

**ANALYTICAL METHOD FOR THE ASSESSMENT OF UNREINFORCED BRICK MASONRY
WALLS SUBJECTED TO ECCENTRIC COMPRESSIVE LOADS**

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ABSTRACT

Slender unreinforced masonry walls are likely to fail due to mechanism formation when they are subjected to eccentric compression loads causing second order bending effects. A practical analytical formulation is herein presented to provide an accurate calculation tool for these cases. The proposed method is compared with Eurocode-6, ACI-530, Southwell Plot method (semi empiric), a Finite Element Analysis and experimental tests. The results show that the proposed method and ACI-530 are accurate at calculating the load-bearing capacity of the most slender walls and the less slender cases respectively. The Southwell Plot method achieves the best performance although its applicability is limited.

KEYWORDS

Brick masonry, Analytical formulation, Second order bending, Standards

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1. INTRODUCTION

Unreinforced brick masonry walls are a common structural typology among existing buildings worldwide. In particular, these elements were extensively used for residential construction in many European countries along the first half of the XXth century. Nowadays, these buildings frequently require rehabilitation interventions. Often, existing buildings are subjected to changes of use requiring the application of higher load levels. These cases require the verification of the loading capacity of the load bearing masonry walls. This verification is also necessary as part of the activities involved by the rehabilitation and maintenance of existing buildings, which are mandatory for economic and environmental reasons and a current research topic [1].

However, it has been observed that the research so far devoted to the experimental characterization, the modelling and the analytical assessment of the structural response of load bearing walls is still very limited. Although masonry structures have received a significant research effort during the last decades, most of this effort has been devoted to in-plane performance of shear walls (see [2–4]) and the seismic capacity of masonry structural systems (see [5–7]).

Due to this limited research oriented to masonry load bearing walls, the assessment of the structural safety of masonry structures is still faces significant difficulties. Taking into account the economic weight of the existing brick-masonry load-bearing supported buildings (e.g. these are over 75% of the existing buildings in Barcelona, Spain [8]), it seems suitable to enhance the knowledge in this area in order to better maintain these structures.

Masonry load bearing walls are subjected to compressive loads transmitted by the roof or the floor slabs, which might be eccentrically supported on the thickness of the wall. Because of this, the calculation of load bearing masonry walls normally requires the consideration of eccentrically applied loads. In this situation, second order bending effects might strongly influence the structural response of the most slender walls and so their load-bearing capacity too.

Experimental campaigns have been performed to study the case of eccentrically compressed brick masonry walls. The huge experimental work carried out by Gross et al. in the Brick Institute of America, which was presented as a recommendation summary [9], is a reference example. Similarly, the researches carried out by Watstein and Allen [10] and Kirtschig and Anstötz [11] have to be highlighted because of the attention allocated to the study of the influence of the test setup, wall slenderness ratio and load eccentricity on the ultimate capacity of brick masonry walls. Finally, the comprehensive work of Kukulsy and Lugez [12] has to be mentioned because it is the basis for the formulation of the current European masonry standard (Eurocode-6 [13]), although their tests were carried out on concrete walls. Nevertheless, most of these investigations were carried out in the 1960's and 1970's and the acquired data is not always complete enough for the possible validation of analytical or numerical tools. More recent experimental researches on this subject are focused on obtaining the necessary data to develop and validate numerical models. This is the case of the campaign carried out by Sandoval et al. [14], who tested small scale (1:4) walls subjected to eccentric

compressive loads in laboratory conditions, or the work by Zuccarello et al. [15] who focused the research on the out-of-plane loading condition.

Using experimental data has allowed the development and validation of accurate numerical models for the assessment of the structural response of masonry elements under different loading and supporting conditions. Among the numerous researches in this area, the work of Lourenço [16] has to be highlighted as a reference for the characterisation of the masonry response. Modelling walls subjected to eccentric compressive loads has been also addressed (see [14,17]). Finally, in the recent years the numerical models have evolved to include the non-linear response, damage simulation and the periodicity of the material, as in the case of the work presented by Salerno and Felice [18]. However, most of the numerical tools recently proposed are complex and computationally demanding, and cannot be used for design purposes. Analytical approaches, less demanding from the calculation point of view, are still necessary for the practical assessment and design of load bearing walls.

The first analytical researches aimed to represent the compressive response of masonry walls go back to the 1930's [19]. One of the first proposals taking into consideration geometrical instability in the calculation of load-bearing masonry walls was made by Haller [20]. Gross et al. [1] presented a formulation resulting from fitting experimental results on hundreds of full-scale tests. Similarly, the experiments of Kirtschig and Anstötz [11] were also used to develop and adjust an analytical formulation for calculating the load-bearing capacity of brick masonry walls against buckling phenomenon. All these researches mainly studied the stability of the walls. In contrast, other researchers focused their investigations on defining the cross section strength. This was the case of the methodology proposed by Yokel [21,22] which set the basis for relating the cross sectional strength with the load-bearing capacity of the wall including the effect of the slenderness and eccentricity. These studies were the basis for later on researches.

