A GENERAL ANALYSIS OF ERRORS IN NOISE MEASUREMENT SYSTEMS +

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ABSTRACT

General and approximated expressions are presented which allow the identification and quantification of errors in noise figure measurements that use the Y-factor method. This formulation is a generalization of that in /2/ to all measurement situations (in particular it is not necessary to include an input isolator). The formulas are utilized to determine the error limits on the noise figure measurement of a microwave low-noise amplifier.

INTRODUCTION

Causes of error in noise figure measurement persist even in modern, microprocessor-controlled measurement systems. Effect of errors is especially important in the characterization of low-noise devices, where departures of indicated noise figure values from actual ones are more significant.

Analysis of such errors has generated a considerable amount of literature. In 1975 Mamola and Sannino presented a generalized analysis of source mismatch effects on noise parameter measurements /1/. The expressions derived there allow the determination of the correct value of the effective input noise temperature of the two-port from the experimental data. In 1981 Strid studied the special but important case in which an ideal ($s_{12}=0$) input isolator is used in the measurement /2/. In 1984 Kuhn studied the influence of the gain variation with input impedance on measurement errors /3/. In the present work general and approximated expressions are derived which allow the identification and quantification of errors in noise figure measurements. This formulation generalizes the one in /2/ to all measurement situations (namely, it is not necessary to include an input isolator) and also it takes into account the second stage correction. Non converting two-ports are assumed. As a practical example, the formulas are utilized to determine the error limits on the noise figure measurement of a microwave low noise amplifier.

NOISE MEASUREMENTS IN REAL CONDITIONS

Figure 1 shows the set-up and quantities involved in a noise figure measurement of a two-port device using a two-states noise source (**). Without loss of generality a 'cold' source temperature $T_c = T_0$ is assumed throughout the paper. The noise figure shown by the instrument (F_{dmeas}) differs from the right F_d value (the value under nominal conditions $T_s=0$) because several departures from an 'ideal' measurement situation occur:

- The noise source reflection coefficient is not the nominal. In addition, it changes between the two states ('cold' and 'hot'). This causes

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(**) See also Glossary of terms below

- G_{tm} , T_{ed} , T_{em} , T_{et} , G_{td} also to change.
- There exist uncertainties in the knowledge of source hot temperature (T_{ha} vs. T_h).
 - The noise meter also contributes to the total read noise powers. To account for this effect a second stage correction using the Friis' formula is commonly done using calibration data.
 - There is instrumentation uncertainty.

Instrumentation uncertainty is not considered in the following derivation but it is added in the final calculations. For each noise source state (1 and 2) the following powers are read by the instrument during measurement (N_1, N_2) and during calibration (N_{1c}, N_{2c}):

$$\begin{aligned} N_1 &= k(T_c + T_{et}(\Gamma_{sc})) \cdot B \cdot G_{tt}(\Gamma_{sc}) \\ N_2 &= k(T_{ha} + T_{et}(\Gamma_{sh})) \cdot B \cdot G_{tt}(\Gamma_{sh}) \\ N_{1c} &= k(T_c + T_{em}(\Gamma_{sc})) \cdot B \cdot G_{tm}(\Gamma_{sc}) \\ N_{2c} &= k(T_{ha} + T_{em}(\Gamma_{sh})) \cdot B \cdot G_{tm}(\Gamma_{sh}) \end{aligned}$$

Noise figures (F_{meas}) are calculated by the instrument using the expression /5/, /6/:

$$F_{meas} = \frac{Y(1 - T_c/T_o) + (T_h/T_o - 1)}{Y - 1} \quad (1)$$

where Y is the Y -factor actually measured by the instrument, defined by:

$$Y = N_2/N_1 \quad (\text{during measurement}) \quad (2a)$$

$$Y = N_{2c}/N_{1c} \quad (\text{during calibration}) \quad (2b)$$

Substituting (2a) in (1) yields

$$F_{tmeas} = \frac{F_t(\Gamma_{sc}) M_c (T_h - T_o)}{M_h (T_{ha} - T_o) + T_o (M_h F_t(\Gamma_{sh}) - M_c F_t(\Gamma_{sc}))} \quad (3) (*)$$

This fundamental expression relates the displayed (measured) noise figure of a chain (formed by the two-port under measurement and the measurement system), to parameters of the measurement set-up and also identifies the causes of error.

