On the Performance Analysis of Asynchronous FH-SSMA Communications

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Abstract—In this paper, a calculation method for assessing the asynchronous frequency-hopping (FH) spread-spectrum multiple-access performance has been envisaged. The method is based upon a Gauss quadrature rule and for this purpose a moment generation algorithm of the random variables involved has been introduced. False alarm and miss probabilities in a basic FH search acquisition configuration and bit error rate in both slow and fast FH binary frequency-shift keyed have been obtained. Different power levels of the multiuser interference, user activity modeled as a Poisson point process and nonfading as well as nonselective Rayleigh fading channels have been considered.

I. INTRODUCTION

FREQUENCY hopping (FH) has become an attractive technique for spread-spectrum multiple-access system (SSMA) for both synchronous and asynchronous environments. However, little effort seems to have been made up to the present time in order to assess the exact performance of the so-called FH-SSMA systems because of the difficulties arising in handling the autointerference. In the synchronous case a recursive calculation of the probability density function (pdf) for a squared envelope of a sum of random phase vectors has been reported [1]. While this technique allows us to carry out a performance analysis in a certain environment, the same would not be appropriate for a random deployment of the users. Furthermore, the receiver noise is not included in the pdf calculation. In the asynchronous case, the difficulties increase and only approximations and bounds of the system performance have been presented in the open literature [2], [3]. This applies to both nonrandom and random users received power.

On the other hand, performance analysis of FH-SSMA systems encompasses both the synchronization of the receiver's hopping pattern to that of an incoming signal, regarding false alarm and miss probabilities, and the operational stage regarding bit error probability. In this paper we intend to go a step further on the subject by presenting a method for the calculation of the above-mentioned probabilities in the presence of asynchronous FH-SSMA communications and receiver noise. In all the cases, the calculation procedure uses a Gaussian quadrature rule (GQR) and for this purpose a moment generation algorithm has been elaborated.

In order to assess the proposed method, we present some results and compare them, whenever it has been possible, to other published results. The presented results are not, of course, exhaustive but they point towards the significant trends. Referring to the synchronization process, a serial search acquisition scheme has been chosen. The proposed method, however, could be identically applied to a matched

filtering acquisition scheme. For the operational stage both fast and slow FH binary frequency-shift keyed (FFH-BFSK and SFH-BFSK) have been considered. In this case, several user deployment statistics and both nonfading and Rayleigh fading environments have been analyzed. When considered, the fading has been assumed to be slow relative to the hopping rate.

II. SYSTEM MODEL

We consider the presence of U simultaneous users operating asynchronously. All the present signals are frequency hopped each τ s according to independent stationary random hopping patterns. The number of available frequency slots is Q .

The probability, conditioned on the existence of only one user, that the arrival time of a tone corresponding to the frequency channel j ($1 \leq j \leq Q$) occurs in a given interval of duration t ($0 \leq t \leq \tau$) is

$$p = \frac{1}{Q} \frac{t}{\tau} \quad (1)$$

This probability could also be interpreted as the probability of having one occurrence in a given interval of duration t where the position of this occurrence is a random variable uniformly distributed on the interval $[0, Q\tau]$.

In the presence of U occurrences, generated by the U users mentioned above, the probability of having V ($V \leq U$) of such occurrences in the given interval of duration t can be approximated, supposing identical and statistically independent users, and $Q\tau \gg t$ (which is always the case in FH systems), by the Poisson distribution [9] as

$$P_V \approx e^{-\lambda t} \frac{(\lambda t)^V}{V!} \quad (2)$$

where

$$\lambda = \frac{U}{Q\tau} \quad (3)$$

Hence, a Poisson point process with rate λ will be retained in our approach to model the arrival times of users in a particular frequency channel.

The above approach allows us to formulate the complex envelope of the so-called autointerference signal present at the IF receiver input as

$$I(t) = \sum_{i=-\infty}^{\infty} \sum_c A_{i,c} \exp(-j\theta_{i,c}) \cdot \text{rect}_\tau(t - l\tau - t_{i,c}) \exp(-j\Delta\omega_c t) \quad (4)$$

$l\tau \leq t \leq (l+1)\tau \quad \text{with } l \text{ integer}$

where $\{t_{i,c}\}$ is a Poisson random point process corresponding to the c th adjacent channel separated $\Delta\omega_c$ from the desired one, $A_{i,c}$ and $\theta_{i,c}$ are, respectively, the amplitude and phase

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Fig. 1. Serial search synchronizer.

contributions of simultaneous users transmitting in the c th adjacent channel, $\theta_{i,c}$ is uniformly distributed over $[0, 2\pi]$, $A_{i,c}$ is distributed according to some specified deployment statistics and

$$\text{rect}_\tau(t) = \begin{cases} 1 & \text{if } 0 \leq t < \tau \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

For the sake of simplicity and because of the filtering effect at IF stages we will not consider the presence of adjacent channels. We will also assume that $l = 0$, then

$$I(t) = \sum_{i=-\infty}^{\infty} A_i \exp(-j\theta_i) \text{rect}_\tau(t - t_i). \quad (6)$$

where we drop $c = 0$ for the desired channel.

A. Serial Search Synchronizer

We will discuss the most basic noncoherent FH acquisition configuration with no information modulation and suited to fast-FH [4]. This synchronizer, shown in Fig. 1, performs an active correlation between the misaligned received and locally generated FH sequences. The energy detector following the IF stage is made up of integrated-and-dump circuits (see Fig. 2), which act as a matched filter due to the rectangular bursts of signal correlation appearing at their inputs. Then, we have at the detector output in absence of the desired signal.

$$M(\tau) = \left| \sum_{i=-\infty}^{\infty} A_i \exp(-j\theta_i) S(\tau - t_i) + n_D(\tau) \right| \quad (7)$$

where

$$S(t) = \frac{1}{2} \text{rect}_\tau(t) * \text{rect}_\tau(t) = \begin{cases} \frac{t}{2} & \text{if } 0 \leq t < \tau \\ \tau - \frac{t}{2} & \text{if } \tau \leq t < 2\tau \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

and

$$n_D(t) = \frac{1}{2} n(t) * \text{rect}_\tau(t) \quad (9)$$

where $(*)$ denotes convolution and $n(t)$ is the complex envelope of the input receiver noise, assumed Gaussian, white, and with $N_0/2$ double-sided spectral density. At this point, we drop τ on the variable $M(\tau)$ for simplicity.

Fig. 2. Energy detector.