In the last years some authors have included the non-linear material response in the analytical approaches. It is the case of the researches presented by De Falco [23,24] or the parabolic description of the compressive response of the masonry used by Mura [25]. However, the tensile strength of the masonry is still not taken into account in most of the proposals. The importance of this parameter in the analytical methods oriented to calculate masonry walls against buckling phenomenon is justified and considered in the research by Lu et al. [26]. Finally, some analytical proposals, like the one presented by Dawe and Liu [27], stand out for its reliability at predicting the buckling failure load, although their application requires the aid of a computer.

In contrast, the friendly formulations included in the standards for calculating the load-bearing capacity of brick-masonry walls have been qualified of conservative by different authors (see [17,28]), especially for the cases with large slenderness or load eccentricity.

With the aim of contributing with a practical calculation tool applicable to the assessment and design purposes, a method for the calculation of unreinforced brick masonry walls under eccentric compressive loads, combining efficiency and accuracy, is herein proposed. In contrast with other available methods, the present one considers the influence of the masonry tensile strength on the capacity of the load bearing walls and takes into account the second order bending deformations. This novel approach has been validated with experimental cases,

compared with numerical results and also compared with the application of different standards (Eurocode-6 and ACI-530).

In addition, a semi empiric method (Southwell Plot) is also used for comparison and the obtained conclusions point out that this particular approach might be suitable for assessing existing load-bearing masonry walls.

2. Methodology

The proposed analytical approach is described in this section along with its application to its comparison with the results obtained from an experimental programme involving tests to failure of 18 unreinforced brick masonry walls eccentrically loaded. These 18 cases correspond to a previous experimental campaign described in [17]. The results obtained with the proposed method are compared with other analytic, semi-empiric and numerical approaches to assess its accuracy. The scope of this research corresponds mainly to slender walls or walls subjected to large eccentricities, for which the mechanism failure mode (bending-buckling failure) is more likely. Nevertheless, the proposed methodology has been used to calculate 16 walls with small eccentricity of the load presented in [11] to complete the study. These results are also compared with numerical and analytical approaches but not with the semi-empiric method because there is no data about the loading process in [11].

2.1. Description of the structural problem and the comparison cases

Only walls with significant width (not pillars) allowing a description by means of a bi-dimensional plain strain model or a one-dimensional beam model, are considered in the present research. Similar hypothesis are implicitly set by the current standards Eurocode-6 [13] and ACI-530 [29] which calculate the load-bearing capacity per unit of width. Moreover, for the proposed analytical approach the walls are described, in a simplified way, as a one-dimensional problem consisting of a compressed beam with the same eccentricity at both ends. The pinned-pinned configuration, with the lower support fixed and the upper support with the vertical displacement free and the rotation free is represented in Figure 1. This structural configuration has been chosen in order to compare the results with experimental campaigns on walls whose boundary conditions were clearly determined, avoiding the uncertainties in this matter. The effective height, H_{ef} , is defined, for these cases, as the vertical distance between axes of rotation of the corresponding hinges.

Different effective heights and variable eccentricities, corresponding to the mentioned 34 (18 + 16) experimental cases, have been considered for the validation of the proposed analytical approach. This range of cases is used to assess the accuracy of the proposed method for different geometric parameters and to define the application range in which it obtains closest results to the experimental ones.

The detailed description of the geometric characteristics of the 18 experimental walls tested and used in this research can be found in [17]. Only the walls with pinned-pinned configuration have been considered because the boundary conditions are clear and the effective height, H_{ef} , can be defined precisely. In addition, only one-wythed walls with solid bricks are studied and compared with analytic results.

The mechanical properties of the masonry and the dimensions of the walls are summarised in Table 1. Three typologies of walls (H , M and S) corresponding to large, medium and small height of the walls have been studied. The load eccentricity (e) of each wall is indicated in the third column of this table. Finally, the effective height, H_{ef} and the width, b , are presented for the different cases. All walls were 132 mm thick, t . The last two columns in Table 1 correspond to the compressive (f_c) and tensile (f_t) strength of the masonry respectively. The average Young's modulus of the masonry, measured as described in [9], was 780 MPa.

Four slenderness ($\lambda = 5.6, 11.1, 18.8$ and 27.7) and two eccentricities cases ($e=0$ and $e=t/8$) of walls with calcium silicate units have been chosen among the results in [11] to validate the proposed approach and cover the cases with less eccentricity. Two walls for each combination were tested and the average results are used for comparison reasons. Thus, 8 comparison cases correspond to these 16 walls. The detailed description of the specimens and materials might be found in [11] and the justified estimation of some parameters ($E = 8540$ MPa, $f_t = 0.3$ MPa) have been previously presented in [17].

