ator characteristic, a voltage linear with frequency over a narrow band. The network can be designed to have an output characteristic which has a relatively low deviation from linearity and a steep gradient over the discriminator bandwidth.

The circuit was analysed using the electrical equivalent circuit of the m.c.f. (Fig. 2). To obtain the theoretical discriminator characteristic, it was necessary to subtract the absolute magnitudes of the two output voltages, which could only be achieved by numerical techniques. We define first a mode spacing, $\Delta\omega$, which is the difference between the antisymmetric-mode and symmetric-mode frequencies of one m.c.f. From the equivalent circuit of Fig. 2, the mode spacing is $\Delta\omega = 1/\omega_0 LC_c$. For a given value of maximum deviation from linearity and mode spacing of each m.c.f., the maximum discriminator bandwidth can be achieved by optimising the filter design parameters.

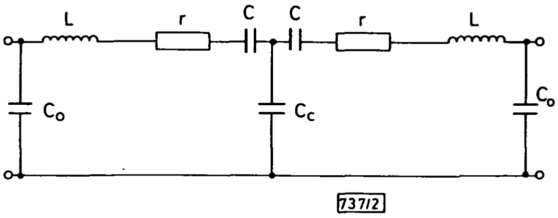

Fig. 2 *Electrical equivalent circuit of one m.c.f.*

As an example, a discriminator was designed to have a 6% maximum deviation from linearity over a 22 kHz bandwidth centred at 10·7 MHz. The theoretical and experimental output characteristics are compared in Fig. 3. The m.c.f.s had a mode spacing of 11·8 kHz and a resonant frequency separation of 29·6 kHz. No spurious modes were observed at frequencies either above or below the range shown; as the rectified output voltages are subtracted, filters with a poor stopband can be tolerated.

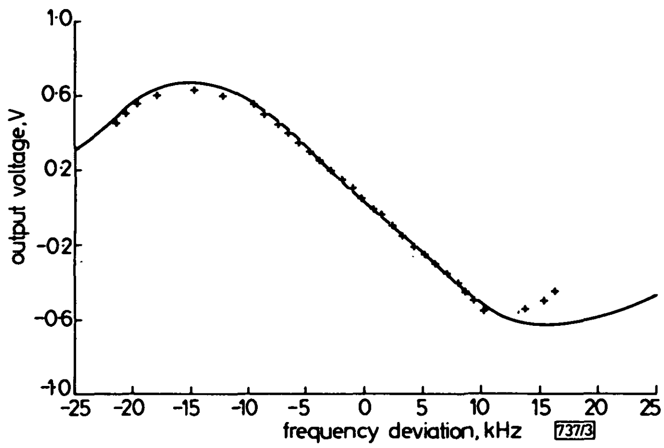

Fig. 3 *Discriminator output characteristic*

— theoretical; + experimental. Centre frequency 10·7 MHz, input signal 1 V r.m.s.

With discriminators designed in this way, smaller deviations from linearity may be achieved over a narrower band, with the same m.c.f. mode spacing, by reducing the resonant frequency separation. Larger bandwidths may be obtained by tuning out the static capacitance (C_0 in Fig. 2) of both m.c.f.s. Without tuning, the discriminator bandwidth is typically 1·3 times the mode spacing for a 2% maximum deviation and 2·2 times the mode spacing for a 10% deviation. With tuning, the bandwidths are 1·7 and 2·6 times the mode spacing for deviations of 2% and 10%. The maximum bandwidth, for a particular value of deviation, is limited by the maximum mode spacing of the m.c.f. that can be achieved (typically 30 kHz at 10·7 MHz). The discriminator bandwidth and the mode spacing can both be frequency scaled, i.e. increasing the mode spacing by 10% increases the discriminator bandwidth by 10%.

A similar design technique can be used to produce discriminators for higher frequencies, using overtone mode m.c.f.s or surface-acoustic-wave-coupled resonator filters[4] which have the same equivalent circuit.