The complex envelope of the desired filtered signal is

$$S_E(\tau) = A \exp(-j\theta) S(\tau - \Delta) \quad (10)$$

where A is the signal amplitude, θ is uniformly distributed on $[0, 2\pi]$ and Δ (being $|\Delta| < \tau$) is the timing offset. The synchronizer, shown in Fig. 1, performs a test every $M_T \tau$ s. Only when K successive tests are 1, the acquisition process ends. Otherwise, the search goes on. Actually, by increasing K , the false alarm probability and the detection probability decrease and the code acquisition time increases.

B. FH-BFSK Scheme

The FH-BFSK demodulator shown in Fig. 3 has two filters matched to the mark and space signals, respectively. Their outputs are envelope-detected and sampled once every data bit time, T_b . A mark-space decision is made according to the larger detector output.

FFH-BFSK where we assume a hopping rate equal to the bit signaling rate, and SFH-BFSK, when two or more bits are transmitted in the time interval between hops, will be considered in our analysis.

The complex envelope of the desired filtered signal is now

$$S_D(T_b) = A \exp(-j\theta) S(T_b) \quad (11)$$

where $T_b = \tau$ for FFH-BFSK and $T_b = \tau/N_b$ for SFH-BFSK where N_b is integer and $N_b \geq 2$.

As we are primarily concerned with multiple-access interference, we do not account for interference between two FSK tones of a given signal. That is, we assume the two detected signals given by

$$|I_D(T_b) + n_D(T_b) + S_D(T_b)|, \text{ for mark} \quad (12)$$

and

$$|I_D(T_b) + n_D(T_b)|, \text{ for space} \quad (13)$$

Fig. 3. BFSK demodulator.

to be statistically independent where

$$I_D(t) = \sum_{i=-\infty}^{\infty} A_i \exp(-j\theta_i) S(t-t_i). \quad (14)$$

III. FALSE ALARM AND MISS PROBABILITIES IN A SERIAL SEARCH SYNCHRONIZER

By referring to Fig. 1, we can formulate the so-called elementary false alarm p_{fa} and miss p_{miss} , probabilities as

$$p_{fa} = \text{Prob} \{ |I_D(\tau) + n_D(\tau)| \geq u \} \quad (15)$$

and

$$p_{miss} = \text{Prob} \{ |S_E(\tau) + I_D(\tau) + n_D(\tau)| < u \} \quad (16)$$

where u is the threshold value at the energy detector output. Then, the false alarm and miss probabilities for one test are

$$P_{FA} = \sum_{j=ID}^{M_T} \binom{M_T}{j} p_{fa}^j (1-p_{fa})^{M_T-j} \quad (17)$$

and

$$P_{MISS} = \sum_{j=0}^{ID-1} \binom{M_T}{j} (1-p_{miss})^j p_{miss}^{M_T-j}, \quad (18)$$

respectively, where ID is the threshold value at the output of the Mod- M_T counter (see Fig. 1). For K tests the false alarm and the miss probabilities are readily found as

$$P_{FA,K} = P_{FA}^K \quad (19)$$

and

$$P_{MISS,K} = 1 - (1 - P_{MISS})^K, \quad (20)$$

respectively.

A. Nonfading Case

From [5] we can formulate the elementary false alarm probability [see (15)] conditioned on the random variable $|I_D(\tau)|$ as

$$p'_{fa} = Q\left(\frac{|I_D(\tau)|}{\sigma_n}, \frac{u}{\sigma_n}\right) \quad (21)$$

where $Q(\cdot, \cdot)$ is the Marcum Q function, and

$$\sigma_n^2 = \frac{N_0 \tau}{4}. \quad (22)$$

A more convenient notation for p_{fa} that includes appropriate parameters as variables, such as the desired and interfering powers as well as the signal-to-noise ratio, can be introduced by defining

$$r = \left| \sum_{i=-\infty}^{\infty} R_i \exp(-j\theta_i) S(\tau-t_i) \right|^2 \quad (23)$$

where

$$R_i = \sqrt{\frac{P_i}{P}} \frac{2}{\tau} \quad (24)$$

$$P = \frac{1}{2} A^2 \quad (25a)$$

and

$$P_i = \frac{1}{2} A_i^2. \quad (25b)$$

Then, from (14), (21) and (23), p_{fa} can be formulated as

$$p_{fa} = \int_0^{\infty} Q\left(\sqrt{2 \frac{E_c}{N_0}} x, \beta\right) f_r(x) dx \quad (26)$$

where

$$\frac{E_c}{N_0} = \frac{P\tau}{N_0}, \quad (27)$$

$$\beta = \frac{u}{\sigma_n} \quad (28)$$

and $f_r(x)$ is the probability density function of the random variable r . Given that $f_r(x)$ is difficult to formulate, a calculation procedure for p_{fa} in (26) requiring only the moments of r is envisaged. Specifically, p_{fa} can be calculated by using a Gaussian quadrature rule (GQR), which guarantees in the area of approximate integration, the highest degree of precision. That is, by choosing N sufficiently large we get

$$p_{fa} \approx \sum_{n=1}^N Q\left(\sqrt{2 \frac{E_c}{N_0}} x_i, \beta\right) w_i \quad (29)$$

where the set $\{w_i, x_i\}$ is called a quadrature rule corresponding to the weight function $f_r(x)$. For this to operate, it is sufficient that the weight function $f_r(x)$ satisfies these conditions [6]:

1) $f_r(x)$ is nonnegative, integrable in $[0, \infty]$ with

$$\int_0^{\infty} f_r(x) dx > 0, \quad (30)$$

2) the integrals

$$\int_0^{\infty} x^K f_r(x) dx \quad (K \text{ integer}) \quad (31)$$

are definite and finite. The pdf, $f_r(x)$ satisfies the above conditions, so the set $\{w_i, x_i\}$ can be evaluated from the $2N + 1$ first moments of the random variable r [7].

In order to assess the complexity of this technique it suffices to say that it basically consists of two steps.

a) Evaluation of the coefficients $(\alpha_n)_{n=1}^N$ and $(\beta_n)_{n=1}^{N-1}$ of the three-term recurrence relationship

$$\begin{aligned} \beta_n P_n(x) &= (x - \alpha_n) P_{n-1}(x) - \beta_{n-1} P_{n-2}(x) \\ n &= 1, 2, \dots, N \\ P_{-1}(x) &= 0 \\ P_0(x) &= 1 \end{aligned} \quad (32)$$

where $P_0(x), \dots, P_N(x)$ are orthogonal polynomials with respect to the random variable r ,

b) generation of a symmetric tridiagonal matrix given by

$$J = \begin{bmatrix} \alpha_1 & \beta_1 & 0 & & 0 \\ \beta_1 & \alpha_2 & \beta_2 & & \\ 0 & \beta_2 & \alpha_3 & & \\ & & & \ddots & \\ 0 & & & \alpha_{N-1} & \beta_{N-1} \\ & & & \beta_{N-1} & \alpha_N \end{bmatrix}. \quad (33)$$

