

PERFORMANCE OF THE OPTICAL AMPLIFIER IN FIBER DIGITAL TRANSMISSION  
SYSTEMS WITH TIMMING UNCERTAINTY

G. Junyent, S. Ruiz-Moreno, S. Ruiz-Boqué, J. Vall-llosera, M.A. Garcia-Sempere

Signal Theory and Communications Dep. ETS. Ing. Telecom., Barcelona

INTRODUCTION

The use of SC optical amplifiers in fiber optic digital transmission systems shows great possibilities. Different published works confirm that its use as a preamplifier can improve the receiver sensitivity [1],[2]. It can also be used as a linear amplifier in long distance links substituting the expensive and complex conventional regenerative repeater [3].

This paper is structured as follows: In Part I we establish a definition of the signal to noise ratio at the photodetector output of a monomode fiber optic digital transmission system including an optical preamplifier. Afterwards, in Part II, the performance introduced by a fiber-optical amplifier chain are studied. Finally in Part III we analyze the degradation due to a random timing offset over the system bit error rate.

1. SNR AT THE PHOTODETECTOR OUTPUT

Let us consider the block diagram of Figure I

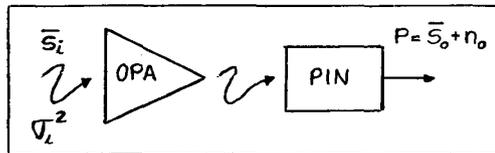


Figure I

where

$$\bar{s}_i = \bar{s} + \bar{n} \quad (1)$$

represents the total photon mean number at the preamplifier input.  $\bar{s}$  is the signal photon mean number and  $\bar{n}$  the noise photon mean number. Let the total number of electrons at the photodetector output be  $P$ . This random variable can be expressed as

$$P = \bar{s}_o + n_o \quad (2)$$

being  $\bar{s}_o = G \eta \bar{s}$  the signal photoelectrons average value at the photodetector output,  $G$  the preamplifier gain,  $\eta$  the quantum

efficiency of the detector and  $n_o$  an additive term which includes the three noise components considered. The first of them is due to the noise contained in the information signal at the preamplifier input ( $\bar{n}$ ), for example if this device is preceded by an optical amplifiers chain. The second component is due to the preamplifier quantum noise. Finally, it must be taken into account the shot noise at the photodetector output due to the power of the incident light as well as to the dark current.

The SNR ratio at the photodetector output can be expressed as

$$SNR_{out} = \bar{s}_o^2 / n_o^2 \quad (3)$$

being

$$n_o^2 = \sigma_p^2 + (\bar{P} - \bar{s}_o)^2 \quad (4)$$

$$\bar{P} = G \eta \bar{s}_i + (G-1) \eta \rho + \bar{d} \quad (5)$$

$$\sigma_p^2 = (G \eta)^2 (\sigma_i^2 - \bar{s}_i) + G \eta \bar{s}_i [1+2(G-1)\eta\rho] + (G-1)\eta\rho [1 + (G-1)\eta\rho] + \sigma_d^2 \quad (6)$$

where  $\bar{P}$  and  $\sigma_p^2$  are the mean and variance of  $P$ ,  $\bar{d}$  and  $\sigma_d^2$  the mean and variance of the electrons due to the dark current and  $\rho$  is the preamplifier spontaneous emission parameter defined as  $\rho = a/(a-b)$  where  $a$  and  $b$  are coefficients related, respectively, to the stimulated emission and absorption phenomena of the preamplifier.

With the definition of  $s_o$  and (4) we can finally express the SNR as

$$SNR_{out} = \frac{(G\eta\bar{s})^2}{\sigma_p^2 + (\bar{P}-\bar{s}_o)^2} \quad (7)$$

2. PERFORMANCE OF THE FIBER-OPTICAL  
AMPLIFIER CHAIN

Let us consider the block diagram of Figure II constituted by  $r$  identical fiber-optical amplifier sections and an optical preamplifier in front of a PIN

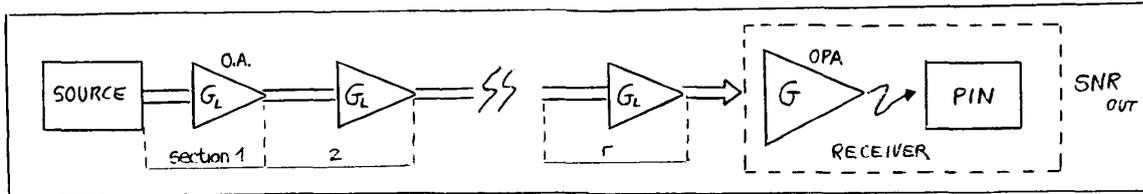


Figure 2.

