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Highlights

Dynamic response of a rotating disk submerged and confined. Influence of the axial gap*Journal of Fluids and Structures* ■ (■■■■) ■■■–■■■Alexandre Presas^a, David Valentin^a, Eduard Egusquiza^a, Carme Valero^a, Ulrich Seidel^b^a Centre de Diagnòstic Industrial i Fluidodinàmica, Universitat Politècnica de Catalunya, Av. Diagonal, 647. ETSEIB. Pab. D+1, 08028 Barcelona, Spain^b Voith Hydro Holding GmbH & Co. KG, Alexanderstraße 11, 89522 Heidenheim, Germany

- The influence of the axial gap on the natural frequencies of a rotating disk submerged and confined has been determined.
- Natural frequencies are determined from the rotating frame experimentally, analytically and numerically.
- The rotating speed of the fluid with respect to the disk has been analysed with a CFD simulation.
- Natural frequencies of the rotating disk are given in a normalised form.

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Dynamic response of a rotating disk submerged and confined. Influence of the axial gap

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ABSTRACT

In this paper, the natural frequencies and mode shapes of a rotating disk submerged and totally confined inside a rigid casing, have been obtained. These have been calculated analytically, numerically and experimentally for different axial gaps disk-casing. A simplified analytical model to analyse the dynamic response of a rotating disk submerged and confined, that has been used and validated in previous researches, is used in this case, generalised for arbitrary axial gaps disk-casing. To use this model, it is necessary to know the averaged rotating speed of the flow with respect to the disk. This parameter is obtained after an analytical discussion of the motion of the flow inside the casing where the disk rotates, and by means of CFD simulations for different axial positions of the disk. The natural frequencies of the rotating disk for the different axial confinements can be calculated following this method. A Finite Element Model has been built up to obtain the natural frequencies by means of computational simulation. The relative velocity of the flow with respect to the disk is also introduced in the simulation model in order to estimate the natural frequencies of the rotating disk. Experimental tests have been performed with a rotating disk test rig. A thin stainless steel disk (thickness of 8 mm, ($h/r < 5\%$) and mass of 7.6 kg) rotates inside a rigid casing. The position of the disk can be adjusted at several axial gaps disk-casing. A piezoelectric patch (PZT) attached on the rotating disk is used to excite the structure. Several miniature and submergible accelerometers have measured the response from the rotating frame. Excitation and measured signals are transmitted from the rotating to the stationary frame through a slip ring system. Experimental results are contrasted with the results obtained by the analytical and numerical model. Thereby, the influence of the axial gap disk-casing on the natural frequencies of a rotating disk totally confined and surrounded by a heavy fluid is determined.

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1. Introduction

Rotating disk-like structures have a relevant interest in many engineering fields such as turbomachinery components, where it is of paramount importance to determine their natural frequencies in order to avoid resonance problems that could cause a critical failure (Egusquiza et al., 2012; Jacquet-Richardet et al., 2013; Ohashi, 1994; Shlyannikov et al., 2001; Tanaka, 2011). The natural frequencies and mode shapes of rotating disk-like structures have been studied in many cases when they are surrounded by low density fluids such as air, where the effect of the fluid itself can be neglected and therefore, the structure is assumed to vibrate in vacuum (Bauer and Eidel, 2007; Campbell, 1924; Heo and Chung, 2004; Lamb and Southwell, 1921; Southwell, 1922). Many influences on the natural frequencies of the rotating disk, such as high rotating speeds, attachment to the shaft, angular misalignment and imperfections have been discussed in the aforementioned researches. Nevertheless, in some applications such as hydraulic turbomachinery, where the rotating disk-like structure is vibrating submerged in a heavy fluid such as water, the effect of the surrounding fluid has to be considered in order to calculate the natural frequencies of the structure. In this case, the concept of the added mass of the surrounding fluid due to the motion of the disk has been extensively used to calculate the natural frequencies of the disk in contact with the fluid (Kwak, 1991; Lamb, 1921; Amabili, 1996; Amabili and Kwak, 1996). The findings of these studies conclude that when the disk is in contact with an infinite water medium, the natural frequencies of the disk are greatly reduced compared to those ones in air due to the added mass effect.

This effect is even more important when the disk is bounded by rigid walls. Particularly interesting is the effect of the axial gaps in the transverse vibration of the disk (Askari et al., 2013; Harrison et al., 2007; Kubota and Ohashi, 1991; Naik et al., 2003; Rodriguez et al., 2012; Valentín et al., 2014). All these studies, which combine analytical, numerical and experimental results conclude that the closer to a rigid surface the vibrating structure is, the lower its natural frequencies are. Although these researches study the influence of the axial and radial gaps structure-casing, the case of a rotating disk-like structure inside the casing is not considered.

The case of a rotating disk surrounded by water has a relevant interest due to the similarity to some kind of hydraulic runners, which are submerged and confined rotating disk-like structures. Nevertheless, the effect of the relative rotation of the flow with respect to the disk-like structure has only been considered in few cases (Kubota and Ohashi, 1991; Presas et al., 2015). Kubota and Ohashi (1991) used an analytical model to predict the natural frequencies of a standing disk with a relative rotation of water with respect to the disk. In that study the water was considered as a rigid body. Therefore, the more similar case to hydraulic turbomachinery, which is a rotating disk which induces a rotating flow with respect to the disk was not discussed. Furthermore, no experimental results were given. Presas et al. (2015) calculated analytically and experimentally the natural frequencies of a rotating and confined submerged disk. In that case, an averaged rotating speed was calculated by means of a CFD simulation for only one axial gap disk-casing. Nevertheless, the natural frequencies and mode shapes were obtained only for one axial distance to the rigid wall and therefore, the effect of varying the axial gap on the dynamic response of the disk was not discussed.

As shown in Kubota and Ohashi (1991) and Presas et al. (2015), to predict the natural frequencies of the rotating disk, the rotating speed of the flow with respect to the disk has to be known and therefore, the flow pattern has to be obtained. Many studies deal with the flow around a rotating disk. One of the first particular solutions of the Navier–Stokes equations for the flow around an infinite rotating disk in a quiescent fluid are attributed to Kármán (1921). Batchelor (1951) studied the flow between two rotating (or one rotating and one stationary) discs with infinite radius. In such study, it was analytically found that, between both discs, there is a large zone where the velocity in the peripheral direction remains constant (flow without viscous effect) and a zone close to both surfaces, where there is a transition layer. Stewartson (1953) studied the same problem as Batchelor (1951). However, in his analytical solution there was not a central core of the flow with homogenous rotating speed. Dijkstra and Van Heijst (1983) showed that both types of flows could be solutions of the governing equations. Brady and Durlöfsky (1987) studied the effect of the radial confinement in finite discs comparing the case of an open end flow in the radial direction and the closed end case. Results confirmed that for a laminar flow between two finite discs, the confined solution (closed end) corresponded to a Batchelor type flow while for the open end corresponded to Stewartson type. The use of numerical simulation and advanced experimental techniques have allowed to study this problem for more complex cases. Lopez (1996) showed numerical and experimental results for finite and rotating discs bounded by a co-rotating wall. Poncet et al. (2005) studied the turbulent flow in a rotor–stator cavity for a high speed rotor. In the aforementioned study, experimental results were provided and compared to the type of solutions found in the previous studies, summarising rather accurately the solutions for the flow that surrounds a rotating disk confined.