The weights $\{w_i\}_{i=1}^N$ are found as the first components of the orthonormalized eigenvectors of this matrix and the abscissas $\{x_i\}_{i=1}^N = 1$ are the corresponding eigenvalues. The coefficients of the three term recursive formula are found through the elements of the matrix L where

$$G = L^T L \quad (34)$$

is the Cholesky decomposition and G is the Gram Matrix such as

$$(G)_{ij} = E[r^{i+j}] \quad i, j = 0, \dots, N. \quad (35)$$

An algorithm to compute these moments has been elaborated and it is outlined in the Appendix. Due to the existence of a delta function $\delta(t)$ in $f_r(x)$, a better GQR behavior is obtained by expressing

$$p_{\text{fa}} = \text{Prob}(i=0) Q(0, \beta) + [1 - \text{Prob}(i=0)]$$

$$\cdot \int_0^\infty Q\left(\sqrt{2 \frac{E_c}{N_0}} x, \beta\right) \cdot f_r(x|i \neq 0) dx \quad (36)$$

where $\text{Prob}(i=0)$ is the probability that the number of Poisson points that appear in 2τ s is equal to zero; that is,

$$\text{Prob}(i=0) = \exp(-2\lambda\tau)$$

due to the 2τ s duration of $S(t)$. The set $\{w_i, x_i\}$ to solve the integral appearing in (36) is now found from the $2N + 1$ moments

$$\int_0^\infty x^n f_r(x|i \neq 0) dx = \begin{cases} \frac{\int_0^\infty x^n f_r(x) dx}{1 - \text{Prob}(i=0)} & \text{if } n \neq 0 \\ 1 & \text{if } n = 0 \end{cases} \quad (37)$$

p_{miss} , at first, can be formulated as

$$\begin{aligned} p_{\text{miss}} &= 1 - \int_0^\infty Q\left(\sqrt{2 \frac{E_c}{N_0}} x, \beta\right) f_{r_s}(x) dx \\ &= 1 - \text{Prob}(i=0) Q\left(\sqrt{2 \frac{E_c}{N_0}} \frac{2}{\tau} S(\tau - \Delta), \beta\right) \\ &\quad - \text{Prob}(i \neq 0) \int_0^\infty Q\left(\sqrt{2 \frac{E_c}{N_0}} x, \beta\right) \\ &\quad \cdot f_{r_s}(x|i \neq 0) dx \end{aligned} \quad (38)$$

where $f_{r_s}(x)$ is the pdf of

$$r_s = \left| \frac{2}{\tau} S(\tau - \Delta) + \sum_{i=-\infty}^{\infty} R_i \exp(-j\theta_i) S(\tau - t_i) \right|^2 \quad (39)$$

and $f_{r_s}(x|i \neq 0)$ is a conditioned pdf of r_s introduced to handle the delta function, $\delta(x - [2/\tau S(\tau - \Delta)]^2)$, appearing in $f_{r_s}(x)$ analogously as it was done in (36) with $\delta(x)$. The GQR fails, however, when evaluating

$$\begin{aligned} \int_0^\infty Q\left(\sqrt{2 \frac{E_c}{N_0}} x, \beta\right) f_{r_s}(x|i \neq 0) dx \\ \equiv \sum_{i=1}^N Q\left(\sqrt{2 \frac{E_c}{N_0}} x_i, \beta\right) \cdot w_i \end{aligned} \quad (40)$$

where the set $\{w_i, x_i\}$ is the quadrature rule corresponding to $f_{r_s}(x|i \neq 0)$ and calculated through the $2N + 1$ moments

$$\begin{aligned} m'_n &= \int_0^\infty x^n f_{r_s}(x|i \neq 0) dx \\ &= \frac{\int_0^\infty x^n f_{r_s}(x) dx - \text{Prob}(i=0) \left[\frac{2}{\tau} S(\tau - \Delta) \right]^{2n}}{1 - \text{Prob}(i=0)}. \end{aligned} \quad (41)$$

This GQR failure occurs because here the roundoff errors generated during the Cholesky decomposition of an ill-conditioned Gram matrix prevents N from reaching a convenient value, that can assure a good convergence in (40). In particular, we found that no convergence was possible for $\lambda\tau \gtrsim 0.3$.

An alternative method for computing P_{miss} has been envisaged and it is presented in the following. From (16) and (7) we can formulate

$$P_{\text{miss}} = \text{Prob}\{|\sqrt{2P} \exp(-j\theta) S(\tau - \Delta) + M \exp(-j\varphi)| < u\} \quad (42)$$

where φ is a random variable uniformly distributed on $[0, 2\pi]$. By observing Fig. 4, we can express

$$P_{\text{miss}} = \int_{\sqrt{2P} S(\tau - \Delta) - u}^{\sqrt{2P} S(\tau - \Delta) + u} \frac{\psi(x)}{\pi} f_M(x) dx \quad \text{if } \sqrt{2P} S(\tau - \Delta) > u \quad (43)$$

Fig. 4. Decision zones for calculating the miss probability.

and

$$p_{\text{miss}} = \int_0^{u - \sqrt{2PS}(\tau - \Delta)} f_M(x) dx + \int_{u - \sqrt{2PS}(\tau - \Delta)}^{u + \sqrt{2PS}(\tau - \Delta)} \frac{\psi(x)}{\pi} f_M(x) dx \quad \text{if } \sqrt{2PS}(\tau - \Delta) < u \quad (44)$$

where $\psi(M)$ is the angle shown in Fig. 4 for a given M value and $f_M(x)$ is the pdf of the random variable M defined in (7). On the other hand,

$$\text{Prob} \{M < x | r\} = 1 - Q \left(\sqrt{2 \frac{E_c}{N_0}} r, \frac{x}{\sigma_n} \right) = F_M(x | r) \quad (45)$$

and

$$f_M(x | r) = \frac{dF_M(x | r)}{dx} = \frac{1}{\sigma_n} \exp \left(-\frac{2 \frac{E_c}{N_0} r + \frac{x^2}{\sigma_n^2}}{2} \right) \cdot I_0 \left(\sqrt{2 \frac{E_c}{N_0}} r \cdot \frac{x}{\sigma_n} \right) \frac{x}{\sigma_n} \quad (46)$$

