underlying mechanisms are usually at play for the generative processes involved in their evolution and growth (M. E. J. Newman, 2005).

2. We use α as a threshold in the range $0 < \alpha \leq 1$ to parameterise the steps of assigning values to the weighted adjacency matrix of our particular synthetic network, with weights $W_{i,j}$ randomly drawn from $p(x)$ if $k \leq \alpha$ and from $q(x)$ otherwise, where k is a random number in the range $0 < k \leq 1$, generated at every step. For $\alpha = 0$, the model fills a weighted adjacency matrix with weights purely coming from an exponential probability density functions of weights, whereas for $\alpha = 1$, the model fills a weighted adjacency matrix with weights solely coming from a power law one.
3. For different values of α , the corresponding normalised strengths are calculated for every node (Eq. 2) to finally obtain the corresponding cumulated strength probability distribution $P_{>}(s_i)$ for each α .

Figure 6 shows averaged values (with standard errors shown as whiskers) for $P_{>}(s_i)$ and for 100 realisations over a network with $n = 100$ nodes using some values of α . Although we observe differences as α varies from 0 to 1, each cumulated strength probability distribution qualitatively follows a cumulated normal distribution (p-value < 0.001 for all of them, results not shown in the text). As a first immediate outcome, our model shows how adjacency matrices filled with weights coming from different probability density functions give rise to qualitatively similar cumulated strength probability distributions. We use the coefficient of variation $c_v = \sigma/\mu$, where σ is the standard deviation and μ is the mean, to quantitatively distinguish the results of our model (Table 4). The coefficient of variation changes from $c_v = 0.53 \pm 0.07$ for the exponential case (i.e., $\alpha = 0.0$) to $c_v = 0.12 \pm 0.01$ for the power law case (i.e., $\alpha = 1.0$), indicating how an increasing probability of having nodes with larger strengths tends to diminish c_v .

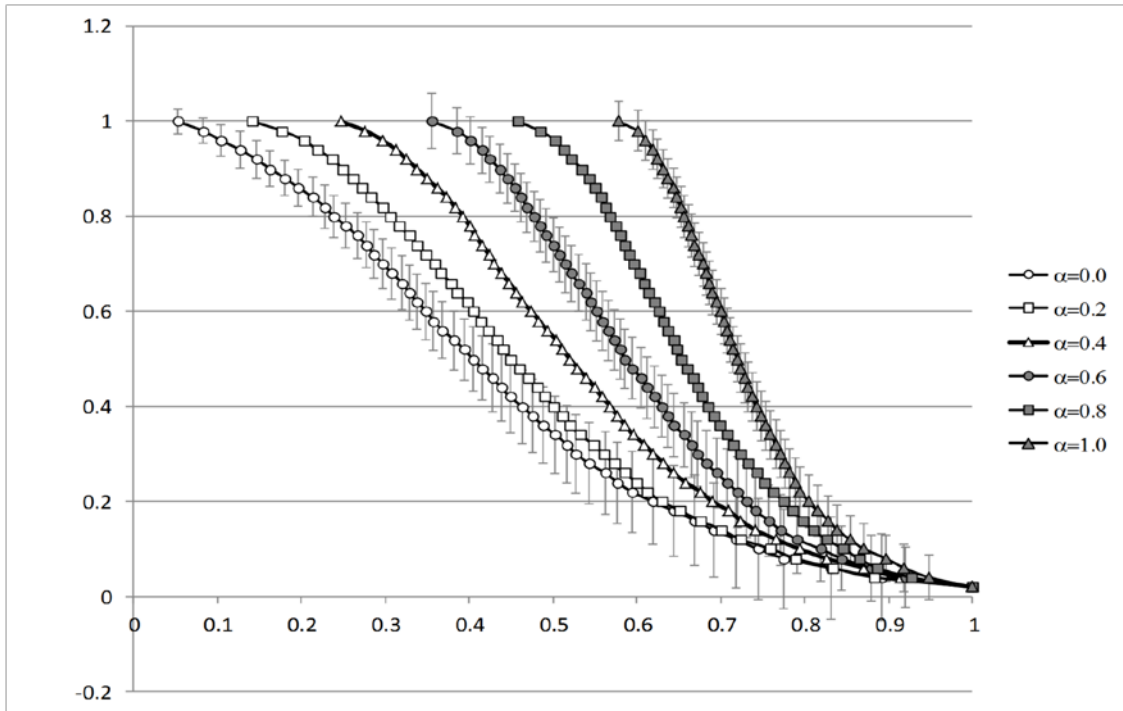


Figure 5: Synthetic strength probability distributions created from weights randomly drawn from exponential (grey) and power law (white) distributions. Standard errors shown as whiskers and only for three distributions to avoid figure cramming.