F_{mmeas} can be obtained in a similar manner by substituting (2b) in (1). The instrument calculates the device gain from /5/:

$$G_{dmeas} = \frac{N_2 - N_1}{N_{2c} - N_{1c}} \quad (4)$$

and then it deembeds F_{dmeas} using the Friis' formula:

$$F_{dmeas} = F_{tmeas} - \frac{F_{mmeas} - 1}{G_{dmeas}} \quad (5)$$

EXPRESSION OF NOISE MEASUREMENT ERROR

Substituting (1), (2), (3) and (4) in (5) the following expression relating the displayed noise figure for the two-port under measurement to

(*) See Glossary of terms below

the parameters of the measurement system is derived:

$$F_t(\Gamma_{sc})M_c(T_h - T_o) + \frac{M_{mh}(T_{ha} - T_o) + T_o M_{mh} F_m(\Gamma_{sh}) - F_m(\Gamma_{sc})M_{mc}T_h}{G_{ad}(0) N/M_o}$$

$$F_{dmeas} = \frac{M_h(T_{ha} - T_o) + T_o(M_h F_t(\Gamma_{sh}) - M_c F_t(\Gamma_{sc}))}{M_h(T_{ha} - T_o) + T_o(M_h F_t(\Gamma_{sh}) - M_c F_t(\Gamma_{sc}))} \quad (6) (*)$$

We are interested in the expected limits for the error, ΔF_d (*). Expressions for the error limits can be obtained by differentiating the Friis' formula

$$F_d(0) = F_t(0) - \frac{F_m(\Gamma_{oo}) - 1}{G_{ad}(0)} \quad (*)$$

yielding

$$\delta F_d = \delta F_t - \frac{1}{G_{ad}(0)} \delta F_m + \frac{F_m(\Gamma_{oo}) - 1}{G_{ad}(0)} \delta G_{ad} \quad (7)$$

Substituting for F_{tmeas} , G_{dmeas} in (7) we obtain, with the lowest order of approximation (which is sufficient if errors are not exceedingly high)

$$\Delta F_d = \left(\frac{F_t(\Gamma_{sc})M_c(T_h - T_o)}{F_t(0)M_h(T_{ha} - T_o)} - 1 \right) F_{tmeas} - \left(\frac{F_m(\Gamma_{sc})M_{mc}(T_h - T_o)}{F_m(\Gamma_{oo})M_{mh}(T_{ha} - T_o)} - 1 \right) \frac{F_{mmeas}}{G_{dmeas}} + \left(\frac{M_h N}{M_{mh} M_o} - 1 \right) \frac{F_{mmeas} - 1}{G_{dmeas}} \quad (8) (*)$$

for which the additional approximations

$$M_h(T_{ha} - T_o) + T_o(M_h F_t(\Gamma_{sh}) - M_c F_t(\Gamma_{sc})) \approx M_h(T_{ha} - T_o)$$

$$M_{mh}(T_{ha} - T_o) + T_o(M_{mh} F_m(\Gamma_{sh}) - M_{mc} F_m(\Gamma_{sc})) \approx M_{mh}(T_{ha} - T_o)$$

that are fulfilled in most practical cases (noise source ENR ≥ 5 dB), have been made.