By noting that

$$f_M(x) = \int_0^\infty f_M(x | r = y) f_r(y) dy \quad (47)$$

p_{miss} can be expressed after substituting (47) into (43) and (44) as

$$p_{\text{miss}} = \int_0^\infty L(y) f_r(y) dy = L(0) \text{Prob} (i=0) + [1 - \text{Prob} (i=0)] \cdot \int_0^\infty L(y) f_r(y | i \neq 0) dy \quad (48)$$

where

$$L(r) = \begin{cases} \int_{\gamma_1}^{\gamma_2} F_1(v, r) dv & \text{if } \gamma_1 > 0 \\ \int_0^{-\gamma_1} F_0(v, r) dv + \int_{-\gamma_1}^{\gamma_2} F_1(v, r) dv & \text{if } \gamma_1 < 0 \end{cases} \quad (49)$$

$$\gamma_1 = \left(1 - \frac{\Delta}{\tau}\right) \sqrt{2 \frac{E_c}{N_0}} - \beta \quad (50)$$

$$\gamma_2 = \left(1 - \frac{\Delta}{\tau}\right) \sqrt{2 \frac{E_c}{N_0}} + \beta \quad (51)$$

$$\frac{\sqrt{2PS}(\tau - \Delta)}{\sigma_n} = \sqrt{2 \frac{E_c}{N_0}} \left(1 - \frac{\Delta}{\tau}\right) \quad (52)$$

$$F_0(v, r) = \exp \left(-\frac{2 \frac{E_c}{N_0} r + v^2}{2} \right) I_0 \left(\sqrt{2 \frac{E_c}{N_0}} r \cdot v \right) v \quad (53)$$

$$F_1(v, r) = \frac{\psi(v\sigma_n)}{\pi} F_0(v, r) \quad (54)$$

with $\varphi(v\sigma_n)$, obtained after a geometrical analysis in the Fig. 4, given by

$$\psi(v\sigma_n) = tg^{-1} \eta \quad \text{if } \gamma_1 > 0 \quad (55)$$

otherwise

$$\psi(v\sigma_n) = \begin{cases} \pi - tg^{-1} \eta & \text{if } -\gamma_1 < v < \sqrt{-\gamma_1 \gamma_2} \\ tg^{-1} \eta & \text{if } \sqrt{-\gamma_1 \gamma_2} < v < \gamma_2 \end{cases} \quad (56)$$

where

$$\eta = \sqrt{\left(\frac{2v(\gamma_1 + \beta)}{\gamma_1 \gamma_2 + v^2} \right)^2 - 1} \quad (57)$$

The GQR can be now used to calculate the integral appearing in (48) as

$$\int_0^\infty L(x) f_r(x | i \geq 1) dx = \sum_{i=1}^N L(x_i) w_i \quad (58)$$

where the set $\{w_i, x_i\}$ coincides with the obtained set from the moments formulated in (37).

B. Rayleigh Fading Case

When A and A_i are Rayleigh distributed random variables, p_{fa} and p_{miss} can be evaluated from (15) and (16), respectively, by using a convenient GQR. In particular, with respect to the p_{fa} calculation, it can be carried out analogously as it was done in (36). In this case, however, we have to set

$$R_i = \frac{1}{\sqrt{2}} \frac{A_i}{\sqrt{P_i}} \sqrt{\frac{P_i}{P}} \frac{2}{\tau} \quad (59)$$

with

$$P = \frac{1}{2} E\{A^2\} \quad (60a)$$

and

$$P_i = \frac{1}{2} E\{A_i^2\} \quad (60b)$$

instead of the R_i previously defined in (24). With regard to the p_{miss} calculation, it can be exploited the Gaussian nature of the term: $S_E(\tau) + n_D(\tau)$ appearing in (16) by writing now

$$p_{\text{miss}} = 1 - \text{Prob} [|I_D(\tau) + S_E(\tau) + n_D(\tau)| \geq u] \\ = 1 - \int_0^\infty Q\left(\sqrt{2\frac{E_c}{N'_0}}x, \beta'\right) f_r(x) dx \quad (61)$$

where

$$N'_0 = N_0 \left(1 + \frac{E_c}{N_0}\right) \quad (62)$$

and

$$\beta' = \beta \frac{N_0}{N'_0} \quad (63)$$

To compute P_{FA} and P_{MISS} , according to (17) and (18), respectively, we have assumed that all the arrival tones fade independently of each other.

IV. DEPLOYMENT STATISTICS

In order to include in our analysis the effects of the random employment of the users, that is, how dependent on the relative location of interferers and the desired transmitter FH-SSMA system performance is, a plausible scenario introduced in [8] for ground-based communications networks has been considered. In that case the interference power-to-desired user power ratio is characterized by the bell-shaped pdf:

$$f_{p_i/p}(x) = \frac{5}{4} m_d^4 \exp\left(-\frac{5}{4} m_d^4 x\right) \quad (64)$$

where $m_d = S_0/r$, S_0 is the peak of the unimodal probability density governing the location of an interferer along a radial distance and r is the distance between the desired transmitter and receiver pair.

Hence, the $2N + 1$ moments of the random variable r needed in the aforementioned GQR formulae can be carried out as

$$E\left\{\left(\frac{P_i}{P}\right)^n\right\} = \frac{4^n n!}{5^n m_d^{4n}} \quad n = 0, 1, \dots, 2N \quad (65)$$

Another more simplistic scenario consisting of only three possible received power levels P_1 , P_2 , and P_3 with appearance probabilities p_1 , p_2 , and p_3 , respectively, has also been considered. In this case,

$$E\left\{\left(\frac{P_i}{P}\right)^n\right\} = \sum_{j=1}^3 p_j \left(\frac{P_j}{P}\right)^n \quad (66)$$

V. SYNCHRONOUS MULTIPLE ACCESS

Up to now we have only dealt with asynchronous FH multiple-access systems. We can also study the synchronous multiple-access situation as a particular case of the asynchronous one. That is, if we consider

$$S_S(t) = \text{rect}_\tau(t) \frac{\tau}{2} \quad (67)$$

instead of $S(t)$ given in (8), and we take it into account when evaluating the $2N + 1$ moments of r , specifically when computing $H_0(b_i)$ [see the Appendix] where

$$E\{R^{2b_i}\} = E\{R_i^{2b_i}\}, \quad (68)$$

we automatically obtain results for the synchronous multiple

access. Obviously, in this case, no considerations have to be made in order to rule out adjacent or intersymbol interference if the tones are spaced n/τ Hz (n integer). Moreover

$$\text{Prob}(i=0) = \exp(-\lambda\tau) \quad (69)$$

due to the τ s duration of $S_S(t)$.

