Elastic FWI of reflection data with a phase misfit function

Jean Kormann, Juan Esteban Rodríguez, Miguel Ferrer, Natalia Gutiérrez,
Josep de la Puente, Mauricio Hanzich and José María Cela^[1]

Computer Applications in Science and Engineering, Barcelona Supercomputing
Center, Jordi Girona 29, 08034 Barcelona, Spain.
`jean.kormann@bsc.es`

Abstract. Full Waveform Inversion of elastic dataset is challenging due to the complexity introduced by free-surface effects or P-S wave conversions among others. In this context, large offsets are preferred for inversion because they favor transmission modes which are more linearly related to P-wave velocity. In this paper, we present an original approach which allows to dynamically select the near offset at each frequency. We illustrate this approach with the inversion of a dataset without density. In order to deal with a more realistic scenario, we next present the inversion with density effects included into the modeling. As inverting density is known to be a hard task, we choose to not invert it. This approach leads to the use of a phase misfit function, which is more connected to the kinematics of the problem than the classic L^2 norm.

Keywords: full waveform inversion, elastic waves, high-performance computing

1 Introduction

Full-waveform inversion (FWI) of seismic datasets allows to retrieve physical properties directly from the seismic traces, once sufficient conditions on the initial guess, data quality and frequency bandwidth are met. Its high-resolving capacity with respect to other imaging methods makes FWI an attractive tool for both academy and industry. Nevertheless, the non-uniqueness of the solution and its high computational cost make performing FWI a challenging task even with modern High Performance Computing architectures.

Typically, in order to reduce the non-linearity of the problem, inversion strategies are used which increase the success ratio of FWI. One of the most widely used strategies is starting with large offsets and selecting (windowing) early arrivals of the P-wave. This strategy favors transmission modes over reflection modes, being the former more linearly related to P-wave velocity [7, 2, 3, 9]. This strategy also facilitates retrieving the lowest possible wavelengths of the model, as the data typically is limited in the low-frequency end. Nevertheless, not always are long offsets available. Analogously, for a fixed offset length, the deepest regions are not well covered with refracting waves. Hence it is of great

Fig. 1. Models and results for the constant-density dataset: V_p (top, right) and V_s (bottom, right) are the target model; V_p (top, left) and V_s (bottom, left) are the FWI results obtained using the DOC strategy and modified L^2 norm.

interest to be able to extract as much information as possible from reflection modes in FWI.

In former studies, using reflection modes for FWI has been considered a challenging task [6]. The incapacity for retrieving the longest wavelengths in the model has stirred an interest in modifying the typical FWI workflows by employing image-domain misfit functions [11] or intertwined FWI/migration algorithms [10, 8].

We show how, with a careful choice of offsets and a phase-based misfit function [4], reflection datasets can be successfully inverted. Furthermore, we exemplify our strategy with elastic FWI instead of acoustic FWI, which is much more challenging in terms of complexity and cost.

2 Theory

Regarding the elastic modeling needed for FWI, we use the time-domain elastic isotropic approach for solving the elastic wave equation:

$$\rho(\mathbf{x})\dot{\mathbf{v}}(\mathbf{x}, t) = \nabla \cdot \boldsymbol{\sigma}(\mathbf{x}, t) + \mathbf{f}_s(\mathbf{x}_s, t), \dot{\boldsymbol{\sigma}}(\mathbf{x}, t) = \mathbf{C}(\mathbf{x}) : \nabla \mathbf{v}(\mathbf{x}, t) \quad (1)$$

where f_s is the source function at position $\mathbf{x}_s$, $\mathbf{v}$ the particle velocity, ρ the density, and $\boldsymbol{\sigma}$ the stress field.

