CROSS SPECTRUM ML ESTIMATE

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ABSTRACT

This work reports how to include general concepts of the one-dimensional MLM procedure in a two-channel problem of cross-spectrum estimation.

It is shown in the sequel that there is no any problem in extrapolating the well-known procedures for auto-spectrum estimation to the cross-spectrum, if the original procedure can be explained as a filter bank analysis procedure.

The resulting cross-spectrum estimate looks formally to satisfy the excellent features which the normalized maximum likelihood procedure, reported previously by the authors, does in the auto-spectrum problem as concerns with resolution a low-side lobe behavior.

1. INTRODUCTION

During the last decade the evolution of autospectrum analysis techniques has been spectacular when compared with crossspectrum procedures. Even right now, authors do not use to pay comparable attention to this important problem. This paper is an attempt to extend to the cross-spectrum problem the potential already shown by parametric methods in autospectrum analysis. It also can be encompassed in previous reported works of Lagunas and Gasull in one and two dimensional spectral estimation procedures based in the use of maximum likelihood filters [1].

It has been shown that the so-called maximum likelihood filters could be considered as matched filters to a pure tone at a given frequency ω_0 in a non-white environment. In other words, the squared magnitude of the filter's output at a given sample N (the length of the ML filter using a FIR design) divided by the power of the filter output corresponding to the environment is maximized. Under this

point of view the ratio test results as formulated below:

being $e(n) = \underline{A}^T \cdot \underline{X}_n$ with $\underline{A}^T = (a(0), \dots, a(Q))$ and

$\underline{X}_n^T = (x(n), \dots, x(n-Q))$. So that, being

$\underline{S}_0 = (1, \dots, \exp(jQ\omega_0))$ we have (1),

Filter's response to signal $\underline{A}^T \cdot \underline{S}_0$

Filter's response to the environment $\underline{A}^T \cdot \underline{X}_n$

$$\text{Detection ratio } |\underline{A}^T \underline{S}_0|^2 / \underline{A}^T \underline{R} \underline{A} \quad (1)$$

Maximizing this detection ratio, will arise to constraint the numerator to be one, and minimizing the denominator quadratic form the resulting filter is shown in (2)

$$\underline{A} = \underline{R}^{-1} \underline{S}_0 / \underline{S}_0^T \underline{R}^{-1} \underline{S}_0 \quad (2)$$

As reported by Capon and Lacoss [2], the previous approach of maximizing the output dynamic range for the signal response with respect the global output noise, is equivalent to maximize the log likelihood ratio to estimate the complex envelope a , of an incoming complex exponential (i.e. $\underline{X}_n = a \underline{S}_0 \exp(j\omega_0 n) + \underline{N}$, being $\underline{N}$ the noise vector). The ML estimate for a , is shown in (3), being $\underline{A}$ from (2).

$$\hat{a} = \underline{A}^T \cdot \underline{X}_n \quad (3)$$

A more realistic approach and well connected with spectral estimation concepts could be provided from a filter bank analysis point of view. Looking at the problem of estimating the power level of the input signal, in a given frequency band P_0 (power of $x(n)$ at the frequency ω_0), by using a filter bank with a band pass filter dedicated to each frequency band; it is reported in [3] that the same filter results,

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being the estimated power level P_o as shown in (4).

$$P_o = 1/S_{oo}^T R^{-1} S_o \quad (4)$$

The above power level is measured with a narrow band pass filter (i.e. the maximum likelihood filter steered at ω_o) which bandwidth have to be estimated in order to provide a spectrum density measure. Using the equal area constraint [4], or the approximation that the actual power spectrum under analysis is flat over the bandwidth of the analysis filter; either of these two procedures results in the same bandwidth estimate, for the equivalent bandwidth (5.a), and the resulting spectral density estimate (5.b)

$$B_e = A^T A = S_o^T R^{-2} S_o / (S_{oo}^T R^{-1} S_o)^2 \quad (5.a)$$

$$S_x(\omega_o) = S_o^T R^{-1} S_o / S_{oo}^T R^{-2} S_o \quad (5.b)$$

It is very important in order to explain the resulting performance as can be seen in [4] and [5], that formula (5.b) can be viewed as a signal to noise ratio in the following way:

If the signal input is $\underline{X}$, where $E[\underline{X}\underline{X}^T] = \underline{R}$, and the noise input is white noise with correlation matrix equal to the $Q \times Q$ square identity matrix; then the signal to noise ratio at the output of the ML filter is:

$$\text{signal } \underline{X} \rightarrow \underline{R} = E[\underline{X}\underline{X}^T] \rightarrow \text{output } \underline{A}^T \underline{R} \underline{A} \quad (6.a)$$

$$\text{noise } \underline{N} \rightarrow \underline{I} = E[\underline{N}\underline{N}^T] \rightarrow \text{output } \underline{A}^T \underline{A} \quad (6.b)$$

$$\text{and } \text{SNR} = \frac{\underline{A}^T \underline{R} \underline{A}}{\underline{A}^T \underline{A}} = \hat{S}_x(\omega_o) \quad (6.c)$$

where $\hat{S}_x(\omega)$ is as defined in (5.b).