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DESIGN OF BANDSTOP FILTERS BY REACTANCE TRANSFORMATION

Indexing terms: Filters, Reactance

The usual method for designing bandstop filters is the reactance transformation from a lowpass prototype. When the filter specifications do not satisfy geometrical symmetry, the choice of stopband central frequency constitutes an interesting problem. This letter shows that the mean square of the stopband edges is the stopband central frequency that leads to the lowest selectivity parameter of the prototype.

The specifications for a bandstop characteristic are shown symbolically in Fig. 1*a*. This implies that for $|\omega| \leq \omega_{p1}$ or $|\omega| \geq \omega_{p2}$, the passband loss should be less than α_p. For $\omega_{a1} \leq |\omega| \leq \omega_{a2}$, the stopband loss should be greater than α_a.

The lowpass–bandstop reactance transformation is[1]

$$\lambda = \frac{s}{s^2 + \omega_\infty^2}$$

where

$$\omega_{p1} < \omega_\infty < \omega_{p2} \tag{1}$$

Its application to the specification in Fig. 1*a* defines the prototype shown in Fig. 1*b*, where

$$\Omega_p = \max\,(\Omega_{p1}, \Omega_{p2}) \tag{2a}$$

$$\Omega_a = \min\,(\Omega_{a1}, \Omega_{a2}) \tag{2b}$$

with

$$\Omega_{p1} = \frac{\omega_{p1}}{\omega_\infty^2 - \omega_{p1}^2}, \qquad \Omega_{p2} = \frac{\omega_{p2}}{\omega_{p2}^2 - \omega_\infty^2} \tag{3a}$$

$$\Omega_{a1} = \frac{\omega_{a1}}{|\omega_\infty^2 - \omega_{a1}^2|}, \qquad \Omega_{a2} = \frac{\omega_{a2}}{|\omega_{a2}^2 - \omega_\infty^2|} \tag{3b}$$

being the transformed frequencies of ω_{p1}, ω_{p2}, ω_{a1} and ω_{a2}, respectively.

The problem that arises is the choice of ω_∞, the centre frequency for the transformation. It is evident that the most convenient value for ω_∞ is that which requires the lowest-order prototype. Considering that the order is an integer, there will be in general a range of values of ω_∞ which provides such a minimum. This range depends on the bandstop filter specifications and the approximation used for the prototype. Therefore its general characterisation is difficult, if not impossible.

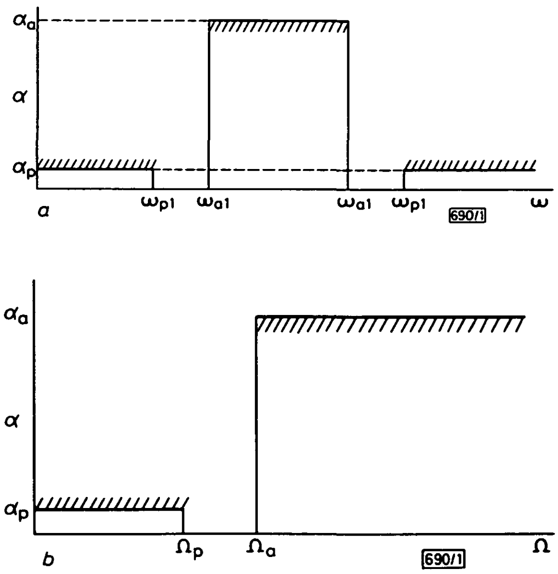

Fig. 1

a Bandstop filter specifications
b Lowpass prototype specifications

It is possible, though, as is shown in this letter, to identify the value of ω_∞ that requires from the selectivity parameter[1] of the prototype

$$K = \frac{\Omega_p}{\Omega_a} \tag{4}$$

the lowest possible value. This information is very convenient, because it allows us to determine the lowest order necessary for the prototype and provides a reference for the range of values of ω_∞ that such a minimum requires.

