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# Cascading failures with local load redistribution in interdependent Watts-Strogatz networks

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Cascading failures of loads in isolated networks have been studied extensively over the last decade. Since 2010, such research has extended to interdependent networks. In this paper, we study cascading failures with local load redistribution in interdependent Watts-Strogatz networks. The effects of rewiring probability and coupling strength on the resilience of interdependent Watts-Strogatz networks have been extensively investigated. It has been found that, for small values of the tolerance parameter, interdependent networks are more vulnerable as rewiring probability increases. For larger values of the tolerance parameter, the robustness of interdependent networks firstly decreases and then increases as rewiring probability increases. Coupling strength has a different impact on robustness. For low values of coupling strength the resilience of interdependent networks decreases with the increment of the coupling strength until it reaches a certain threshold value. For values of coupling strength above this threshold, the opposite effect is observed. Our results are helpful to understand and design resilient interdependent networks.

*Keywords*: Cascading failures; Watts-Strogatz network; interdependent networks

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## 1. Introduction

Since the small-world network model [1] and the scale-free network model [2] were introduced at the end of the last century, the study of complex networks has attracted an increasing amount of attention in many fields, such as network modelling [3,4], synchronization [5,6], cascading failures [7,8] and optimization [9,10,11]. The importance of robustness against cascades in many real networked systems, has led researchers to explore a number of important aspects of cascading failures such as cascading modeling [12,13], cascade control and defense [14,15,16,17], cascading analysis in real-world networks [18,19], among others. However, most previous studies have focused on the case of a single or an isolated network, ignoring that many infrastructure net-

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works depend on each other. Examples of such infrastructures include power grid and communication networks, where a communication network controls a power grid network and the power grid network in turn provides power to the communication network [20]. Owing to this coupling relationship they can be modeled as interdependent networks [21,22,23].

In two interdependent networks $A$ and $B$, a node in network $A$ is interdependent on its coupling node in network $B$ when the node in network $A$ needs the coupling node from network $B$ to function properly, and vice versa. Failure in one network can cause a cascade of failures that propagates to the other network. This may trigger a recursive process of cascading failures that can catastrophically disintegrate both networks. A realistic example of cascading failures of interdependent networks is the blackout in Italy on 28 September 2003. The failure of the power stations caused a breakdown of the communication control system, which in turn led to further failure in power stations [24].

Cascading failures in interdependent networks is a hot topic recently drawing a great deal of attention [25,26,27,28,29,30,31]. In a pioneering work by Buldyrev et al. [20], a percolation model was developed to research the robustness of interdependent networks subject to cascading failures. They found that a broader degree distribution increases the vulnerability of interdependent networks to random failure, which is opposite to how a single network behaves. Parshani et al. [32] explored the case of two partially interdependent networks. They found that decreasing the coupling strength between two networks leads to a change from a first order percolation phase transition to a second order percolation transition at a critical point. Huang et al. [33] investigated the robustness of interdependent networks under malicious attack on high-degree and low-degree nodes. The result shows that interdependent networks are difficult to defend by protecting the high-degree nodes. These research papers mainly focused on the structure of interdependent networks, while the load redistribution in interdependent networks was not considered.

Currently, there are load redistributions in many real networks and some researchers have investigated the cascading failures of loads in interdependent networks [34,35,36,37,38]. In this scenario, when some nodes fail in network $A$, their loads are redistributed within the same network which can further lead to the overload of more nodes in network A. Accordingly, the dependent nodes in network $B$ can also break down. Brummitt et al. [39] investigated the effects of coupling strength on the cascading failures of loads in interconnected networks with a sand pile model and found that some interdependence is beneficial as it supresses the largest cascades in both networks. Tan et al. [40] studied the cascading failures of loads in interdependent Erdös-Rényi (ER) random networks [41] and Barabási-Albert (BA) scale-free networks [2]. They found that interdependent ER networks are robust under either random failures or intentional attacks, but interdependent BA networks are only robust under random failures but fragile under intentional attacks. Therefore, previous works primarily focused on interdependent or interconnected regular lattice networks [42,43], random networks [37,38,40,44] and scale-free networks [34,37,38,40]. How-

ever, it is known that many real-world systems, which often have high clustering and short characteristic path length, are small-world networks [1]. There is still a lack of unified understanding of the vulnerability of interdependent small-world networks. In this paper, we analyze in detail the cascading failures with local load redistribution in interdependent Watts-Strogatz (WS) small-world networks [1]. Numerical results show that the resilience of interdependent WS networks is greatly affected by both rewiring probability and coupling strength.