For low noise amplifiers, designed to have a noise figure near the minimum, it is reasonable to assume that slight deviations of the source impedance from the nominal value do not produce large variations of the actual noise figure /3/, so that expression (8) can be approximated by

$$\Delta F_d = \left(\frac{M_c(T_h - T_o)}{M_h(T_{ha} - T_o)} - 1 \right) F_{tmeas} - \left(\frac{M_{mc}(T_h - T_o)}{M_{mh}(T_{ha} - T_o)} - 1 \right) \frac{F_{mmeas}}{G_{dmeas}} + \left(\frac{M_h N}{M_{mh} M_o} - 1 \right) \frac{F_{mmeas} - 1}{G_{dmeas}} \quad (9) (*)$$

Expression (9) indicates that for low-noise amplifier measurement the prevalent causes of error are the source impedance variation due to the noise source switching and the ENR uncertainty, and it is thought to be useful in a R&D laboratory environment. It also shows that including a well characterized isolator at the noise source output results in lower error limits, in agreement with /4/. Expression (9) should not be used for two-ports having an input isolator; in that case $F_t(\Gamma_{sc})/F_t(0)$ is known /2/

$$\frac{F_t(\Gamma_{sc})}{F_t(0)} = \frac{|1 - \Gamma_{sc} S_{11}^a|^2}{1 - |\Gamma_{sc}|^2} \quad (\text{assuming ideal isolator, } s_{21}^a = 0)$$

(*) See Glossary of terms below

and (8) leads to less pessimistic limits than (9), according to /2/. Note that (9) makes only use of F_{tmeas} , F_{mmeas} , G_{dmeas} which can be directly read during calibration and measurement.

PRACTICAL EXAMPLE

Noise-figure measurement of a low-noise narrowband microwave amplifier are considered. The amplifier is used in the 2nd IF section (2.62 GHz) of 20-30 GHz earth stations receivers and it was designed to meet very low noise requirements (around 1 dB NF). It has two stages with NEC's NE75083 MES-FET's. The measurement system is based upon a HP 8970 B noise figure meter and HP 346B noise sources. Four measurements were made. The first two used a noise source with ENR = 15 dB. The third and fourth used a noise source with ENR = 5 dB. The second and fourth added an input isolator. Experimental results for F_{dmeas} were:

- (a) - $F_{dmeas} = 1.17$ dB (ENR 15 dB, no input isolator)
- (b) - $F_{dmeas} = 1.28$ dB (ENR 15 dB, with input isolator)
- (c) - $F_{dmeas} = 1.07$ dB (ENR 5 dB, no input isolator)
- (d) - $F_{dmeas} = 1.35$ dB (ENR 5 dB, with input isolator)

The error limits were then calculated using (9) for (a) and (c) and (8) for (b) and (d) in the following manner (see Table I). $|\Gamma_{sc}|$, $|\Gamma_{sh}|$ and ENR uncertainty were known from noise source manufacturer's data. Instrumentation uncertainty were also known from noise figure meter manufacturer's data. $|\Gamma_i|$ was measured. Thus it was possible to calculate M_c/M_h uncertainty. In a similar way, M_{mc}/M_{mh} , M_h/M_{mh} and N/M_o could have been calculated from $|\Gamma_m|$ (manufacturer's data) and $|\Gamma_o|$ (measured). However, as G_{dmeas} is high (29 dB), the contribution of the second and third terms in (9) is negligible. Combining M_c/M_h uncertainty with ENR and instrumentation uncertainties (in both worst case and RSS manner) the error limits were obtained. Figure 2 shows the measured noise figures and the computed error limits for F_d . Using transistor manufacturer's data, the approximation $F_t(\Gamma_{sc})/F_t(0) \approx 1$ was calculated to affect the estimation of the error limits by less than ± 0.07 dB in case (a) and less than ± 0.02 in case (c).

CONCLUSIONS

A general expression relating the noise figure of a two-port displayed by a noise measurement system that uses a Y-factor method, to the various parameters appearing in a real measurement situation is presented. Simplified expressions are derived to estimate error limits in a noise figure measurement of low-noise amplifiers even those not including isolators at their inputs. These expressions only require measured data for F_m , G_d , $|\Gamma_i|$ and $|\Gamma_o|$ and manufacturer's data for the rest of the parameters. Calculations show that errors can be as high as ± 0.74 dB when measuring a 1 dB NF amplifier with no input isolator and that approximation $F_t(\Gamma_{sc})/F_t(0) \approx 1$ is justified for amplifiers with noise figure near the minimum. In this case, the measurement errors are dominated by mismatches.