In the hereafter sections cross-spectral concepts will be reported starting from the one channel concepts described before.

II. CROSS-SPECTRUM ANALYSIS

From a filter bank analysis point of view the problem of getting a cross-spectral estimate from two data sample records $x(n)$ and $y(n)$, with $n=0, N-1$; can be based on the design of two narrow-band pass filters $\underline{A}_x$ and $\underline{A}_y$ steered at the same frequency ω_o . Thus the cross-spectrum power of the signal under processing can be inferred from the cross-correlation of the residuals at the zero lag.

The procedure is summarized in Fig. 2. Of course, a fine estimate of the power level $P_{xy}(\omega_o)$ depends on the bandwidth, in other words, we should try to reduce as much as possible such bandwidth in order to arise to better and better estimates for the cross-spectrum density estimate.

Fig 2. Basic scheme to obtain a cross-spectrum power level estimate.

The above comment about the analysis bandwidth can be encompassed in a ML filter concept by reading that reducing this parameter is equivalent to match the filter transfer response to the signals features, in order to avoid leakage from lateral frequency bands when measuring $P_{xy}(\omega_o)$. If this goal is achieved, approximately, we can say that $P_{xy}(\omega_o)$ could be inferred directly from $E[e_x^*(n) e_y(n)]$, and the cross-spectrum $S_{xy}(\omega_o)$ will satisfy (7).

$$P_{xy}(\omega_o) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \underline{A}_x^*(\omega) \underline{A}_y(\omega) S_{xy}(\omega) d\omega \quad (7)$$

Thus, considering that the cross-spectrum is flat over the cross-bandwidth of both filters the estimate shown in (8) could be derived

$$\hat{S}_{xy}(\omega_o) = P_{xy}(\omega_o) / \underline{A}_x^T \underline{A}_y \quad (8)$$

Some comments are worthwhile to remark from the above paragraphs. First at all, note that this approach allows us to design built-in spectrum analyzers with one or two channels processing to obtain auto-spectrum and cross-spectrum density estimates, auto and cross power level estimates, time delay estimation, coherence, transfer function analysis, coherence, signal to noise ratio, etc. These functions are very familiar in instruments based in a Fourier (FFT) processor where both filters are as shown in (9),

$$\underline{A}_{-x}^{\omega_o} = \underline{A}_y^{\omega_o} = (1, e^{-j\omega_o}, \dots, e^{-jQ\omega_o})^T \quad (9)$$

or, in a block processing way, two FFT corresponding to the two data sample records.

The key point is to consider the use of the power tools we have currently in one-dimensional problems to the philosophy briefly described in Figure 2. Probably, most of the cases, the potential of very well-known parametric procedures in hardly reduced because their fundamental limitations in explaining their objective, behavior and performance, from a filter bank analysis point of view. This is no longer the case for the ML estimate as described in [2] and [3]. In Section III the procedure which results from the use of ML filters as $\underline{A}_x$ and $\underline{A}_y$ will be provided.

III. ML CROSS-SPECTRAL ESTIMATE

Considering that an ML filter designed from the signal corresponding to each channel achieves the desired narrow band residuals $e_x(n)$ and $e_y(n)$ in order that $P_{xy}(\omega_0)$ can be obtained from their zero lag cross correlation, the power level estimate results:

$$P_{xy}(\omega_0) = E[e_x(n)e_y(n)] = \underline{A}_x^T E[\underline{X}\underline{Y}^T] \underline{A}_y \quad (10)$$

being

$$\underline{R}_{xy} = E[\underline{X}\underline{Y}^T] \quad (11)$$

and using the detailed formulae for the ML filters, the final power level estimate is given by (12)

$$P_{xy}(\omega_0) = \frac{\underline{S}_{o=xx}^{-1} \underline{R}_{xy} \underline{S}_{o=yy}^{-1}}{(\underline{S}_{o=xx}^{-1} \underline{S}_{o=xx}) (\underline{S}_{o=yy}^{-1} \underline{S}_{o=yy})} \quad (12)$$

At the same time, using (7) and assuming a flat behavior in the cross-bandwidth, (8) allows to obtain (13) as a cross-power spectrum estimate.

$$\hat{S}_{xy}(\omega_0) = \frac{\underline{S}_{o=xx}^{-1} \underline{R}_{xy} \underline{S}_{o=yy}^{-1}}{\underline{S}_{o=xx}^{-1} \underline{S}_{o=xx}} \quad (13)$$

From the above formula results clear that basic concepts of section I are yet preserved regardless the extension of the procedure to the two channel problem. It can be easily checked out than under a common input, both (12) and (13) will arise to the auto-power level and autospectrum estimate of the single input signal. As a side result note that (13) is like a quotient which compares the cross-correlation matrix of the two data records againsts the case of a common white noise input (i.e. $\underline{R}_{xy} = \underline{I}$ in such case). Under a time delay or cross-spectrum estimation problem when both inputs are white noise with a pure delay between each other, the reported estimate reduces to the Fourier procedure.