The paper is organized as follows. In the next section we present the cascading load model of interdependent WS networks in detail. In Section 3, simulation results and the corresponding theoretical analysis are provided. Finally, the work is summarized in Section 4.

## 2. The Model

Since many real-life interdependent networked systems are of high clustering and short characteristic path length, we consider the case of two Watts-Strogatz (WS) small-world networks [1] labeled A and B. The WS network starts from a regular ring lattice, with each node connected to its $\langle k \rangle$ nearest neighbors, where $\langle k \rangle$ is the average degree of the network. With a rewiring probability $p$, each link is rewired to a node chosen randomly over the entire ring avoiding link duplication [1]. For $p = 0$, the initial ring lattice is unchanged and the network becomes completely random when $p = 1$. For small values of $p$ ($0 < p \ll 1$), the network achieves high clustering and short characteristic path length, i.e., the network exhibits a small-world structure. In our model we set network $A$ and $B$ with the same number of nodes ($N_A = N_B = 500$) and the same average degree ($\langle k_A \rangle = \langle k_B \rangle = \langle k \rangle = 4$). The size of the interdependent WS network is therefore $N = N_A + N_B = 1000$.

In interdependent networks, there are two kinds of links: connectivity links and dependency links. i) The connectivity links make nodes function cooperatively as one network. ii) The dependency links reflect interactions between interdependent networks, i.e., the functioning of node $A_i$ in network $A$ depends on the corresponding node $B_i$ in network $B$, and vice versa. In our model, each node in network $A$ depends on only one node in network $B$. These one-to-one bidirectional dependency links are established randomly, i.e., a randomly selected node $A_i$ from network $A$ is connected to a randomly selected node $B_i$ from network $B$. The fraction of dependency between network $A$ and $B$ is represented by the coupling strength $r$, and $r$ is defined as the ratio between the number of dependency links and the number of nodes $N_A$ in network $A$. It is clear that the coupling strength $r$ is in the range from 0 to 1.

The load of node $i$ can be denoted by its betweenness [12], which is characterized as the total number of shortest paths running through $i$ [45]:

$$b_i = \sum_{j \neq l \neq i} \frac{\sigma_{jl}(i)}{\sigma_{jl}}, \tag{1}$$

where $\sigma_{jl}$ is the number of shortest paths linking each pair of nodes $j$ and $l$, and $\sigma_{jl}(i)$ is the number of shortest paths going from $j$ to $l$ and passing through node

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$i$. The capacity of a node is the maximum load that the node can process [12]:

$$C_i = (1 + \alpha)b_i, \qquad (2)$$

where $\alpha$ ($\alpha \geq 0$) is a tolerance parameter and $b_i$ is the betweenness of node $i$ in the original network. Obviously, the value of $\alpha$ is related to the capability of the nodes to handle the load. A larger $\alpha$ value corresponds to a higher ability of the nodes to resist load perturbations.

Many load redistribution mechanisms have been proposed to study cascading failures of loads in complex systems [12,13,14,46,47,48]. Following common practices, we use the local load redistribution rule [47,48,49]. The load of the failed node $i$, defined by $F_i$, is redistributed to its nearest active neighbors. The additional load $\Delta F_j$ received by the neighboring node $j$ is proportional to its degree $k_j$:

$$\Delta F_j = F_i \frac{k_j}{\sum_{l \in \Gamma_i} k_l}, \qquad (3)$$

where $\Gamma_i$ is the set of neighboring nodes of node $i$. If $F_j + \Delta F_j > C_j$, then node $j$ has failed, inducing the redistribution of the load of $F_j + \Delta F_j$ and probably further breakdown of other vulnerable nodes. To evaluate the robustness of the interdependent networks we use the relative size of the largest component [12,20,34] (connected by both connectivity and dependency links) $G = N'/N$, where $N$ and $N'$ are the size of the largest component before and after cascading, respectively.

At this stage we adopt the commonly used malicious attack strategy where the highest load node $A_i$ is attacked initially [12,17]. When node $A_i$ fails all links connecting to it are removed simultaneously. The load of $A_i$ is redistributed to its nearest non-failed neighbors in network $A$, resulting in fragile nodes in network $A$ to collapse. Moreover, because of the interdependence between two networks the failed node $A_i$ causes the breakdown of its dependent node $B_i$ in network $B$ even though node $B_i$ is still connected via connectivity links in network $B$. These will lead to an iterative process of cascading failures and the process will continue until there are no more casualties in either network. As a final step, the current relative size of the largest connected component $G$ is computed.

