ANALYSIS OF INHOMOGENEOUS AND ANISOTROPIC WAVEGUIDES BY THE FINITE ELEMENT METHOD

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ABSTRACT

Two numerically efficient procedures for the analysis of arbitrarily-shaped inhomogeneous and anisotropic waveguides showing no spurious modes have been considered. The first one is based on a transverse magnetic-field formulation, while the second one is straightforwardly derived from the first-order Maxwell curl equations and involves all the components of the electric and magnetic fields. In both cases, the final step is in most cases to solve a symmetric complex generalized eigensystem. The commercial mathematical libraries -such as IMSL and EISPACK- usually consider only the general and hermitian cases, but not the symmetric one. In this paper, an extension of the symmetric real case to the symmetric complex case has been developed in order to reduce the time consumed in the solving process.

FINITE ELEMENT FORMULATION

To analyze non-magnetic inhomogeneous and anisotropic lossy waveguides, we can start from the vectorial wave equation for $\vec{H}$:

$$\nabla \times ([\epsilon]^{-1} \nabla \times \vec{H}) - k^2 \vec{H} = 0$$

where k is the free-space wavenumber.

Application of the standard finite element method via a Galerkin procedure gives a matrix equation of the form [1]:

$$[S]\{H\} - k^2[T]\{H\} = \{0\}$$

The divergence-free constraint for the magnetic field can be written

$$H_z = \gamma^{-1} \left(\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} \right), \quad \gamma = \alpha + j\beta$$

and, after a Galerkin procedure:

$$\{H\} = [D]\{H_t\}, \quad \{H_t\} = \begin{Bmatrix} H_x \\ H_y \end{Bmatrix}$$

Combining the two matrix equations, a complex quadratic eigenvalue problem arises [1]:

$$\gamma^2[A]\{H_t\} + \gamma[B]\{H_t\} + [C]\{H_t\} = \{0\}$$

that can be reduced to a complex eigensystem.

An analogous formulation can be obtained for non-dielectric materials in terms of the electric field.

In the general inhomogeneous and anisotropic lossy waveguide problem, the solutions must satisfy [2]

$$\sum_e \int \int \left\{ \vec{E}_{test}^* \cdot (j \nabla \times \vec{H} + \omega \epsilon_0 \hat{\epsilon} \vec{E}) + \vec{H}_{test}^* \cdot (-j \nabla \times \vec{E} + \omega \mu_0 \hat{\mu} \vec{H}) \right\} dx dy = 0$$

for all admissible test functions $\vec{E}_{test}$ and $\vec{H}_{test}$ where the summation is extended over all elements.

The final eigensystem can involve the three components of the electric and magnetic fields or only the transversal components of them, being in both cases of the form:

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$$[P] \begin{Bmatrix} E \\ H \end{Bmatrix} = j\gamma [Q] \begin{Bmatrix} E \\ H \end{Bmatrix}$$

where the matrices are sparse if we involve all the components of the fields.

For all isotropic and some anisotropic cases, the eigensystem may be reduced still more, comprising only the transversal components of the electric or the magnetic field.

In both formulations, the matrices appearing in the final eigensystems are usually symmetric complex (not hermitian) for most problems. However, the commercial mathematical libraries (for instance IMSL, EISPACK or MATLAB) do not consider this special case, thus requiring the use of the general complex routine. In this paper, we have developed a very efficient routine for the symmetric complex generalized eigensystem problem. This routine is an extension of the symmetric real case and consists basically of the following steps: modified Cholesky decomposition, modified Householder tridiagonalization and modified QZ algorithm.

The following tables show the consumed time in an HP-9000 Model 835 using the IMSL (general complex case) and the proposed routine for 50 and 100 equations with arbitrary coefficients, in which we can observe the high time reduction achieved.

Number of unknowns: 50	Time (seconds)
IMSL	51
Proposed	21

Number of unknowns: 100	Time (seconds)
IMSL	290
Proposed	174

NUMERICAL EXAMPLES

Using the first finite element formulation stated above, we have analyzed two inhomogeneous but isotropic waveguides (examples A and B), comparing the obtained results with analytical and other author's results, respectively. The third example presented is an anisotropic lossy waveguide, analyzed by means of the second finite element formulation stated above. In all cases we have solved the final eigensystem with the proposed routine, obtaining very good results in a very reasonable time.

A. Inhomogeneous lossy-free rectangular waveguide.

The following table shows the exact and computed phase constant β for a rectangular waveguide of dimensions 2x1 inch, loaded with a dielectric slab of relative permittivity $\epsilon_r = 2.56$ and dimensions 2x0.33 inch along one of the widest walls. The relative error and the final eigensystem size are also shown for a number of frequencies and modes.

f(GHz)	Mode	exact [3]	computed	error (%)	Number of unknowns
6.948	TM_{10}^y	92.08	93.05	1.04	196
8.106	TM_{10}^y	137.65	138.46	0.58	196
9.264	TM_{10}^y	177.28	178.20	0.51	196
10.42	TM_{10}^y	214.86	216.06	0.56	196
11.58	TM_{10}^y	251.94	253.57	0.64	196
15.14	TM_{10}^y	370.66	373.55	0.77	332
	TM_{20}^y	280.55	287.89	2.55	332
17.67	TM_{10}^y	459.93	463.86	0.85	332
	TM_{2c}^y	390.35	398.13	1.95	332

B. Rectangular waveguide centrally loaded with a slab having parabolic variation in its permittivity.

This example consists of a rectangular waveguide of width 2 cm centrally loaded with a dielectric slab of width 1 cm, whose relative permittivity varies as $\epsilon_r(x) = 1 + 32(0.5 + x)(0.5 - x)$, being x the distance to the center in cm. The table shows the computed phase constant β of the dominant mode for two frequencies and the results obtained by other authors.

f(GHz)	Present paper	Other authors [4]	difference (%)	Number of unknowns
3.581	88.79	92.31	3.96	228
4.775	177.55	188.57	5.84	228

C. Anisotropic lossy waveguide.

We have also analyzed an anisotropic lossy waveguide of dimensions $W \times h$, being $h/W = 0.4454$ with the complex tensor permittivity

$$\epsilon_r = \begin{bmatrix} 11.86 - j0.80 & 0 & 0 \\ 0 & 20.83 - j3.16 & 0 \\ 0 & 0 & 11.86 - j0.80 \end{bmatrix}$$

The table shows the computed normalized propagation constant $\gamma W = (\alpha + j\beta)W$ for the fundamental and first higher order modes for several frequencies and the results obtained by other authors with a different formulation. In all cases, the number of unknowns was $N=70$.

Fundamental mode		
$k_0 W$	Present paper	Other authors [5]
2	0.735 + j8.60	0.736 + j8.63
4	1.40 + j18.0	1.40 + j18.0
6	2.08 + j27.3	2.08 + j27.4
8	2.77 + j36.5	2.78 + j36.6

First higher order mode		
$k_0 W$	Present paper	Other authors [5]
2	0.944 + j6.70	0.947 + j6.73
4	1.47 + j17.2	1.47 + j17.2
6	2.13 + j26.7	2.13 + j26.8
8	2.80 + j36.1	2.81 + j36.2

CONCLUSIONS

The application of the finite element method to the analysis of arbitrary waveguides leads to complex generalized eigensystems where the matrices involved are, in most cases, symmetric. In this paper, a very efficient routine to solve this kind of problems has been developed and several examples has been analyzed using this improvement, obtaining very good results in a very reasonable time.

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