An interesting feature of tracking delay problems is that both autocorrelation matrix which appear in the estimate could be determined with accuracy from a long time period and considered to be invariant in the tracking problem, so that only the cross correlation matrix $\underline{R}_{xy}$ must be up to dated to renew the estimate.

The main problem to be faced when using a cross-correlation matrix in a quadratic form like $\underline{S}_{o=xy}^{-1}$, or in general, when the ML approach is handled steems from the fact that they do not allow to recover the correlation or cross-correlation data lags with the same values we enter when the procedure is started. In other

words, it can be said that ML procedures are not coherent with the data second order functions. No matter that some criticism could be made about the importance in preserving, as constraints in an estimation procedure, values which have to be estimated again from the given data sample; the herein described procedure seems to be valuable even when cross or autocorrelation extrapolation is our main concern as occurs in a time delay estimation problem. Without extensive experience on it and from a few simulations, an average and overlap procedure using successive estimates with previously delayed data records seems to prelude the languidness of (13) in the extrapolation of the cross-correlation estimate.

IV. CONCLUSIONS AND PERSPECTIVE

It has been reported how ML filters can be used in cross-spectrum problems at it was in autospectrum. Note also, that from a realistic point of view, an ML filters based spectrum analyzer looks to be the only approach which provides almost simultaneously power level and power density estimates. This is done without the cumbersome integration process associated with an ME spectral estimate which, is clear, cannot be performed by a non-expertized user avoiding the implementation of maximum entropy techniques in wide range applications spectrum analyzers.

The second interesting feature of the ML spectrum analyzer is its intrinsic character of measuring power directly in the frequency domain, with no needs of equally spaced sensors in chart-graphics for biomedical applications, sonar, etc. and, as a consequence [3], its easy extension for 2-D spectral estimation. The mentioned extension results with no comparison in computational load and a very competitive performance with currently reported methods in this field.

The last feature to apprise the potential and the interest of data-dependent filter bank analysis is briefly described in this paper. Cross-spectrum estimation is probably right now one of the most promising fields in applying parametric procedures for signal analysis. The ML approach shows that we have the possibility of getting cross-power levels and the corresponding cross-spectrum density.

Further efforts will be devoted to the non-coherent behavior of ML estimates with the data correlation and cross-correlation functions in order to further improve the results the authors got in time-delay estimation. No matter these results are better than the corresponding ones from the DFT approach, their quality is not in a right agreement with the degree of quality shown by the procedure in the frequency domain.

V. A SIMULATION EXAMPLE

In order to check the performance of the reported procedure a simple example was

carried out using the previous formulae. Two data records of 32 samples length were under consideration. Record $x(n)$ consist in two real sinusoids with normalized frequencies of 0.1 and 0.25 buried in white noise with a signal to noise ratio of 20 dB each. Record $y(n)$ was essentially of the same structure but with - normalized frequencies of 0.25 and 0.4.

Fig 2. Numerator term $\underline{S}_{R=x}^{T-1} \underline{R}_{xy=y}^{-1} \underline{S}$

In Fig 2 it is shown the numerator term where we can see how the 3 tones are clearly distinguished, denominator to estimate power level (i.e. $\underline{S}_{R=x}^{T-1} \underline{S} \underline{S}_{R=y}^{T-1} \underline{S}$) is shown in Fig 3. It results that this term solves what is in common between the two records. The resulting power estimate can be seen in Fig 4.

Fig 3. Denominator terms for power level $\underline{S}_{R=x}^{T-1} \underline{S} \underline{S}_{R=y}^{T-1} \underline{S}$.

If the cross-spectrum density is desired the cross-bandwidth estimate should be used In Fig 5 it can be viewed the resulting cross-spectrum density estimate and in Fig 6 the 32 point DFT estimate.

As in autospectrum or two dimensional spectral estimation the reader can see how - resolution is increased from the ML power level estimate with respect to the ML power density estimate.

Fig 4. Resulting power level estimate (quotient of plots #1 and #2).

Fig 5. Cross spectrum estimate from plot #4 after normalization.

Fig 6. 32-point DFT cross spectrum estimate.

VI. REFERENCES

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