In this paper, the natural frequencies of a rotating disk confined for different axial distances with respect to the casing have been experimentally obtained and contrasted with the results obtained with an analytical model and with numerical simulation. For the analytical model, a formulation, that has been used in previous researches (for one axial position of the disk with respect to the rigid wall), is used here generalised for the different axial positions of the disk. To analytically calculate the natural frequencies the rotating speed of the flow with respect to the disk has to be known. Therefore, the study of the surrounding flow pattern has been made for the different axial gaps disk-casing with a CFD model and comparing with the analytical and experimental solutions found in previous researches. To experimentally study the topic, a rotating disk test rig has been used. It consists of a heavy disk with several miniature accelerometers that measure the response of the disk from the rotating frame. The disk has been excited with a Piezoelectric patch (PZT). Signals are

Fig. 1. Model of a totally confined disk rotating in water at low rotating speeds.

transmitted from the rotating to the stationary frame with a slip ring system. Finally, a numerical FE model combined with the results of the CFD, is used to address the problem completely.

2. Theoretical background

2.1. Simplified analytical model for rotating discs confined and submerged in heavy fluids rotating at low speeds

The natural frequencies and mode shapes of a rotating disk confined in a heavy fluid are discussed in this section. The disk that rotates at low rotating speeds is confined and surrounded with a heavy fluid (water). Due to the rotation of the disk, a rotating velocity of the surrounding fluid with respect to it is induced as shown in Fig. 1.

Firstly, considering an annular disk rotating in vacuum and neglecting effects of rotating inertia and effects of shear deformation (low rotating speeds), the following equation for the transverse vibration can be deduced as for a standing disk (Belvins, 1984; Lamb and Southwell, 1921):

$$\rho_D h_D \frac{\partial^2 w}{\partial t^2} + D \left[\frac{\partial}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right]^2 w = 0, \quad (1)$$

w is the transverse deformation of the disk defined for $r_{in} \leq r \leq r_{out}$ and $0 \leq \theta \leq 2\pi$, where r is the radial coordinate and θ is the angular coordinate. In Eq. (1), $D = \frac{E h_D^3}{12(1-\nu^2)}$ is the bending stiffness of the disk, with E and ν being the Young and Poisson modulus of the material respectively.

The disk is clamped at its centre. Therefore

$$w = \frac{\partial w}{\partial r} = 0 \text{ for } r = r_{in}. \quad (2)$$

To consider the effect of the surrounding fluid on the transverse vibration of the disk, usually potential flow is considered for the surrounding flow (Kubota and Ohashi, 1991; Amabili and Kwak, 1996; Ohashi, 1991, 1994). In this case two fluid fields are considered (Fig. 1) as in other studies (Askari et al., 2013; Kubota and Ohashi, 1991; Kubota and Suzuki, 1984; Presas et al., 2015). Therefore, in cylindrical coordinates, the potential ϕ has to accomplish

$$\begin{cases} \nabla \phi_{up}(r, \phi_{up}, z) = 0 \\ \nabla \phi_{down}(r, \phi_{down}, z) = 0. \end{cases} \quad (3)$$

Due to the relative rotation of the flow with respect to the disk for the upper flow, the angular coordinate is ϕ_{up} and for the lower flow ϕ_{down} . They are related to θ (angular coordinate for the disk) according to the following equation:

$$\theta = \phi_{up} + \Omega_{up} t \text{ \& } \theta = \phi_{down} + \Omega_{down} t. \quad (3)$$

The boundary conditions for the potential flow associated to the defined problem are

$$\begin{cases} \frac{\partial \phi_{up}}{\partial z} \Big|_{z=H_{down}+H_{up}+h_d} = \frac{\partial \phi_{up}}{\partial r} \Big|_{r=r_{casing}} = \frac{\partial \phi_{up}}{\partial r} \Big|_{r=r_{in}} = 0 \\ \frac{\partial \phi_{down}}{\partial z} \Big|_{z=0} = \frac{\partial \phi_{down}}{\partial r} \Big|_{r=r_{casing}} = \frac{\partial \phi_{down}}{\partial r} \Big|_{r=r_{in}} = 0 \end{cases} \quad (5)$$

$$\begin{cases} \frac{\partial \phi_{up}}{\partial z} \Big|_{z=H_{down}+h_d} = \frac{\partial \phi_{down}}{\partial z} \Big|_{z=H_{down}} = \frac{\partial w}{\partial t} \text{ for } r_{in} \leq r \leq r_{out} \\ \frac{\partial \phi_{up}}{\partial z} \Big|_{z=H_{down}} = \frac{\partial \phi_{down}}{\partial z} \Big|_{z=H_{down}} \text{ and } \phi_{down} = \phi_{up} \text{ for } r_{out} \leq r \leq r_{casing}. \end{cases} \quad (6)$$

For standing discs in contact with water (where $\theta = \phi_{up} = \phi_{down}$), the AVMI factors β_{nm} for each mode, which relates the natural frequencies in air (ω_{nm}) and in water (ω_{Fnm}) are usually calculated (Askari et al., 2013; Kwak, 1991; Amabili and Kwak, 1996)

$$\omega_{Fnm} = \frac{\omega_{nm}}{\sqrt{1 + \beta_{nm}}}, \quad (7)$$

where n is the number of nodal diameters and m the number of nodal circles of the corresponding mode shape. For the case of a rotating disk submerged, as commented in Presas et al. (2015), the real flow pattern cannot be represented with potential flow due to the viscous effects. Therefore, in that study, a simplified model to determine the diametrical modes (mode shapes with $m=0$) was used.

This simplified model has been developed and introduced by Kubota and Ohashi (1991) where a flow that rotates uniformly with respect to a standing disk is presented. In that study, no experimental results were given. Presas et al. (2015) used the same model compared with experimental results for one axial position of the disk, commenting the adequate shape ratios of the disk and calculating the rotating speed of the fluid with respect to the disk (Ω_{up} and Ω_{down}) by means of a CFD model.