2.2. Second order bending analytical approach

An analytical method aimed to calculate the load-bearing capacity of unreinforced brick masonry walls likely to fail by mechanism formation is herein presented. The main features of this method are the consideration of the second order bending effects and the adoption of a non-null tensile strength of masonry. On the whole, the proposed method consists of calculating the axial-bending response of the walls and comparing it with an axial-bending failure criterion to obtain the load-bearing capacity. The eccentricity-caused second order deformation is calculated by assuming an elastic response of the material. It has to be noted that this hypothesis is not conventional for masonry structures. In fact, the compression response of the masonry is usually assumed to be plastic and a rectangular stress diagram is used in most of the common methods for the verification of masonry walls. However, for slender or eccentrically loaded walls, the second order deformations, which represent the geometric non-linearity of the problem, might be more influential on the load-bearing capacity than the material non-linear response in compression. In addition, the assumption of a linear elastic response allows considering the second order deformations in an easy and effective way. It has to be highlighted that the proposed analytical method is mainly oriented to calculate those walls whose failure mode is associated with a mechanism formation (large slenderness or eccentricity of the applied load). The accuracy of the obtained results (see section 3) also justifies the hypothesis of compressive linear response of the masonry.

The bending moment, M , at any section of an eccentrically compressed beam can be calculated according with (Eq. 1), where N is the applied axial load, e_0 is the initial eccentricity of the load (constant along the beam's length, or by analogy along the wall's height) and v is the lateral deformation of the beam due to the eccentrically applied load.

$$M = -N \cdot (v + e_0) \quad (\text{Eq. 1})$$

A differential equation (Eq. 2) relates the bending moment, M , with the lateral deformation, v , along the wall's height, following the variable x . Thus, for each applied compressive load, N ,

the corresponding deflection, v , can be calculated at any point along the wall's height. The solution of this differential equation evaluated at mid-height, $H_{ef}/2$, is the expression (Eq. 3) which relates the applied axial load, N , with the lateral deformation of the wall at mid-height, v . The solution is evaluated at mid-height because it is the point of maximum deflection for the considered structural configuration. Thus, at this point the bending moment is maximum and mid-height is the most critical section to be checked.

$$M = EI \cdot \frac{\partial^2 v}{\partial x^2} \rightarrow EI \cdot \frac{\partial^2 v}{\partial x^2} + N \cdot v = -N \cdot e_0 \quad (\text{Eq. 2})$$

$$v \left(x = \frac{H_{ef}}{2} \right) = e_0 \cdot \left(\frac{1}{\cos \left(\frac{H_{ef}}{2} \cdot \sqrt{\frac{N}{EI}} \right)} - 1 \right) \quad (\text{Eq. 3})$$

This deflection, v , has to be added to the initial eccentricity, e_0 , in order to obtain the total lateral deformation, e_{total} , of the wall for each value of the applied load, N , as shown in (Eq. 4). This calculation procedure has been previously used by other researchers like [30].

$$e_{total} \left(x = \frac{H_{ef}}{2} \right) = e_0 \cdot \left(\frac{1}{\cos \left(\frac{H_{ef}}{2} \cdot \sqrt{\frac{N}{EI}} \right)} \right) \quad (\text{Eq. 4})$$

In this equation, e_0 is the real eccentricity at mid-height at the beginning of the loading process, H_{ef} is the effective height of the wall and E and I are the Young's modulus of the masonry and the modulus of inertia of the cross section respectively.

Thus, for each value of the applied load, N , it is possible to calculate the theoretical applied bending moment by multiplying the axial load, N , by the total eccentricity, e_{total} . Gathering the data for different load steps an axial-bending interaction curve might be drawn. This type of plot shows the theoretical evolution along a hypothetic loading process.

In order to calculate the load-bearing capacity of each considered wall, it is necessary to intersect the axial-bending response curves with the curves representing the axial-bending stress limit criterion. These curves, which are associated with the stress limit criterion, are calculated by assuming the following hypotheses:

- Linear strain distribution along the cross section thickness.
- Linear stress distribution in both tensile and compressed areas.
- Failure occurs when the maximum compressive stress reaches the masonry compressive strength (f_c) or when the tensile stress reaches the masonry flexural strength. The masonry flexural strength is evaluated as the flexural strength of the masonry joints (f_t).

The first two hypothesis are a direct consequence of the previously mentioned and justified assumption that the material behaves linear elastic. The failure criteria in compression is commonly used for calculations in which the structure is only subjected to normal stresses, which is this study case. Finally, the hypothesis that the masonry fails in flexion when the flexural strength of the masonry joints is achieved is the consequence of assuming that the joints are the weakest point of the masonry in a bending configuration. This fact is certain for all the studied cases and this is the most common hypothesis for current masonry structures the research is focused on.