## 3. Simulation Results and Discussions

Since the property of the WS network is mainly determined by the rewiring probability $p$, we firstly explore in detail the effect of the rewiring probability $p$ on the robustness of interdependent WS networks. Figure 1(a) shows the relative size of the largest connected component $G$ as a function of the tolerance parameter $\alpha$ with different values of rewiring probability ($p = 0.01$, 0.1 and 1). Here we set the coupling strength $r = 0.5$. One can see that the value of $G$ increases as the tolerance parameter $\alpha$ increases. For low values of $\alpha$, the value of $G$ under rewiring probability $p = 0.01$ is the largest and has smaller similar values for $p = 0.1$ and $p = 1$. For high values of $\alpha$, the smallest value of $G$ is found for $p = 0.1$ and the largest

for $p = 1$, with a middle value of $G$ for $p = 0.01$. These results indicate that the rewiring probability $p$ plays an important role in the robustness of interdependent WS networks.
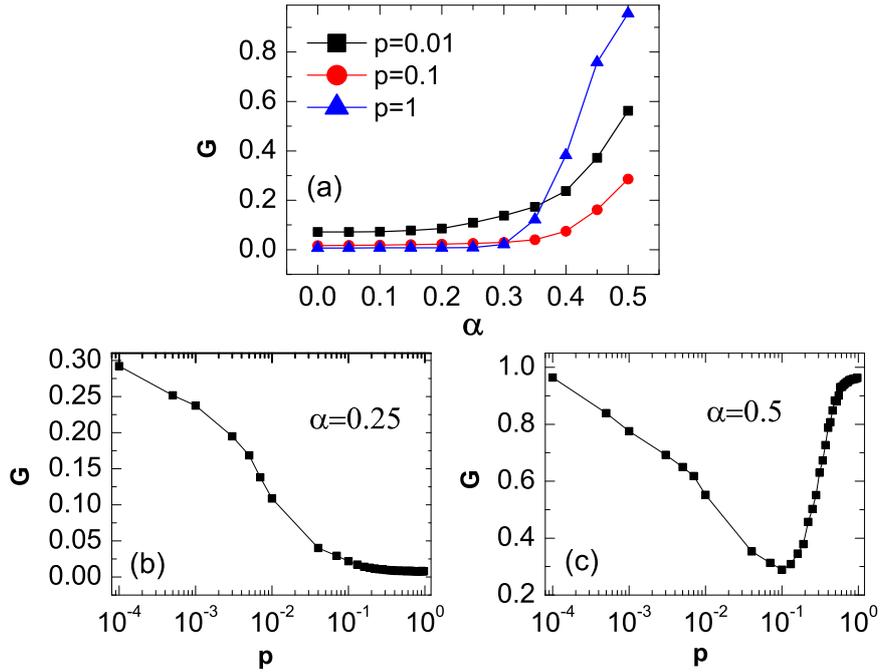


Fig. 1.   (Color online) (a) The relative size of the largest connected component $G$ as a function of the tolerance parameter $\alpha$. $G$ as a function of the rewiring probability $p$ under different values of $\alpha$: (b) $\alpha = 0.25$, (c) $\alpha = 0.5$. The coupling strength is set to $r = 0.5$ and each figure is averaged over 1000 independent realizations.

To further verify these results, we have plotted the relationship between $G$ and the rewiring probability $p$ for different values of $\alpha$. Figure 1(b) shows the relationship between $G$ and $p$ under a low value of $\alpha$ ($\alpha = 0.25$). This shows that the value of $G$ decreases with the increment of $p$. To explain this interesting phenomenon we illustrate the structure of two small interdependent WS networks before and after cascading (Fig. 2). Under $p = 0$, the degree of each node is the same (Fig. 2(a)), and although the value of $\alpha$ is low, the interdependent WS networks can survive after cascading (Fig. 2(b)). For $p = 1$, however, the joint effect of differences in node degree and the low value of $\alpha$ (Fig. 2(c)) causes the complete destruction of the interdependent WS networks (Fig. 2(d)).