VI. NUMERICAL RESULTS FOR A SERIAL SEARCH SYNCHRONIZER

In the following results, a good GQR convergence was obtained at least up to the first three significant digits and $N \leq 9$. Some results have been found illustrating the behavior of the one test miss and false alarm probabilities under reasonable circumstances. That is, we have chosen M_T , I_D , and β parameters so that the resulting false alarm probabilities are centered around a typical value as $p_{\text{fa}} = 10^{-3}$. Smaller false alarm probabilities can be obtained setting $K > 1$ in the search process control here considered. Fig. 5 shows P_{FA} and P_{MISS} in the nonfading case. The $\lambda\tau$ value has been chosen to be 0.025 that corresponds on the average to a 2.5 percent occupation of the Q available frequencies. The P_{FA} variation is significant for different values of the deployment distribution parameter m_d . So the greater the power interference is, the greater the P_{FA} . It can be noted how the false alarm rate hardly changes by increasing E_c/N_0 in the nonfading case. That could be expected inasmuch as the presence of an interfering term is the first cause of a false hit, while the noise has only a second-order effect. The slight increase of P_{FA} with E_c/N_0 is due to the larger detection probability of the interfering received signals for the fixed normalized detection threshold, β .

As regards to the P_{MISS} , Fig. 5 shows its behavior only for $m_d = 1$ because in the presence of different deployment statistics, p_{MISS} hardly changes (at most 0.2 dB error in E_c/N_0 for $m_d = 0.4$) even for low E_c/N_0 values. That means that the energy added to the useful signal by the multiuser interference at the here-assumed low $\lambda\tau$ value is not significant to produce an appreciable diminution of the P_{MISS} obtained for $\lambda\tau = 0$.

Fig. 6 shows p_{FA} and p_{MISS} in the Rayleigh fading case. In order to maintain P_{FA} values around 10^{-3} in most cases, different M_T , I_D , and β parameters have been selected. Now a great dispersive effect in the P_{FA} value is observed. Specifically for $m_d = 0.6$, p_{FA} takes a rather high value if compared to the P_{FA} resulting for $m_d = 1$. This phenomena could not be pointed out with a more simplistic interference model that would not take into account a deployment statistics. Also, a more pronounced increase of p_{FA} with respect to E_c/N_0 can be now noted. This greater dispersive effect in the p_{FA} behavior when compared to the results shown in Fig. 5 are due to both the higher M_T value and the higher dependency of P_{fa} with respect to E_c/N_0 because of the Rayleigh fading present in the interfering tones. P_{MISS} is also reasonably insensitive to m_d variations, although a slight p_{miss} decrease for $m_d = 0.6$ appears now. A synchronous multiple access is also shown for the same user activity and $m_d = 1$. It can be noted that the P_{FA} values are slightly greater than in the asynchronous case. P_{MISS} variation, however, is hardly noticeable.

VII. BIT ERROR PROBABILITY IN FH-BFSK

We have calculated the bit error rate (BER) in FFH-BFSK and SFH-BFSK systems for the nonfading and Rayleigh fading cases.

A. FFH-BFSK Nonfading Case

We can take advantage of the miss probability calculation in the serial search acquisition ($\Delta = 0$) by observing that

$$\text{BER} = \text{Prob} \{ |\sqrt{2PS}(\tau) + M_1 \exp(-j\varphi_1)| < |M_2 \exp(-j\varphi_2)| = M_2 \} \quad (70)$$

where $M_1 \exp(-j\varphi_1)$ and $M_2 \exp(-j\varphi_2)$ correspond to non-

Fig. 5. Serial search synchronizer performance in the nonfading case and several deployment statistics.

Fig. 6. Serial search synchronizer performance in the Rayleigh fading case and several deployment statistics.

desired signal samples appearing at the output of the mark and space channels, respectively, M_1 and M_2 are statistically distributed the same as M [see (7)] and φ_1 and φ_2 are uniformly distributed over $[0, 2\pi]$. The four random variables are mutually independent.

By denoting $u = M_2$, we can formulate

$$\text{BER} = \int_0^\infty p_{\text{miss}}(x) f_\beta(x) dx \quad (71)$$

where p_{miss} given by (48) can be now used if we take into account its dependency on the normalized threshold β . For this purpose $p_{\text{miss}}(\beta)$ has been introduced in the above expression.

A more appropriate formulation results from defining

$$\beta^2 = \frac{M_2^2}{\sigma_n^2} = 2 \frac{E_c}{N_0} r_2 \quad (72)$$

where

$$r_2 = \left| \sum_{i=-\infty}^{\infty} \sqrt{\frac{P_i}{P}} \exp(-j\theta_i) \frac{2}{\tau} S(\tau - t_i) + \sqrt{\frac{2}{P}} \frac{1}{\tau} n_D(\tau) \right|^2 \quad (73)$$

Then

$$\begin{aligned} \text{BER} &= \int_0^\infty p_{\text{miss}} \left(\sqrt{2 \frac{E_c}{N_0}} x \right) f_{r_2}(x) dx \\ &\approx \sum_{i=1}^N p_{\text{miss}} \left(\sqrt{2 \frac{E_c}{N_0}} x_i \right) \cdot w_i \end{aligned} \quad (74)$$

where the set $\{x_i, w_i\}$ is the quadrature rule corresponding to $f_{r_2}(x)$. The moments of r_2 are readily found through the moments of r and the moments of $|\sqrt{2/P}(1/\tau)n_D(\tau)|^2$. Finally, $p_{\text{miss}}(\sqrt{2(E_c/N_0)}x_i)$ can be evaluated from (48).

A more convenient formulation in the point of view of GQR convergence for $E_c/N_0 \gg 1$ is handled by writing

$$\begin{aligned} \text{BER} &= \left[\int_0^\infty p_{\text{miss}} \left(\sqrt{2 \frac{E_c}{N_0}} x \right) f_{r_2}(x|i=0) \right] \text{Prob}(i=0) \\ &+ \left[\int_0^\infty p_{\text{miss}} \left(\sqrt{2 \frac{E_c}{N_0}} x \right) f_{r_2}(x|i \neq 0) \right] \text{Prob}(i \neq 0) \end{aligned} \quad (75)$$

and computing the GQR sets corresponding to $f_{r_2}(x|i=0)$ and $f_{r_2}(x|i \neq 0)$, respectively.

B. FFH-BFSK Rayleigh Fading Case

In this case, the BER can be computed as follows. The signals detected in the mark and space branches are

$$\begin{aligned} r_m(\tau) &= A \exp(-j\theta) S(\tau) + \sum_{i=-\infty}^{\infty} A_i \\ &\cdot \exp(-j\theta_i) S(\tau - t_i) + n_1(\tau) \end{aligned} \quad (76)$$

and

$$r_s(\tau) = \sum_{l=-\infty}^{\infty} A_l \exp(-j\theta_l) S(\tau - t_l) + n_2(\tau), \quad (77)$$

respectively. $n_1(\tau)$ and $n_2(\tau)$ are mutually independent and distributed the same as $n_D(\tau)$.