The limit criteria calculated in this way is represented by two straight lines in the axial-bending space. These graphs depend on the geometry of the cross section (thickness, t and width, b) and on the masonry compressive strength (f_c) and the flexural strength of the masonry joints (f_t).

The maximum axial force (N) and bending moment (M) combination that a rectangular section can resist, taking into account the maximum compressive strength (f_c) of the masonry, is calculated, under the previous hypothesis, with the following equation (Eq. 5).

$$M = \frac{f_c b t^2}{6} + \frac{N t}{6} \quad (\text{Eq. 5})$$

Similarly, the maximum axial force (N) and bending moment (M) combination that a rectangular section can resist taking into account the maximum flexural strength (f_t) of the masonry is calculated, under the previous hypothesis, with the following equation (Eq. 6).

$$M = \frac{f_t b t^2}{6} - \frac{N t}{6} \quad (\text{Eq. 6})$$

The procedure to obtain the load-bearing capacity by intersecting the calculated theoretical response with the imposed failure criteria is shown in Figure 2. It has to be remarked that, in this figure, dimensionless variables are defined for comparison purposes. Thus, the axial force, N , is divided by the compressive strength of the cross section under uniform compression ($N_u = b t f_c$), and the calculated bending moment, M , is divided by $N_u \cdot t / 8$. The cases of walls W#2, W#3 and W#5 (defined in Table 1) are represented in Figure 2 as an example.

2.3. Other approaches for comparison

The accuracy of the proposed analytical approach, which considers the second order bending effects, is analysed by comparing its results with the results obtained by applying Eurocode-6 and ACI-530 formulations. Similarly, the Southwell plot method and the numerical model proposed in [17] are used to compare with the results of the proposed analytical method.

2.3.1. Standards

Two standards have been considered for comparison purposes: Eurocode-6 [13] and ACI-530 [29]. The main difference between them relay on the way these codes deal with the axial, axial-bending and buckling phenomena. ACI-530 considers these three phenomena independently whereas Eurocode-6 considers these three failure causes together in a unique formulation. Since the formulations of the standards are used for the purpose of comparison

with other methods and with experimental results, no safety factors are considered in the calculations.

The formulation proposed into Eurocode-6 for the calculation of the load-bearing capacity of unreinforced brick masonry walls subjected to eccentric compressive loads is based on the comparison of the axial strength of the cross section with the applied axial loads. This comparison has to be carried out at the ends of the wall (only affected by eccentricity) and at the most unfavourable section within the central fifth of the wall (affected by eccentricity and slenderness). For all studied cases the most restrictive conditions correspond to the mid-height section. The boundary conditions, the slenderness of the wall, the eccentricity of the load, the geometric irregularities and the out-of-plane loading effects are taken into account in the analytical approach proposed into Eurocode-6.

It has to be highlighted that EC-6 code does not consider the tensile strength of the masonry leading to conservative results (see section 3). The masonry tensile strength has been shown to influence significantly on the capacity of walls showing large slenderness and / or eccentric loading [14,26,28].

The formulation for calculating the load-bearing capacity of masonry walls subjected to eccentric axial loads of ACI 530 [21] considers the possibility of calculating the limit stress instead of the ultimate load. In this code, the stress distribution is based on the elasticity equations (linear stress distribution) but the limit stress is defined with an empirical criteria. Each phenomenon (axial, axial-bending and buckling) is analysed with an independent formulation. For the tested walls, two possible failure modes were possible: the axial-bending and the buckling. Thus, both modes have to be checked to obtain the load-bearing capacity of the wall, which is obtained as the resulting minimum capacity.

Like EC-6, ACI-530 does not consider the tensile strength of the masonry in the calculation of the load-bearing capacity.

2.3.2. Southwell Plot method

In the early 1930's Southwell [31] proposed a hybrid method consisting on using experimentally determined data about the structural response of imperfect columns to predict their load-bearing capacity. This procedure is commonly used to estimate the buckling critical load of a compressed element taking into account the load eccentricity in an implicit way and requires an elastic response of the structure. The work by Bažant and Cedoline [32] offers a detailed description of the application of Southwell Plot method.

The Southwell Plot method consists on assuming that there is a relationship (Eq. 7) between the lateral deflection, h , and the applied load, N , which depends on the critical load, P_{cr} and the initial mid-height lateral deformation, a . Thus, applying Southwell plot method requires measuring the response of the structural element (applied load and corresponding lateral deformation) for low or moderate loads. The application of the method is possible in structures showing lateral deflections due to eccentrically applied loads or geometric imperfections. Finally, the relationship between h and h/N is obtained as a straight line (Eq. 7). The slope of the line describing the relation between h and h/N is the load-bearing capacity of the wall due to the buckling failure.

$$\frac{h}{N} \approx \frac{1}{P_{cr}} h + \frac{a}{P_{cr}} \quad (\text{Eq. 7})$$

The applicability of the Southwell Plot method depends on the scattering of the measured data. For this reason, this method is not suitable for those cases with large dispersion in the measurements or inconsistent experimental data. In fact, during the current research it has been observed that the points of the Southwell Plot (h versus $\frac{h}{N}$ graph) should be linear fitted with a regression coefficient greater than 0.8 to obtain reliable results. More scattered data results in a meaningless value of the load-bearing capacity. The Southwell Plot linear fitting for the M wall series is shown in Figure 3.

