TABLE I

Algorithm/ Operations	Direct Computation of S_k (1)	FFT		EFFT	
		Directly	Using (16)–(18)	Directly	Using (16)–(18)
Complex Multiplications (approx.)	$\frac{1}{2}N(m+n+1)$	$N(\log_2 N + \frac{1}{2})$	$\frac{1}{2}N(\log_2 N + 2)$	$\frac{1}{2}N(\log_2 m' + \log_2 n' + 1)$	$\frac{1}{2}N(\log_2 m' + \log_2 n' + 1)$
Complex Additions (approx.)	$\frac{1}{2}N(m+n)$	$2N \log_2 N$	$N \log_2 N$	$N(\log_2 m' + \log_2 n')$	$\frac{1}{2}N(\log_2 m' + \log_2 n')$

numbers. In order to further increase computational efficiency, the real sequence of the ARMA parameters can be alternately stored in real and complex arrays, and subsequently unscramble the result. To apply this procedure for ARMA spectral estimation, a complex parameter sequence C_i is defined:

$$C_i = p_{2i} + j p_{2i+1}, \quad i = 0, 1, \dots, \frac{M}{2} - 1. \quad (16)$$

Now by applying the EFFT algorithm,

$$Z_k = \sum_{i=0}^{M/2-1} C_i W_{N/2}^{ik} \quad (17)$$

can be computed based on

$$\begin{aligned} g_{1k} &= \frac{1}{2} (Z_k + Z_{N/2-k}^*) \\ g_{2k} &= \frac{1}{2} (Z_k - Z_{N/2-k}^*) \\ g_k &= (g_{1k} + W_N^k g_{2k}) \\ g_{N/2-k}^* &= (g_{1k} - W_N^k g_{2k}), \\ k &= 0, 1, \dots, \frac{N}{4} - 1 \end{aligned} \quad (18)$$

where (*) expresses the complex conjugate. By computing g_k 's using expressions (16)–(18), the number of parameters and spectral estimates is only half of that in (7) for $k = N - 1$. The number of complex multiplications and additions in (17) is approximately $\frac{1}{4}N(\log_2 M - 1)$ and $\frac{1}{2}N(\log_2 M - 1)$, respectively, which is considerably less than by using (7) directly.

In order to obtain all spectral estimates of an ARMA (n, m) , process we proceed computing the following quantities:

$$\begin{aligned} g_{x,k} &= \sum_{i=0}^{m'-1} \exp(-j2\pi ik/N), \quad k = 0, 1, \dots, \frac{N}{2} - 1 \\ g_{y,k} &= \sum_{i=0}^{n'-1} y_i \exp(-j2\pi ik/N) \end{aligned}$$

where

$$\begin{aligned} x_0 &= y_0 = -1 \\ x_i &= \theta_i, \quad i = 1, 2, \dots, m \\ x_i &= 0, \quad i = m+1, \dots, m' - 1 \\ y_i &= \phi_i, \quad i = 1, 2, \dots, n \\ y_i &= 0, \quad i = n+1, \dots, n' - 1 \end{aligned} \quad (20)$$

and N , m' , and n' are integers satisfying the requirements of the radix-2 EFFT algorithm. Generally, a few zeros have to be added into the parameter sequence to make m' and n' a power of 2. Therefore, (1) can be written as

$$S_k = \sigma_a^2 \frac{g_{x,k} g_{x,k}^*}{g_{y,k} g_{y,k}^*}, \quad k = 0, 1, \dots, \frac{N}{2} - 1. \quad (21)$$

The computational efficiency of the different algorithms which can be used to compute (1) is given in Table I.

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An Improved Maximum Likelihood Method for Power Spectral Density Estimation

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Abstract—In this paper, we present a new procedure for spectral analysis; it represents a tradeoff between the high resolution provided by the maximum entropy method (MEM) and the low sidelobe characteristic of the maximum likelihood method (MLM). The approach that we follow is to introduce a slight modification of the second technique to obtain from the ML filter the true spectral density, not the power level as the original procedure does.

The proposed method can be used both in one-dimensional and two-dimensional problems without a remarkable increase of computational load relative to the original MLM.