With respect to the misfit function, instead of the classic L^2 norm we prefer a locally normalized version L^2_{mod} which has shown to be more robust in our testing. This function is defined as:

$$L^2_{\text{mod}} = \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^n \left[\frac{u_j^i(\mathbf{m})}{\max_j(|u_j^i(\mathbf{m})|)} - \frac{u_j^i(\mathbf{m}_{\text{real}})}{\max_j(|u_j^i(\mathbf{m}_{\text{real}})|)} \right]^2, \quad (2)$$

where N is the number of receivers, n the number of samples of each trace, u the measurement obtained from the current model $\mathbf{m}$ or the real model $\mathbf{m}_{\text{real}}$, in which case we call the measurement data. The search direction $\mathbf{p}_k$ at the

k^{th} iteration is obtained by means of a non-linear conjugate gradient method, is given by

$$\mathbf{p}_k = \nabla \epsilon_k + \beta_k \mathbf{p}_{k-1} . \quad (3)$$

The Polak-Ribière criterion is used for obtaining β_k . Finally, a line search algorithm finds the optimum α_k and then updates the models according to

$$\mathbf{m}_k = \mathbf{m}_{k-1} + \alpha_k \mathbf{p}_k . \quad (4)$$

We apply a two-step preconditioning: At a first stage, we apply a change of variable, and choose to work with $\tilde{\mathbf{m}} = \log_e(\mathbf{m})$ instead of $\mathbf{m}$. This is equivalent to multiplying the gradients of λ and μ , the Lamé parameters, by the parameters themselves. The last step compensates for geometrical spreading by using the preconditioner proposed by [5].

3 Results

In the following we present the inversion of a model based upon the 3D SEG / EAGE Overthrust model [1]. We chose a 2D slice of the model and devised an acquisition geometry involving sources located at 50 m deep, and with a spacing of 200 m starting from the inline location $x=1050$ m, resulting in a total of 71 shots. We have simulated 10 s of traces using a Ricker wavelet with central frequency at 10 Hz. Furthermore, we used a heterogeneous density model for the data and included a free-surface condition. The V_s and ρ models were set proportional to V_p by a factor of 0.5 and 0.41, respectively. A dataset with constant density was also generated, for checking purposes. No preprocessing of the data has been done to remove ground roll or window seismic phases. The data has been used in the FWI as it is.

The FWI starting models were obtained by applying a Gaussian smoothing to the target models (see Fig. 1, right panels) with a correlation length of 500 m. Our FWI uses a multi-grid and multi-scale strategy, sweeping with low-pass filters from low to high frequencies. We want to stress that the computational grid does not match the receiver and source grids, as the grid is re-generated, and the model consequently interpolated, at each frequency band. Last but not least, we didn't fix any part of the model during the inversion and we haven't used any kind of regularization in our inversions.

3.1 Dynamic Offset Control

In order to validate our FWI against other approaches [2], we first used the dataset obtained with a constant density model, with a value set to 1000 kg/m^3 . Our approach for FWI consists in constraining the maximum offset to the longest wavelength of the compressional wave at the present frequency maximum. We call this approach Dynamic Offset Control (DOC). In this way, the maximum receiver offset is 3530 m for the lowest frequency of 1.7 Hz. This strategy has the

side effect of reducing the computational domain as we go into higher frequencies while minimizing the effect of the surface wave on the gradients. Last but not least, by selecting only near-offset traces, we are able to avoid cycle skipping between synthetic and data and can use the whole trace length in our FWI. This leads to a fast and reliable inversion scheme. For this test we can rely on our modified L^2 norm defined in Equation 2. Table 3.1 summarizes the multi-scale, multi-grid strategy used for this case.

FWI strategy				
1.7 Hz	2.5 Hz	3.4 Hz	5.2 Hz	7.3 Hz
72x352	96x483	124x606	160x800	196x975
45.0 m	33.0 m	25.8 m	20.0 m	16.3 m
10	10	10	10	10

Table 1. Multi-grid and multi-scale parameters for 2D FWI in the constant density case; from top to bottom: cut-off frequency, mesh size, spatial discretization, and number of iterations.

Figure 1 shows the inverted models after 50 iterations divided among 5 frequencies. We observe that the inversion is successful, with a good resolution of all the features, even for the deepest layers. Reflectors show a good continuity and the fracture zone is clearly delineated. We remark, once more, the very small dataset and the straightforward FWI scheme used.