We define the new Gaussian random variable $n'_1(\tau)$ as

$$n'_1(\tau) = A \exp(-j\theta) S(\tau) + n_1(\tau). \quad (78)$$

Hence, after a little algebra, the BER conditioned to the r_1 and r_2 values can be written as [5]

$$\begin{aligned} P_b(r_1, r_2) &= \frac{v^2}{1+v^2} \\ &\cdot \left[1 - Q \left(\sqrt{\frac{E_c/N_0}{2+E_c/N_0}} 2r_1, \right. \right. \\ &\quad \left. \left. \sqrt{\frac{E_c/N_0}{2+E_c/N_0}} 2r_2 \right) \right] \\ &+ \frac{1}{1+v^2} Q \left(\sqrt{\frac{E_c/N_0}{2+E_c/N_0}} 2r_2, \right. \\ &\quad \left. \sqrt{\frac{E_c/N_0}{2+E_c/N_0}} 2r_1 \right) \end{aligned} \quad (79)$$

where

$$v^2 = \frac{E\{|n_1'(\tau)|^2\}}{E\{|n_2(\tau)|^2\}} = \frac{E_c}{N_0} + 1 \quad (80)$$

and r_1, r_2 are mutually independent random variables statistically distributed the same as r given in (23) with R_i defined in (59). Finally,

$$\text{BER} \approx \sum_{i=1}^N \sum_{j=1}^N P_b(x_i, x_j) w_i \cdot w_j \quad (81)$$

where the set $\{x_i, w_i\}$ is the quadrature rule corresponding to the weight function $f_i(x)$. Actually, as we did before, we handle conveniently $P_b(r_1, r_2)$ in order to use $f_i(x|i \neq 0)$ as the weight function. Then, a good convergence of the BER is guaranteed.

C. SFH-BFSK

Expression (48) is not longer applicable for SFH-BFSK, and an additional analysis is now required for the BER calculation. As above, both nonfading and Rayleigh fading environments will be considered. In the latter case, only fading which is slow relative to the hopping rate will be retained. In that case, when a random error-correcting code is to be employed, the coded symbols are first interleaved prior the modulator and the detector output symbols are deinterleaved so that the channel be memoryless.

In order to analyze the BER of a SFH-BFSK, we have introduced the Fig. 7, wherein f'_l and f_l , $l = (\cdot, -1, 0, 1, \cdot)$, denote the frequencies of an interfering signal and the useful signal, respectively, and t_i refers to a particular arrival time in the Poisson point process involved in the multiuser interference modeling. Then, as it can be seen in Fig. 7, three multiuser interference cases arise. Case a) occurs when two consecutive identical interfering bits, belonging to the same hop interval, hit the $(0, T_b)$ interval corresponding here to the desired tone of frequency f_0 . Then $f'_0 = f'_1 = f_0$. By defining a binary variable $a_i \in (0, 1)$, the probability for the above event to happen is

$$P_a = \text{Prob} \{a_i = 1\} = \frac{1}{2} \left(1 - \frac{1}{N_b}\right). \quad (82)$$

Case b) arises when the two previously mentioned consecutive bits are different. Then, $f'_0 = f_0$ or $f'_1 = f_0$. The probability for this to happen is

$$P_b = \text{Prob} \{b_i = 1\} = \frac{1}{2} \left(1 - \frac{1}{N_b}\right) \quad (83)$$

where now a binary random variable $b_i \in (0, 1)$ has been defined. Finally, case c) occurs when t_i coincides with a hop change instant. Then the two consecutive interfering bits belong to different hop intervals. The appearance probability for this case is

$$P_c = \text{Prob} \{c_i = 1\} = 1 - P_a - P_b = \frac{1}{N_b} \quad (84)$$

where $c_i \in (0, 1)$ is a binary random variable introduced to characterize this last event.

The BER calculation can now be accomplished simply by using instead of the random variable r the new random

Fig. 7. Timing diagram for asynchronous SFH-BFSK.

variable r' given by

$$r' = \left| \sum_{i=-\infty}^{\infty} a_i R_i \exp(-j\theta_i) S_s(T_b - t_i) \right. \\ \text{case a} \\ \left. + \sum_{i=-\infty}^{\infty} b_i R_i \exp(-j\theta_i) S'(T_b - t_i) \right. \\ \text{case b} \\ \left. + \sum_{i=-\infty}^{\infty} c_i R_i \exp(-j\theta_i) S(T_b - t_i) \right|^2 \quad (85)$$

where $S'(t)$ is given by

$$S'(t) = \begin{cases} t/2 & 0 \leq t \leq T_b \\ 0 & \text{otherwise} \end{cases} \quad (86)$$

and refers to case b) with $f_0 = f'_1$ and $f_0 \neq f'_0$. Identical statistical characterization for r' would be obtained by assuming $f_0 \neq f'_1$ and $f_0 = f'_0$.

The moments of r' can be readily computed because of

$$a_i + b_i + c_i = 1 \quad (87)$$

and consequently

$$E[a_i^l b_i^m c_i^k] \neq 0 \quad (88)$$

only if $l = m = k = 0$. Furthermore, each interference term involved in r' has the same analytical structure as that corresponding to r . Hence, a simple recursive procedure can be made up for obtaining the $2N + 1$ moments of r' needed to calculate the BER from the GQR technique.

VIII. NUMERICAL RESULTS FOR FH-BFSK SCHEMES

The same good GQR convergence behavior obtained with the previously analyzed serial search synchronizer applies to the following results. Fig. 8 shows the BER in the nonfading case for $\lambda\tau = 0.05$ and FFH-BFSK signaling. The influence of the deployment parameter m_d can be appreciated. Interferers with equal power and three power levels, respectively, are also shown for the sake of comparison. In all the analyzed cases, BER curves tend to be flat for $E_c/N_0 \lesssim 10$ dB. Clearly, in that signal-to-noise margin the channel becomes interference limited.

When the interfering power is much greater than the desired signal power, that is, $P_i \gg P$ in (24), the BER can be approximated by

$$\text{BER} \approx (1 - e^{-2\lambda\tau})^2 \frac{1}{2} + (1 - e^{-2\lambda\tau}) e^{-2\lambda\tau} \quad (89)$$

Fig. 8. FFH-BFSK bit error rate in the nonfading case and several deployment statistics.

where the first term applies when the interference hits both the mark and space channels and the second term applies when only the nondesired channel is hit. For $2\lambda\tau = 0.1$ it results $\text{BER} \approx 0.09$. This value would correspond to that obtained for $m_d = 0.4$ and $E_c/N_0 \geq 10$ dB as Fig. 8 shows. So, decreasing m_d will not alter appreciably this asymptotic BER value. Through Fig. 8 it is concluded that a significant BER disagreement between two extreme situations such as $m_d = 0.4$ and $m_d = 1.4$ do exist when nonequal interfering powers are involved. In that sense, it is worthy to point out that if coding is used, the differences between the smaller BER obtained can be readily significant for the different deployments. As a matter of interest, Fig. 9 shows the behavior of a concatenated coding scheme with a (15, 9) Reed-Solomon outer code and a (7, 4) binary Hamming inner code when used in the nonfading environment.