I. INTRODUCTION

Several published works in spectral analysis compare MEM [1] to MLM [2]. It seems clear that these methods provide basically different results: MEM involves power spectral den-

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Fig. 1. Decomposition of the input power spectral density in a small band around ω_0 ($\tilde{S}_x(\omega)$) and an interfering part ($\hat{S}_x(\omega)$).

sity (watts/hertz), whereas MLM involves a power level estimate (watts) [3], [4]. Then we cannot compare them in a proper way without a prior modification. Note, for example, that for a sinusoid, MEM tends to indicate its power spectral density, an infinite value, while MLM would show an estimate of its power.

In the next section, in view of the above comments, we introduce a modification of MLM in order to obtain an estimate of power spectral density from the output of the ML filter.

A. Modified MLM

To maintain simplicity, we will explore the possibility of measuring power spectral density with an ML filter in a 1-D context, but we must indicate that the resulting modification can be easily extended to 2-D applications, as we will see later.

Referring back to the original philosophy applied to the design of a simultaneous spectrum analyzer [5], we are dealing with a bandpass FIR filter centered at ω_0 that has to minimize the interference produced by the other frequency bands and to offer a unity transfer function at ω_0 . Decomposing the spectrum $S_x(\omega)$ under analysis in the form

$$S_x(\omega) = \hat{S}_x(\omega) + \tilde{S}_x(\omega) \quad (1)$$

where $\hat{S}_x(\omega)$ and $\tilde{S}_x(\omega)$ are the distortion and the explored parts of the spectrum, as one can see in Fig. 1, the problem can be formulated as the design of an FIR filter which does not alter $S_x(\omega)$ and which minimizes the output due to $\hat{S}_x(\omega)$.

The power of the input signal contained in the explored band is

$$P_o = (1/2\pi) \int_{\omega_0 - \epsilon}^{\omega_0 + \epsilon} \tilde{S}_x(\omega) d\omega. \quad (2)$$

Assuming that to measure P_o is equivalent to measuring the power of a spectral line centered at ω_0 with an interference $\hat{S}_x(\omega)$, we can rewrite (1) in the form

$$S_x(\omega) = P_o \delta(\omega - \omega_0) + \hat{S}_x(\omega). \quad (3)$$

If we represent the desired FIR filter by the vector $A^H = [a(0), \dots, a(Q)]$ (H indicates Hermitian transpose), and if we impose P_o as its output power and the minimization of the power of the interference at the output, the design consists of

$$\text{minimize } (1/2\pi) \int_{-\pi}^{\pi} \hat{S}_x(\omega) |A(\omega)|^2 d\omega \quad (4a)$$

$$\text{under } A^H S_o = 1 \quad (4b)$$

where $S_o = |1, \exp(j\omega_0), \dots, \exp(jQ\omega_0)|^H$ and $A(\omega)$ is the transfer function of the filter.

Note that, according to constraint (4b), we can use $S_x(\omega)$ in (4a) since the difference between the two integrals is a constant P_o . By introducing this equivalence and using Parseval's relationship, (4a) and (4b) can be rewritten in the form

$$\text{minimize } A^H R A \quad (5a)$$

$$\text{under } A^H S_o = 1 \quad (5b)$$

where R is a $(Q+1) \times (Q+1)$ autocorrelation matrix of the process under analysis.

Note that the theoretical limit for $\min A^H R A$ is P_o (when the filter completely removes the interference). Then we can use an estimate for P_o of the form

$$\hat{P}_o = A^H R A. \quad (6)$$

Since the solution to (5a) and (5b) is the well-known formula

$$A = R^{-1} S_o / S_o^H R^{-1} S_o \quad (7)$$

we have for the estimate of the power at ω_0

$$\hat{P}_o = 1 / S_o^H R^{-1} S_o. \quad (8)$$

Now, if we are interested in estimating the spectral density $S_x(\omega_0)$, we can use the relationship existing between $S_x(\omega_0)$ and $A(\omega)$:

$$\hat{P}_o = (1/2\pi) \int_{-\pi}^{\pi} S_x(\omega) |A(\omega)|^2 d\omega \quad (9)$$