That model assumes that the deformation of the disk and the motion of the flow in the radial direction can be neglected compared to the motion in the peripheral direction. Therefore, the differential coefficients in the radial direction of Eq. (1) are neglected and the governing equation of the rotating disk vibrating submerged and confined with a heavy surrounding fluid can be represented at one characteristic radius

$$\rho_D h_D \frac{\partial^2 w}{\partial t^2} + \frac{D^*}{r_o^4} \frac{\partial^4 w}{\partial \theta^4} = p_{r_o}. \quad (8)$$

In Eq. (8), w is the axial deformation of the disk represented in the characteristic radius r_o . D^* is a parameter that depends on the geometry and material of the disk and has the same units as the stiffness D . p_{r_o} is the pressure that the surrounding fluid (that rotates with respect to the disk) exerts on the disk surface at radius r_o .

The motion of the disk in r_o becomes then

$$w = \sum_{n=-2}^{+\infty} A_n e^{jn\theta} e^{j\omega_n t} + \sum_{n=-2}^{-\infty} A_n e^{jn\theta} e^{j\omega_n t}. \quad (9)$$

As shown in Presas et al. (2015), Eqs. (8) and (9) are only valid to calculate the natural frequencies of diametrical modes (modes with $m=0$) with more than one nodal diameter ($|n| \geq 2$) of annular thin discs with high outer to inner radius ($\frac{h_D}{r_{out}} < 0.1$ and $\frac{r_{in}}{r_{out}} < 0.2$). These modes are usually the first modes found experimentally in disk-shaft assemblies (Presas et al., 2015; Valentín et al., 2014) and the most prone to be excited in case of hydraulic runners (Egusquiza et al., 2012).

The equation of the potential flow under these assumptions becomes

$$\begin{cases} \frac{1}{r_o^2} \frac{\partial^2 \phi_{up}}{\partial \theta^2} + \frac{\partial^2 \phi_{up}}{\partial z^2} = 0 \\ \frac{1}{r_o^2} \frac{\partial^2 \phi_{down}}{\partial \theta^2} + \frac{\partial^2 \phi_{down}}{\partial z^2} = 0, \end{cases} \quad (10)$$

With boundaries

$$\begin{cases} \frac{\partial \phi_{up}}{\partial z} \Big|_{z=H_{down}+H_{up}+h_d} = 0 \\ \frac{\partial \phi_{down}}{\partial z} \Big|_{z=0} = 0 \\ \frac{\partial \phi_{up}}{\partial z} \Big|_{z=H_{down}+h_d} = \frac{\partial \phi_{down}}{\partial z} \Big|_{z=H_{down}} = \frac{\partial w}{\partial t} \end{cases} \quad (11)$$

Using the energy equation in the non-stationary form (White, 1979), the fluid dynamic pressure exerting to the disk can be calculated as

$$\begin{cases} p_{r_o} = p_{up} - p_{down} \\ p_{up} = -\rho_F \frac{\partial \phi_{up}}{\partial t} \Big|_{z=H_{down}+h_d} \\ p_{down} = -\rho_F \frac{\partial \phi_{down}}{\partial t} \Big|_{z=H_{down}} \end{cases} \quad (12)$$

The analytical expressions of $p_{up}(z=H_{down}+h_d)$ and $p_{down}(z=H_{down})$ are obtained from Kubota and Ohashi (1991).

Substituting the analytical expressions of p_{r_o} and w (Eq. (9)) in Eq. (8) the natural frequencies ω_n are obtained (Presas et al., 2015)

$$\begin{aligned} & \left(\left[\coth\left(\frac{nH_{up}}{r_o}\right) + \coth\left(\frac{nH_{down}}{r_o}\right) \right] \frac{\rho_F r_o}{n} + \rho_D h_D \right) \omega_n^2 + \left(\left[\coth\left(\frac{nH_{up}}{r_o}\right) 2n\Omega_{up} + \coth\left(\frac{nH_{down}}{r_o}\right) 2n\Omega_{down} \right] \frac{\rho_F r_o}{n} \right) \omega_n \\ & + \left(-\frac{D^*}{r_o^4} n^4 + \left[\coth\left(\frac{nH_{up}}{r_o}\right) n^2 \Omega_{up}^2 + \coth\left(\frac{nH_{down}}{r_o}\right) n^2 \Omega_{down}^2 \right] \frac{\rho_F r_o}{n} \right) = 0 \text{ for } |n| \geq 2. \end{aligned} \quad (13)$$

The following conclusions can be extracted after analysing Eq. (13) in detail. To consider the case of a standing disk (independently of the surrounding flow), both upper and lower rotating speeds must be 0 ($\Omega_{up} = \Omega_{down} = 0$) and therefore, the second term of the equation is 0. In this case, the same solution ω_n is obtained for each pair of positive and negative n . Substituting each pair of n and ω_n in Eq. (9), the resulting mode shape is a standing wave with n nodal diameters.

In case of a rotating disk (made with a high density material), rotating in low density medium (such as air), the second term can be neglected (since $\frac{\rho_F}{\rho_D} \approx 0$) and therefore, the same conclusions as before can be stated.

Fig. 2. Flow around a rotating disk radially confined.

When the disk rotates in a high density medium (such as water), the second term cannot be neglected. In this case, a different solution ω_n is obtained for the positive n and for the negative n . Substituting each n and ω_n in Eq. (9), the mode shapes are determined. The mode shape corresponding to the lower ω_n in frequency (positive n) corresponds to a travelling wave travelling in the same direction than the disk. The higher ω_n in frequency (negative n) corresponds to a travelling wave travelling in the opposite direction than the disk.

To analyse the influence of H_{up} and H_{down} (axial gap) in the natural frequencies of the disk, the dependence of Ω_{up} and Ω_{down} on these variables has to be known.

2.2. Influence of the axial confinement on Ω_{up} and Ω_{down}

In order to use Eq. (13), it remains to determine Ω_{up} and Ω_{down} . Eq. (13) is deduced assuming that the disk has a surrounding and confined potential flow that rotates as a rigid body (uniformly) with respect to it. Nevertheless, the case of study is a rotating disk (confined with a non-rotating casing) that induces a motion of the flow with respect to it. In this case, the real flow pattern is characterised by a central core, where the fluid is moving as a rigid body, and a transition layer close to the boundaries (Brady and Durlofsky, 1987; Lopez, 1996; Poncet et al., 2005).

The objective of this section is to determine an averaged value of Ω_{up} and Ω_{down} by studying the flow around the rotating disk (Fig. 1). These can be analytically obtained and by means of CFD simulations.

2.2.1. Analytical

The flow pattern around a rotating disk has been studied in many cases for laminar and turbulent flows and for different boundary conditions (Batchelor, 1951; Brady and Durlofsky, 1987; Dijkstra and Van Heijst, 1983; Kármán, 1921; Stewartson, 1953). To study the flow around the rotating and confined disk (Fig. 1), again the fluid field between the disk and the upper rigid wall and the fluid field between the disk and the lower rigid wall are considered. Each flow field can be considered as the flow between a rotating and stationary disk radially confined (Brady and Durlofsky, 1987) (Fig. 2).