Figure 1(c) shows $G$ as a function of $p$ under a high value of $\alpha$ ($\alpha = 0.5$). In constrast to the previous case, as $p$ increases the value of $G$ decreases first and increases afterwards. It is well known that network homogeneity has an important
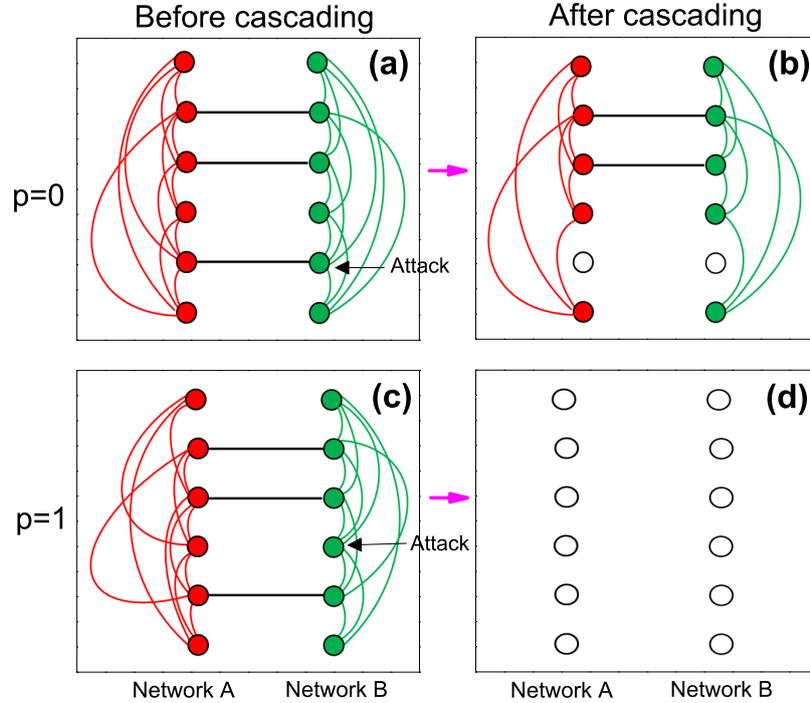
Fig. 2. (Color online) Structures of the interdependent WS networks before cascading with different rewiring probabilities $p$: (a) $p = 0$, (c) $p = 1$. Structures of the interdependent WS networks after cascading with different rewiring probabilities $p$: (b) $p = 0$, (d) $p = 1$. Here the average degree of each network $\langle k \rangle = 4$, the size of the interdependent WS network $N = 12$, $N_A = N_B = 6$, the coupling strength $r = 0.5$ and the tolerance parameter $\alpha = 0.25$. The straight black lines are dependency links.

impact on network robustness against cascading failures of loads [12]. The more homogeneous the network is the more robust the network will be against cascading load failures [12,15]. It is known that network homogeneity can be measured through network polarization [50]:

$$\pi = \frac{b_{max} - \langle b \rangle}{\langle b \rangle}, \tag{4}$$

where $b_{max}$ and $\langle b \rangle$ are the maximum and the average values of betweenness in the network. Obviously, the smaller the value of $\pi$ the less heterogenous the network is. Figure 3 shows the polarization $\pi$ of network $A$ as a function of the rewiring probability $p$. This shows that the relationship between $\pi$ and $p$ is non-monotonic. With the increment of rewiring probability, $\pi$ increases until it achieves a maximum value, then decreases as $p$ keeps increasing. Due to the comparatively homogeneous network structure, the value of $\pi$ for $p = 0$ and $p = 1$ is relatively small. By

comparing the values of $\pi$ (Fig. 3) with the values of $G$ (Fig. 1(c)), we can see that, for the same value of $p$, the value of $G$ has a negative correlation with the value of $\pi$. Consequently, for high values of $\alpha$, the robustness of interdependent WS networks is affected primarily by the homogeneity of the WS networks.
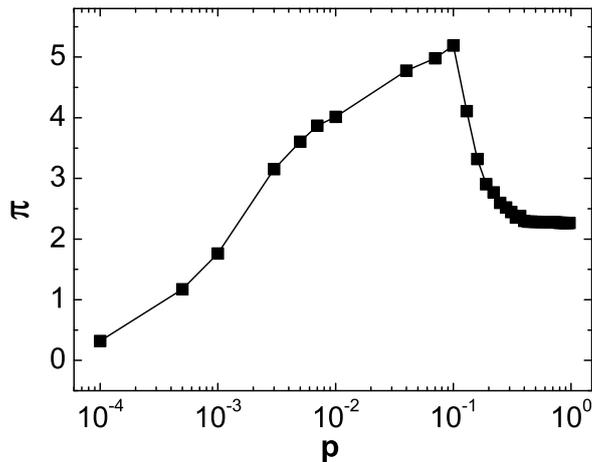


Fig. 3.   (Color online) The polarization $\pi$ of network $A$ as a function of the rewiring probability $p$. Here we set $N_A = 500$, and each figure is averaged over 1000 independent realizations.