photodiode. The mean and variance of the photon number at the link input are  $\bar{n}_1$  and  $\sigma_1^2$  respectively. If we assume that the amplifiers exactly compensate the overall losses of each section, the mean and variance at the output of the r-th stage are given, respectively, by

$$\bar{n}_{r+1} = \bar{n}_1 + r\rho_L(G_L-1) \quad (8)$$

and

$$\sigma_{r+1}^2 = \sigma_1^2 + 2r\rho_L(G_L-1)\bar{n}_1 + \rho_L r(G_L-1) [1 + r\rho_L(G_L-1)] \quad (9)$$

where r is the line amplifiers number and  $G_L$  and  $\rho_L$  are their gain and spontaneous emission parameter, respectively.

Identifying

$$\bar{s}_i = \bar{n}_{r+1}, \quad \bar{s} = \bar{n}_1 \quad (10)$$

$$\bar{n} = r(G_L-1)\rho_L, \quad \sigma_i^2 = \sigma_{r+1}^2$$

From (7) we can calculate the signal to noise ratio,  $SNR_{OUT}$ , at the photodetector output as

$$SNR_{OUT} = \left\{ \frac{\sigma_{r+1}^2 - \bar{n}_{r+1}}{\bar{n}_1^2} + \frac{\bar{n}_{r+1} [1+2\eta\rho(G-1)]}{G\eta\bar{n}_1^2} + \frac{(G-1)\eta\rho [1+(G-1)\eta\rho]}{(G\eta\bar{n}_1)^2} + \frac{\sigma_d^2 + [r\eta\rho_L(G_L-1)G+(G-1)\eta\rho+\bar{d}]^2}{(G\eta\bar{n}_1)^2} \right\}^{-1} \quad (11)$$

which is represented in Figure 3 as a function of the number of optical amplifiers.

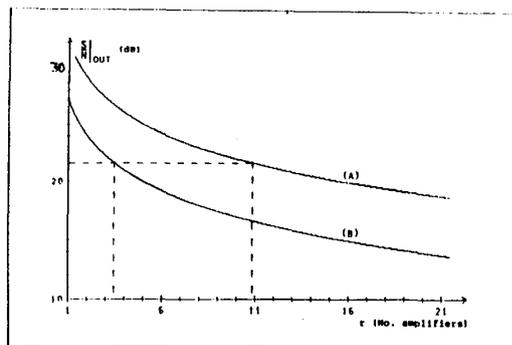


Figure III

a)  $G = 15$  dB, b)  $G = 20$  dB

### 3. INFLUENCE OF THE TIMING UNCERTAINTY

The received optical signal after being photodetected and electrically amplified is integrated over a bit duration of T seconds. The integrator timing is provided by the synchronization system, so, if we consider a Poisson statistics for the light entering to the first optical amplifier the integrated electrons count are:

for a "1" emitted  $n_1 = n_s + n_b$

for a "0" emitted  $n_0 = n_b$

where s and b stand for signal and background respectively and:

$$n_s = G\eta \bar{n}_1 \quad (\bar{n}_1 \text{ is the mean photon number at the link input})$$

$$n_b = [G\eta r\rho_L(G_L-1) + \eta\rho(G-1) + \bar{d}] \quad (12)$$

An accurate evaluation of system performance in terms of the error probability becomes difficult particularly for large signal counts [4]. An approximation consisting in represent the pdf by a single Gaussian process is used, being the variance:

$$\sigma^2 = n_s [2r\rho_L\eta G(G_L-1) + 1 + 2\eta\rho(G-1)] + n_b + [G\eta\rho_L\eta(G_L-1)]^2 + 2\eta^2\rho\rho_L rG(G-1)(G_L-1) + \sigma_T^2 / (eB)^2 \quad (13)$$

where  $\sigma_T^2$  represents the receiver thermal noise, e is the electron charge and  $B=1/T$ .

The optimum threshold for the case of ON-OFF signaling is given by

$$N_{CT} = (y)^{-1} \left[ \left\{ (1+yn_1)(1+yn_0) \left( 1 + \frac{1}{n_1-n_0} \ln \left[ \frac{1+yn_1}{1+yn_0} \right] \right) \right\}^{1/2} - 1 \right] \quad (14)$$

being  $y = (eB)^2 / \sigma_T^2$

If a time offset of  $\Delta$  seconds (Figure IV) exists during a bit period, only a portion of the signal energy is available in the integrator output, and part of the signal energy may contribute to the adjacent interval.