To obtain the flow pattern, cylindrical coordinates are used. Therefore, the velocity of the particles is decomposed in the radial, tangential and axial directions which are v_r , v_ϕ , v_z respectively. The governing equations of the problem are the Navier–Stokes equations of the steady motion of the flow, which is symmetrical about the axis $r=0$ (Batchelor, 1951).

In case of one rotating disk at $z=0$ and one standing disk at $z=H$, the following boundary conditions are considered

$$\begin{aligned} v_z = v_r = 0 \text{ and } v_\phi = \Omega_{\text{disk}} \cdot r \text{ at } z = 0 \\ v_z = v_r = v_\phi = 0 \text{ at } z = H. \end{aligned} \quad (14)$$

As pointed by Brady and Durlofsky (1987), when the flow is radially delimited (Fig. 2), the solution is a Batchelor-type flow (Batchelor, 1951). A detailed way to solve these equations can be found in this reference. The resulting flow is characterised by a transition layer close to the disk and close to the rigid wall and a central core with a constant angular speed v_ϕ . Further theoretical, experimental and numerical studies confirmed this flow characteristic for a radial delimited flow (Lopez, 1996; Poncet et al., 2005).

To determine an averaged value of Ω_{up} and Ω_{down} , the tangential velocity v_ϕ is considered. According to the references mentioned, this component of the velocity is much greater than the other components for the analysed case (flow radially confined). An averaged $\bar{v}_\phi(r)$ can be obtained for each radial position

$$\bar{v}_\phi(r) = \frac{1}{H} \int_0^H v_\phi(r, z) dz. \quad (15)$$

As in Poncet et al. (2005), the entrainment coefficient of the fluid $K(r)$ is defined as the ratio between the averaged value of the tangential velocity of the flow and the tangential velocity of the disk at the same radius

$$K(r) = \frac{\bar{v}_\phi(r)}{\Omega_{\text{disk}} \cdot r}. \quad (16)$$

It has to be noticed that $K(r)$ is defined for each radial position.

To obtain an averaged value of the rotating speed of the fluid, $\overline{\Omega}_{fluid} \cdot \bar{v}_\varphi(r)$ is averaged along the radial coordinate

$$\overline{\Omega}_{fluid} = \frac{1}{r_{out}} \int_0^{r_{out}} \frac{\bar{v}_\varphi(r)}{r} dr = \frac{\Omega_{disk}}{r_{out}} \int_0^{r_{out}} K(r) dr. \quad (17)$$

Therefore, the averaged entrainment coefficient $\bar{K}$ can be defined as

$$\bar{K} = \frac{\overline{\Omega}_{fluid}}{\Omega_{disk}} = \frac{1}{r_{out}} \int_0^{r_{out}} K(r) dr. \quad (18)$$

Poncet et al. (2005) compared experimental values of $\bar{K}$ with other analytical studies and concluded that, for a wide region of Reynolds numbers, 0.4 can be taken as a reference value for the entrainment coefficient $\bar{K}$ of the flow between a rotating and stationary frame radially delimited, i.e.

$$\bar{K} = \frac{\overline{\Omega}_{fluid}}{\Omega_{disk}} \approx 0.4 \text{ for } 1 \cdot 10^6 < Re = \frac{\Omega_{disk} \cdot r_{out}^2}{\vartheta} < 4.5 \cdot 10^6. \quad (19)$$

where ϑ is the kinematic viscosity of the fluid. This value of $\bar{K}$ can be used to calculate the rotating speed of the fluid with respect to the stationary frame ($\overline{\Omega}_{fluid}$) for the field above and below the disk (Fig. 1).

In order to calculate the natural frequencies of the disk with the simplified model and particularly with Eq. (13), it has to be noticed that Ω_{up} & Ω_{down} are referred to the rotating speed of the fluid with respect to the disk (Fig. 1). Using the relation between Ω_{up} & Ω_{down} (velocity of the fluid with respect to the disk) and $\overline{\Omega}_{fluid}$ (velocity of the fluid with respect to the stationary frame):

$$\overline{\Omega}_{up} = \overline{\Omega}_{down} = \Omega_{disk} - \overline{\Omega}_{fluid} = \Omega_{disk} - \bar{K} \Omega_{disk} = (1 - \bar{K}) \Omega_{disk} \approx 0.6 \cdot \Omega_{disk} \quad (20)$$

Therefore, $0.6 \cdot \Omega_{disk}$ can be used as a reasonable approximation for Ω_{up} and Ω_{down} in Eq. (13), if the conditions in Eq. (19) are fulfilled.

2.2.2. CFD

$\overline{\Omega}_{up}$ and $\overline{\Omega}_{down}$ (Fig. 1) can be also obtained more precisely by means of CFD simulations. In this case, Navier–Stokes Equations are solved numerically in all the fluid mesh domains. The fluid mesh has to be defined for the flow delimited by the rotating disk and the rigid casing (Fig. 1). A rotating speed component has to be introduced on the disk surface.

Once the solution is obtained for a number of discrete points (nodes) the problem can be studied in an r–z section, due to the axis symmetry of the flow. The value of $\bar{K}$ can be then obtained as explained in Eqs. (15)–(18), in the discrete version of these equations.

3. Experimental procedure

To analyse the problem from the experimental point of view a rotating disk test rig has been used. The disk is connected to a variable speed motor and its velocity can be adjusted from 0 to 8 Hz. The disk can be excited with a piezoelectric patch and its response can be measured with miniature and submergible accelerometers screwed on the disk.

3.1. Test rig description

A scheme of the test rig used is shown in Fig. 3. The disk is connected to the motor (Mavilor MLV-072) through a driving belt. The rotating speed of the disk can be adjusted with a precision of 1/300 Hz. The motor uses a resolver (Mavilor Resolver

Fig. 3. Test rig used.

Fig. 4. Disk with installed sensors.

Table 1
Configurations used.

$H_{\text{up}}/r_{\text{out}}$	$H_{\text{down}}/r_{\text{out}}$
0.05	0.485
0.1	0.435
0.15	0.385
0.2	0.335

2T8) to control the rotating speed. The rotating speed in water is limited to 8 Hz in order to avoid damages of the motor due to high power consumption. The vibrations of the motor are isolated from the rest of the test rig through rubber mounting isolators. The casing is made of stainless steel.

The disk is a thin disk with a thickness of $h_d/r_{\text{out}} < 5\%$ and $r_{\text{in}}/r_{\text{out}} < 20\%$ and it weighs approximately 7.6 kg. For the excitation, a piezoelectric patch PI-876A12 is used. The patch is glued on the disk with an epoxy component (LOCTITE 454). To measure the response of the disk, seven miniature and submersible accelerometers Dytran 3006A with a sensitivity of 100 mV/g have been screwed to the disk. It is checked that, after the installation of all these sensors, the natural frequencies of the disk in air do not vary (sensors weight less than 1% of the mass of the disk all together). The installed sensors and actuators on the disk are shown in Fig. 4.