2.3.3. Numerical approach

The numerical approach which has been used to compare with the proposed analytical method and assessing its accuracy is fully described in [17]. This is a simplified bidimensional micromodel which uses a bilinear cohesive zone model material to represent the tensile response of the masonry joints and assumes a linear elastic perfect plastic response of the masonry in compression. The boundary conditions correspond to the pinned-pinned configuration and the load is indirectly applied as an imposed vertical descending displacement of the top of the wall. In addition, large displacements are considered for all cases to allow developing the large lateral deflection associated with the mechanism formation failure mode.

3. Results

The results of the calculations of the load-bearing capacity of the 18 tested walls are summarised in Table 2. The geometric type of each wall is indicated in the first column. The columns third to seventh present the load-bearing capacities calculated by using Eurocode-6, ACI-530, Southwell Plot method, the proposed analytical method based on the second order deflection calculation and the Finite Element Method described in section 2.3.3 respectively. The experimental results, which are the reference values, are presented in the eighth column. Finally, the last five columns show the absolute value of the average relative error of each calculation method in comparison with the experimental results.

Observing the values in Table 2 it is noticed that the methodology proposed in Eurocode-6 provides very conservative results in all cases and especially for the more slender walls (H wall series). More specifically, Eurocode-6 predicts that the studied walls would not be able to resist any significant load. In addition, it has to be highlighted that Southwell Plot method could not be applied in 2 out of the 8 cases of the H wall series walls and in 2 out of 3 cases of the S wall series walls.

The average of the absolute value of the relative errors for each considered calculation method and each walls' typology is shown in Figure 4. In all cases, Eurocode-6 (EC-6) is the less accurate one and Southwell Plot method the most accurate one. Among the other three calculation procedures, the herein proposed method is more accurate than ACI-530 or the FEM for the most slender walls (H wall series), whereas the numerical model is better than the

other two for the M wall series cases and the standard ACI-530 provides the better accuracy for the less slender walls (S wall series).

By analysing the results of Table 2 it is observed that Eurocode-6 is always conservative (it always predicts lower load-bearing capacities than those experimentally obtained). ACI-530 provides conservative errors for most of the cases of the H wall series and all the cases of the M and S wall series, whereas Southwell Plot is balanced with estimated load-bearing capacities over and below the experimental ones. The proposed analytical method is balanced for the walls of the H wall series but conservative for the M and S wall series. Finally, the numerical method is balanced for the H wall series walls but conservative for the rest of the cases.

The results for the 8 experimental cases (corresponding to 16 walls) presented in the bibliography [11] and chosen to extend the range with cases with smaller eccentricities, are summarised in Table 3. It has to be reminded that Southwell Plot results are not available because the bibliographic data does not contain the loading information. The results in Table 3 show that the proposed method accurately predicts the load-bearing capacity experimentally obtained. In fact, the proposed method is more accurate than the considered standards (Eurocode-6 and ACI-530) overall. In more detail, it can be noticed that ACI-530 is little more accurate than the proposed method for the cases with no eccentricity, but for the ones with $e=t/8$, for which the second order effects are more significant, the proposed method is better than ACI-530.

4. Discussion

The results for the tested walls in the previous section indicate that the Southwell Plot method is the most accurate among the compared ones for all considered slenderness. However, this calculation procedure requires experimental data which might be not available in some cases (as in designing phase) or might be not good-conditioned enough to apply the calculation (see cases of walls W#1, W#5, W#18 and W#19). Therefore, the applicability of this method is not granted for all cases. The semi empiric orientation of Southwell Plot method explains its better accuracy. This method considers the real structure response, which indirectly includes all the geometric irregularities, the real effect of the load eccentricity and the boundary conditions.

The proposed analytical method (based on the consideration of second order bending effects) is the most accurate for H wall series (tested walls, Table 2) among the methods which are applicable to design purposes (this excludes Southwell Plot method). In addition, the second order bending calculation is better than the other two analytical approaches for the cases with non-zero eccentricity considered from [11] (Table 3). These results support the hypothesis that the second order bending effects are highly significant for the most slender walls or the ones with more eccentricity. In addition, the decreasing accuracy of this method for the less slender walls makes it evident that it may not be suitable when the second order bending effects are almost negligible. Finally, the accurate results obtained for the H wall series and all considered walls from [11], support the assumption of a linear stress distribution in the wall sections and justifies the consideration of the tensile strength in the calculation procedure.