Theorem 1: The value of ω_∞ that provides the lowest selectivity parameter for the prototype satisfies

$$\min\,(\omega_{p1}\omega_{p2},\, \omega_{a1}\omega_{a2}) \le \omega_\infty^2 \le \max\,(\omega_{p1}\omega_{p2},\, \omega_{a1}\omega_{a2})$$

Proof: Let us assume that

$$\omega_\infty^2 \le \min\,(\omega_{p1}\omega_{p2},\, \omega_{a1}\omega_{a2}) \tag{5}$$

It is easy to check in eqn. 3 that

$$\Omega_{p1} \ge \Omega_{p2} \qquad \Omega_{a1} \ge \Omega_{a2}$$

Therefore, following eqns. 2, 3 and 4, we get

$$K = \frac{\Omega_{p1}}{\Omega_{a2}} = \frac{\omega_{p1}}{\omega_{a2}} \frac{\omega_{a2}^2 - \omega_\infty^2}{\omega_\infty^2 - \omega_{p1}^2}$$

For ω_∞^2 satisfying eqns. 1 and 5, K will take the lowest value when ω_∞^2 is maximum, that is

$$\omega_\infty^2 = \min\,(\omega_{p1}\omega_{p2},\, \omega_{a1}\omega_{a2})$$

To complete the proof we have to proceed in a similar way with the upper limit.

Corollary: If $\omega_{p1}\omega_{p2} = \omega_{a1}\omega_{a2}$ (geometrical symmetry), $\omega_\infty^2 = \omega_{p1}\omega_{p2} = \omega_{a1}\omega_{a2}$ provides the lowest selectivity parameter for the prototype.

Theorem 2: If $\omega_\infty^2 = \omega_{a1}\omega_{a2}$, the selectivity parameter of the prototype take the lowest possible value.

Proof: Let us assume that

$$\omega_{a1}\omega_{a2} < \omega_{p1}\omega_{p2}$$

According to theorem 1, the value of ω_∞ that provides the lowest K satisfies

$$\omega_{a1}\omega_{a2} \le \omega_\infty^2 \le \omega_{p1}\omega_{p2} \tag{6}$$

In this case, it is easy to prove that

$$\Omega_{p1} \ge \Omega_{p2} \qquad \Omega_{a2} \ge \Omega_{a1}$$

which, following eqns. 2 and 3, leads to

$$K = \frac{\Omega_{p1}}{\Omega_{a1}} = \frac{\omega_{p1}}{\omega_{a1}} \frac{\omega_\infty^2 - \omega_{a1}^2}{\omega_\infty^2 - \omega_{p1}^2}$$

Except for $\omega_\infty^2 = \omega_{p1}^2$, we have that

$$\frac{dK}{d\omega_\infty^2} > 0$$

Therefore, for ω_∞^2 satisfying eqn. 6, K is minimum for $\omega_\infty^2 = \omega_{a1}\omega_{a2}$.

By a similar procedure, we can show that, if $w_{a1}w_{a2} > w_{p1}w_{p2}$, an identical result holds. This completes the proof.

It is interesting to mention that a similar result can be obtained in the bandpass case. When identical minimum losses are specified for both stopbands, the mean square of the passband edges is the central frequency of the passband that provides the lowest value for the selectivity parameter of the lowpass prototype.

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OPTICAL DEMULTIPLEXER USING CONCAVE GRATING IN 0·7–0·9 μm WAVELENGTH REGION

Indexing term: Communications, Integrated optics

Experimental results of a demultiplexer using an aberration corrected concave grating for an optical system are described. The demultiplexer has 10 channels and a wavelength spacing of 20 mm from 0·7 to 0·9 μm wavelengths. Insertion losses and crosstalk were less than 2·5 dB and −30 dB, respectively, for all channels.

Introduction: Optical-fibre transmission technology has developed rapidly owing to improvements in optical fibres, light sources and passive devices. Among them, wavelength-division-multiplexing (w.d.m.) transmission is considered a promising approach for future systems—especially for subscriber loops and large-capacity transmission systems.

The wavelength-division-multiplexing (w.d.m) technology,