The module NI-9263 combined with the amplifier OEM-835 sends the signal to the patch with the desired voltage range (–100V to 250 V). With the slip ring Michigan S10, the accelerometer and patch signals are transmitted from the rotating to the stationary frame. A Bruel&Kjaer Type 3038 (frequency acquisition of 25.6 KHz) is used.

The radial gap disk-casing is $(r_{\text{casing}} - r_{\text{out}})/r_{\text{out}} = 0.035$. The axial gap disk-casing can be varied according to Table 1.

Therefore, after changing the axial position of the disk with respect to the wall, the influence of the axial gap on the natural frequencies of the rotating disk submerged and confined can be evaluated.

3.2. Excitation of the disk and signal analysis

To determine the first diametrical modes of the disk, which are similar to the diametrical modes of some kind of hydraulic runners and the most prone to be excited (Egusquiza et al., 2012; Kubota and Ohashi, 1991; Tanaka, 2011), the disk has been excited with a slow sweep signal. The excitation band is selected from 0 to 1200 Hz. Such a signal can be described as

$$y = A \sin(\alpha \cdot t^2). \quad (21)$$

The sweep rate α can be selected with the time of the excitation and setting $\frac{\alpha t_{\text{end}}}{2\pi}$ to 1200 Hz. The sweep has to be slow enough in order to have a good resolution in frequency, when applying the FFT, without losing information.

The Short Time Fourier Transform (STFT) is applied to the complete time signals of the patch P-0 and the accelerometers. For each average, a time signal of 4 s (resolution of 0.25 Hz), weighted with a Hanning window, is used. The window is translated 0.2 s in each average step. With the peak hold method, which takes the maximum amplitude for each frequency

in all the averages obtained with the STFT analysis, one single spectrum is obtained for the exciter P-0 and one for each accelerometer. The natural frequencies are obtained by the peaks of the accelerometers spectra and the mode shapes are obtained by the relative phase of the accelerometers with respect to P-0. This procedure is explained in detail in Presas et al. (2015).

4. Numerical simulations

4.1. CFD

For the presented simulation in this paper, Ansys Fluent® v14.5 is used to model the fluid domain. To solve the set of equations, a pressure based-double precision solver has been selected with a second order upwind discretization schemes on the transport equations (momentum, energy and turbulence).

The simulations have been performed in steady state for different rotating speeds and different axial positions of the disk with respect to the casing (varying H_{up} and H_{down}).

The geometrical model has been modelled with a 3D structured hexahedral grid. The nodes in contact with the rigid walls have been fixed and a rotating speed has been imposed on the disk (Ω_{disk}). Therefore, the nodes in contact with the disk have the same velocity as the disk surface ($v_\phi = \Omega_{disk} \cdot r$).

A mesh sensitivity study has been performed. In this case, the averaged $\overline{\Omega}_{fluid}$ (Eq. (17)) is selected to compare each mesh tested. The optimal mesh (which has an affordable time consuming) has approximately $7 \cdot 10^5$ elements and less than 1% of discrepancy between the densest mesh tested ($2 \cdot 10^6$ elements).

4.2. Finite element model

To contrast the analytical and experimental natural frequencies of the rotating disk submerged and confined, a Finite Element Model (FEM) of the test rig has been made. The natural frequencies of the disk for the different rotating speeds and different confinements (varying H_{up} and H_{down} according to Table 1) are obtained. Acoustic-structural coupling simulations have been carried out. The rotating speed of the fluid with respect to the disk can be defined considering that all the fluid moves as a rigid body with respect to the disk. Therefore, the results for $\overline{\Omega}_{up}$ and $\overline{\Omega}_{down}$ obtained from the CFD simulation are introduced in the model (as in the analytical model).

Ansys Workbench® software (release 14.5) has been used to carry out the numerical modal analyses. The nodes of the solid parts in contact with the fluid are defined as a Fluid Structure Interaction (FSI) interface. The bottom and the top of the shaft are fixed in the axial direction. A view of the model is shown in Fig. 5. A mesh sensitivity study has been performed using the first diametrical modes of the disk as the comparing variable. Density of the mesh is changed in the axial and radial directions. The final mesh used has approximately $7 \cdot 10^4$ elements and has less than 1% error with respect to the densest mesh tested (about 10^6 elements). More details about this model can be found in Presas et al. (2015).

Fig. 5. Detail of the finite element model.

Table 2
Natural frequencies (Hz) of the disk attached to the shaft in air.

	$f_{exp.}$	$f_{an.}$	$f_{num.}$	Discrepancy (%) (f_{exp} vs. f_{an})	Discrepancy (%) (f_{exp} vs. f_{num})
$n=2$	258	257.47	261	0.21	1.16
$n=3$	589	579.18	574.93	1.66	2.39
$n=4$	1032.5	1029.65	1009.7	0.27	2.2

Table 3
Natural frequencies (Hz) of the disk submerged and confined ($H_{up}/r_{out}=0.05$).

	$f_{exp.}$	$f_{an.}$	$f_{num.}$	Discrepancy (%) (f_{exp} vs. f_{an})	Discrepancy (%) (f_{exp} vs. f_{num})
$n=2$	127	123.76	134.7	2.55	6.06
$n=3$	321	340.92	336.36	6.21	4.78
$n=4$	642.25	671.99	645	4.63	0.43

Fig. 6. Influence of the axial gap on the natural frequencies of the standing disk. (a) Analytical model. (b) Mode $n = \pm 2$. (c) Mode $n = \pm 3$. (d) Mode $n = \pm 4$.

5. Results and discussion

5.1. Stationary disk submerged and confined. Influence of the axial gap

The first several natural frequencies and mode shapes of the disk have been obtained experimentally, analytically and by means of numerical simulation. The natural frequencies of the stationary disk attached to the shaft in air and submerged and confined inside the casing for $H_{up}/r_{out}=0.05$ are compared in Tables 2 and 3.

The parameters r_o , D^* of the analytical model can be selected in several ways according to the studies found. Kubota and Ohashi (1991) and Presas et al. (2015) used $r_o = \sqrt{r_{in} \cdot r_{out}}$ and adapted the parameter D^* to their experimental set-up. Nevertheless Kubota and Suzuki (1984) used the expression $r_o = \frac{r_{in} + r_{out}}{2}$.