Fig. 10 shows BER results in the Rayleigh fading case for $\lambda\tau = 0.05$ and FFH-BFSK signaling. As before, the equal power and $m_d = 1$ scenarios lead to similar BER results, while remarkable BER differences arise between two situations such as $m_d = 0.6$ and $m_d = 1.4$. Also, similar coding effect is noted (see Fig. 11).

Fig. 12 shows the BER in the nonfading case for $\lambda T_b = 0.05$ and SFH-BFSK signaling with $N_b = 20$. This figure also shows the upper (P_u) and lower (P_L) bounds on BER introduced in [2] and given by

$$P_L = P_0(1 - \hat{P}) \quad (90)$$

and

$$P_u = P_L + \frac{1}{2} \hat{P} \quad (91)$$

where

$$P_0 = \frac{1}{2} e^{-E_b/2N_0} \quad (92)$$

E_b denotes bit energy and

$$\hat{P} \approx 1 - \left\{ 1 - \frac{1}{q} \left(1 + \frac{1}{N_b} \right) \right\}^{K-1} \quad (93)$$

where K is the number of simultaneous users and q the number

Fig. 9. FFH-BFSK bit error rate of a concatenated coding scheme: (15, 9) Reed-Solomon outer code and (7, 4) binary Hamming inner code (nonfading case).

Fig. 10. FFH-BFSK bit error rate in the Rayleigh fading case and several deployment statistics.

of channels as defined in [2] and here chosen to be $K = 25$ and $q = 250$, respectively. It can be noted that for large q and $K \gg 1$,

$$\hat{P} \approx 2\lambda T_b \left(1 + \frac{1}{N_b} \right) \quad (94)$$

where

$$\lambda T_b = \frac{K}{2q} \quad (95)$$

as defined in (3) for $U = K$, $Q = 2q$ and T_b instead τ . The obtained BER results for this SFH-BFSK scheme at any m_d value are slightly lower than those obtained for the FFH-BFSK scheme shown in Fig. 8. Actually, the greater N_b is, the lower BER is. This fact is illustrated in Table I for $m_d = 1$ and the same $\lambda T_b = 0.05$.

Fig. 11. FFH-BFSK bit error rate of a concatenated coding scheme: (15, 9) Reed-Solomon outer code and (7, 4) binary Hamming inner code (fading case).

Fig. 12. SFH-BFSK bit error rate in the nonfading case and several deployment statistics.

TABLE I
BER FOR A SFH-BFSK SYSTEM VERSUS THE N_b VALUE IN A NONFADING ENVIRONMENT WITH $m_d = 1$.

E_b/N_0	$m_d = 1, \lambda\tau = 0.05$				
	$N_b = 2$	$N_b = 5$	$N_b = 10$	$N_b = 20$	
7	5.37	5.30	5.29	5.29	$\times 10^{-2}$
8	3.80	3.76	3.75	3.75	$\times 10^{-2}$
9	2.92	2.88	2.87	2.86	$\times 10^{-2}$
10	2.52	2.49	2.48	2.47	$\times 10^{-2}$
11	2.44	2.41	2.39	2.39	$\times 10^{-2}$

Fig. 13. SFH-BFSK bit error rate in the Rayleigh fading case and several deployment statistics.

Fig. 13 shows the BER in the fading case for $\lambda T_b = 0.05$ and SFH-BFSK signaling with $N_b = 20$. The same P_L and P_u bounds above mentioned are also shown with

$$P_0 = \frac{1}{2 + \frac{E_b}{N_0}} \quad (96)$$

where $\bar{E}_b$ denotes mean bit energy. As in the above nonfading BFH-BFSK case, the BER values obtained are also slightly lower than these obtained with a FFH-BFSK scheme.

As approximate BER results of the SFH-BFSK scheme are available in the literature [2] for systems with equal power received signals and for both nonfading and Rayleigh fading cases, we have used these results as a comparison basis to those obtained in our analysis. As Tables II and III show good agreement between both results can be observed. In particular, the remarkable coincidence shown up for the Rayleigh fading environment could be explained because of the more precise analysis undertaken in [2] for this case.

IX. CONCLUSIONS

We have given a calculation procedure for determining the asynchronous FH-SSMA performance based upon a GQR. To illustrate the procedure a FH serial search acquisition configuration and both SFH-BFSK and FFH-BFSK signaling schemes have been analyzed under reasonable users activity and deployment statistics. False alarm and miss probabilities for the acquisition process and BER for the operational stage within a desired accuracy can be proved. It is concluded that the here proposed method could be used in many cases of practical interest as an alternative to bounds or approximation techniques previously proposed.

APPENDIX

The n th moment of the random variable r is given by

$$E(r^n) = E \left\{ \left[\left(\sum_{i=-\infty}^{\infty} R_i \exp(-j\theta_i) S(\tau - t_i) \right) \cdot \left(\sum_{k=-\infty}^{\infty} R_k \exp(-j\theta_k) S(\tau - t_k) \right)^* \right]^n \right\} \quad (A1)$$

TABLE II
BER COMPARISON FOR A SFH-BFSK SYSTEM IN A NONFADING ENVIRONMENT AND EQUAL-POWER SIGNALS. $\bar{P}_A$ DENOTES THE APPROXIMATE BIT ERROR RATE OBTAINED IN [2]

$K = 25,$	$q = 250,$	$N_b = 20$
P_0	$\bar{P}_A$	BER
0.100	0.118	1.14×10^{-1}
0.050	0.070	6.64×10^{-2}
0.030	0.051	4.72×10^{-2}
0.020	0.042	3.74×10^{-2}
0.010	0.032	2.75×10^{-2}
0.005	0.027	2.23×10^{-2}

TABLE III
BER COMPARISON FOR A SFH-BFSK SYSTEM IN A RAYLEIGH FADING ENVIRONMENT AND EQUAL-POWER SIGNALS. $\bar{P}_A$ DENOTES THE APPROXIMATE BIT ERROR RATE OBTAINED IN [2]

