MICROVAWE IMAGING OF MULTILAYER CYLINDERS USING OPTIMIZATION TECHNIQUES

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Introduction

The reconstruction of weak-scattering objects from the scattered field measured in cylindrical geometries, can be solved by using the Born approximation [1]. However, first order algorithms often fail to reconstruct biological bodies due to their high contrast in permittivity [2].

The reconstruction of arbitrary strong-scattering objects is still an open area of research. The paper presents a new approach to solve this problem using optimization techniques. The algorithm is based on the minimization of the mean squared error between the measured and calculated scattered fields from the reconstruction.

In this work the object is assumed to have cylindrical symmetry and we are interested in the radial profile of permittivity. The validity of the proposed algorithm is investigated both numerically and experimentally.

Direct method: diffraction of circular concentric cylinders

The theoretical scattering solution for a wave incident normally on a lossy dielectric multilayer circular cylinder can be obtained using the Richmond's recursive method [3].

For a TM mode, the E-field outside the cylinder is the sum of the incident and scattered.

$$E_z(r, \phi) = \sum_{n=-\infty}^{n=\infty} (\alpha_n J_n(k_0 r) + c_n H_n^{(2)}(k_0 r)) e^{jn\phi} \quad (1)$$

In the layer m the field is

$$E_{z,m}(r, \phi) = \sum_{n=-\infty}^{n=\infty} (A_{mn} J_n(k_m r) + B_{mn} Y_n(k_m r)) e^{jn\phi} \quad (2)$$

Matching the tangential fields across the boundary, the recursion equation is obtained

$$\begin{pmatrix} A_{m+1,n} \\ B_{m+1,n} \end{pmatrix} = \frac{\pi r_m}{2\mu_m} \begin{pmatrix} U_{mn} & V_{mn} \\ V_{mn} & X_{mn} \end{pmatrix} \begin{pmatrix} A_{m,n} \\ B_{m,n} \end{pmatrix} \quad (3)$$

$$U_{mn} = \mu_m k_{m+1} J_n(k_m r_m) Y_n'(k_{m+1} r_m) - \mu_{m+1} k_m J_n'(k_m r_m) Y_n(k_{m+1} r_m)$$

$$V_{mn} = \mu_{m+1} k_m J_n'(k_m r_m) J_n(k_{m+1} r_m) - \mu_m k_{m+1} J_n(k_m r_m) J_n'(k_{m+1} r_m)$$

$$W_{mn} = \mu_m k_{m+1} Y_n(k_m r_m) Y_n'(k_{m+1} r_m) - \mu_{m+1} k_m Y_n'(k_m r_m) Y_n(k_{m+1} r_m)$$

$$X_{m,n} = \mu_{m+1} k_m Y_n'(k_m r_m) J_n(k_{m+1} r_m) - \mu_m k_{m+1} Y_n'(k_m r_m) J_n'(k_{m+1} r_m)$$

The boundary condition in the external (m=M) and the first layers ,gives

$$C_n = j B_{M+1,n} \quad A_{M+1,n} - j B_{M+1,n} = \alpha_n \quad B_{1n} = 0 \quad (4)$$

The problem can be solved starting with the first layer and $A_{1n} = 1$ and normalizing after all the coefficients.

Inverse problem

The inverse problem can be solved using a optimization technique combined with the direct method. The process is outlined below.

The error function is a quadratic sum of the difference between the measured fields and the fields calculated with the estimated parameters. The error is multiplied by a weighting factor, taking into account the validity region for the results.

$$e(\phi, \psi) = \sum W'(\psi) |E_{i(mes)}^s - E_{i(tho)}^s|^2 \quad (5)$$

The optimization process is based in the **simplex** algorithm, that needs multiple evaluation of the error function [4], we reject the methods of gradient because it is necessary to compute the derivative of this function.

Numerical results

To simulate the reconstruction procedure, we take an object consisting of two concentric cylinders immersed in water ($\epsilon = 77.27$ $\tan \delta = 0.1195$), this body is characterized with the radius, dielectric constant, and loss tangent of the two layers as shown in the first row of table 1. The measured scattered fields on a 20 cm diameter semicircular line (fig. 1) are simulated adding gaussian noise to the fields calculated with the direct method at 2.45 GHz.

Table 1 shows the reconstruction for several S/N ratios in the field measurement; with S/N > 20 dB the accuracy is remarkable and 5 dB S/N is enough to retrieve the object with an error below 10 % in all parameters.

The results have been obtained with a threshold error of 10^{-8} , requiring around 150 iterations. The computation time is 15 minutes on a Hewlett-Packard series 9000/840 computer.

S/N	R_1 (cm)	ϵ_1	$\tan \delta_1$	R_2 (cm)	ϵ_2	$\tan \delta_2$
theory	3.0000	32.00	0.200	5.000	62.580	0.050
∞	2.9996	31.97	0.208	5.002	62.581	0.049
40	2.9999	32.01	0.198	5.001	62.581	0.049
30	3.0038	32.07	0.194	5.005	62.662	0.051
20	2.9936	32.21	0.202	5.003	62.622	0.051
15	2.9591	32.65	0.236	5.015	62.450	0.048
10	2.9955	33.58	0.211	5.012	62.080	0.040
5	2.8942	35.31	0.150	4.905	59.330	0.058

Experimental results

A tomographic microwave system [5] operating at 2.45 GHz has been used to measure the scattered field produced by a dielectric cylinder. Fig. 2 shows a diagram of the system, consisting of a circular array of 64 antennas, a coherent microwave transmitter/receiver and control circuitry. Each antenna can be addressed as emitter or receiver using a switching matrix. The cylinder is placed in the center of the geometry and the scattered field produced on an incident cylindrical wave are measured by the half array opposite the emitter element. Fig. 3 shows the comparison between the measured scattered field and the theoretical field corresponding to the optimized cylinder $\epsilon = 61.62$ $\tan \delta = 0.38$ $R = 2.23 \text{ cm}$, the reconstruction error is negligible for the radius and below 10% for the dielectric constant and loss tangent.

References

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Figure 1.

Figure 2. Measurement set-up

Figure 3. An experimental gel cylinder optimization. The measured scattered field is shown in solid line. in dotted line the calculated field from the optimized cylinder.