3.2 Including density into the dataset

In a more challenging test, we used the dataset generated with a heterogeneous density model. The inversion, however, will only invert the P- and S-wave velocities. As our density model in FWI is homogenous and constant throughout the inversion, we cannot expect to explain completely the data with our FWI. We remark that density is rarely well known and is very challenging to invert with FWI. Our goal here is concentrating on the wave kinematics which can be explained in terms of P- and S-wave velocity while ignoring the dynamic part which has a strong impact on the amplitude of each phase. To that goal we must use a misfit function which focuses on correcting phases instead of amplitudes. The classic L^2 would behave poorly in this case. Our modified L^2_{mod} norm in Equation 2 is better suited as the normalizations will help focusing on phase changes. Nevertheless, a proper phase-focused misfit function was presented by [5] and has been applied in the present FWI exercise. This phase misfit function relies on time-frequency analysis, and is obtained by the cross-correlation of the Gabor transform between the synthetics and the data.

The combination of DOC and phase misfit in FWI results in better phase correspondence between synthetic and data. This allowed us to use larger frequency jumps in our FWI strategy without suffering from cycle-skipping. Assuming that the starting frequency is the same as the one used in the previous section, we can

Fig. 2. Models and results for the heterogeneous density elastic dataset: V_p (top, right) and V_s (bottom, right) are the target model; V_p (top, left) and V_s (bottom, left) are the FWI results obtained using the DOC strategy and phase misfit.

perform FWI by doubling the frequency at each frequency iteration; a strategy which is summarized in Table 2.

FWI strategy		
1.7 Hz	3.4 Hz	6.8 Hz
48x238	92x457	160x800
67.5 m	34.9 m	20.0 m
15	20	20

Table 2. Multi-grid and multi-scale parameters for 2D FWI in the heterogeneous density case; from top to bottom: cut-off frequency, mesh size, spatial discretization, and number of iterations.

Figure 2 presents the FWI results for this heterogeneous-density dataset after 55 iterations divided in 3 frequencies. We observe that the inversion results are very good, even at larger depth. The upper part is very well recovered, with no clear footprint of the acquisition geometry. Nevertheless, the effects of density are visible, especially when comparing Figures 1 and 2. We loose the continuity of some reflectors, especially for the finest structures. We also observe some over-estimation of the velocities, especially for the reflector lying at depth 1000 m and range 6 km. Nevertheless, we observe a smooth convergence and stability during the inversion: misfit reduction at 6.8 Hz reaches 26 per cent after 20 iterations (see Fig. 4).

In Fig. 3 we pick three velocity profiles for both V_p (top) and V_s (bottom) at ranges 6, 10, and 14 km. We can see a very good agreement between target and inverted profiles, especially for V_s velocities, as expected due to the S-wave having a higher resolving power. There is also a good correlation of the low-frequency content, and the velocity amplitudes are almost correctly recovered by the method. We remark that this result has been obtained from data that included density effects while ignoring density in our elastic FWI scheme, and using only very short offsets.

Fig. 3. Top (from left to right): P-wave velocity profiles at range 6, 10 and 14 km respectively; Bottom (from left to right): S-wave velocity profiles at range 6, 10 and 14 km respectively;

4 Summary and discussion

We have shown an elastic FWI successfully working on data dominated by reflections, using only near-source offsets. For this purpose, we have introduced an offset selection method called Dynamic Offset Control (DOC). We have applied DOC to 2D elastic datasets and demonstrated that we can recover accurately the elastic models even when we include density in the data. Furthermore, DOC allows us to reduce the maximum aperture of each shot in FWI, hence mitigating the computational cost increase when moving into higher frequencies, which hints at making 3D elastic inversion more feasible. A key ingredient to our FWI results is using a phase misfit function [5], that is more naturally connected to the kinematic behavior of the waves. We believe that density is a proxy for amplitude-dominant phenomena which we do not include in our FWI scheme, and postulate that DOC with phase misfits might work well also in case of having attenuation in the data. Finally, we want to stress that we used a rather conventional FWI workflow, with no data windowing, phase selections or ground roll removal. Neither we used regularization or layer freezing.

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Fig. 4. Normalized misfit for the 3 inverted frequencies.

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