and considering that $S_x(\omega)$ is approximately flat in the narrow band of $A(\omega)$ around ω_0 , the previous formula can be approximated in the form

$$\hat{P}_o \simeq \tilde{S}_x(\omega_0) (1/2\pi) \int_{-\pi}^{\pi} |A(\omega)|^2 d\omega = \tilde{S}(\omega_0) A^H A \quad (10)$$

from which we have an estimate for $S(\omega_0)$:

$$\tilde{S}(\omega_0) = \hat{P}_o / A^H A. \quad (11)$$

This is an estimator that can be extended to all frequencies, and then, we have the general expression

$$\tilde{S}(\omega) = S^H R^{-1} S / S^H R^{-2} S. \quad (12)$$

The obtained estimator allows its direct comparison to other methods which measure power spectral density as it does.

It is easy to prove that the proposed estimate converges, in the distributional sense, to the true spectral density as the dimension of the correlation matrix goes to infinity.

Formula (12) can be extended easily to 2-D spectral estimation problems.

Fig. 2 shows an example of two sinusoids in white noise in order to prove that the modified MLM (c) represents tradeoff between classical MLM (b) and MEM (a). The length of the data sequence is 64 and the signal-to-noise ratio is 10 dB for each sinusoid. The horizontal line in the figure represents the noise level and the vertical lines the position of the sinusoids in the frequency range. Also, their magnitudes with respect to the noise level are indicated in the plot.

II. APPLICATION TO 2-D PROBLEMS

Given a filter support in the 2-D image denoted by a vector X^T (where T indicates transpose) and the filter response denoted by a vector A^H (with the usual order of vector elements), we can apply the previous formulas (with R adequately defined) to derive the modified ML estimator.

To better describe this extension, let us assume that the filter support is a 2×2 matrix (see Fig. 3).

Fig. 2. Two sinusoids in white noise, 10 dB each. (a) MEM (10), (b) MLM (10), and (c) modified MLM order of 10. The data length is 64 samples; the noise level together with the exact peak locations and their magnitude are indicated in the plots.

Fig. 3. ML filter support in a 2-D example.

In this case, the output of the ML filter will be

$$e(n, m) = A^H X \quad (13)$$

where

$$X^T = [x(n, m), x(n-1, m), x(n, m-1), x(n-1, m-1)] \quad (14a)$$

$$A^H = [a(0, 0), a(1, 0), a(0, 1), a(1, 1)] \quad (14b)$$

Now, in order to have a unity transfer function at some se-

Fig. 4. Two sinusoids in white noise, 3 dB each. (a) and (b) are the MLM and modified MLM from a 5×5 correlation support.

lected frequency pair $(\omega_{01}, \omega_{02})$, we need to force

$$A^H S = 1 \quad (15)$$

where $S^H = [1, \exp(j\omega_{01}), \exp(j\omega_{02}), \exp(j(\omega_{01} + \omega_{02}))]$. The procedure follows in the same fashion as in the 1-D case; the quantity to be minimized is

$$E \|e(n, m)\|^2 = A^H R A \quad (16)$$

where $R = E[XX^H]$. The rest of the process coincides with the 1-D case.

Fig. 4 depicts an example showing the behavior of the proposed procedure compared to MLM, both using the same original data sample and correlation support.

The exact autocorrelation values corresponding to an ML filter with support 3×3 (see Fig. 3), applied over a 2-D signal containing real sinusoids in white noise, were used in the example. The frequencies of the sinusoids were $(0.1, 0.1)$ and $(0.2, 0.32)$, respectively; the signal-to-noise ratio was 3 dB for each.

III. AN INTERPRETATION AND SOME FURTHER POSSIBILITIES

It should be remarked that the use of the energy of the impulse response of the filter $A^H A$ in normalizing the estimator, when we demand a true power spectral density evaluation [see (11)], is just a procedure to employ the effective bandwidth of the ML filter:

$$B_w^e = (1/2\pi) \int_{-\pi}^{\pi} |A(\omega)|^2 d\omega / |A(\omega)|_{\max}^2 \quad (17)$$

as a normalizing value; note that, given the constraint of the minimization, (17) can be expressed as

$$B_w^e = A^H A. \quad (18)$$

Then we have the alternative expression for (11):

$$\tilde{S}_x(\omega_0) = \hat{P}_0 / B_w^e. \quad (19)$$

More efficient evaluations of the effective bandwidth will better estimate the actual power spectral density or they will further improve the resolution. In any case, note that B_w^e is more accurate than the classical $1/Q$ of MLM for power spectral density estimation.