The numerical model, which has been introduced for comparison reasons, becomes the most accurate one for M wall series (tested walls), and all series from bibliography [11], among the methods for designing purposes (excluding the Southwell Plot method). This fact meets the aim of the numerical model of being balanced as mentioned in [17].

In relation with the analytical methodologies included in the standards ACI-530 and Eurocode-6, it has to be remarked that both codes are conservative for the most slender cases (H wall series). This fact meets the requirement of assuring the safety conditions of the structural masonry systems with guarantees. However, this evidence might also point out the need of considering the tensile strength in the calculation procedures as a suitable way to provide more accurate results. In addition, it has to be remarked that ACI-530 is the most accurate method for S wall series among the studied procedures. This evidence points out that the formulation included in ACI-530 is optimised for low slenderness cases. Comparing the results in Table 3, it can be said that ACI-530 also works better for the cases with small load eccentricity. Thus, the methodology proposed in ACI-530 seems more suitable for those cases which are not expected to fail due to bending/buckling phenomena. Finally, it has been observed that EC-6 is conservative for all tested cases.

5. Conclusions

According with the analysis of the obtained results and taking into account the bibliographic references presented in section 1 it can be concluded that there is not a simple, practical and accurate analytical method to calculate the load-bearing capacity of unreinforced brick masonry walls subjected to eccentric axial loads and considering different slenderness configurations. The complexity of this structural problem and the requirement of simple and practical calculation methodologies for practitioners make it necessary to consider different analytical methods. The applicability range for each studied method mainly depends on the geometric configuration of the structural problem and the possibility of testing the corresponding wall.

In the case of calculating an existing wall to assess its structural safety it is recommendable to apply the Southwell Plot method because of its best accuracy. However, the applicability of this method is limited by the possibility of performing the required experimental test. In addition, this method is more reliable for medium and large slenderness cases (H and M series).

The herein proposed analytical method, which considers the second order bending effects, seems suitable for walls with large slenderness in a design step.

In contrast and according with the obtained results, ACI-530 should be applied for calculating the load-bearing capacity of the less slender walls and the cases with little eccentricity of the load, among the studied methodologies.

The proposed analytical method and ACI-530 show better accuracy than the considered Finite Element Analysis for the H and S wall series of the tested walls respectively. In addition, these analytical methodologies require less computational resources than the numerical approach.

Finally, it has to be highlighted that the herein presented analytical formulation achieves good accuracy in the aimed application range (the most slender walls likely to fail by mechanism formation and the cases with non-zero eccentricity from [11]). It can be also concluded that including the tensile strength in the formulation has given place to an improved accuracy.

To sum up, it might be concluded that the proposed analytical approach is suitable for the designing step of load-bearing walls with great slenderness or large eccentricity of the load. In contrast, the Southwell Plot method is recommended, when applicable, to assess already-built walls. If there is not experimental data available, the proposed method based on the calculation of second order bending deformations is also suitable to assess walls with great slenderness or eccentricity of the load, for which it performs better than the current standards.

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7. References

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Type	Wall	e (mm)	H_{ef} (mm)	b (mm)	f_c (MPa)	f_f (MPa)	
<i>H</i>	W#1	5.6	2947	900	18.2	0.23	
	W#2	19.6	2927	900	12.9	0.36	
	W#3	1.2	2922	870			
	W#5	7.0	2917	900			
	W#6	33.7	2857	875	13.7		
	W#7	13.1	2872	885			
	W#8	-7.6	2942	879			
	W#9	21.5	2892	888			
	<i>M</i>	W#10	0.2	1865	888		10.8
W#11		13.9	1892	870			
W#12		1.6	1841	882			
W#13		31.8	1861	900			
W#14		21.2	1861	900			
W#15		29.0	1845	895			
W#16		19.5	1860	894			
<i>S</i>	W#18	20.8	1197	871	10.8	0.36	
	W#19	30.3	1220	871			
	W#20	33.0	1221	868			