Presas et al. (2015) showed that selecting $r_o = \sqrt{r_{in} \cdot r_{out}}$ and adapting D^* to match the experimental and analytical value of the natural frequency $n=2$ for the stationary disk in air, also gives accurate results for higher modes $n=3-6$. In that study, it is also shown that for the confined disk in water with short radial gaps to the casing (a gap of less than approximately 5% of the outer radius has an influence on the natural frequencies Askari et al., 2013; Valentín et al., 2014)) the value of r_o can be slightly increased to consider the added mass effect of this radial gap.

Fig. 7. Amplitude of the modes $n = \pm 2$. (a) Standing disk in air. (b) Rotating disk in air ($\Omega_{disk} = 8$ Hz). (c) Standing disk in water ($H_{up}/r_{out} = 0.05$). (d) Rotating disk in water ($H_{up}/r_{out} = 0.05$, $\Omega_{disk} = 8$ Hz).

In this paper, for the analytical results of the disk in air $r_o = \sqrt{r_{in} \cdot r_{out}}$ and D^* is selected to minimise the square error between analytical and experimental results in air for the considered modes $|n| = 2-4$. For the analytical results of the submerged and confined disk, the same stiffness D^* as in air is used (as done by Presas et al. (2015)) and r_o is slightly increased (approximately an increase of 2.5% with respect to r_{out}) in order to consider the radial gap (minimising the discrepancy between experimental and analytical results for the natural frequencies of the submerged-confined and standing disk).

Fig. 6 shows the influence of the axial confinement on the studied natural frequencies (standing disk). Experimental, numerical and analytical values for the ratio $f_{water(standing\ disk)}/f_{air}$ are compared here. As in other studies shown (Kubota and Ohashi, 1991; Kubota et al., 1985; Valentín et al., 2014), the closer the disk to a rigid surface is, the lower their natural frequencies are. It is seen that for lower modes the added mass effect of the surrounding fluid is higher as predicted by the analytical model and confirmed by the numerical and experimental results. For the used test rig, there is a wide range where the influence of the axial gap is negligible, i.e. the values of the natural frequencies of the disk do not change when changing the axial position (Fig. 6).

5.2. Rotating disk confined and submerged in water

As deduced analytically by Kubota and Ohashi (1991) and demonstrated experimentally in Presas et al. (2015), when the surrounding fluid rotates with respect to the disk, two natural frequencies which correspond to two opposite travelling waves appear for each unique diametrical mode (standing wave) in the standing case.

5.2.1. Frequency modulation

The modulation in frequency is clearly seen in Fig. 7. The amplitude of the FRF (A-X/P-0) is shown in this figure. For the standing case in air (Fig. 7a), the amplitude of four of the installed accelerometers is shown. For the mode $n = \pm 2$ the accelerometers A-0, A-90, and A-180 have the maximal amplitude while the accelerometer A-210, which is close to a nodal diameter, has a low amplitude. For the rotating case in air (Fig. 7b), there is a slight increase in the natural frequency due to

Fig. 8. Phase of the modes $n = \pm 2$. (a) Standing disk in air. (b) Rotating disk in air ($\Omega_{disk} = 8$ Hz). (c) Standing disk in water. (d) Rotating disk in water ($\Omega_{disk} = 8$ Hz).

Fig. 9. Experimental and theoretical wave. (a) Mode $n = \pm 2$, rotating disk air ($\Omega_{disk} = 8$ Hz). (b) Mode $n = +2$, rotating disk water ($\Omega_{disk} = 8$ Hz). (c) Mode $n = -2$, rotating disk water ($\Omega_{disk} = 8$ Hz).

rotation as predicted by some studies (Bauer and Eidel, 2007) and the amplitude behaviour of the accelerometers is the same as in the standing case. In this case, since the density of the fluid is low, the difference in the added mass effect between the forward and the backward wave is not relevant and only one peak is seen. For the standing case in water

Fig. 10. Tangential velocity at r_0 . $\Omega_{disk}=8$ Hz and $H_{up}/r_{out}=0.05$. (a) Upper fluid. (b) Lower fluid.

Fig. 11. $K(r)$ for the upper and lower fluid.

(Fig. 7c), both the frequency and the amplitude are reduced due to the added mass effect. The different effect of the added mass in the forward and backward wave is seen in Fig. 7d, where the disk rotates in water. Two peaks are seen in this case.

5.2.2. Mode shapes

The mode shapes can be analysed with the phase of the available accelerometers installed on the rotating frame. As seen in Fig. 8a–c, which correspond to the phase plot of (Fig. 7a–c) all the points move in phase or in counterphase to each other. For the rotating disk in water (Fig. 7d), the phase of the accelerometers indicates that not all the points are moving in phase or in counterphase.

To see the wave pattern of the rotating disk in air and in water, viewed from the rotating frame, the plot in Fig. 9 is used. The motion of the accelerometers installed on the disk is shown for different times. The amplitude and phase of all accelerometers are normalised with respect to A-0 in each case (A-0 has amplitude 1 and phase 0°). Fig. 9a shows the motion of the accelerometers for the peak of the mode $n = \pm 2$ (Fig. 7b). In this case, it is seen that all the points pass through their maxima/minima at $t=0$ and $t=T/2$ while for $t=T/4$ and $t=3T/4$ they pass through 0. All of these measured points are moving in phase or in counterphase to each other, which indicates that they are moving as a standing wave. Fig. 9b shows the motion of the accelerometers for the first peak in Fig. 7d (mode $n=2$). As seen in this case, the measured points describe a travelling wave which is moving in the same direction than the rotating direction of the disk. Fig. 9c shows the second peak Fig. 7d (mode $n=-2$) which is a travelling wave moving in the opposite direction than the disk.

5.3. Influence of the axial gap on the rotating speed of the fluid

The rotating speed of the fluid with respect to the disk is obtained by means of a CFD simulation and compared with the analytical values summarised in Poncet et al. (2005). The results are calculated for every $r/r_{out} \approx 0.5\%$ and $h/h_{tot} \approx 0.1\%$ in the radial and axial directions respectively.

To obtain Ω_{up} & Ω_{down} the tangential velocity component is considered and averaged according to Eq. (15). Fig. 10 shows the tangential velocity profile, obtained with the CFD simulation of the test rig model, for the flow below and above the disk at the averaged radius r_0 ($\Omega_{disk}=8$ Hz, $H_{up}/r_{out}=0.05$). In both cases, the coordinate $z=0$ represents the node in contact with the disk surface, while $z=H_{up}/r_{out}$ and $z=H_{down}/r_{out}$ are in contact with the rigid walls.

Fig. 12. (a) $\bar{K}$ when varying Ω_{disk} ($H_{up}/r_o = 0.05$). (b) when varying H_{up}/r_o ($\Omega_{disk} = 8$ Hz).

Fig. 13. f_{diff} and f_{centre} .

