Multimode Equivalent Network Representation for Multiple Arbitrarily Shaped posts in H-Plane Waveguide

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ABSTRACT

This paper presents a method for the calculation of the S-parameter of arbitrary shape and composition post or group of posts in guide. The method segments the problem into regions that are characterized by their respective generalized admittance matrix (GAM). Analytic formulation as well as a boundary integral formulation based on wave function are used in this characterization. Our method is specially suitable for its incorporation to real time CAD tools because only the changing parameters must be computed. This method also permits the simple combination of different numerical techniques.

I. INTRODUCTION

The rigorous study of the dielectric posts in guide is of constant interest in the design of filters. A number of analysis have been performed in recent years facing more and more complex and interesting problems each time [1-4]. In the methods applied to date, the region containing the post is analysed together with the post itself. This implies that in case of any modification in form, position or electric characteristics of the post, the problem would need to be analysed from the beginning, thus limiting its systematic computation. The method that we propose segments the problem into regions. Each region is analysed independently by means of a generalized admittance matrix (GAM). This method has a double advantage. The first one is that one can use the technique that is best suited to the characteristics of the post. The second one is that it avoids having to calculate once more those elements which were not modified. The problem we intend to solve is depicted in fig. 1. In a rectangular waveguide there is a post generally not centered and of arbitrary cross-section and composition. The most convenient segmentation is the one which establishes a circular contour around the post, because the scattered field can be better expressed in terms of cylindrical waves. On the other hand, the propagation of these waves in the waveguide can be expressed more adequately by means of the solution modes in it.

II. INTEGRAL FORMULATION FOR THE ADMITTANCE OF TRANSITION REGIONS I AND III

A modified version of the boundary integral equation based on the wave function proposed by Kishi and Okoshi [5] is used in the full-wave description of homogeneous regions by means of GAM. Regions I and III are homogeneous source free areas where Green's second identity can be

applied on the auxiliar vector potencial $\vec{A} = \psi \hat{e}$ and the electric field $\vec{E}$. After some manipulations, the following identity is obtained

$$\oint_C (j\omega\mu\psi\vec{J}_s + \vec{M}_s \times \nabla\psi) \cdot \hat{z}dC = 0 \quad (1)$$

being $\vec{J}_s = \hat{n} \times \vec{H}_t$ and $\vec{M}_s = \vec{E}_t \times \hat{n}$. The solution of the two-dimensional homogeneous wave equation in a waveguide is a linear combination of modes. Then identity (1) should be simultaneously valid to all the solutions. We therefore have a group of N integral equations. Expanding the equivalent currents in the contour as a linear combination of orthogonal functions, the integral operator is transformed into a matricial one

$$[H] \cdot [J]^C = [U] \cdot [M]^C \quad (2)$$

being

$$[Y] = [H]^{-1} \cdot [U] \quad (3)$$

III. ANALYTICAL FORMULATION FOR THE ADMITTANCE IN THE CENTRAL REGION

In region II two accesses can be observed, as depicted in figure 2. The post or the group of posts are generally not centered. Therefore the wave function that is used to characterize the accesses 3 and 4 is

$$\psi_{II} = \sum_{n=-M}^M [a_n J_n(k\rho) + b_n H_n^{(2)}(k\rho)] e^{jn\phi} \quad (4)$$

where the spectrum of the incident and reflected waves are related by means of a reflection coefficient matrix.

$$[b_n]^C = [R_c] \cdot [a_n]^C \quad (5)$$

When the post is centered, $[R_c]$ represents its reflection coefficient. However, when the post is not centered it is necessary to transform the incident and reflected waves, and then a new reflection coefficient is obtained.

$$[R_c] = [T_H] \cdot [R_c] \cdot [T_J] \quad (6)$$

where $[T_H]$ and $[T_J]$ are translation matrices for Hankel and Bessel functions respectively, obtained from Graf's Addition Theorem for Bessel functions. Now, we can establish a relationship between $\vec{E}$ and $\vec{H}$ in accesses 3 and 4 by means of

$$H_\phi = -\frac{j\omega\epsilon}{k^2} \frac{\partial E_z}{\partial \rho} \quad (7)$$

As the wave function is separable for accesses 3 and 4, the four matrix elements $[Y_{ij}]$ in the generalized admittance matrix can be obtained independently. Then, expanding the electric and magnetic field and using the inner product, after some manipulations, self and mutual admittances can be expressed in matrix form as

$$[Y_v] = [W_v] \cdot [Y_{II}] \cdot [G_v] \quad (8)$$

where

$$w_{\phi n} = \frac{2}{\pi} \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} h_{\lambda_n}(\phi) e^{jn\phi} d\phi \quad (9)$$

$$g_{nl} = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} m_{nl}(\phi) e^{-j n \phi} d\phi \quad (10)$$

$$[Y_H] = \frac{j}{\eta} ([J_a] + [H_a][R_c])^{-1} \cdot ([J_a] + [H_a][R_c]) \quad (11)$$

$[J_a]$ and $[H_a]$ are diagonal matrices containing Bessel and Hankel modes evaluated at $\rho=a$. When a set of posts is considered, the group can be characterized by a reflection coefficient. In this way, the previous formulation for the isolated post can be used. Enforcing the relationships among incident and reflected spectra on each post, and after some manipulations, the following reflection coefficient for the whole set of posts can be obtained

$$[R_c] = [T_w] \cdots [T_n] \cdots [T_{m-1}] \begin{bmatrix} [I] & \cdots & -[R_1][T_n] & \cdots & -[R_1][T_{m-1}] \\ \vdots & & \vdots & & \vdots \\ -[R_n][T_n] & \cdots & [I] & \cdots & -[R_n][T_{m-1}] \\ \vdots & & \vdots & & \vdots \\ -[R_w][T_w] & \cdots & -[R_w][T_n] & \cdots & [I] \end{bmatrix}^{-1} \begin{bmatrix} [R_1][T_n] \\ \vdots \\ [R_n][T_n] \\ \vdots \\ [R_w][T_w] \end{bmatrix} \quad (12)$$

where $[T_{ij}]$ are traslation matrices, $[R_i]$ is the reflection coefficient of each post and $[I]$ is the identity matrix.

When the three regions have been characterized by their GAMs, the next step is to get the global admittance matrix. They can be connected by imposing the continuity conditions in the common surfaces.

IV. RESULTS

Figure 3 shows a plot of the S parameters as a function of permittivity for a circular post of $R = 0.75a$. The frequency of work is $f_c/0.7$. These values are compared with the ones calculated by Abdulnour [4] and show great agreement. A set of three dielectric posts has been computed to solve for the S_{21} parameter. The values obtained by the generalized network model are compared against those obtained by Hsu and Auda [2] using FEM in figure 4. Both curves show the same tendency although they are slightly displaced.

V. CONCLUSIONS

A systematic method for the analysis of posts in waveguide has been proposed using the network theory and the concept of generalized admittance.

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Figure 1. Description of the problem.

Figure 2. Region surrounding the posts to be characterized analytically.

Figure 3. S parameters for a centered cylindrical dielectric post vs. dielectric constant.

Figure 4. S_{21} parameter for a set of three cylindrical dielectric posts vs. normalized frequency.