GLOSSARY OF TERMS

F_{dmeas} : Measured Noise Figure of the Device
 F_{mmeas} : Measured Noise Figure of the Noise Meter
 F_{tmeas} : Measured Noise Figure of the Chain
 G_{dmeas} : Measured Gain of the Device
 ΔF_d = $F_{dmeas} - F_d(0)$
 F_d : Device Noise Figure
 F_m : Noise Figure of the Noise Meter
 F_t : Noise Figure of the Chain
 G_{ad} : Device Available Gain
 G_{tt} : Chain Transducer Gain
 G_{tm} : Meter Transducer Gain
 T_{et} : Effective Input Noise Temperature of the Chain
 T_{em} : Effective Input Noise Temperature of the Meter
 T_h : Noise Source Temperature (Calibration Value)
 T_{ha} : Noise Source Temperature (Actual Value)
 $\Gamma_{sc,h}$: Noise Source Reflection Coefficient (Cold/Hot)
 Γ_m : Input Reflection Coefficient of the Noise Meter
 Γ_i : Two-Port Input Reflection Coefficient (during Measurement)
 Γ_o : Two-Port Output Reflection Coefficient (during Measurement)
 Γ_{oo} = Γ_o when $\Gamma_s=0$

$$f(\Gamma_s, \Gamma_i) = \frac{(1 - |\Gamma_1|^2)(1 - |\Gamma_2|^2)}{|1 - \Gamma_1 \Gamma_2|^2} \quad \text{Mismatch Coefficient}$$

M_c = $f(\Gamma_{sc}, \Gamma_i)$
 M_h = $f(\Gamma_{sh}, \Gamma_i)$
 M_{mc} = $f(\Gamma_{sc}, \Gamma_m)$
 M_{mh} = $f(\Gamma_{sh}, \Gamma_m)$
 N = $f(\Gamma_{oo}, \Gamma_m)$
 M_o = $f(0, \Gamma_i)$
 k : Boltzmann's constant
 B : System Bandwidth (much narrower than that of the Two-Port)

REFERENCES

- /1/ G. Mamola, M. Sannino
"Source mismatch effects on measurements of linear two-port noise temperatures"
IEEE trans. on I.M., vol IM-24 n.3, sept. 1975, pp. 239-242
- /2/ E. Strid
"Noise measurement for low-noise GaAs FET amplifiers"
Microwave Systems News, nov. 1981, pp. 62-70
- /3/ N.J. Kuhn
"Curing a subtle but significant cause of noise figure error"
Microwave Journal, june 1984, pp. 85-98
- /4/ E. Strid
"Noise measurement checklist eliminates costly errors"
Microwave Systems News, dec. 1981, pp. 88-107
- /5/ "Noise figure measurements. Principles and applications"
Hewlett-Packard seminar, 1982
- /6/ "Fundamentals of RF and Microwave Noise Figure Measurements"
Hewlett-Packard Application Note 57-1, July 1983

Noise Source ENR(dB)	$ \Gamma_{sc} $	$ \Gamma_{sh} $	$ \Gamma_i $ measured		M_c/M_h uncertainty(dB)		$ \Gamma_M $ (W.C.)
			a,c	b,d	a,c	b,d	
15 dB	0.03	0.04	0.56	0.061	± 0.34	± 0.02	0.26
5 dB	0.01	0.01			± 0.11	± 0.006	

ENR uncertainty: ± 0.3 dB (W.C.), ± 0.1 dB (RSS)

Instrumentation uncertainty: ± 0.1 dB

TABLE I - Parameters intervening in ΔF_d