It is seen that this type of flow is a Batchelor type flow as confirmed by Brady and Durlofsky (1987) for a rotating disk radially confined. Close to the disk surface and to the wall there is a transition layer where the velocity changes. In the middle there is a central core where the fluid moves as a rigid body. The entrainment coefficient of the fluid $K(r)$ is obtained for each radius (Eq. (16)). Fig. 11 shows the variation of the $K(r)$.

It is seen that K increases from 0.32 to 0.45 for the upper fluid while the behaviour for the lower fluid is the opposite. These slight variations of K in the radial direction are also commented by Poncet et al. (2005). The averaged value $\bar{K}$ can be calculated averaging $K(r)$ from r_{in} to r_{out} as in Eq. (17) (also see Fig. 11).

To have $\bar{\Omega}_{up}$ & $\bar{\Omega}_{down}$ for each confinement and rotating speed, the variation of $\bar{K}$ within these two parameters is analysed. Fig. 12a shows that the value of $\bar{K}$ is stabilised at about 0.4, which is the reference value given (Poncet et al., 2005). In that study, the experiments were carried out for $1 \cdot 10^6 < Re = \frac{\Omega_{disk} r_{out}^2}{\nu} < 4.5 \cdot 10^6$. In the present case, $Re \approx 1 \cdot 10^6$ corresponds to $\Omega_{disk} = 4$ Hz and therefore, up to this velocity, both studies are comparable.

The variation of $\bar{K}$ with the axial position of the disk (varying H_{up}/r_o and H_{down}/r_o) is seen in Fig. 12b for a constant speed of 8 Hz. In this figure, it is appreciated that the values of $\bar{K}$ (for $\Omega_{disk} = 8$ Hz, $Re \approx 2 \cdot 10^6$) remain nearby constant and approximately around 0.4 which is the reference value mentioned by Poncet et al. (2005).

From this study it can be seen that, for a wide range of velocities of the disk and axial positions the value 0.4 can be taken as a reference value to obtain the rotating speed of the fluid with respect to the stationary frame. Therefore, with the CFD model of the flow it has been confirmed the factor $0.6 \cdot \Omega_{disk}$ can be applied to obtain $\bar{\Omega}_{up}$ & $\bar{\Omega}_{down}$ (see Eq. (20)).

5.4. Influence of the axial gap on the natural frequencies of the rotating disk in water

To compare the numerical results of the natural frequencies of the rotating disk confined and submerged obtained with the three methods (analytical, numerical and experimental), these will be represented through the parameters $f_{centre} = (f_{n+} + f_{n-})/2$ and $f_{diff} = f_{n-} - f_{n+}$ as made by Presas et al. (2015) (Fig. 13).

The value of f_{centre}/f_{air} for the pair of modes $n = \pm 2$ and $H_{up}/r_{out} = 0.05$ is shown in Fig. 14a. According to the results this value remains nearby constant for the rotating speeds range tested.

Fig. 14. (a) f_{centre}/f_{air} and f_{diff} for the modes $n = \pm 2$ and $H_{up}/r_{out} = 0.05$.

Fig. 15. f_{diff}/Ω_{disk} for the modes $n = \pm 2$ and $H_{up}/r_{out} = 0.05$.

Table 4

Change in % between f_{centre} (8 Hz) and f_{centre} (0 Hz) (or $f_{water(standing\ disk)}$).

H_{up}/r_o	$n = \pm 2$	$n = \pm 3$	$n = \pm 4$
0.05	−1.74	−0.51	−3.34
0.1	−3.11	−4.21	−3.61
0.15	−1.74	−0.2	−0.7
0.2	−4.79	−3.72	−4.15

The dependence of f_{diff} on Ω_{disk} is shown in Fig. 14b. As seen in this figure, there is a linear increase of f_{diff} for increasing Ω_{disk} , which physically means that for increasing rotating speed the difference of the relative velocity of the flow with respect to the forward and to the backward wave increases and therefore, the difference in the added mass effect between both positive and negative mode also increases.

In order to have a representative value for each confinement and mode independently from the rotating speed, the value f_{diff}/Ω_{disk} is used. As seen in Fig. 15, the value f_{diff}/Ω_{disk} remains nearby constant for increasing Ω_{disk} as f_{diff} has a linear dependence on Ω_{disk} . This value is slightly lower for low rotating speeds of the disk due to the small differences in $\bar{K}$ for these lower speeds (Fig. 12).

Therefore, with f_{centre}/f_{air} and f_{diff}/Ω_{disk} obtained for each diametrical mode, the natural frequencies can be analysed for each axial position independently from the rotating speed.

5.4.1. Influence of the axial gap on f_{centre}

As shown in Fig. 14a, the value of f_{centre} remains nearby constant for increasing rotating speed and therefore, this value is approximately equal to the value of $f_{water(standing\ disk)}$ represented in Fig. 6. Hence the value of f_{centre} for the different axial positions can be obtained with good accuracy with Fig. 6 independently from Ω_{disk} , for the range of velocities tested.

Fig. 16. Influence of the axial gap on f_{diff}/Ω_{disk} . (a) Analytical model. (b) Mode $n = \pm 2$. (c) Mode $n = \pm 3$. (d) Mode $n = \pm 4$.

Fig. 17. Modes $n = \pm 3$ for $\Omega_{disk}=8$ Hz. Comparison $H_{up}/r_{out}=0.05$ and $H_{up}/r_{out}=0.2$.

Nevertheless, as pointed in some studies (Hengstler, 2013; Presas et al., 2015) there is a slight decrease in the experimental values of f_{centre} for high rotating speeds. Table 4 shows the variation of f_{centre} comparing $\Omega_{disk}=0$ and $\Omega_{disk}=8$ Hz for the different axial positions and modes considered.

This decrease is lower than 5% for all the cases and therefore, as mentioned before, the value of f_{centre} can be obtained from the standing disk case (Fig. 6). For the numerical and analytical results this change is less than 0.1% for all the modes and confinements tested.

5.4.2. Influence of the confinement on f_{diff}

As shown in Fig. 15, f_{diff}/Ω_{disk} remains nearly constant independently from Ω_{disk} . For the numerical, analytical and experimental results the value of f_{diff}/Ω_{disk} is taken for $\Omega_{disk}=8$ Hz, where the flow pattern is stabilised (Fig. 12).