Table 1 Geometric and mechanical properties of the considered cases

Type	Wall	N_{max} [kN]					Error [%]					
		EC-6	ACI-530	Southwell	2 nd order	FEM	Experimental	EC-6	ACI-530	Southwell	2 nd order	FEM
H	W#1	0.0	117.0	(*)	115.5	91.8	172.3	100.0	30.9	(*)	33.0	45.8
	W#2	0.0	53.8	63.6	63.7	95.0	68.7	100.0	18.1	3.3	7.2	44.6
	W#3	0.1	88.9	176.0	142.4	132.7	136.7	99.9	33.5	31.5	4.1	0.8
	W#5	0.0	76.2	(*)	37.2	116.0	242.7	100.0	68.2	(*)	13.4	51.6
	W#6	0.0	50.8	29.0	86.1	58.6	32.8	100.0	69.4	3.3	37.4	95.4
	W#7	0.0	114.3	168.7	106.0	116.5	137.5	100.0	15.2	25.3	19.8	13.5
	W#8	0.0	150.4	152.0	58.8	181.0	132.2	100.0	16.3	17.5	47.8	39.9
	W#9	0.0	82.1	107.8	72.9	92.7	112.7	100.0	25.2	1.8	87.4	15.6
	M	W#10	66.6	224.3	320.1	373.0	333.2	425.7	84.3	47.1	24.5	12.4
W#11		13.1	96.2	319.1	159.4	161.4	372.9	96.5	74.1	14.0	57.2	56.5
W#12		73.2	285.4	413.4	352.4	352.8	472.8	84.5	39.4	12.3	25.5	25.1
W#13		0.2	44.5	91.9	60.4	88.0	85.4	99.8	46.9	9.7	29.3	5.1
W#14		3.9	119.8	558.0	104.3	140.8	520.3	99.2	76.9	7.6	80.0	72.8
W#15		0.5	53.3	260.5	65.3	87.7	238.5	99.8	77.5	10.0	72.6	63.0
W#16		5.5	161.7	540.5	115.2	168.7	410.0	98.6	60.4	32.4	71.9	58.7
S	W#18	118.9	297.2	(*)	238.0	362.4	804.4	85.2	63.0	(*)	70.4	54.9
	W#19	34.6	141.0	(*)	59.2	169.0	687.1	95.0	79.4	(*)	91.4	75.4
	W#20	21.9	111.3	165.1	44.9	72.0	153.6	85.6	27.0	8.2	70.8	52.8

(*) The available experimental data do not allow performing the calculation using Southwell Plot method as explained in section 2.3.2.

Table 2. Results of the different calculation methods considered in the research compared with the experimental results and absolute value of the relative error for each tested case

e (mm)	λ	N_{max} [kN]					Error [%]			
		EC-6	ACI-530	2 nd order	FEM	Experimental	EC-6	ACI-530	2 nd order	FEM
0	5.6	1233	1376	1301	1349	1400	11.9	1.7	7.1	3.6
	11.1	1104	1301	1186	1274	1248	11.5	4.3	4.9	2.2
	18.8	809	1111	964	1181	1115	27.5	0.4	13.5	6.0
	27.7	391	770	662	697	663	40.9	16.2	0.1	5.2
14.3 (t/8)	5.6	973	793	789	981	1000	2.7	20.7	21.1	1.9
	11.1	784	767	752	853	880	10.9	12.8	14.5	3.0
	18.8	424	697	591	611	530	20.0	31.5	11.5	15.2
	27.7	115	463	307	373	255	54.9	81.6	20.4	46.3

Table 3. Results of the different calculation methods considered in the research (except Southwell Plot) compared with the experimental results (average values from two walls for each case) and absolute value of the relative error for each case from the bibliography [11]

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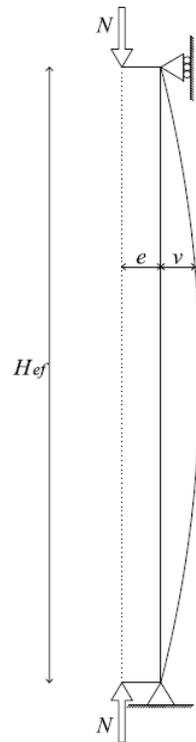