$K = 5, q = 100, N_b = 10$			
$\bar{P}_b/N_0$	$\bar{P}_A$	BER	
6	1.72	1.71	($\times 10^{-1}$)
8	1.26	1.26	($\times 10^{-1}$)
10	9.02	9.01	($\times 10^{-2}$)
12	6.36	6.35	($\times 10^{-2}$)
15	3.81	3.79	($\times 10^{-2}$)
20	1.87	1.85	($\times 10^{-2}$)
40	-	0.91	($\times 10^{-2}$)
∞	0.92	-	($\times 10^{-2}$)

For $n \geq 1$

$$\begin{aligned}
 E(r^n) &= E \left\{ \left[\sum_{i_1=-\infty}^{\infty} R_{i_1} \exp(-j\theta_{i_1}) S(\tau - t_{i_1}) \right. \right. \\
 &\quad \cdots \sum_{i_n=-\infty}^{\infty} R_{i_n} \exp(-j\theta_{i_n}) S(\tau - t_{i_n}) \left. \right] \\
 &\quad \cdot \left[\sum_{k_1=-\infty}^{\infty} R_{k_1} \exp(j\theta_{k_1}) S^*(\tau - t_{k_1}) \right. \\
 &\quad \cdots \sum_{k_n=-\infty}^{\infty} R_{k_n} \exp(j\theta_{k_n}) S^*(\tau - t_{k_n}) \left. \right] \Big\} \\
 &= \sum_{i_1=-\infty}^{\infty} \cdots \sum_{i_n=-\infty}^{\infty} \sum_{k_1=-\infty}^{\infty} \cdots \sum_{k_n=-\infty}^{\infty} E \left[\prod_{l=1}^n R_{i_l} R_{k_l} \right. \\
 &\quad \cdot \exp(-j\theta_{i_l}) \exp(j\theta_{k_l}) S(\tau - t_{i_l}) S^*(\tau - t_{k_l}) \Big]. \quad (A2)
 \end{aligned}$$

The above expression can be simplified using that

$$\begin{aligned}
 E \left[\prod_{l=1}^n R_{i_l} R_{k_l} \exp(-j\theta_{i_l}) \right. \\
 \left. \cdot \exp(j\theta_{k_l}) S(\tau - t_{i_l}) S^*(\tau - t_{k_l}) \right] = 0 \quad (A3)
 \end{aligned}$$

if

$$G_i^A(t_{i_l}) \neq G_k^B(t_{i_l}) \quad l = 1, \dots, n$$

where $G_i^A(t_{i_l})$ is the number of times that a given t_{i_l} shows up in the set $A = (t_{i_1}, \dots, t_{i_j}, \dots, t_{i_n})$ and $G_k^B(t_{i_l})$ is the number of times that a given t_{i_l} shows up in the set $B = (t_{k_1}, \dots, t_{k_j}, \dots, t_{k_n})$.

Grouping all the nonzero terms [$G_i^A(t_{i_l}) = G_k^B(t_{i_l})$] so that in each group the set A have $n, n-1, \dots, 1$ different elements, respectively, we can write

$$\begin{aligned}
 E(r^n) &= \sum_{b_1=1}^n F(b_1, 0, \dots, 0) E(R^{2b_1}) \\
 &\quad \cdot E \left[\sum_{i_1=-\infty}^{\infty} S^{2b_1}(\tau - t_{i_1}) \right] + \cdots + \sum_{b_1=1}^n \sum_{b_2=1}^{b_1} \cdots \\
 &\quad \cdot \sum_{b_j=1}^{b_{j-1}} F(b_1, \dots, b_j, 0, \dots, 0) \\
 &\quad \cdot [E(R^{2b_1}) \cdots E(R^{2b_j})] \\
 &\quad \cdot \left\{ E \left[\sum_{i_1=-\infty}^{\infty} S^{2b_1}(\tau - t_{i_1}) \right] \cdots \right. \\
 &\quad \cdot E \left[\sum_{i_j=-\infty}^{\infty} S^{2b_j}(\tau - t_{i_j}) \right] \Big\} + \cdots + \sum_{b_1=1}^n \sum_{b_2=1}^{b_1} \cdots \\
 &\quad \cdot \sum_{b_n=1}^{b_{n-1}} F(b_1, \dots, b_n) [E(R^{2b_1}) \cdots E(R^{2b_n})] \\
 &\quad \cdot \left\{ E \left[\sum_{i_1=-\infty}^{\infty} S^{2b_1}(\tau - t_{i_1}) \right] \right. \\
 &\quad \cdot \cdots E \left[\sum_{i_n=-\infty}^{\infty} S^{2b_n}(\tau - t_{i_n}) \right] \Big\} \quad (A4)
 \end{aligned}$$

where

$$E(R^{2b_j}) = E(R^{2b_j}) \quad (A5)$$

$F(b_1, \dots, b_j, \dots, b_n)$ gives the total number of different combinations of nonzero terms with as many different time instants in A as b_j different from zero; b_j is the number of times that the same time instant t_{i_j} appears in A

$$F(b_1, \dots, b_n) = \frac{F_1(b_1, \dots, b_n) [F_2(b_1, \dots, b_n)]^2}{F_3(b_1, \dots, b_n)} \quad (A6)$$

$$F_1(b_1, \dots, b_n) = \begin{cases} 1 & \text{if } \sum_{j=1}^n b_j = n \\ 0 & \text{if } \sum_{j=1}^n b_j \neq n \end{cases} \quad (A7)$$

$$F_2(b_1, \dots, b_n) = \prod_{j=1}^n \binom{a_j}{b_j}, \quad (\text{A8})$$

$$a_1 = n, \quad (\text{A9})$$

$$a_{j+1} = a_j - b_j, \quad (\text{A10})$$

$$F_3(b_1, \dots, b_n) = \prod_{j=1}^n g_j!, \quad (\text{A11})$$

g_j is the number of times that the integer j , $1 \leq j \leq n$, appears in the set $(b_1, \dots, b_n)$.

From [9],

$$E \left[\sum_{i=-\infty}^{\infty} S^{2b}(\tau - t_i) \right] = \lambda \int_{-\infty}^{\infty} [S(t)]^{2b} dt, \quad (\text{A12})$$

and

$$E(r^n) = \sum_{b_1=1}^n \sum_{b_2=0}^{b_1} \dots \sum_{b_n=0}^{b_{n-1}} F(b_1, \dots, b_n) \cdot \left[\prod_{i=1}^n H_0(b_i) \right], \quad n \neq 0 \quad (\text{A13})$$

where

$$H_0(b_i) = \begin{cases} 1 & \text{if } b_i = 0 \\ \lambda E(R^{2b_i}) \int_{-\infty}^{\infty} |S(t)|^{2b_i} dt & \text{if } b_i \neq 0 \end{cases}. \quad (\text{A14})$$

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