When cascading is triggered in interdependent networks nodes can fail because of either overload or loss of dependency. The fraction of dependency between networks is denoted by the coupling strength $r$. Understanding how robustness is affected by the coupling strength is a challenge when designing resilient systems. Next we will investigate the effect couple strength has on the vulnerability of interdependent WS networks.

Figure 4(a) shows the relation between the relative size of the largest connected component $G$ and the tolerance parameter $\alpha$ under different values of coupling strength ($r = 0.01, 0.1, 0.5$ and 1). This shows that the robustness of interdependent WS networks increases as $\alpha$ does for all values of coupling strength. The value of $G$ under $r = 0.01$ is the largest. The robustness of the interdependent WS networks with $r = 0.1$ is larger than that of $r = 1$, and the value of $G$ under $r = 0.5$ is the smallest. These results show that the coupling strength $r$ has a notable impact on the robustness of interdependent WS networks.

To further investigate the effect of the coupling strength $r$ in more detail, the relationship between $G$ and $r$ with $\alpha = 0.2, 0.5$ and 0.6 is depicted in Fig. 4(b). For low values of coupling strength, increasing this coupling strength makes interdependent WS networks less robust against cascading failures. As the dependency
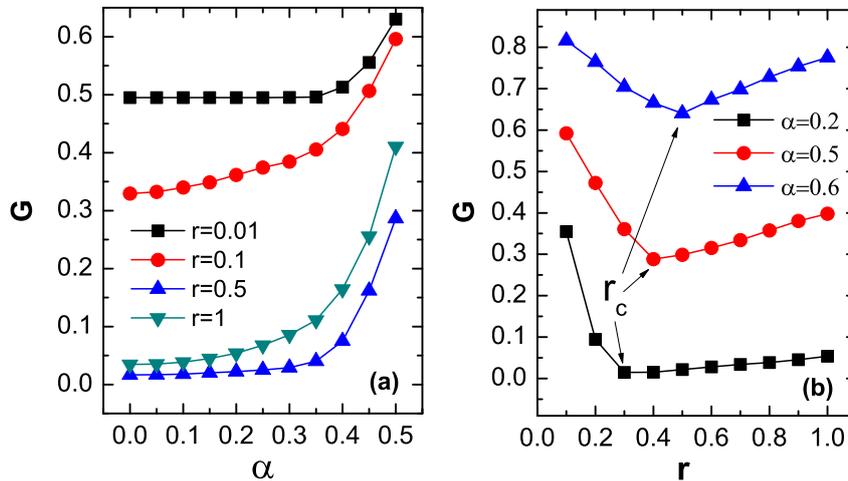
Fig. 4.   (Color online) (a) The relative size of the largest connected component $G$ as a function of the tolerance parameter $\alpha$ with different values of the coupling strength $r$. (b) $G$ as a function of the coupling strength $r$ under different values of $\alpha$. The critical coupling strength $r_c$ is the critical value of $r$ corresponding to the lowest value of $G$. Here the rewiring probability $p = 0.1$, and each figure is averaged over 1000 independent realizations.

links can cause an iterative process of cascading failures between two networks, the interdependent networks will become more fragile as coupling strength increases. For moderate values of coupling strength the robustness of interdependent WS networks is weak and there is a lower value of $G$. For high values of coupling strength, however, increasing the coupling strength can improve the resilience of the interdependent WS networks. In the model, the robustness of interdependent networks is measured by the largest mutually connected component and a large amount of dependency links can make network $A$ and $B$ connect tightly. Thus, the robustness of the interdependent networks is enhanced when the number of dependency links is large enough. This result illustrates that dependency links can also provide benefits, i.e., dependency links can balance all damage when coupling strength goes beyond a certain threshold $r_c$, where $r_c$ corresponds to the lowest value of $G$.

## 4.  Conclusion

We propose a cascading load model in interdependent Watts-Strogatz networks with a local load redistribution mechanism. The impacts of rewiring probability and coupling strength on the robustness of interdependent WS networks have been investigated extensively. Results show that under small values of tolerance parameter $\alpha$ increasing the rewiring probability will make interdependent WS networks more vulnerable. In the case of large values of $\alpha$, a non-monotonic relationship

exists between the rewiring probability and the robustness of interdependent WS networks. Moreover, the correlation between the coupling strength and the resilience of interdependent WS networks is not monotonous.

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