The Bit decisions will be influenced by the adjacent bit (the adjacent when  $\Delta > 0$  and the former when  $\Delta < 0$ ) and we obtain after mathematical analysis:

REFERENCES

- [1] Y.Yamamoto, H.Tsuchiya, "Optical Receiver Sensitivity improvement by semiconductor Laser Preamplifier" *Elect. Lett.* March 1980, Vol.16, No.6.
- [2] D.M.Fye, "Practical Limitations on Optical Amplifier Performance", *J.of Lightwave Tech.*, August 1984, Vol.LT.2, No.4.
- [3] G.Zeidler and D.Schicketanz, "Use of Laser Amplifiers in a glass-fiber Communication System", *Siemens Fors. Entwickl.Ber.Bd.2*, No.4, 1973.
- [4] Midwinter, J.E. "Optical Fibres for Transmission", Wiley Chichester, 1979.
- [5] R.Gangopadhyay, D.Datta, "Performance of an optical receiver employing an avalanche photodetector in the presence of timing uncertainty", *Journal of the Institution of Electronic and Radio Engineers*, Vol.55, No.2, February 1985.

$$P_e(\epsilon) = \frac{1}{2} + \frac{1}{8} \left[ \operatorname{erf} \left( \frac{N_{cr} - n_1}{\sqrt{2} \sigma_T} \right) - \operatorname{erf} \left( \frac{N_{cr} - n_0}{\sqrt{2} \sigma_0} \right) + \operatorname{erf} \left( \frac{N_{cr} - n'_1}{\sqrt{2} \sigma'_1} \right) - \operatorname{erf} \left( \frac{N_{cr} - n'_0}{\sqrt{2} \sigma'_0} \right) \right] \quad (15)$$

where  $n'_1 = n_s(1-\epsilon) + n_b$   $n'_0 = n_s\epsilon + n_b$

and the general expression for the variances is the same we have seen before but for  $n_1, n_0, n'_1$  and  $n'_0$ .

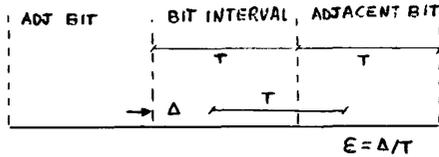


Figure IV

The bit error probability  $P_e(\epsilon)$  increases with increasing values of timing offset, and the degradation in the system performance is worse for high values of the signal electron counts  $n_s$ . In a practical system the timing error is a random variable which statistics depend on the PLL and on the received signal [5]. So for SNR ratios greater than 3 it is possible to approximate the pdf of  $\epsilon$  for:

$$p(\epsilon) = \frac{\exp(\text{SNR} \cos(2\pi\epsilon))}{I_0(\text{SNR})} \quad (16)$$

where  $I_0(\text{SNR})$  is the imaginary Bessel function.

The average error probability is obtained by averaging  $P_e(\epsilon)$  over the statistics of random timing error  $p(\epsilon)$  as:

$$P_e = \int_{-1/2}^{1/2} P_e(\epsilon) p(\epsilon) d\epsilon \quad (17)$$

4. CONCLUSION

We have studied the signal to noise ratio at the photodetector output for a transmission system including on-line optical amplifiers as well as an optical preamplifier. The effect of timing error on the performance of a direct detection PIN receiver has also been studied. This allows an accurate design of the optical link in terms of the bit error rate desired ( $10^{-9}$  is required for optical communications). This implies that for a system with no timing error the signal to noise ratio at the photodetector output must be of 21.6 dB. With timing uncertainty a higher SNR is necessary and, for a given value of  $n_s$ , this allows the design of the link in terms of the gain of the optical amplifiers and number of them in a more realistic way.

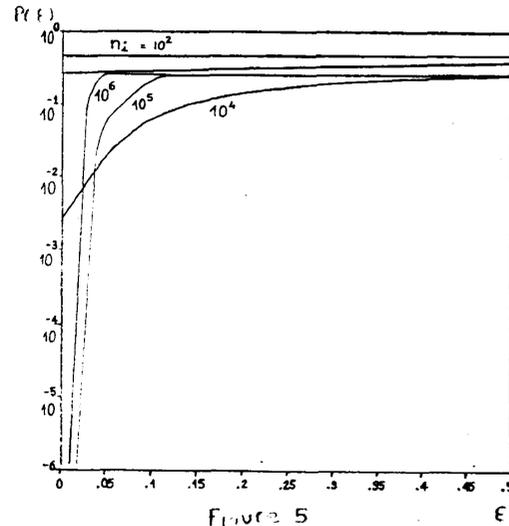


Figure 5