From Fig. 16 it can be seen that the diametrical mode and the axial confinement have a great influence on the value of f_{diff} . This value physically represents the difference in the added mass effect of the rotating flow with respect to the positive and the negative wave. Generally experimental and numerical results match very well to each other, while the analytical

Fig. 18. Influence of two close rigid walls on $f_{\text{centre}}/f_{\text{fair}}$ and $f_{\text{diff}}/\Omega_{\text{disk}}$ of the modes $n = \pm 2$, $n = \pm 3$, $n = \pm 4$. Variation of $H_{\text{up}}/r_{\text{out}}$ for constant $H_{\text{down}}/r_{\text{out}} = 0.3$, 0.15, 0.1, 0.05, 0.02, 0.01.

model has the trend to predict lower values of f_{diff} , especially for the modes $n = \pm 4$. This difference can be attributed to the radial gap effect, which is not directly considered in the analytical model. Valentín et al. (2014) compared experimental and analytical natural frequencies obtained with the model of Kubota and Suzuki (1984) for a standing disk (which does not consider the radial gap) and concluded that the error (between analytical and experimental results) when the radial gap is not considered, is higher for higher diametrical modes ($n = \pm 4$ and $n = \pm 5$) than for lower diametrical modes ($n = \pm 2$).

Nevertheless, if the natural frequencies of the rotating disk are estimated through the analytical model, a difference of one unit in $f_{\text{diff}}/\Omega_{\text{disk}}$ (which is approximately the maximum difference between the analytical and experimental value in all the tested cases (Fig. 16)), means that the value of f_{n+} is approximately 4 Hz overestimated and the value of f_{n-} is 4 Hz underestimated for a rotating speed of 8 Hz. For the natural frequencies $n = \pm 4$ of the current test rig, this difference represents less than 1%.

It can be also observed that for higher modes the value $f_{\text{diff}}/\Omega_{\text{disk}}$ is higher. A physical explanation is that there is an increased difference in the added mass effect of the fluid with respect to the forward and the backward wave for higher modes, as for these modes the spinning speed of the waves is higher (Hengstler, 2013).

For a disk with close rigid walls, the added mass effect is higher (see Fig. 6 and first term of Eq. (13)) and therefore, the difference in the added mass effect is also higher (second term of Eq. (13)). For this reason, the value $f_{\text{diff}}/\Omega_{\text{disk}}$ increases, which means more separated frequencies for each pair of n , when the disk is closer to a rigid wall. This can be clearly appreciated for the mode $n = \pm 3$, comparing the case of $H_{\text{up}}/r_{\text{out}} = 0.05$ and $H_{\text{up}}/r_{\text{out}} = 0.2$ (Fig. 17) for a rotating speed $\Omega_{\text{disk}} = 8$ Hz in both cases. The peaks corresponding to $H_{\text{up}}/r_{\text{out}} = 0.05$ appear more separated than the peaks for $H_{\text{up}}/r_{\text{out}} = 0.2$.

5.4.3. Analytical study of a disk close to a rigid wall in both sides

For the used test rig, the axial gap which had an influence is only H_{up} . As seen in Figs. 6 and 16 for this test rig and positions available the influence of H_{down} is negligible since for $H_{up}/r_{out}=0.2$ (farthest distance to the rigid walls) f_{centre} and f_{diff}/Ω_{disk} are stabilised. Furthermore, with this test rig it is not possible to study the case of a disk close to a rigid wall in both sides.

This case has been studied analytically (Fig. 18).

Each line represents a constant value of H_{down}/r_{out} , while H_{up}/r_{out} ranges from 0.01 to 0.3. As seen in Fig. 18, there is only a slight difference between the line of $H_{down}/r_{out}=0.3$ and $H_{down}/r_{out}=0.15$, which indicates that up to 0.3, the influence of increasing H_{down}/r_{out} is negligible. Therefore, the line of $H_{down}/r_{out}=0.3$ corresponds to the test rig used. The values of $H_{up}/r_{out}=0.3$ for $H_{down}/r_{out}=0.3$ are the values for an axial position where the axial gap has no influence; i.e. if the axial distances are increased, the natural frequencies do not change.

Departing from the case of $H_{down}/r_{out}=0.3$, generally, it is seen that reducing this gap has a greater effect on the lower diametrical modes than on the higher diametrical modes (closer lines in this case). For $0.1 < H_{down}/r_{out} < 0.3$ the effect of this decrease is more important for higher values of H_{up}/r_{out} . Nevertheless, when $H_{up}/r_{out} < 0.05$ the influence of $H_{down}/r_{out}=0.1-0.3$ is almost negligible.

For configurations where the disk is very close to a rigid wall ($H_{down}/r_{out}=0.01-0.05$), there is a clear influence of this wall especially for higher H_{up}/r_{out} configurations (more separated lines).

For $H_{down}/r_{out}=0.01$ it can be seen that the effect of increasing or decreasing H_{up}/r_{out} is not as relevant as in the former cases. In this case, there is a lower decrease in f_{centre}/f_{air} and a lower increase in f_{diff}/Ω_{disk} when comparing the values for $H_{up}/r_{out}=0.3$ to the values for $H_{up}/r_{out}=0.01$.

6. Conclusion

The influence of the axial gap on the natural frequencies of a rotating disk submerged and confined with close rigid walls has been studied analytically, experimentally and by means of computational FEM. The analysis has been performed from the rotating frame for rotating speeds in range of actual hydraulic turbomachinery. Conclusions of this study could be extrapolated to more complex disk-like structures, such as some kind of hydraulic impellers, which behave as disk-like structures for their first vibration modes and rotate submerged and confined with very short distances to the casing. Only the diametrical modes of the disk (modes with n nodal diameters and no nodal circles), which are similar and comparable in both structures (disk and hydraulic impeller), have been considered in this paper.

As in previous studies, the analytical, numerical and experimental model confirms the following conclusion. Due to the different added mass effect of the surrounding rotating fluid with respect to the forward and to the backward modes, for each natural frequency in the stationary case (standing wave), two natural frequencies appear in the rotating case (travelling waves). The central value of these two natural frequencies remains nearby constant when increasing the rotating speed of the disk (for the rotating speeds range tested). The difference between these two natural frequencies increases linearly with the rotating speed of the disk. For a disk rotating in the rotating speeds range tested and surrounded by a low density fluid (air), this effect is negligible.

With respect to previous studies, this paper investigates the effect of the axial gap on the natural frequencies of the rotating disk. This effect has been analysed in the past for standing discs but never for rotating discs. The axial gap has a great influence on the difference between the two natural frequencies. When the disk is close to a rigid wall, the difference between these two natural frequencies increases. This difference is higher for higher diametrical modes.

The central value of the two natural frequencies has the same behaviour as the natural frequency obtained for the standing disk. Therefore, when reducing the axial distance between the disk and a rigid wall, the central value decreases due to the increasing added mass effect. For lower diametrical modes the relative reduction in the central value, with respect the corresponding natural frequency in air, is higher.

The analytical model that has been validated in previous studies, is used here for an arbitrary axial gap disk-casing, considering the influence of the axial gap on the relative velocity of the surrounding flow with respect to the rotating disk. It has been shown that for a wide range of rotating speeds and axial positions of the disk, the velocity of the flow with respect to the disk is proportional to the rotating speed of the disk with the same constant factor (entrainment coefficient of the fluid).

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