Figure 1. Deflection of an eccentrically compressed column

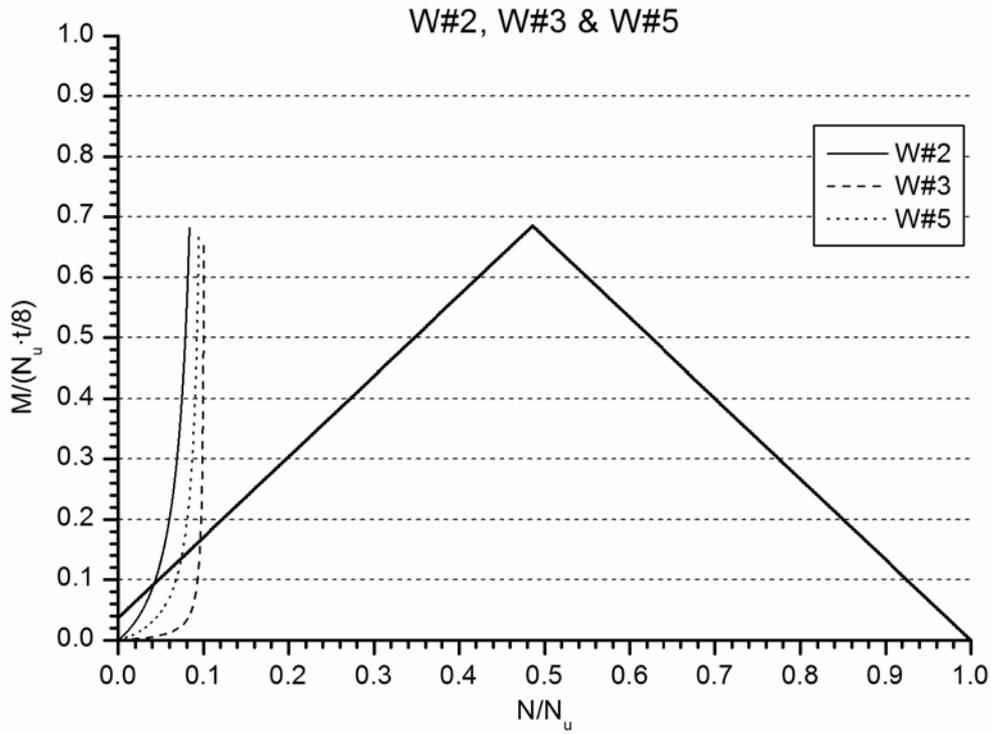


Figure 2. Second order axial-bending response intersected with the failure criteria

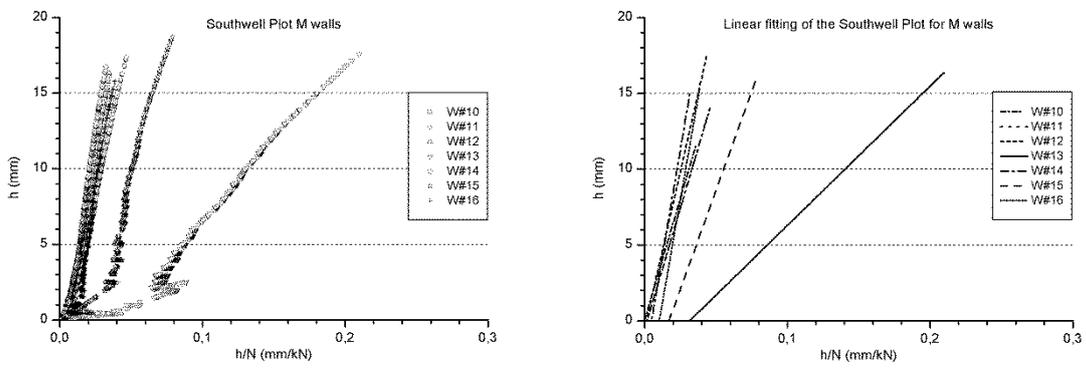


Figure 3. Southwell Plot and linear fitting for the M wall series

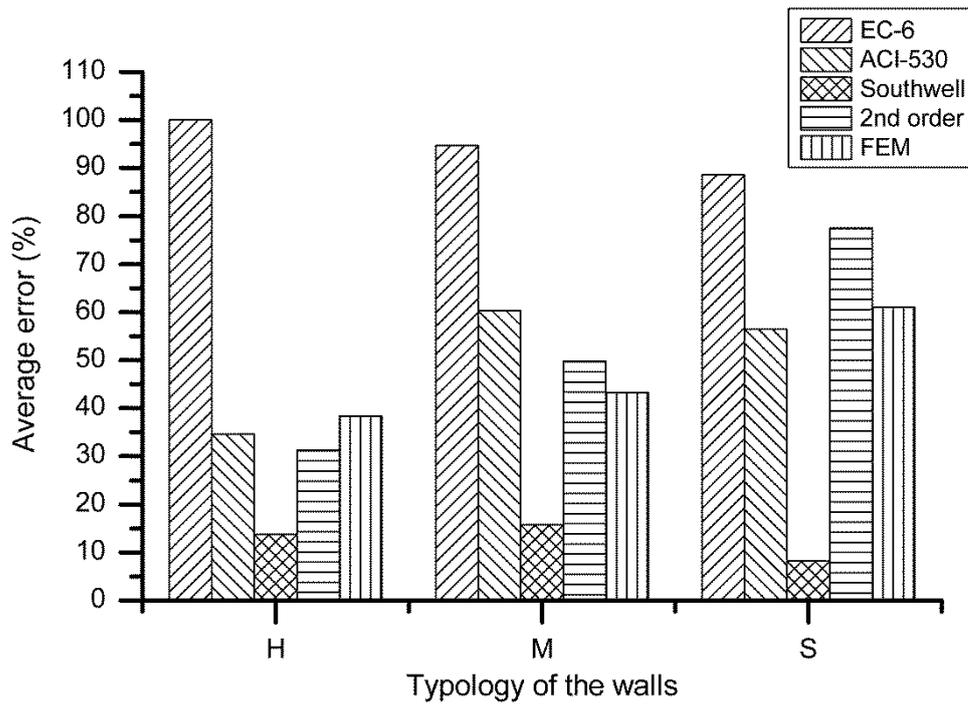


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