It is worthwhile to remark that selecting other bandwidth estimates or areas for the ML filter will result in an increased complexity, obtaining, in most of the cases, the same resolution in the final estimate. The approximation suggested by the authors in (10) seems to be a tradeoff between the two concepts previously mentioned.

IV. CONCLUSIONS

We have introduced a simple modification of MLM to estimate power spectral densities, and no power levels. This new formulation has a simple interpretation as a normalization of ML estimators by the effective bandwidth of the ML filter.

With this approach, we establish a tradeoff between resolution and low sidelobe level, the main characteristics of MEM and MLM, respectively; we will obtain more resolution in detecting lines than MLM and lower sidelobe levels than MEM.

The method can be extended to 2-D problems without substantial modification, yet preserving the above-mentioned properties.

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Image Display Techniques Using the Cosine Transform

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Abstract—Transform coding has the ability to rapidly display a picture of low resolution by inverse transformation of a few of the low sequency coefficients only. Picture resolution is then progressively built up as

more coefficients are received and decoded. In this paper, different sequences of transmitting the coefficients are investigated to find one which achieves the best subjective quality with the fewest coefficients transmitted and requiring the least overhead information. Five schemes are examined on two source images with different statistics to compare the effectiveness of the schemes to achieve the above objectives on each of the images.

I. INTRODUCTION

In a low bit rate image coding system such as the slow scan TV and picture videotex system, the time taken for an image to fill the screen can be considerable. Therefore, the way in which the image is progressively built up from the moment the first pixel is received until the last pixel is displayed is important. Differential pulse code modulation (DPCM) processes the image a line or a few lines at a time, depending on whether one-dimensional or two-dimensional prediction is used. Therefore, at the receiver, the image is decoded and displayed line by line from top to bottom. The viewer will have no idea about the full content of the image until nearly all the lines are received and displayed.

Transform coding [1]–[3], apart from achieving a greater bit rate compression ratio than the DPCM, possesses the ability to produce a blurred but recognizable image upon transmitting and inverse transforming at the receiver the first few coefficients of higher magnitudes. Since the viewer is being given a "preview" of what he is going to see if the full set of coefficients is received and converted into the pixel domain, he will have the option of terminating the transmission of further coefficients if the image is of little use to him, thus effecting savings in time and cost.

In this paper, different sequences of transmitting the transform coefficients for image display are investigated. Different source images are used in order to ascertain how the source statistics affect the selection of the transmission sequence. In Section II, details of different sequences of coefficient transmission are described. Results of the simulations are presented in Section III, while the following section contains a discussion of the results. A summary of the paper is given in the concluding section.

II. IMAGE DISPLAY TECHNIQUES

An image transformation can be viewed as a decomposition of the image data into a generalized two-dimensional spectrum. Each spectral component in the transform domain corresponds to the amount of energy of the spectral function within the image. Typically, the first (DC) coefficient defines the average brightness of the picture block and is therefore positive, and the next few (AC) coefficients are large since, in most images, low frequency components predominate. The AC coefficients give an indication of the amount of detail in the picture block. For example, the presence of large amount of vertical detail will manifest itself in large AC coefficients along the vertical edge of the transform block. Transmission of different coefficients gives visual effects of different kinds. The objective of this investigation is to find a transmission sequence that not only give the best subjective effect, but needs only the minimum number of coefficients.

A. Scheme I

In this scheme, the order in which the coefficients are transmitted is based on the hierarchy of the coefficients used in reconstructing the image. The position in the hierarchy occupied by a coefficient is found by ranking it among the rest of the coefficients according to the normalized mean-square error (NMSE) of its reconstructed image when only that coefficient together with the DC coefficient are transmitted. NMSE is

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