Table 4 also shows real c_v values for Plaça de la Virreina (Vs) and Vila de Gràcia (Gn), obtained from a fitting of a cumulated normal distribution (Figure 5(a), dashed lines). Although both cases differ slightly, our model sets Vs closer to the exponential case than Gn. From a network's structural point of view, the exponential probability distribution in the case of Vs implies more trivial underlying generative mechanisms than in the case of Gn. Particularly, the correlation between answers for questions related with some needs in the case of Gn is considerably stronger than in Vs, thus making (a) the sub-local level correlations between needs much more loose and (b) the underlying social network at this level less clearly dominated by particular needs.

Finally, Figure 5(b) shows that for $\hat{s}_i < 0.15$, the value of $P_{>}(s_i)$ for questions related with Affection and Security attains $0.18 < P_{>}(s_i) < 0.22$ in contrast with questions related with

the remaining needs, where $P_{>}(s_i) < 0.13$. When all questions are considered, the probability of having nodes connected and classified under the needs Affection and Security is similar, suggesting a more relevant social importance of these two particular needs and at both spatial levels. One difference, though, is the value of \hat{s}_i at which these similar values of $P_{>}(s_i)$ are attained; much lower in the case of Vs than in Gn. Thus, Vs withstands a much more abrupt transition than Gn for these two same needs. Although we cannot account for a plausible explanation of this fact based on sociological facts, there exists a topological explanation for this result and it comes from the difference in strength distribution between Vs and Gn, as previously commented. Since the probability of having nodes classified under the needs Affection and Security is the same in both networks, a network with a more homogenous distribution of correlation strengths among questions, and thus needs, such as Vs, will necessarily have to generate this level of connectivity between the questions related with these two needs much more suddenly than a more inhomogeneous (i.e., hub-dominated) network such as Gn.

Table 4: Results for the coefficient of variation of the mean values of the model and parameter α running from $\alpha = 0.0$ (exponential weight distribution) to $\alpha = 1.0$ (power law weight distribution), compared to our real study cases Plaça de la Virreina square (Vs) and Vila de Gràcia neighbourhood (Gn).

α	c_v
0.0	0.53 ± 0.06
0.2	0.41 ± 0.05
0.4	0.33 ± 0.05
0.6	0.24 ± 0.03
0.8	0.18 ± 0.01

1.0	0.12 ± 0.01
Square (Vs)	0.42
Neighbourhood (Gn)	0.38

In our case study, and with this methodology, the map of subjacent connexions between categories can be observed thanks to a process of direct enquiry to the actors of our urban system. The results of this process are used as a proxy of how this community builds its particular social contract based on preferences. Our numerical model shows that connectivity patterns are a function of spatial and temporal constraints, together with the strength of the different connexions between categories.

In order to explain how these connectivity patterns can be used to design strategies to act on the function and aspect of a system, in our case an urban context at two spatial levels, we need to move into the social sphere of sustainability science. Under the H-SD paradigm, one main assumption is that the presence of all needs (or categories) is equally important to avoid any form of human poverty. In this case, Identity dominates the global need connectivity on both scales, which is somehow an expected trend considering the idiosyncratic neighbourhood of Gràcia with a close-knitted social fabric that allows high levels of public engagement. While Affection and Security are the most connected needs on both scales, different probabilities of occurrence are observed for questions belonging to some particular human needs and at different scales. These probabilities are a function of how satisfiers are perceived. A dissimilar probability of occurrence implies a different presence of satisfiers for the same needs. From the results of our two case studies, it is possible to devise more operational ways to balance the presence of those less-connected needs and to promote integration policies and actions in this direction. For example:

- (1) Identity and Security needs, with the highest strengths, dominate the overall connectivity pattern of the network. It implies, on the one hand, that these needs are well defined at the social level and, on the other, that they are also more difficult to alter or modify. Thus, Identity and Security related aspects, such as sense of belonging, customs or social environment, can be used to strengthen weakly connected needs. For instance, allowing well established social organisations, groups and local NGO's to design and encourage participatory thinking on new spaces for artistic expression to strengthen Creativity, or meditation to strengthen Spirituality, within the urban boundaries.
- (2) Affection (a need with a high percentage of correlation strength among questions according to the probability distributions on both scales) and its existential categories (related with respect, generosity, family and relationships with nature) can be adopted in workshops and hands-on activities as a driving force for social and cultural change that may also lead to rewarding interactions in environments such as schools, universities, communities, thus increasing satisfaction of another poorly connected need like Participation.

With the help of this network representation of how need connectivity is structured, the presence or absence of needs (or categories) appears in a clearer way. In this particular case, the notion of place can be better conceptualised from a more embedded point of view, so as to meet the 'grand challenges' of sustainable adaptations and transitions (Marsden, 2013). In order to accommodate the particular heterogeneity and diversity of places and scales, the values and preferences that support future and converging visions of a place must be determined. This calls for value-laden stances of future generations to be included in visioning processes. It should start from an educational level: an egalitarian and integrative view of needs from an inclusive foundation. This can lead to a sense of caring (for people,

environment, the future, etc.) and it would overcome generic institutional barriers in implementing transition strategies.

Of course communities do not only make decisions based on people's perceptions of needs, but because of events, problems, or actions in the community or even a broader urban system or SES. In a sense, things that are more global to the community may have local consequences, with this scalar shift being termed a process of glocalisation in the broad literature (Martin, McCann, & Purcell, 2003), involving a simultaneous globalisation and localisation of things and processes. At the same time events in the community may motivate plans of action that are not necessarily based on perceptions of needs in the community per se. How a process of assessment of needs can be improved to encompass and address these issues remains thus an open question and it gives several different alternatives for further work.

Introducing a spatial and temporal perspective to this analysis is also fundamental. An important issue would be to reproduce this kind of analysis in other temporal and spatial lines, since temporal perspective raises public awareness of inter-generational phenomena (i.e., trade-off between short-term gains and long-term concerns) and spatial perspective brings an emphasis to intra-generational equity (Kajikawa, 2008; Martens, 2006).

5 Conclusions

This paper presents a methodology based on weighted networks and dependence coefficients aimed at revealing connectivity patterns between categories. The methodology is applied to a case study where categories correspond to human needs and in order to better understand the social dimension of sustainability. The manner in which this paper uncovers how a community builds its particular social framework based on preferential needs or existential categories has not yet been investigated in the research literature. It addresses this particular

dimension of social science theory from an empirically informed bottom-up perspective and with the particular objective of devising new strategies to cope with the complexity of place-making process in general and urban-making ones in particular.

The motivation to developing this comparative analysis on two spatial levels is to shed light on how needs are perceived (i.e., connected) on different scales, to suggest possible reasons for similarities and differences, and to propose, rather than postulate, how it can be used to answer some research questions in social science in general, and also related with that part of sustainability science more linked to social issues in particular. Although it is acknowledged that sustainability science is still characterised by the quest of how to move from complex-system thinking to transformational change, there are techniques coming from other fertile ideas that can be transversally transferred. Here a methodology based on complex networks and maximal information methods was used to reveal the appearance and importance of the various human needs in the urban context. The consequences of this analysis can be used in implementing transformational projects in places. In particular, it presents an approach to define the function and aspect of sustainable human-environment systems (i.e., visions or desirable future states) and also introduces a tentative, yet viable, way to transition urban systems from their current state to a more sustainable one. The aim is to incorporate emerging models and conceptualisations on collective dynamic interactions to integrate social development and sustainability. The final objective is to reveal ways to improve social capacity to guide interactions between nature and society towards more sustainable trajectories.

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